

1 Misunderstandings Related to Einstein's Principle of Equivalence
2 and
3 Einstein's Theoretical Errors on Measurements

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15 **Abstract**

16 Einstein's 1911 preliminary application of the equivalence between acceleration and Newtonian uniform gravity to derive gravi-
17 tational redshifts was commonly but mistakenly regarded to be the same as Einstein's principle of equivalence, which has its founda-
18 tion in theorems of Riemannian geometry and special relativity. Consequently, limitations due to Newtonian gravity shown in such
19 derivations were incorrectly believed as that Einstein's equivalence principle should not only be inadequate to derive the bending of
20 light, but also have questionable validity itself. Moreover, in Einstein's theory of measurements, his instruments being in free fall
21 states are incorrectly ignored. Consequently, there are theoretical errors and inconsistency, for example, as shown in the case of
22 Einstein's uniformly rotating disk. However, after the theoretical errors are identified and rectified, the uniformly rotating disk
23 would be an example that illustrates the importance of Einstein's equivalence principle and the inadequacy of Pauli's version. The
24 crucial role of Einstein's equivalence principle in the time dilation and space contractions is thus illustrated.

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28 *Philip Morrison of MIT, who passed away on 22 April 2005, was the coauthor of the first draft.

29 **Key Words:** Einstein's Principle of Equivalence, Preliminary Application of Equivalence, Euclidean-like Structure, E-R Space,
30 Einstein-Minkowski Condition.

1. Introduction

It is generally agreed, as pointed out by Einstein [1], Eddington [2], Pauli [3], Weinberg [4], Misner, Thorne & Wheeler [5], and Straumann [6] that Einstein's principle of equivalence is the theoretical foundation of general relativity. However, as Einstein [7] saw it, few understand the crucial role of Einstein's equivalence principle in terms of physics. On the other hand, Einstein insisted, throughout his life, on the fundamental importance of this principle to his general theory of relativity [7].

For instance, Einstein's equivalence principle [8] played a crucial role on Einstein's theory of measurements and his notion of curved space. However, Whitehead [9, p. 83] objected,

“By identifying the potential mass impetus of a kinematic element with a spatio-temporal measurement Einstein, in my opinion, leaves the whole antecedent theory of measurement in confusion, when it is confronted with the actual conditions of our perceptual knowledge. The potential impetus shares in the contingency of appearances. It therefore follows that measurement on his theory lacks systematic uniformity and requires a knowledge of the actual contingent physical field before it is possible.”

Einstein is universally accepted as a genius. However, a genius is often not without faults. Whitehead remarked,

“But the worst homage we can pay to genius is to accept uncritically formulations of truths which we owe to it.”

From this remark, Whitehead ingeniously foresaw problems existing in current theory of general relativity. It is in this spirit of Whitehead that this paper on Einstein's theoretical errors on measurements and related problems is written.

It has been shown that Whitehead's objection on Einstein's theory of measurements is well justified [10, 11]. His theory of measurements forced him [8] to propose an interim assumption, the so-called “covariance principle” that has also been proven to be over extended [12]. Fortunately, it is also found that his problematic theory of measurements is only due to Einstein's oversights rather than an intrinsic problem of general relativity [10-12]. In fact, it has been shown [11] that the theoretical framework of general relativity has given a definite physical meaning to space-time coordinates.

It will be shown that Einstein's theory of measurements is inconsistent, but is not an integral part of general relativity. Moreover, it is fundamentally incorrect that Einstein's 1911 application of his earlier notion of equivalence be regarded the same as Einstein's principle of equivalence of 1921. Einstein's uniformly rotating disk [1, 8] will serve as an example.

2. Einstein's Equivalence Principle and its Preliminary.

Einstein's equivalence principle states the equivalence between a uniformly accelerated system K' and a system K at rest that processes a gravitational field where all bodies are equally and uniformly accelerated. However, although this principle formally stated in his book, “The Meaning of Relativity”, it has a long history starting from 1907 [4]. Unfortunately, this long history seems to have become a source of misunderstanding. The 1911 preliminary application of his notion of equivalence incorporated Newtonian gravity only [8]. Then gravitational redshifts were derived, but a derivation of the light bending failed. Thus, many believed

1 Einstein's equivalence principle alone could be used to derive the gravitational redshifts only and this preliminary application of
 2 equivalence has been mistaken as Einstein's equivalence principle. However, the problem may have arisen from the preliminary
 3 application to Newtonian gravity rather than Einstein's equivalence principle itself [13].

4 Einstein regards that a consequence of his equivalence principle is the *Einstein-Minkowski condition* that the local space of a par-
 5 ticle under gravity must be locally Minkowskian [1, 8], from which he obtained the time dilation and space contractions. However,
 6 others often regarded this condition as non-essential [7], although Einstein's used this condition in his 1916 initial paper [8] on gen-
 7 eral relativity and his book, "The Meaning of Relativity" [1]. Apparently, many have missed this important point.

8 Historically, the idea of equivalence between inertial mass and gravitational mass goes back as far as Galileo. Then mathemati-
 9 cal theorems [14] show that the local space of a particle under the influence of gravity only is locally constant, but not necessarily
 10 Minkowskian. However, in special relativity, such a local space is Minkowskian. Thus, the Einstein-Minkowski condition is nec-
 11 essary. *In fact, the Einstein-Minkowski condition is the accurate new form of Einstein's Equivalence principle and this is what*
 12 *Einstein used in his subsequent calculations* [15]. Although Einstein's equivalence principle is stated clearly, it is still unclear be-
 13 cause a verified example for the Einstein-Minkowski condition has not been provided.

14 Nevertheless, Pauli's "infinitesimal"- principle of equivalence [3] was commonly but mistakenly regarded as Einstein's principle,
 15 although Einstein strongly objected to this version as a misinterpretation [7]. Pauli's [3] version is as follows:

16 "For every infinitely small world region (i. e. a world region which is so small that the space- and time-variation of gravity can
 17 be neglected in it) there always exists a coordinate system $K_0 (X_1, X_2, X_3, X_4)$ in which gravitation has no influence either in the
 18 motion of particles or any physical process."

19 Thus, Pauli regards the equivalence principle as just the mathematical existence of locally constant spaces, which may not be locally
 20 Minkowskian. The main difference is that Einstein claimed that in a free fall a natural process would generate the co-moving local
 21 Minkowski space. Moreover, in disagreement with mathematical theorems, Pauli incorrectly extended the removal of uniform grav-
 22 ity with a uniform acceleration to the removal of gravity in general by means of a coordinate transformation.

