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Einstein's Physical Space of an Accelerated Frame and Uniform Gravity (Einstein's Equivalence Principle)

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Abstract

Einstein's equivalence principle was first presented in terms of the equivalence of uniform gravity and an accelerated frame of reference. Nevertheless, attempts to present the initial form of Einstein's equivalence principle in terms of a space-time metric had failed, and Pauli's version has become the substitution of Einstein's principle. However, it is found that previous failures and the substitution are due to misinterpretation of general relativity. Moreover, in Einstein's physical space, the frame of reference necessarily has dual structures, a Riemannian metric and the Euclidean-like structure that emerges from a measuring method complimentary to Einstein's. This leads to recognizing the uniqueness of the physical gravitational gauge for a given frame of reference. Then, the space-time metric of an accelerated frame is calculated to confirm Einstein's equivalence principle. Concurrently, the crucial role of Einstein's equivalence principle in general relativity and Einstein's predictions is further clarified. It is pointed out also that Paulis version is the root of theoretical inconsistency and disagreements with observations. Based on existing evidences, it is predicted that the Schwarzschild solution would be rejected by an experiment on local light speeds.

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1. INTRODUCTION

It is generally agreed, as pointed out by Einstein [1], Eddington [2], Pauli [3], Weinberg [4], Misner, Thorne and Wheeler [5], Straumann [6], and Yu [7], that Einstein's equivalence principle is the theoretical foundation of general relativity. However, a surprising fact is, as Einstein [8] saw it, that few like Eddington [2] understand the crucial role of Einstein's equivalence principle in terms of physics adequately. Einstein insisted, throughout his life, on the fundamental importance of this principle to his general theory of relativity [8]. However, Einstein's insistence on this point has created a puzzle for many [8] because of inadequate understanding of the physics particularly in connection with Einstein's predictions (see Sections 2 and 3).

This is confirmed, for instance, by the (invalid) calculations of Tolman [9] and Fock [10] (see Section 4). In addition, Einstein's theory is often misunderstood, misinterpreted and then unjustifiably criticized by his followers. For instance, Synge [11] professed his misunderstandings on Einstein's equivalence principle as follows:

> "...I have never been able to understand this principle....Does it mean that the effects of a gravitational field are indistinguishable from the effects of an observer's acceleration? If so, it is false. In Einstein's theory, either there is a gravitational field or there is none, according as the Riemann tensor does or does not vanish. This is an absolute property; it has nothing to do with any observer's world line....The Principle of Equivalence performed the essential office of midwife at the birth of general relativity....I suggest that the midwife be now buried with appropriate honours and the facts of absolute spacetime be faced."

However, Einstein's equivalence principle states only that the effects of an accelerated frame are equivalent to a related uniform gravity [1,12]. Therefore, a gravitational field need not relate to a non-vanishing curvature, nor be equivalent to the effects of an observers acceleration. Nevertheless, Bergmann [13] misleadingly illustrates Einstein's equivalence principle with "Einstein's elevator" in the gravity of the earth that Einstein [14] stated as not equivalent to an accelerated frame. Perhaps, this is where Synge picked up part of his misinterpretation since Einstein wrote a very impressive "Forward" for Bergmann's book.

Moreover, Einstein [8] explained to Laue, "What characterizes the existence of a gravitational field, from the empirical standpoint, is the non-vanishing of the Γ^l_{ik} (field strength),

not the non-vanishing of the R_{iklm}." and no gravity is a special case of gravity. This view is crucial because it justifies Einstein's to conclusion that the geodesic equation is also the equation of motion of a massive particle under the influence of only gravity.

Currently, misunderstanding persists among relativists. For instance, Thorne [15] criticized Einstein's principle as follows:

> "In deducing his principle of equivalence, Einstein ignored tidal gravitation forces; he pretended they do not exist. Einstein justified ignoring tidal forces by imagining that you are (and your reference frame) are very small."

The notion that gravity in every infinitely small world region can be removed by a coordinate transformation[1)] actually came from Pauli [3] and Einstein objected this as a misinterpretation [8]. Moreover, Einstein in his letter to A. Rehtz [16] wrote:

> "The equivalence principle does not assert that every gravitational field (e.g., the one associated with the Earth) can be produced by acceleration of the coordinate system. It only asserts that the qualities of physical space, as they present themselves from an accelerated coordinate system, represent a special case of the gravitational field."

