On Invalidity of the "Covariance Principle" and Deflection of Light to Second Order

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Abstract

Bodennor & Will claimed that results of deflection of light to second order are the same for different gauges. However, it is found that their calculation is based on only coordinate systems that have the same frame of reference. Moreover, although the deflection of light to second order have the same form, if expressed in the "impact parameter" b, the value for each individual gauge are actually different since b is different in terms of the measurable closest distance "d" of the given gauge. Apparently, they regarded b (instead of d) as the fundamental measurement, but they do not have a theory of measurement to support their implicit assumption. It is also clear that they do understand Einstein's equivalence principle. Moreover, not every result obtained in their coordinate systems can be expressed as mathematically invariant. For instance, the "event of horizon" is different for different gauges although the frame of reference is the same, and can even be an arbitrary constant for a special gauge. The deflection of light to second order involves what is the correct physical gauge, and thus is a far more difficult problem.

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1. Introduction

Einstein's principle of general relativity states "The law of physics must be of such a nature that they apply to systems of reference in any kind of motion. Einstein later extended this principle to unrestricted mathematical covariance and called it as the "principle of covariance" [1, 2]. He stated, "The general laws of nature are to be expressed by equations which hold good for all systems of co-ordinates, that is, are co-variant with respect to any substitutions whatever (generally co-variant)." Consequently, many accepted Lorentz manifolds are not valid in physics [3, 4]. Moreover, based on that both the Schwarzschild and the harmonic solution produced the same first order deflection of a light ray, Einstein [2] remarked, "It should be noted that this result, also, of the theory is not influenced by our arbitrary choice of a system of coordinates."

However, the coordinate systems of these solutions are very special type, and thus logically the result is actually very far from the validity of an arbitrary choice of a system of coordinates. If all the results must be invariant respect to any such choice, then general relativity can produce nothing but scalars.[1)] Since an angle of deflection is clearly not a scalar, what Einstein meant should only be covariance.

Speaking out against the "principle of covariance", Zhou [5] pointed out, "The concept that coordinates don't matter in the interpretation of Einstein's theory ... necessarily leads to mathematical results which can hardly have a physical interpretation and are therefore a mystification of the theory." Moreover, the "covariance principle" is in conflict with Einstein's equivalence principle. Based on this principle, Einstein pointed out that the effects of space contractions and time dilation are measurable [1, 2]. Thus, for a given frame of reference, there is at most one gauge is physically valid among diffeomorphic metrics [6, 7]. Thus, although the Schwarzschild solution[2)] is diffeomorphic to the harmonic solution, at most one of them is valid in physics, and thus they are not equivalent in physics.

Einstein proposed the "covariance principle" because he was unable to give a meaning to coordinates of a physical space [1]. Unexpected by Einstein, his equivalence principle was practically substituted by Pauli's version [8] in spite of strong objections from Einstein [9]. The popularity of this substitution was assisted by the incorrect belief that Einstein's equivalence principle is not needed or only approximately valid [10, 11]. The former is, in part, due to that the general case of obtaining space contractions and the time dilation has not been adequately studied [12, 13]. Thus, the crucial role of Einstein's equivalence principle was not understood. Due to conceptual errors, efforts to obtain a metric for uniform gravity had failed [11, 14].

However, as pointed out by Morrison, the "covariance principle" is invalid because it disrupts the necessary physical continuity from special relativity to general relativity [14, 15]. In fact, Einstein's "principle of covariance" has no theoretical basis in physics or observational support beyond what is allowed by the principle of general relativity [15]. Nevertheless, Bodenner & Will [16] claimed that their "covariance" results of the deflection of light to second order further illustrates the "principle of co-

"principle of covariance" and the belief that coordinates in general relativity are completely arbitrary. Such a claim is clear incorrect since it is in disagreement with the rigorous proof mentioned above. In this paper, it will be shown what went wrong in their considerations. Surprisingly, it is found that they actually have not attempted to prove their claim.

2. Does the Calculation of Bodenner & Will Support the "Covariance principle"?

The 2003 paper of Bodenner & Will [16] has essentially the same content as in the 1999 paper of Gérard & Piereaux [17]; we follow the better notation of the latter paper. Their calculation is based on the assumption that the metric for a static, spherical coordinate system takes the form,

$$ds^2 = A^2(r)\, c^2dt^2 - B^2(r)dr^2 - D(r)^2\, r^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \tag{1}$$

In other words, this coordinate system is attached to a static Sun with spherical symmentry mass distribution. First, this coordinate system admits a spherical symmetry mass distribution, and thus cannot be arbitrary. [3)] In fact, the spherical coordinate system (r, θ, φ) is necessarily defined on the Euclidean-like structure [6, 7] of the frame of reference, and therefore this system is very special among arbitrary coordinate systems. Since this system is very restricted, any result derived from such a coordinate system is not an evidence to support the "covariance principle". Thus, Will has made invalid claims again [18], and this manifests his questionable logic.