23 Straumann [6] claimed that Einstein's principle of equivalence would be, "In any arbitrary gravitational field no local experiment
 24 can distinguish a freely falling non-rotating system (local inertial system) from a uniformly moving system in the absence of a gravi-
 25 tational field." He recognized that the local space in a free fall is locally Minkowskian, but failed to see that such a metric is not the
 26 same metric in the absence of a gravitational field. Similarly, Will [16] claimed, "Equivalence came from the idea that life in a free
 27 falling laboratory was equivalent to life without gravity. It also came from the converse idea that a laboratory in distant empty
 28 space that was being accelerated by a rocket was equivalent to one at rest in a gravitational field."

29 Pauli, Straumann, and Will overlooked (or disagreed with) Einstein's [8, p. 144] remark, "For it is clear that, e.g., the gravita-
 30 tional field generated by a material point in its environment certainly cannot be 'transformed away' by any choice of the system of

1 coordinates...” Pauli’s misunderstanding has far reaching consequences in theoretical physics since it leads to the acceptance of un-
 2 physical solutions as valid in physics [17]. It was based on Pauli’s version that Logunov and Mestvirishvili [18] showed that gen-
 3 eral relativity would lead to inconsistency related to the radiation formula. Moreover, the local distance formula of Landau & Lif-
 4 shitz [19] would give results in disagreement with observation and be even incompatible to their own claims [15].

5 Pauli’s version is popular, in part, because it imposes no requirement on coordinates other than the proper metric signature.
 6 This is compatible with the situation that the space-time coordinates are ambiguous although Pauli’s version does not provide the
 7 physics for the time dilation and space contractions [15]. However, both Einstein’s equivalence principle and the principle of gen-
 8 eral relativity require a clear meaning of coordinates, and thus cannot be rigorously defined [9, 20]. Whitehead [9] and Fock [20]
 9 regarded this ambiguity as intrinsic and thus rejected general relativity as a physical theory.

10 However, the above objections of Whitehead and Fock pertain only to Einstein’s oversights but are actually irrelevant to general
 11 relativity. It has been shown that the physical meaning of space-time coordinates has already been included in the theoretical
 12 framework of general relativity [10, 11] although Einstein overlooked this [1, 8]. The failed calculations of Fock [20] and Tolman
 13 [21] on the metric of a uniform gravity are due to misconceptions that identified a frame of reference with a Euclidean subspace [15]
 14 although the metric of a rotating disk would show clearly that such an identification is incorrect [11, 22].

15 In Newtonian theory, one may define uniform gravity as “a homogeneous field is characterized by the fact that any part of it is
 16 representative of the whole [23].” However, an intrinsic difference from Newtonian theory is that the gravitational potential plays a
 17 crucial role in physics and gravity is generated from a space-time metric instead of a scalar potential. Unfortunately, in terms of a
 18 space-time metric, Einstein’s notion of uniform gravity was not clearly illustrated.

19 Historically, in the 1911 preliminary application of Einstein’s notion of equivalence, the notion of curved space-time had not yet
 20 been proposed. Fock [20], who failed to obtain the space-time metric for a uniform gravity, actually based his calculation essen-
 21 tially on a Newtonian notion of gravity and thus should not have concluded that Einstein’s equivalence principle was at fault.²⁾
 22 Nevertheless, Fock has won many converts, including the Wheeler school [25]. Fortunately, a direct derivation of the Maxwell-
 23 Newton Approximation, which is independent of Einstein’s equation, shows that Fock was incorrect [13, 26].

24 Another problem was that Einstein and other theorists seemed to have concluded mistakenly that any metric representing a uni-
 25 form gravity was static [20, 25]. Thus, uniform gravity, though intuitively simple in Einstein’s equivalence principle, became theo-
 26 retically complicated in general relativity. Moreover, a major problem was the ambiguity of space-time coordinates, which arose
 27 due to an inadequate application of Einstein’s equivalence principle in Einstein’s theory of measurements [10].
 28

29 **3. Confusions and Criticisms on Einstein’s Equivalence Principle**

1 Because the physical meaning of the space-time coordinates had not been provided, Einstein's equivalence principle was often
 2 misunderstood, and consequently its crucial role in general relativity was overlooked. For instance, Synge [14] professed his mis-
 3 givings about Einstein's equivalence principle as follows:

4 "...I have never been able to understand this principle...Does it mean that the effects of a gravitational field are indistin-
 5 guishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravita-
 6 tional field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has
 7 nothing to do with any observer's world line...The Principle of Equivalence performed the essential office of midwife at
 8 the birth of general relativity...I suggest that the midwife be now buried with appropriate honours and the facts of absolute
 9 spacetime be faced."

10 From these statements, it is clear that Synge does not fully understand the physics underlying Einstein's equivalence principle (see
 11 also § 6 & Appendix A). In fact, Einstein's equivalence principle states only that the effects of an accelerated frame are equivalent
 12 to a related uniform gravity [1, 8]. Furthermore, Einstein [27] stated that the gravity of the earth is not equivalent to an accelerated
 13 frame, although Bergmann [28] confused Einstein's equivalence principle with "Einstein's elevator".

14 Moreover, a gravitational field need not be related to a non-vanishing curvature. As Einstein [7] explained to Laue, "What
 15 characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^l_{ik} (field strength), not
 16 the non-vanishing of the R_{iklm} ," and no gravity is therefore just a special case of gravity. This view is crucial because it justifies
 17 that the geodesic equation is also the equation of motion of a massive particle under the influence of only gravity.

18 Nevertheless, misunderstandings continued. Thorne [29] criticized Einstein's principle for ignoring tidal gravitational forces.
 19 However, Einstein had already clarified this issue in a letter to A. Rehtz [7]:

20 "The equivalence principle does not assert that every gravitational field (e. g. , the one associated with the Earth) can be
 21 produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present
 22 themselves from an accelerated coordinate system, represent a special case of the gravitational field."

23 Thus, Einstein's principle is proposed for a *physical space*, where all physical requirements are sufficiently satisfied.