Here, Einstein has made clear that his principle is proposed for a physical space, where all physical requirements are sufficiently satisfied. Moreover, a similar explanation on the gravity of the earth has been provided in his popular book, "Relativity" [14].

Einstein explained the initial form of his equivalence principle in terms of the uniform gravity and acceleration clearly in 1911 and in 1916 [12]. Einstein assumed that the mechanical equivalence of an inertial system $K(x, y, z)$ under a uniform gravitational field, which generates a gravitational acceleration γ (but, system K is free from acceleration), and a system $K'(x', y', z')$ accelerated by γ in the opposite direction, can be extended to other physical processes. Based on this assumed equivalence, Einstein [12] derived the gravitational red shifts. He found also that his equivalence principle is compatible with the Doppler effects and even the notion of photon. Thus, the equivalence principle has been firmly established on the ground of universality of physics, although the formula for light speeds under gravity needed improvement. This is independent of the need of a Riemannian space[2)], which is additionally based on the principle of general relativity and special relativity [1].

A connection between gravity and a curved Riemannian space was established in 1915 [17]. However, a space-time metric that corresponds to a uniformly accelerated frame of reference remains to be clarified (see Section 4). This incompleteness contributed to a speculation that the deficiency is intrinsic and thus added difficulty in understanding Einstein's theory. Moreover, such speculation is supported by the invalid calculations of both Tolman [9] and Fock [10] (see Sections 3 and 4) that failed in relating the accelerated frame to a metric of uniform gravity, as Einstein's principle requires.

Recently, such a speculation has been proven incorrect because the field equation is intimately related to Einstein's equivalence principle. The Maxwell-Newton Approximation [18-20] (the same as the linearized field equation for weak gravity due to massive matter) that produced an accurate bending of light has been derived with Einstein's equivalence principle together with the notion of a curved space, if Newtonian theory is taken as a form of first order approximation 3). Thus, Einstein's equivalence principle is fully compatible with the notion of a curved Riemannian space-time.

However, the question of how the curved space is related to an accelerated frame remains a puzzle. In this paper, it will be shown that failures of Tolman and Fock are due to the conceptual error that identified a frame of reference with only a Euclidean subspace. This conceptual error is related to the fact that the physical meaning of space-time coordinates was not clear in general relativity (see Sections 2 and 3), and for such a deficiency Whitehead [21] rejected general relativity as a physical theory. This deficiency has far reaching consequences, namely: 1) it leads to the substitution of Einstein's equivalence principle with Pauli's version which requires merely the proper signature of the metric; and 2) it also leads to the substitution of the principle of general relativity with the mathematical covariance. These substitutions, which have no support by any observational evidence, seemed theoretically necessary because both principles of Einstein require a clear meaning of the space-time coordinates.

Unfortunately, Pauli's version leads to acceptance of unphysical solutions [22], theoretical inconsistency [23,24], and disagreements with special relativity and related observations [12,20]. Moreover, these substitutions necessarily lead to the incorrect misinterpretations [8], and the invalid belief that space-time coordinates have no physical meaning [25]. Based on such a belief, most theoretical inconsistency could be ignored simply because any calculation is necessarily based on a coordinate system. However, such a belief has a fatal fault, in

addition to be in disagreement with physical principles [20,26], because it contradicts with the obvious fact, the existence of non-scalar physical quantities such as vectors [27].

Nevertheless, Einstein's predictions can be reproduced mathematically without considering Einstein's equivalence principle directly and explicitly [1]. Because of currently inadequate understanding of the physics in Einstein's theory (see Section 2), some theorists therefore believed that Einstein's assumption of satisfying Einstein's equivalence principle seemed non-essential for Einstein's predictions [8,28,29]. Moreover, some theorists were happy with the new mathematical freedom because physical requirements other than the metric signature could be ignored. Pauli's version is very popular among advocates of black holes [15,30,31] whose justification seems to need Pauli's version (see Section 5).