From metric (1), for a weak field, the deflection angle is derived [16, 17] as

$$\alpha(b) = \frac{4GM}{c^2 b}\left[1 + \frac{15\pi}{16}\frac{GM}{c^2 b}\right] + O\left(\frac{m^3}{b^3}\right), \qquad \text{where} \quad b = \frac{D(r_0)}{A(r_0)} r_0 \qquad \text{and} \qquad m = \frac{GM}{c^2} \tag{2}$$

Here, $\kappa = G/c^2 = 7.425\text{x}10^{-29}\text{cmg}^{-1}$, M is the total mass and r_0 is the closest (Euclidean-like) distance from the center. Gérard & Piereaux [17] called "b" as the impact parameter. Then, the deflection angle α(b) is independent of whether the Schwarzschild, the harmonic or the isotropic gauge is used. For this case, covariance is physically meaningful, only if it is invariant since a deterministic measurement is unique for a given frame of reference.

However, this invariance in (2) is only superficial mathematics. To be explicit, the deflection angle is respectively,

$$\alpha(b) = \frac{4m}{d_s}\left[1 + (\frac{15\pi}{16} - 1)\frac{m}{d_s}\right] + O\left(\frac{m^3}{d_s^{\,3}}\right), \qquad b = \frac{D(d_s)}{A(d_s)} d_s = \frac{d_s}{(1 - 2m/d_s)^{1/2}} \tag{3a}$$

$$\alpha(b) = \frac{4m}{d_h}\left[1 + (\frac{15\pi}{16} - 2)\frac{m}{d_h}\right] + O\left(\frac{m^3}{d_h^{\,3}}\right), \qquad b = \frac{D(d_h)}{A(d_h)} d_h = d_h \frac{(1 - m/2d_h)^3}{(1 - m/2d_h)} \tag{3b}$$

$$\alpha(b) = \frac{4m}{d_I}\left[1 + (\frac{15\pi}{16} - 2)\frac{m}{d_I}\right] + O\left(\frac{m^3}{d_I^{\,3}}\right), \qquad b = \frac{D(d_I)}{A(d_I)} d_I = d_I \frac{(1 - m/2d_I)^3}{(1 - m/2d_I)} \tag{3c}$$

where d_s, d_h, and d_I are respectively the closest distance (in their respective Euclidean-like structure) [16]. Thus, the deflection of light to second order is not gauge invariant, (assumed the experimental accuracy of second order can be reached) a question is which of (3a), (3b), or (3c) should be applied. Then, if r_0 represent d_i of the gauge, one has

$$b \approx m + r_0, \qquad \text{or} \qquad 2m + r_0 \tag{4}$$

depending on what gauge is valid for reality. Moreover, when a number is obtained for the closest distance r_0 from a measurement, in principle, one has to decide what the actual gauge is in reality. Since the paper of Bodennor & Will [16] does not provide an answer, this type of restricted covariance is essentially not only useless but also misleading in physics. A similar problem would occur in the issue of gravitational redshifts. The crucial question is which formula should be used.

One might argue that b is the ratio of angular momentum to energy for the photon, which can be measured at infinity. However, this is only a mean of evasion, but not a valid counter argument. Since the angular momentum of the photon must be measured in term of the distance of the Euclidean structure and time of the coordinate system, this is a much more complicated operation than measuring the shortest distance r_0. Moreover, the angular momentum also involves r_0.

However, (2) suggests that parameter "b" could be obtained from a metric with D(r) = A(r), i.e., a metric of the form,

$$ds^2 = A^2(R)\, c^2dt^2 - B^2(R)\, dR^2 - A^2(R)R^2(d\theta^2 + \sin^2\theta d\varphi^2). \tag{5}$$

with

$$R^2 = r_S^3/(r_S - 2m), \qquad A^2(R) = (1 - 2m/r_S), \qquad \text{and} \qquad B(R) = (r_S - 2m)/(r_S - 3m) \tag{6}$$

Thus, it seems that b could be obtained directly. However, metric (4) would mean that the local light speed remains c in the θ-direction, and that the local light speed would be larger than c in the R-direction if $2m < r_s < 2.25m$. Thus, this metric does not satisfy Einstein's equivalence principle, [5] and is not realizable. In short, while claiming the impact parameter b is measurable, Bodenner & Will [16] neglected a crucial fact that there is no way to measure b except through r_0.