24 Nevertheless, Zel'dovich & Novikov [22] believed the equivalence principle meant only that a particle followed the geodesic.
 25 Landau & Lifshitz [19] incorrectly believed that, in an accelerated frame everybody (with different speeds) had the same acceleration.
 26 Wald [30] and Ohanian & Ruffini [25] regarded the equivalence principle as merely the equivalence of inertial mass and gravita-
 27 tional mass. Thus, it is useful to clarify the theoretical foundations of Einstein's equivalence principle.

28 29 **4. The Physical Meaning of Space-time Coordinates**

1 Einstein “showed” that it is necessary to abandon Euclidean geometry because, for a rotating reference system, the time intervals
 2 and spatial distances in non-Galilean systems cannot be determined (with his method of measurement) by means of a clock and rigid
 3 standard measuring rod. To clarify this, one must distinguish between the measurements based on coordinates and those based on
 4 the metric of a space (see Appendix A).

5 As shown by Weinberg [4], “For both Euclidean and non-Euclidean geometry the ‘model’ is provided by the theory of real num-
 6 bers.” Descartes’ analytic geometry shows that if a point is identified with a pair of real numbers (x_1, x_2) and the distances between
 7 two points (x_1, x_2) and (X_1, X_2) is identified as $d(x, X) = [(x_1 - X_1)^2 + [(x_2 - X_2)^2]^{1/2}$, then all of the Euclid’s postulates can be proved
 8 as theorems about real numbers. However, if a different metric $d'(x, X)$ is defined, one obtains a non-Euclidean geometry in Des-
 9 cartes’ coordinates. Thus, the meaning of the coordinates in a non-Euclidean geometry is independent of the metric, since the coor-
 10 dinates remain the same as in Euclidean geometry.

11 In general relativity, the invariant line element is

$$12 \quad ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \quad (1)$$

13 where $g_{\mu\nu}$ is a general space-time metric in a Riemannian space. Since $g_{\mu\nu}$ is not a constant metric, one cannot hope to derive from
 14 (1) a simple distance formula as in Euclidean geometry where the spatial distance $d(P_1, P_2)$ of two points P_1 and P_2 is still

$$15 \quad d(P_1, P_2) = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}. \quad (2)$$

16 However, *in a different way*, the Euclidean-like structure (2), which deals with global distance between two points in a frame of ref-
 17 erence, is necessarily preserved within the Riemannian space-time [11].²⁾

18 To illustrate this, let us examine the Schwarzschild solution [1] of Riemannian space (x, y, z, t) ,³⁾

$$19 \quad ds^2 = (1 - 2M\kappa/\rho) c^2 dt^2 - (1 - 2M\kappa/\rho)^{-1} d\rho^2 - \rho^2 d\theta^2 - \rho^2 \sin^2\theta d\varphi^2, \quad (3)$$

20 where

$$21 \quad \rho^2 = x^2 + y^2 + z^2, \quad x = \rho \sin\theta \cos\varphi, \quad y = \rho \sin\theta \sin\varphi, \quad \text{and} \quad z = \rho \cos\theta. \quad (4)$$

22 In addition, κ is a coupling constant, and M is the total mass. Note that metric (3) alone does not tell what the frame of reference is.
 23 Then eq. (4) clarifies that the frame has a spherical coordinate system, and therefore has a Euclidean-like structure that satisfies the
 24 Pythagorean theorem. This illustrates that the Euclidean-like structure is included in Einstein’s Riemannian space. However, in a
 25 free fall, its local spaces being locally Minkowskian is assumed only, but not proven [1, 8].

26 To understand Einstein’s measurement, we must clarify what “measure” means in relation to Einstein’s equivalence principle.
 27 In Einstein’s theory, the measuring instruments are resting but in a free fall state [1, 8]. From Einstein’s equivalence principle, time
 28 dilation and local space contraction are obtained. Based on such measurements that would create a problem of circular logic, Ein-
 29
 30

stein believed, “In the general theory of relativity, space and time cannot be defined in such a way that differences of the spatial coordinates can be directly measured by the unit measuring-rod, or differences in the time coordinates by a standard clock”.

Moreover, since his measuring rods at different points, though all are at rest, would be in different free fall states, this would be an impossible situation for measuring to an extended object. However, if the measuring instruments are attached to the frame of reference, since the measuring instruments and the coordinates being measured are under the same influence of gravity, a Euclidean-like structure emerges as if gravity did not exist [10, 11]⁴⁾. (For a more detailed discussion, see reference [12].) *Moreover, the space contractions have a physical meaning only in terms of the Euclidean-like structure [15] (also see Appendix A).*

5. Uniformly Rotating Disk and Riemannian Space-Time

Although Einstein initially conceived his theory by considering a linear acceleration, the exposition of his theory started by considering a rotation [8]. In a free fall of Einstein’s rotating disk, its local spaces are, in fact, (not just assumed) Minkowskian. Specifically, he considered a Galilean (inertial) system of reference $K(x, y, z, t)$ and a system $K'(x', y', z', t')$ in a uniform rotation Ω relatively to K . The origins of both systems and their axes of z and z' coincide. The flat metric of K is,

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dz^2 \quad \text{where} \quad x = r \cos \phi, \quad y = r \sin \phi, \quad (5)$$

in the cylindrical coordinate system. For reason of symmetry⁵⁾, a circle around the origin in the x - y plane of K may at the same time be regarded as a circle in the x' - y' plane of K' . Einstein argued that if circle is measured from K' , because of Lorentz contraction, the circumference would be greater than $2\pi r'$ although the so measure radius $r' = r$. Moreover, Einstein claimed [8],

“An observer at the common origin of co-ordinates would therefore see it lagging behind the clock beside him.... So, he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be.”

Thus, Einstein defined a physical space-time coordinate system together with a metric that relates to local clock rates and local spatial measurements.⁶⁾ In other words, a physical space-time coordinate system is not arbitrary.

Let us compare his claims with his uniformly rotating disk. According to Einstein, the transformation to a uniformly rotating reference frame (x', y', z') ⁷⁾ with angular velocity Ω has the form [15, 19, 22],

$$x = x' \cos \Omega t - y' \sin \Omega t, \quad y = x' \sin \Omega t + y' \cos \Omega t, \quad \text{and} \quad z = z', \quad (6a)$$

or

$$r = r', \quad z = z', \quad \text{and} \quad \phi = \phi' + \Omega t \quad (6b)$$

Note that (6a) implies

$$r' = r, \quad x' = r \cos \phi', \quad \text{and} \quad y' = r \sin \phi', \quad (7)$$

1 Thus, (7) together with $z = z'$ means K' also has the Euclidean-like structure just as the frame for the Schwarzschild solution.