In this paper, after addressing the related conceptual problems, an appropriate metric form for an accelerated frame will be derived (see Section 4). To this end, let us first identify the existing misconceptions.

2. THE PHYSICAL MEANING OF SPACE-TIME COORDINATES, AND EINSTEIN'S EQUIVALENCE PRINCIPLE

Einstein showed that in general relativity it is necessary to abandon Euclidean geometry because, for a rotating reference system, the time intervals and spatial distances in non-Galilean systems cannot just be determined by means of a clock and rigid standard measuring rod. However, this does not mean the abandonment of the Euclidean-like structure of space coordinates. To clarify this, one must distinguish the coordinates and the metric function of a mathematical space[4)].

As shown by Weinberg [4], "For both Euclidean and non-Euclidean geometry the 'model' is provided by the theory of real numbers." Descartes analytic geometry shows that if a point is identified with a pair of real numbers (x_1, x_2) and the distances between two points (x_1, x_2) and (X_1, X_2) is identified as $d(x, X) = [(x_1 - X_1)^2 + [(x_2 - X_2)^2]^{1/2}$, then all of the Euclids postulates can be proved as theorems about real numbers. However, if a different metric $d'(x, X)$ is defined, one obtains a non-Euclidean geometry in Descartes' coordinates. Thus, the meaning of the coordinates in a non-Euclidean geometry can be independent of the metric. However, if a non-Euclidean geometry such as the surface of a sphere is embedded in a Euclidean geometry of higher dimension, then the meaning of the coordinates of the

non-Euclidean geometry is determined by the higher dimensional space. In short, one must determine what is the meaning of coordinates for a curved space, if they are independent of the metric.

In general relativity, the invariant line element is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{1}$$

where $g_{\mu\nu}$ is a general space-time metric in a Riemannian space. Since $g_{\mu\nu}$ is not a constant metric, one cannot hope to derive from (1) a simple distance formula as in Euclidean geometry that the spatial distance $d(P_1, P_2)$ of two points P_1 and P_2 is still

$$d(P_1, P_2) = [(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2]^{1/2}, \tag{2}$$

However, in a different way, the Euclidean-like structure (2) is necessarily preserved within the Riemannian geometry.

To illustrate this, let us examine the Schwarzschild solution [1] of Riemannian space (x, y, z, t),

$$ds^2 = (1 - 2M\kappa/\rho)dt^2 - (1 - 2M\kappa/\rho)^{-1} d\rho^2 - \rho^2 d\theta^2 - \rho^2 sin^2\theta d\phi^2, \tag{3}$$

$$x = \rho\ sin\theta\ cos\phi, \quad y = \rho\ sin\theta\ sin\phi, \quad z = \rho\ cos\theta, \quad and \quad \rho^2 = x^2 + y^2 + z^2, \tag{4}$$

where κ is a coupling constant, and M is the total mass. Since the metric is defined in terms of ρ, θ, and ϕ, the Riemannian space is actually defined in terms of the Euclidean characteristics (4). This illustrated that this Euclidean-like structure (x, y, z) is included in Einstein's Riemannian space. Moreover, since Einstein [12] defined the velocity of light "in the sense of Euclidean geometry", the Euclidean-like structure is also supported by the observed light deflection. The subspace (x, y, z) is called the frame of reference. Thus, Einstein's notion of Riemannian space-time has a frame of reference with the Euclidean-like structure.

To understand the Euclidean subspace (x, y, z) in terms of physical measurements, we must first clarify what measure means in relation to Einstein's equivalence principle. In Einstein's theory, the measuring instruments are resting but in a free fall state [1]. From Einstein's equivalence principle, time dilation and space contraction are obtained. Based on such measurements, Einstein believed, "In the general theory of relativity, space and time cannot be defined in such a way that differences of the spatial coordinates can be directly

measured by the unit measuring-rod, or differences in the time coordinates by a standard clock." However, if the measuring instruments are resting and are attached to the frame of reference, since the measuring instruments and the coordinates being measured are under the same influence of gravity, a Euclidean-like structure emerges as if gravity did not exist. Thus, the Euclidean-like structure is an integral part of general relativity (see also Section 3).