Moreover, an implicit assumption in their paper is that all the gauges are valid in physics. This misled them to believe that things can be solved with a modification of the definitions. However, physics is not a matter of just definitions alone as in mathematics. [6] It will be shown that there are more such gauges that are invalid in physics.

3. A Counter Example for Covariance

Apparently, there are other unphysical metrics of form (1). Consider the metric of the following form,

$$ds^2 = A^2(R)\, c^2dt^2 - dR^2 - G(R)^2\, R^2\, (d\theta^2 + \sin^2\theta\, d\varphi^2), \tag{7}$$

where H(R) and G(R) are functions of R, which is $[X^2 + Y^2 + Z^2]^{1/2}$. This metric is diffeomorphic to the isotropic solution,

$$ds^2 = [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]\, c^2dt^2 - (1 + M\kappa/2r)^4\, (dx^2 + dy^2 + dz^2) \qquad (8)$$

where $\kappa = G/c^2$ ($G = 6.67 \times 10^{-8}$ erg cm/gm^2), M is the total mass of a spherical mass distribution with the center at the origin of the frame of reference K (x, y, z), and $r = [x^2 + y^2 + z^2]^{1/2}$. The related diffeomorphism [19] is

$$R(r) = r - M\kappa \ln(M\kappa/2r) - (M\kappa)^2/4r + M\kappa C_0, \qquad r > M\kappa/2 \qquad (9a)$$

and

$$A(R) = (1 - M\kappa/2r)/(1 + M\kappa/2r), \qquad RG(R) = (1 + M\kappa/2r)^2\, r, \qquad (9b)$$

where $M\kappa C_0$ (= R[Mκ/2]) is a non-negative constant. These two metrics share the same frame of reference and lead to the same deflection angle of a light ray [19]. However, G(R) is smaller than 1 for sufficiently large R, and this means "space expansion" instead of the usual space contraction if Einstein's equivalence principle were valid for this metric.

Moreover, an "event of horizon" for metric (7) corresponds to the arbitrary constant $M\kappa C_0$, whereas, for the Schwarzschild, the harmonic and the isotropic metrics, their different event horizons correspond to each other in a definite manner. This illustrates beyond doubt that the notion of general covariance is invalid in physics.

From, eq. (9), according to formula (2), we have

$$\alpha(b) = \frac{4m}{d_u}\left[1 + \frac{m}{d_u}\left(\frac{15\pi}{16} - 2 + C_0 - \ln\frac{m}{2d_u}\right)\right] + O\left(\frac{m^3}{d_u^{\,3}}\right), \qquad b \approx d_u\left[1 + \frac{m}{d_u}\left(2 - C_0 + \ln\frac{m}{2d_u}\right)\right], \qquad (10)$$

where d_u is the closest distance for this gauge. For the sun, we have $m/d_u \approx 2.12\text{x}10^{-6}$. Thus the logarithmic term should not be neglected. In addition, there is C_0, the arbitrary constant. Thus, the second order term is clearly not gauge invariant.

4. Discussions and Conclusions

This analysis further establishes that general relativity is not a gauge invariant theory. This can be traced back to Einstein's equivalence principle.[7)] Moreover, this explicit calculation and analysis unequivocally rejects the "covariance principle". Based on Einstein's equivalence principle, it has been shown [6, 7] that the theoretical framework of general relativity has given a definite physical meaning to space-time coordinates. Thus, the criticisms of Whitehead [20] and Fock [11] are actually toward Einstein's oversight, but not general relativity. However, Will [21] invalidly regards the "covariance principle" as an integral part of general relativity because Will does not understand it. Since physical quantities such as vectors and tensors are measurable and their components are expressed in terms of coordinates. The claim of arbitrariness of physical coordinates, if put Zhou's [5] statement in a clear language, just make no sense in physics.

Bodennor and Will [16] also have a problem in distinguishing physics from mathematics. [6] They seem to believe that physics is just a matter definition. For instance, they claim that the impact parameter b, instead to the closest distance r_0 , is directly measurable in physics. However, they failed to show how this is feasible. They have a restricted covariant formula, but fail to show how this can be or should be applied in comparison with observation, that requires a choice of the gauge.