2 Then a metric in terms of the coordinates in $K'(x', y', z')$ can be obtained from

$$3 \quad dr = dr', \quad dz = dz', \quad \text{and} \quad d\phi = d\phi' + \Omega dt \quad (8)$$

4 which are simply a consequence of (6b). The transformed metric in system $K^*(x', y', z', t)$ would have the following form,

$$5 \quad ds^2 = (c^2 - \Omega^2 r^2) dt^2 - 2\Omega r^2 d\phi' dt - dr^2 - r^2 d\phi'^2 - dz'^2 \quad (9)$$

6 The system $K^*(x', y', z', t)$ with metric (9) satisfies Pauli's version of equivalence principle, but the light speed could be larger than

7 c . However, according to Einstein, the issue of local clock rates is not clearly since "t" is related to local clocks resting at (x, y, z) .

8 Nevertheless, according to the local distance formula of Landau & Lifshitz [19], the local distance in the ϕ' -direction is

$$9 \quad (1 - \Omega^2 r^2/c^2)^{-1/2} r' \int_0^{2\pi} d\phi'. \quad (10a)$$

10 and thus

$$11 \quad \oint ds = (1 - \Omega^2 r^2/c^2)^{-1/2} r' \int_0^{2\pi} d\phi' = 2\pi r' (1 - \Omega^2 r^2/c^2)^{-1/2} > 2\pi r' \quad (10b)$$

12 would be the circumference of the circle with radius r' . (Eq. (10b) is inconsistent with the Euclidean-like structure eq. (7).) How-

13 ever, according to the "standard" arguments given by Ohanian and Ruffini [25],⁸⁾ they would have claimed that there is no space

14 contractions in the presence of gravity. Obviously, these theorists disagree with each other. Still, they are different from Einstein

15 because they believed Pauli's version as adequate. The main difference between their arguments and Einstein's [1, 8] is that

16 Einstein saw the need of assuming the validity of his equivalence principle [1, 8] that would make the Einstein-Minkowski condition

17 applicable, while they ignore his principle. An advantage of this case is that the Einstein-Minkowski condition is certainly satisfied

18 since the local space in free fall has a Minkowski metric. Thus, the validity of these claims can be checked.

19

20 **6. Einstein's Equivalence Principle and Einstein's Notion of Local Time**

21 To see the meaning of local time and Einstein's equivalence principle for this case, one needs to find out what the local spaces

22 are in free falls. In Einstein's calculations [1, 8] such local spaces being locally Minkowskian is assumed only.

23 Consider a particle P resting at (r', ϕ', z') of $K^*(r', \phi', z', t)$. The local space of P is (dr', dX, dz', dT) which has a Minkowski

24 metric. In K, P has a position (r, ϕ, z) and its local space $(dr, rd\phi, dz, dt)$ has the Minkowski metric in K. These two local spaces

25 have a relative velocity $r\Omega$ in the ϕ -direction in K. Since $r' = r$, and $z' = z$, we have $dr' = dr$, and $dz' = dz$.

26 Then, the two local spaces relate to each other by the Lorentz transformation in special relativity as follows:

$$27 \quad rd\phi = [1 - (r\Omega/c)^2]^{-1/2} [dX + r\Omega dT], \quad (11a)$$

28 and

1
$$cdt = [1 - (r\Omega/c)^2]^{-1/2} [cdT + (r\Omega/c)dX]; \quad (11b)$$

2 or
3
$$dX = [1 - (r\Omega/c)^2]^{-1/2} [rd\phi - r\Omega dt], \quad (11c)$$

4 and
5
$$dT = [1 - (r\Omega/c)^2]^{-1/2} [dt - (r\Omega/c^2) rd\phi] \quad (11d)$$

6 From (11c) and (8), one obtains

7
$$dX = [1 - (r\Omega/c)^2]^{-1/2} rd\phi', \quad (12a)$$

8 and
9
$$dT = [1 - (r\Omega/c)^2]^{1/2} \{dt - [1 - (r\Omega/c)^2]^{-1} (r\Omega/c^2) rd\phi'\} \quad (12b)$$

10 Eq. (12a) shows that, *the local distance in the ϕ' -direction is measured from a local space at a free fall*, and therefore its is not
11 measured in $K'(x', y' z')$. If the circumference of a circle is measured to obtain (12), although the measuring rods are at rest, they
12 must be in a different free fall state at each point, an impossible situation to execute. Thus, the local distance of Landau & Lifshitz
13 [19] is not a local distance in K' as they claimed, and thus (12a) can be consistent with the Euclidean-like structure (7).

14 In (12b) we have replace $d\phi$ with $d\phi'$ because a local clock at K' has $d\phi' = 0$. Now, it is clear that the term for space contrac-
15 tions in the ϕ' -direction can be generated by completing the square of the $dt d\phi'$ and the dt^2 terms, i. e. ,

16
$$ds^2 = (c^2 - \Omega^2 r^2) \{dt - [1 - (r\Omega/c)^2]^{-1} (r\Omega/c^2) rd\phi'\}^2 - dr^2 - [1 - (r\Omega/c)^2]^{-1} r^2 d\phi'^2 - dz'^2 \quad (13)$$

17 This suggests a local time dt' , and locally the metric is,

18
$$ds^2 = (c^2 - \Omega^2 r'^2) dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2 d\phi'^2 - dz'^2. \quad (14)$$

19 Then one would have

20
$$dT = [1 - (r\Omega/c)^2]^{1/2} dt' \quad \text{where} \quad cdt' = cdt - (r\Omega/c)rd\phi'[1 - (r\Omega/c)^2]^{-1}. \quad (15)$$

21 The local clock in K' is identified with $d\phi' = 0$ and its rate is $[1 - (r\Omega/c)^2]^{1/2} dt$, and at $r = 0$, one has $dt' = dt$. Thus, eq. (15) is con-
22 sistent with the argument of Landau & Lifshitz [19] that for a clock resting at the frame of reference

23
$$ds = c d\tau, \quad \text{and} \quad d\tau = [g_{00}]^{1/2} dx^0, \quad (16)$$

24 where x^0 is the time variable of the metric.