To make a useful distinction, such a class shall be called the Einstein Spaces named after its creator. However, an Einstein space would be a physical space that models reality only if Einstein's equivalence principle is satisfied. The frame of reference plays a crucial role in the theory of general relativity, as pointed out by Fock [10], in particular to Einstein's equivalence principle, but Fock incorrectly identified the frame of reference with a Euclidean subspace that is related to the metric. Nevertheless, as pointed out by Whitehead [20], Einstein was unable to define the coordinates in general relativity precisely. This is perhaps a main reason that leads to the popular acceptance of Pauli's version of the equivalence principle. Pauli [3] regards the equivalence principle essentially as just the existence of local Minkowski spaces as follows:

> "For every infinitely small world region (i.e. a world region which is so small that the space- and time-variation of gravity can be neglected in it) there always exists a coordinate system $K_0(X_1, X_2, X_3, X_4)$ in which gravitation has no influence either in the motion of particles or any physical process."

Einstein strongly objected Pauli's version as reported in details by Norton [8]. The notion of acceleration with respect to a frame of reference is essential in Einstein's equivalence principle [10]. It has been shown that static acceleration may not exist for a non-constant metric, and this situation leads to inconsistent in physics [19].

Since other related physical considerations are ignored, Pauli's version is actually only a mathematical statement of the Lorentz manifold [11]. A noted difference from Pauli's version is that Einstein requires additionally: i) "the special theory of relativity applies to the case of the absence of a gravitational field [12, p.115]"; ii) "he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be [2, p.116]"; and iii) a local Minkowski space is obtained by choosing the acceleration. Einstein [12, p.118] wrote, "... we must choose the acceleration of the infinitely small ('local') system

of coordinates so that no gravitational field occurs; this is possible for an infinitely small region." Thus, Einstein's equivalence principle requires a clear physical meaning of the space-time coordinates, which define acceleration and a frame of reference. For Einstein's equivalence principle, special relativity is naturally a special case, but this is not the case for Pauli's version.

Many believed incorrectly that Einstein's predictions do not depend on his equivalence principle [10,11,28,29]. For instance, the gravitational red shift was regarded as derivable with a coordinate-free method [6,25]. The fact is, however, that a valid space-time coordinate system, which satisfied Einstein's equivalence principle, must be implicitly used [1,27]. Although the deflection angle of a light ray can be derived directly without referring to the local light speeds, a physical coordinate system is necessary such that the deflection angle can be defined. For the perihelion of Mercury, a physical coordinate system is also needed to define perihelion although the connection between perihelion and Einsteins equivalence principle is not direct and obvious. Moreover, these three predictions are expressed in terms of the Euclidean-like structure [1,27], whose existence depends on a satisfaction of Einstein's equivalence principle. Thus, Einstein's predictions are inextricably related to Einstein's equivalence principle and a physical space-time coordinate system (see also Section 5).

In fact, the physical meaning of space coordinates has been used right at the beginning implicitly in symmetry considerations[5]. Now, the physical meaning of space-time coordinates has been identified, and the experiment on local light speeds would make the crucial role of Einstein's equivalence principle even clearer [26,27].

Pauli's version is inadequate in physics because it requires only the existence of Local Minkowski space and ignores Einstein's notion of physical space [1,12,16]. It is based on Paulis version that Logunov and Mestvirishvili [23] showed that general relativity would lead to inconsistency related to the radiation formula. Moreover, the local distance formula of Landau and Lifshitz [32], which is based on Pauli's version, would give results in disagreement with observation and even incompatible to their own claims 6) [33]. Thus, it is necessary to re-establish Einsteins equivalence principle on a firm theoretical ground.