Moreover, if Will believes that the fundamental distance is measured in term of b instead of r_0, at least he should have developed his theory of measurement to justify such a claim. Will often does not make enough efforts to see his assumptions have a good ground to be valid [18]. Note that the observed light bending [1, 2] supports that r_0 is a fundamental measurement because light speeds must be defined in terms of the Euclidean-like structure [12, 14, 22].

More important, Bodennor and Will considered the question of covariance is just a problem in mathematics; they did not check whether the metric is valid in physics. For instance, since the "event of horizon" is not invariant, and even arbitrary, it certainly does not make sense that the "covariance principle" is valid. Their paper shows clearly that, in addition to Will's book [21], their knowledge in general relativity is also out dated.

Another major problem is that, like many others, Will [21] does not understand that Einstein's equivalence principle is incompatible with Newtonian gravity.[8] Thus Will did not realize that the bending of light to the first order is only a test of Einstein's equivalence principle [23]. However, the second order involves the question of what is the correct physical gauge [6, 7], and thus is a far more difficult problem.

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ENDNOTES

1) An absurd claim of having proved all measurable quantities are scalars was actually made by some theorists [24].

2) Zhou [5, 25] has proposed that the harmonic gauge condition to be necessary for an asymptotically flat metric, and thus rejected the Schwarzschild solution. He also correctly argued that the "covariance principle" is not valid in physics.

3) The principle of causality [26] assumes that the causes of phenomena are identifiable. This principle is commonly used in symmetry consideration in electrodynamics. In general relativity, Einstein and other theorists have used this principle implicitly on symmetry considerations [1, 8, 10, 13, 27-33]. Thus, the physical meaning of space coordinates has been used

not only subsequently, but also right at the beginning [1]. Some other consequences are that parameters unrelated to any physical cause in a solution is not allowed and that a dynamic solution must be related to a dynamic source.

4) This argument was endorsed by the editorial of Am. J. Phys., but the editorial also conceded that the her journal may not be the best place to discuss such an issue (November 8, 2005).

5) A satisfaction of Einstein's equivalence principle requires that a time-like geodesic must represent a physical free fall and in particular the co-moving local space must be Minkowski. This means that the manifold of space-time models the reality and all (not just some) physical requirements are sufficiently satisfied. Thus, the equivalence principle may not be satisfied in a Lorentz Manifold [34], which implies only the mathematical existence of local Minkowski spaces.

6) A theory in physics must be supported by experiments, in addition to being logically self-consistent [14].

7) Einstein's equivalence principle implies the physical existence of a local Minkowski space, and this is stated in Eq. (106) in Einstein's book "The Meaning of Relativity". Some claimed instead that gravity in a small region can always be transformed away, in spite of Einstein objected as a misinterpretation [9]. For instance, Will [35] incorrectly claimed, "Equivalence came from the idea that life in a free falling laboratory was equivalent to life without gravity." Apparently, Will overlooked (or disagreed with) Einstein's [1, p. 144] remark, "For it is clear that, e.g., the gravitational field generated by a material point in its environment certainly cannot be 'transformed away' by any choice of the system of coordinates...". Will's crucial mistake [21] is his belief in equivalence of gravity and acceleration, whereas Einstein proposed only the equivalence of a uniform gravity and an accelerated frame [1, 2].

8) In Newtonian theory, the potential for a uniform gravity is static, similar to an electric field. It turns out, however, that in general relativity the metric related to uniform gravity has to be time-dependent. Unfortunately, it was not recognized that such a time-dependence is necessary although a uniform acceleration cannot last indefinitely [14]. Like many others, Schiff [36] has mistaken the 1911 preliminary application of equivalence between acceleration and Newtonian gravity as Einstein's equivalence principle of 1921 [2]. Such an error has led to the incorrect belief that Einstein's equivalence principle, instead of Newtonian gravity, was the cause to the failure in deriving a valid deflection of light [29].

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Dear Sir:

I would like to submit my paper, On Deflection of Light to Second Order and Invalidity of the "Principle of Covariance", to your journal. The copyright of this paper, if accepted, will be automatically transferred to your journal.

The purpose of this submission, as indicated by its title, is to rectify the errors in the article of Bodennor & Will, "Deflection of Light to Second Order: a Tool to Illustrate the Principles of General Relativity," American Journal of Physics in August 2003. Moreover, it is pointed out that the second order is a far mote complicated problem that needs further investigation. The late response is due to that I was unaware this article until it is referred to me by others. I have tried to resolve this issue with the publishing journal. However, it turns out that this paper is beyond the scope of American Journal of Physics.

I shall be glad to answer any questions that you may have. Thank you for your kind attention. I am looking forward to hearing from you.

Sincerely yours,

C. Y. Lo