25 For a clock rest at K' , according to Einstein [1, 8], the observed clock rate is dT . Thus, if observed from K , we have

26
$$dt = [1 - (r\Omega/c)^2]^{-1/2} dT \quad (17)$$

27 Eq. (17) is the same result as derived from (11b) in special relativity with $dX = 0$.

1 Thus, in terms of the final measurable results, metric (14) would be considered just as a midwife that identifies the space contrac-
2 tions and the time dilation clearly. Since (14) is derived from Einstein's equivalence principle, its crucial role is now clear.

3 Then, metrics (1) and metric (14) relate each other with the following relations,

$$4 \quad d\phi = d\phi' + \Omega dt, \quad dr' = dr, \quad dz' = dz \quad (8)$$

5 and

$$6 \quad cdt' = cdt - (r\Omega/c)r d\phi' [1 - (r\Omega/c)^2]^{-1}. \quad (15)$$

7 If one believes that t' is a global variable, then one integrated them, and would get

$$8 \quad \phi = \phi' + \Omega t, \quad r' = r, \quad z' = z \quad (18a)$$

9 and

$$10 \quad ct' = ct - (r\Omega/c)r\phi' [1 - (r\Omega/c)^2]^{-1} \quad (18b)$$

11 if (15) were integrable. However, (18b) is not valid in physics. A problem is that $\phi' \pm 2\pi$ is the same position, but t and t' would not
12 be the same. The physical reason is, as shown in (15), that dt' is related to different inertial systems at different r and t , and thus (15)
13 is not integrable.⁹⁾ (This would not be clear if the Einstein-Minkowski condition is not considered [11].) Moreover, if the metric
14 has only diagonal elements, then it would be invariant under the exchange of $\phi' \leftrightarrow -\phi'$. According to Synge [14; p. 309], this
15 would mean no rotation. Thus, dt' should be considered as representing the local time only.

16 Nevertheless, in this consideration, there is no need to exclude the possibility that the relations in eq. (8) are approximations since
17 being approximately valid would not change the main physical conclusion. As long as Einstein's notion of a uniformly rotating disk
18 is approximately valid, a space contraction and a time dilation would be obtained from the Lorentz transformation.

19 In short, Einstein's theory of measurement is inconsistent because Einstein incorrectly saw space contractions, which are obtained
20 from a local space at free fall, as measured in the frame of reference. Since the Einstein-Minkowski condition is satisfied,¹⁰⁾ Ein-
21 stein's notion of local time and local clocks are supported.

22 7. Discussions and Conclusions

24 Currently, uniform gravity is actually understood in terms of Newtonian gravity. And uniform gravity is used as a local ap-
25 proximation for gravity. In fact, this was what Einstein [8] did in his 1911 derivation of the gravitational redshifts. Nevertheless,
26 Einstein's arguments for such a derivation are essentially valid because the result is only a first order approximation. First, because
27 of the Euclidean-like structure, for this situation a frame of reference of a curved space-time can be treated as if a Euclidean space.
28 Second, since light normally travels much faster than the velocity of an accelerated frame, the potential for a uniform gravity could
29 be considered as if static. Thus, the status of Einstein's equivalence principle is further strengthened.

1 However, raising such an approximation to the level of a physical principle, which results in the equivalence of gravity and accel-
2 eration, is far more serious. As Einstein pointed out, this is misleading in physics [7], and in fact mathematically incorrect [8].
3 Related theorems in Riemannian space have shown that the local space of a “freely falling” particle is locally constant [14], but gen-
4 erally cannot be a constant metric even in a very small region. Einstein proposed that such a local metric must be locally Min-
5 kowskian (the Einstein-Minkowski condition), and claimed this to be a consequence of his equivalence principle [1, 8]. Since Ein-
6 stein used only the Einstein-Minkowski condition in his subsequent calculations, it is what his principle actually is. However, Ein-
7 stein did not illustrate it with examples. Thus, some theorists [15, 26] did not recognize the inadequacy of Pauli’s version.

8 Another major problem in Einstein’s theory is that the space-time coordinates are ambiguous, and Einstein’s “covariance princi-
9 ple” discontinuously separates special relativity from general relativity [12]. Because of this, Whitehead [9] rejected general rela-
10 tivity as a physical theory and Fock [20] rejected even both of Einstein’s principles. On the other hand, Einstein’s predictions agree
11 with observations very well and few of his peers had taken a critical look at his theory and analyzed it thoroughly. Moreover, be-
12 cause of such ambiguity, theorists tried to make physical sense out of just the solutions of Einstein’s equation. However, as Einstein
13 pointed out, mathematics may not be related to the physical reality [24]. Since Einstein’s equivalence principle implies measurable
14 time dilation and space contractions, a physical space is not just any Riemannian geometry [10, 11, 32].

15 Currently, possible mathematical and/or physical restrictions on Einstein’s theory are often inadvertently ignored [10, 11, 15, 26].
16 This is because physical requirements often depend on the meaning of coordinates [26]. The root of such problems is that Einstein
17 was unable to clarify the physical meaning of space-time coordinates of a curved space-time.⁴⁾ Following superficially the existing
18 mathematical framework of Riemannian geometry embedded in a flat space, Einstein believed that the coordinates were defined in
19 terms of the space-time metric. Thus, a problem of circular definition was created. Moreover, if a physical space-time were
20 embedded in a high dimensional flat space, one must provide a physical meaning for such a space.

21 It did not occur to Einstein or Dirac [33] that the definition of space-time coordinates is necessarily independent of the space-time
22 metric. Weinberg [4] illustrated, however, a curved space need not be embedded in a higher dimensional flat space. In fact, the
23 theoretical framework of general relativity makes this very unlikely since a frame of reference must have the Euclidean-like structure.
24 In other words, in a physical space there are dual mutually complementary structures, the Euclidean-like structure and the space-time
25 metric. Then, it would be possible to clarify the meaning of Einstein’s equivalence principle [10].

26 However, the meaning of Einstein’s equivalence principle was commonly mistaken [34] to be the same as the preliminary appli-
27 cation of equivalence with Newtonian gravity [8]. This is fundamentally incorrect since the gravitation potential is a scalar in New-
28 tonian theory, but is a second rank tensor in general relativity. A uniform gravity must be time-dependent because of Einstein’s
29 equivalence principle, but can be static in Newtonian theory. Moreover, the essence of Einstein’s equivalence principle is the re-

1 sulting local metric in a free fall to be Minkowskian. However, many theorists focus their attention on uniform gravity as a local
2 approximation. It is unfortunate that Einstein's equivalence principle, though correct, was not well understood.