3. THE PRINCIPLE OF GENERAL RELATIVITY, UNIFORM ROTATION, AND RIEMANNIAN SPACE-TIME

Einstein considered a Galilean (inertial) system of reference $K(x, y, z, t)$ and a system $K'(x', y', z', t')$ in uniform rotation Ω relatively to K. The origins of both systems and their axes of Z permanently coincide. (The gravity of this example would be useful for the calculation of the gravity related to a uniformly accelerated frame.) For reason of symmetry[5)], a circle around the origin in the x-y plane of K may at the same time be regarded as a circle in the x'-y' plane of K'. Then, according to special relativity, in the x-y plane and the x'-y' plane, the metrics of K and K' [1,34] are respectively the following:

$$ds^2 = c^2dt^2dr^2 - r^2d\phi^2 - dz^2, \quad where \;\; x = rcos\phi, \;\; y = rsin\phi, \tag{5}$$

and

$$ds^2 = (c^2 - \Omega^2 r'^2)dt'^2 - dr'^2 - (1 - \Omega^2 r'^2/c^2)^{-1} r'^2 d\phi'^2 - dz'^2 \tag{6}$$

Then,

$$\oint ds = (1-\Omega^2 r'^2/c^2)^{-\frac{1}{2}} r' \int_0^{2\pi} d\phi' = 2\pi r'(1-\Omega^2 r'^2/c^2)^{-\frac{1}{2}}, \;\; x' = r'cos\phi', \;\; y' = r'sin\phi'. \tag{7}$$

would be the circumstance of a circle of radius r' (= r) for an observer in K. Moreover, as Einstein pointed out, "an observer at the common origin of co-ordinates would therefore see it lagging behind the clock beside him." So, he will be obliged to define time in such a way that the rate of a clock depends upon where the clock may be [14]. Thus, Einstein also defined a physical space-time coordinate system together with its metric that is related to local clock rates and local spatial measurements.

To illustrate Einstein's equivalence principle and the notion of Einstein space, let us first derive metric (6). Consider the coordinate transformation to the uniformly rotating disk [34], in terms of Newtons notion of "absolute time" as follows:

$$x = x'cos\ \Omega t - y'sin\ \Omega t, \quad y = x'sin\ \Omega t + y'cos\ \Omega t, \quad and \;\; z = z' \tag{8a}$$

or

$$r = r', \quad z = z', \quad \phi = \phi' + \Omega t, \tag{8b}$$

in cylindrical coordinate systems of K and K', where Ω is the angular velocity. We take advantage of the fact that one can start with an arbitrary coordinate system of a Riemannian

space. Then, from (5) the resulting metric has the following form,

$$ds^2 = (c^2 - \Omega^2 r'^2)dt^2 - 2\Omega r'^2 d\phi' dt - dr'^2 - r'^2 d\phi'^2 - dz'^2. \tag{6'}$$

However, the mathematical system $K^*(x', y', z', t)$ is not a physical space-time coordinate system for the uniformly rotating disk K because what measured in a resting local clock is time t' but not time t. In other words, metric (6) together with its coordinates K^* is not a space-time coordinate system, as Einstein defined, that can be used for physical measurement.

Nevertheless, metric (6') alone can be used to derive metric (6), which some claimed as incompatible with (5). To obtain a physical coordinate system including the time t' of the rotating disk, a comparison of (6) and (6') leads to,

$$d\phi' = d\phi - \Omega dt, \tag{9a}$$

and

$$cdt' = [cdt - (r\Omega/c)rd\phi][1 - (r\Omega/c)^2]^{-1}. \tag{9b}$$

Thus, it is necessary to modify the time coordinate t'. Relation (9) makes clear that metric (6) is related to the flat metric (5).

The time dilation and the spatial contraction in general relativity [1,12] are results due to comparisons with a clock and a measuring rod in relatively rest at the beginning of a free fall. To verify this, consider a particle P resting at (r', ϕ', z'). Then, P has the velocity of Ωr in the ϕ'-direction, which is denoted by dx''. It follows that the Lorentz coordinate transformation is,

$$rd\phi = [1 - (r\Omega/c)^2]^{-1/2}[dx'' + r\Omega dt'']; \tag{10a}$$

and

$$cdt = [1 - (r\Omega/c)^2]^{-1/2}[cdt'' + (r\Omega/c)dx'']. \tag{10b}$$

Then,

$$rd\phi' = [1 - (r\Omega/c)^2]^{1/2}dx''; \quad and \quad cdt' = [1 - (r\Omega/c)^2]^{-1/2}cdt'' \tag{11a}$$

and

$$ds^2 = c^2 dt''^2 - dx''^2 - dr'^2 - dz'^2. \tag{11b}$$

These are exactly the time dilation and spatial contraction. This illustrates that a particle resting at K', can attached to a local Minkowski space. Thus, this is also an example of Einstein's version of infinitesimal equivalence principle.