3 Einstein's equivalence principle implies that the time dilation and space contractions are measurable [1]. However, such meas-
4 urements seem to be trivial since Einstein addressed only the diagonal metrics or metrics without a crossing space-time element.
5 This creates a false impression that the Einstein-Minkowski condition is trivial.¹⁰⁾ For instance, Synge [14], who is an excellent
6 mathematician, failed to investigate the physics of Einstein's equivalence principle.

7 Among the textbooks, only Landau & Lifshitz [19] first took the trouble of addressing the important issue of space contractions
8 for the general case. Unfortunately, because their arguments are based on Pauli's version, the derivation of their formula for the
9 local distance is not faultless [15]. There are problems such as implicit assumptions and that the physical meaning of the related
10 local time is not clear. Note that almost all the textbooks in the West ignored their work [4-6, 21, 25-30, 33]. Moreover, theorists,
11 who do not understand Einstein's notion of local time [1, 8], rejected the formula of Landau & Lifshitz [19] totally.⁹⁾

12 In this paper, the metric for a uniformly rotating disk is derived rigorously with the Einstein-Minkowski condition. Then the
13 physical meanings of the metric and the local time are clear. For the given assumptions of Einstein, validity of the metric is firmly
14 established since such a calculation is based on special relativity. Concurrently, the arguments of Ohanian & Ruffini [25] for the
15 space contractions⁸⁾ are proven invalid, but the calculated space contractions of Landau & Lifshitz [19] are valid for the case of the
16 rotating disk. Thus, the crucial role of the Einstein-Minkowski condition in the space contractions is illustrated. More important,
17 the errors in Einstein's theory on measurements are rectified. In general relativity, the local distance is measured in a free fall local
18 space, although in the embedded space of a spherical surface, the distance is measured by attachments.

19 It is meaningful to make the importance of Einstein's equivalence principle be understood in this year of celebrating the birth of
20 relativity. This would give added confidence to the metric for a uniform gravity to illustrate Einstein's equivalence principle such
21 that one may understand more clearly, what Einstein's notion of uniform gravity is. Although the time dependency is clear now,¹¹⁾
22 it was difficult to see this intuitively right from the beginning.

24 Acknowledgments

25 P. Morrison of MIT was the coauthor of the first draft (Appendix B). However, he passed away four days before the submission.
26 According to the laws, his coauthorship can be recognized only in a different form such as an acknowledgment. This paper has the
27 same conclusions of the first draft although this version provides more details such that it can be easier to be understood. The au-
28 thors gratefully acknowledge stimulating discussions with S. -J. Chang, A. Napier, and A. J. Coleman and for his insightful remarks
29 on Whitehead's outstanding work. Special thanks are to the referees, Taksek Chan, and Jim Markovitch for valuable comments and
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1

2 **Appendix A: Some Remarks on General Relativity and Riemannian Space.**

3 In a physical Riemannian space, the Euclidean-like structure [10] deals with a distance between two points in the frame of refer-
 4 ence, whereas the metric provides the invariance among space-time coordinate systems with different frames of reference. The met-
 5 ric relates to the time dilation and space contractions that can be measured in a local space at free fall. These clarify a physical
 6 space having a very distinct geometry that is compatible with the experiment on light bending [1, 8].

7 In the initial development of Riemannian geometry, however, the metric was identified formally with the notion of distance in
 8 analogy as the case of the Euclidean space. Such geometry is often illustrated with the surface of a sphere, a subspace embedded in
 9 a flat space [33]. Then, the distance is determined by the flat space and can be measured with a static method. For a general case,
 10 however, the issue of measurement was not addressed, before Einstein's theory.

11 In general relativity, the local distance represents the space contraction, which is measured in a free fall local space (see § 6).
 12 Thus, this is a dynamic measurement since the measuring instrument is in a free fall state under the influence of gravity, while the
 13 Euclidean-like structure determines the static distance between two points in a frame of reference. Einstein's error is that he has
 14 mistaken this dynamic local measurement as a static measurement.¹²⁾ Consequently, Einstein and subsequent theorists believed that
 15 a space-time metric could be regarded as defining a locally measured distance as in the case of an embedded subspace [33].

16 However, there is no compelling reason in mathematics (or physics) to consider the indefinite metric $g_{\mu\nu}$ as related to the locally
 17 measured distance. The structure of a Riemannian space allows that the notion of "global distance" function is defined on the
 18 coordinates being used, but is independent of the metric. On the other hand, the notion of a metric is necessarily defined in terms of
 19 the coordinates. Zhou [38] would probably be the first who was aware of the essence of such a new structure.

20 This mathematical structure is clearer once Einstein's general relativity has been better understood [12]. Thus, in a sense, the
 21 development of general relativity clarifies the notion of Riemannian space further. The Euclidean-like structure is essentially
 22 independent of the transformation of coordinates. However, such a structure is necessary for defining light speeds.

23 Einstein stated that the light speed is measured "in the sense of Euclidean geometry [1]." Moreover, *all Einstein's predictions*
 24 *are in terms of the Euclidean-like structure.* For instance, a ray of light, traveling at a shortest distance Δ from the sun of mass M ,
 25 will be deflected, in all by an amount [1, 8] $M\kappa/2\pi\Delta$. Moreover, the secular rotation of the elliptic orbit of the planet in the same
 26 sense as the revolution of the planet, amounting in radius per revolution to $24\pi^3 a^2/(1 - e^2)c^2T^2$. In addition to Δ , e the numerical
 27 eccentricity and a the semi-major axis of the planetary orbit in centimeters are defined in terms of the Euclidean-like structure, and T
 28 the period of revolution in seconds is defined in terms of the time of a "quasi-Minkowskian space" [4].

1 A few years later, Whitehead with his own theory [39] and much later Fock [20] with the harmonic solution, obtained the same
 2 predictions. Unfortunately, both Whitehead and Fock saw the physics in terms of philosophy instead of measurements, and failed to
 3 see that their desired space-time coordinates [10, 11] are due to Einstein's equivalence principle.

4 Misner et al [5], as shown in their eq. (40. 14) [5; p. 1107], got a physically incorrect conclusion on the local time of the earth in
 5 the solar system because they did not understand Einstein's equivalence principle¹⁾ and related theorems in Riemannian space.
 6 Ohanian & Ruffini [25; p. 198] had the same problems as shown in their eq. (50). It should be noted, however, that Eddington [2],
 7 Liu [31], Straumann [6], Wald [30] and Weinberg [4] did not make the same mistake.

9 **Appendix B: Morrison and General Relativity**

10 On April 22, P. Morrison closed his eyes forever, and thus ended our association of 15 years of our research in the field of gen-
 11 eral relativity that Einstein established with his accurate predictions. In these fifteen years, we had very fruitful research results.
 12 After solving the fundamental problem of the physical meaning of space coordinates, our efforts culminated to our joint paper,
 13 "Misunderstandings Related to Einstein's Principle of Equivalence and Einstein's Theoretical Errors on Measurements". In this
 14 paper, the physical meaning of the notion of local distance is clarified with Einstein's equivalence principle in terms of measurements.

15 The foundation of our association is the conviction, in agreement with Einstein, that Einstein's equivalence principle is funda-
 16 mentally correct and is physically distinct from Pauli's version, although many theorists commonly, but mistakenly believe that Ein-
 17 stein equivalence principle is not needed and/or not accurately valid. We follow the tradition of MIT starting from N. Rosen that
 18 general relativity is essentially correct, but there are fundamental problems to be resolved. For instance, like Einstein, Oppenheimer,
 19 Weinberg, Weisskopf, and Yilmaz, we judged that the notion of black holes is far from settled.

20 Morrison is famous for his exceptional ability to penetrate to the core of an issue and explain it clearly in terms of physics. In
 21 numerous occasions, I have benefited from our discussions. Although I may have started a discussion by explaining a certain issue,
 22 at the end I found his explanation to be better than mine. For instance, I explained to him why Einstein's "covariance principle" is
 23 invalid according to many physical facts as well as the observation of Zhou Pei-Yuan. I expected him to question me intensively,
 24 but he simply listened and asked only a few questions. Then he later remarked to me that the "covariance principle" is physically
 25 invalid because it disrupts the necessary physical continuity from special relativity to general relativity.

26 Morrison would be very persistent in following an issue of interest to the very end. For example, I explained to him why the
 27 Nobel Committee was correct in questioning the existence of a dynamic solution of the two-body problem. I have found that there
 28 is simply no dynamic solution, and the calculation of the radiation of binary pulsars is actually based on a modified Einstein equation.
 29 From then on, he questioned me from different views for almost two months. Finally, he stopped this questioning after I pointed out
 30 that such a solution would necessarily violate the principle of causality. Later, I understood that he went to Princeton University at

1 least twice to discuss the calculation of the binary pulsar with Professor J. H. Taylor who eventually told Professor Morrison that the
2 credit of this calculation should go to T. Damour.

3 Currently, many theorists have identified the 1911 preliminary application of equivalence between acceleration and Newtonian
4 gravity as Einstein's equivalence principle formally proposed in 1921. This would add to the confusion created by Pauli's version
5 and the incorrect explanation of the Wheeler school that later rejected Einstein's equivalence principle. In our discussion, Professor
6 Morrison pointed out that this identification was probably incorrect since this preliminary application would lead to disagreements
7 with results based on Einstein's equivalence principle. His remark was the starting point of our joint paper that performs an essen-
8 tial rectification of general relativity.

9 Professor Morrison was very modest in spite of his outstanding in theoretical understanding in physics. Once, he remarked that
10 he would not be a good partner for such discussions because he had not written a paper on general relativity. I was totally unpre-
11 pared for such a remark. I replied, "Then, at least you are not wrong." Nevertheless, Professor Morrison did not find my off
12 hand remark offensive.

13

14 **Endnotes**

15 1) Misner, Thorne, & Wheeler [5; p. 386] falsely claimed that Einstein's equivalence principle is as follows:

16 "In any and every local Lorentz frame, anywhere and anytime in the universe, all the (Nongravitational) laws of phys-
17 ics must take on their familiar special-relativistic form. Equivalently, there is no way, by experiments confined to in-
18 finitely small regions of spacetime, to distinguish one local Lorentz frame in one region of spacetime frame any
19 other local Lorentz frame in the same or any other region."

20 However, this is only an alternative version of Pauli's because the Einstein-Minkowski condition, which requires that the local
21 space in a free fall must have a local Lorentz frame, is missing.

22 2) An existence of the Euclidean-like structure, due to Einstein's equivalence principle, is necessary for a physical space [10, 11].

23 Experimentally, such a structure has been verified by the observed bending of light, which is bent in comparison with a "straight
24 line" in the structure. So far as measurements are possible in principle, the Euclidean-like structure is operationally defined in
25 terms of spatial measurements essentially the same as Einstein defined the frame of reference for special relativity [35]. Since the
26 attached measuring instruments and the coordinates being measured are under the influence of the same gravity, a Euclidean-like
27 structure emerges from such measurements as if gravity did not exist. However, such a coordinate system could be restricted due
28 to physical considerations such as that the velocity of light in vacuum is the maximum of particle speeds.

29 3) The Schwarzschild solution in quasi-Minkowskian coordinates [4; p. 181] is the following:

$$30 \quad ds^2 = (1 - 2M\kappa/\rho)c^2dt^2 - [(1 - 2M\kappa/\rho)^{-1} - 1] \rho^{-2}(xdx + ydy + zdz)^2 - (dx^2 + dy^2 + dz^2), \quad (3')$$

1 where

$$2 \quad \rho^2 = x^2 + y^2 + z^2, \quad x = \rho \sin\theta \cos\varphi, \quad y = \rho \sin\theta \sin\varphi, \quad \text{and} \quad z = \rho \cos\theta. \quad (4)$$

3 Coordinate transformation (4) tells that the space coordinates satisfy the Pythagorean theorem. The Euclidean-like structure
4 represents this fact, but avoids confusion with the notion of a Euclidean subspace, determined by the metric. Clearly, metric (3')
5 and the Euclidean-like structure (4) is complementary to each other in the Riemannian space. These space-time coordinates
6 forms not just a mathematical coordinate system since a light speed ($ds^2 = 0$) is defined in terms of dx/dt , dy/dt , and dz/dt [8].

7 4) Currently, a common misunderstanding is that any Riemannian space with a proper signature would be a valid physical space
8 [12]. Obviously, not every Riemannian space has a Euclidean-like structure. Moreover, even a Riemannian space with the
9 Euclidean-like structure is not necessary a physical space [15]. In principle, a physical space must sufficiently satisfy all the
10 physical requirements including Einstein's equivalence principle [10]. In practice, however, the physical requirements would be
11 understood better as physics progresses. In physics, things are not always defined perfectly from the outset, as they might be as in
12 mathematics [10]. Moreover, as Einstein [36] once remarked, "If you want to find out anything from the theoretical physicists
13 about the methods they use, I advise you stick closely to one principle: don't listen to their words, fix your attention on their
14 deeds." Following his advice, it is found that, from his metric solutions, the space coordinates are characterized by

- 15 i. the Euclidean-like distance function $d(P_1, P_2) = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}$, and
- 16 ii. the frame of reference chosen before a solution is obtained, and
- 17 iii. the time coordinate related to local clock rates through orthogonality to the space coordinates.

18 The coordinates are independent of the gravity, and Einstein's predictions are related to the Euclidean-like distance instead of the
19 metric distance. Thus, Einstein's coordinate system has very specific physical implications. *It thus follows that physical co-*
20 *variance is limited to what the principle of general relativity allows* [12]. To define a curved space as having a non-zero curva-
21 ture tensor is inadequate. In this paper, a curved space is defined as that some geodesics in the frame are curves.

22 5) The time-tested assumption [26] that phenomena can be explained in terms of identifiable causes is called the *principle of cau-*
23 *sality*. In general relativity, Einstein and subsequent theorists have used this principle implicitly on considerations of symmetry
24 [1-7]. Thus, the physical meaning of space coordinates has been used right at the beginning [10, 11].

25 6) According to Einstein [1], in principle, it should be possible to place instruments locally in a physical coordinate system to do
26 measurements. However, such a requirement would put restrictions on a physical coordinate system. For example, the coordi-
27 nate system K' is restricted by $(c^2 - \Omega^2 r'^2) > 0$ since no particle (and thus no clock) should have a speed larger than c .

28 7) A frame of reference is physical realizable if its coordinates are part of the coordinate system of a physical space, or equivalently
29 physical measurements can be performed in such a frame.

30 8) The "standard" arguments given by Ohanian and Ruffini [25; p. 164] is as follows:

1 “... and the spacetime interval becomes $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$. This expression determines the spacetime distances.
 2 For example, a coordinate displacement dx^1 along the x-axis has a length $\sqrt{(-g_{11})} dx^1$, that is, the measured distance
 3 differs from dx^1 by a factor $\sqrt{(-g_{11})}$. Likewise, a coordinate time (t time) displacement dx^0 has a duration $\sqrt{g_{00}} dx^0$
 4 when measured by the proper time of a clock at rest.”

5 They (including Landau & Lifshitz [19]) failed to recognize that a local differential distance is measured from a local space at free
 6 fall; whereas the distance $d(P_1, P_2)$ of two points in a frame has decided already by the Euclidean-like structure.

7 9) In general, if a metric has a non-zero irreducible space-time cross element, we can synchronize clocks along any open curve, but
 8 it is not possible to synchronize all the local clocks along a closed contour [19, 31]. Thus, (15) is not integrable. However, the
 9 validity of (14) is based only on the Einstein-Minkowski condition (i.e., special relativity for the case of the uniformly rotating
 10 disk). Nevertheless, some theorists have mistakenly claimed (14) as invalid because (15) is not integrable.

11 10) As point out by Straumann [6; p. 83], “Briefly, we may say that gravity can be locally transformed away. This is a well known
 12 fact to anyone who has watched space flight on television.”

13 11) For a system $K(x, y, z, t)$ uniformly accelerated with a in the x-direction to an inertial system $K'(x', y', z', t')$, the metric is,

$$14 \quad ds^2 = (c^2 - 2U) dt^2 - (1 - 2U/c^2)^{-1} dx^2 - (dy^2 + dz^2), \quad \text{and} \quad c^2/2 > U(x, t) \geq 0. \quad (E1)$$

15 where

$$16 \quad U(x, t) = (at')^2/2 = a[x'(t') - x], \quad \text{then} \quad dU = -a dx + av dt, \quad (E2)$$

17 where $v(t') = at'$ and $x'(t') = x + at'^2/2$ [21]. Here $v(t')$ and $x'(t')$ are calculated with x being fixed [37]. Moreover, it is clear
 18 that this metric provides a uniform gravity since $\partial U/\partial x = -a$. However, a constant acceleration should not lead to a speed larger
 19 than c . Thus, it is necessary to have the restriction, $c^2/2 > U(x, t) = (at')^2/2$. It should be noted also that dt in (E1) is a local
 20 coordinate of local time. ⁹⁾ Fock [20], however, assumed incorrectly the metric is of the static form, $ds^2 = g_{\tau t}(x) c^2 dt^2 - dx^2 -$
 21 $dy^2 - dz^2$, with a Euclidean subspace.

22 12) Perhaps, this error arises from confusing mathematics and physics. This confusion is due to that the mathematically defined local
 23 distance would be measured physically only in a free fall local space in a different dynamic situation.

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Résumé

10 L'application préliminaire d'Einstein en 1911, de l'équivalence entre l'accélération et la gravité uniforme newtonienne pour
11 dériver "redshifts" gravitationnels était considéré ordinairement et aussi par erreur, pareil comme le principe d'équivalence d'Einstein,
12 qui a sa fondation dans les théorèmes de géométrie et de la relativité spéciale de Riemannian. Par conséquent, les limitations en rai-
13 son de la gravité newtonienne montrées dans telles dérivations ont été inexactement crues comme ce principe d'équivalence
14 d'Einstein doit être inadéquat pour dériver le courber de lumière, et aussi avoir la validité discutable lui-même. De plus, dans la
15 théorie de mesures d'Einstein, ses instruments dans les états de chute libres sont inexactement négligés. Par conséquent, il y a des
16 erreurs et l'inconsistance théoriques, par exemple, selon le cas de disque uniformement tournant d'Einstein. Cependant, après les er-
17 reurs théoriques sont identifiés et sont rectifiés, le disque uniformement tournant serait un exemple qui illustre l'importance de prin-
18 cipe d'équivalence d'Einstein et l'insuffisance de version de Pauli. Le rôle crucial de principe d'équivalence d'Einstein dans la dilata-
19 tion de temps et les contractions spatials est ainsi illustré.