However, for the coordinate system $K^*(x', y', z', t)$, the question of time dilation is complicated because Einstein's equivalence principle is not applicable. Nevertheless, let us assume the Einstein's equivalence principle could be applied to K^*. Mathematically, for a particle P resting at K^*, the state vector of P is $(0, 0, 0, dt)$. According to (9), P is also resting at K' with a state vector $(0, 0, 0, dt')$. Then the local Minkowski space for P is identical to (11b). It thus follows that

$$dx'' = [1 - (r\Omega/c)^2]^{-1/2} r d\phi', \tag{12a}$$

and

$$dt'' = [1 - (r\Omega/c)^2]^{1/2} dt - [1 - (r\Omega/c)^2]^{-1/2} (r\Omega/c^2) r d\phi'. \tag{12b}$$

Thus,

$$dt = [1 - (r\Omega/c)^2]^{-1/2} dt'' \tag{12c}$$

would be considered as the time dilation since a clock rest at K^* has $d\phi' = 0$. The problem of this derivation is that the parameter t is not related to the local clocks for the frame $K'(x', y', z')$.

Since metric (6') satisfies Pauli's "equivalence principle", Pauli's version is clearly inadequate in physics. This calculation confirms that Einstein's equivalence principle is applicable only to a physical space. *Thus, in spite of general covariance, the freedom toward the physical space-time coordinate systems that can be used for physical interpretation is severely limited.*[7)]

The directional spatial contraction as indicated in metric (6), is measured with a resting measuring rod in the state of free fall. However, if a spatial measurement is performed with a measuring rod which is attached to the frame $K'(x', y', z')$, it would appear as Euclidean. In fact, it is based on this implicit assumption that the cylindrical coordinate system is well defined in K'. Thus, as shown in examples (3) and (6), the distance in terms of the Euclidean-like structure is necessary and complementary to the metric for the Riemannian space that produces the local distance. The system $K^*(x', y', z', t)$ has a Euclidean subspace, but the time t is not associated with the frame $K'(x', y', z')$. Although K^* is diffeomorphic to K, K^* is not a physical space-time since it fails the physical requirement of local time. Thus, diffeomorphic manifolds may not be equivalent in general relativity as Wald [25] believed and Logunov and Mestvirishvilli [23] objected.

Thus, an Einstein space is a Riemannian space with a Euclidean-like structure, and is a

physical space if it sufficiently satisfies all physical requirements, including Einstein's equivalence principle. As in special relativity, the Euclidean characteristics are not invariants. However, if Pauli's version instead of Einstein's equivalence principle were considered, since coordinates have no physical meaning, it would not be possible to derive the metric (6) for the rotating disk and thus obtain the relation (7). Thus, Einstein's equivalence principle is essential in the theory of general relativity.

4. UNIFORM ACCELERATION AND EINSTEIN'S EQUIVALENCE PRINCIPLE IN RIEMANNIAN SPACE

The analysis of a rotating disk suggests that there are similarities with respect to the case of a uniform acceleration. Based on similarity to the case of the rotating disk, the metric for the case of uniform acceleration would be

$$ds^2 = (c^2 - v^2)dt'^2 - (1 - v^2/c^2)^{-1}dx'^2 - dy'^2 - dz'^2, \tag{13}$$

where $v(t')$ is the relative velocity of the coordinate systems in the x'-direction. Metric (13) has a Euclidean-like structure as if v were zero. In other words, for the acceleration in the x-direction, the metric would have the following form,

$$ds^2 = (c^2 - 2U)dt'^2 - (1 - 2U/c^2)^{-1}dx'^2 - (dy'^2 + dz'^2), \quad and \quad c^2/2 > U(x', t') \geq 0. \tag{14}$$

Note that a uniform acceleration cannot exist forever, otherwise the resulting speed would exceed the velocity of light.

It follows that a uniform acceleration must be started at some time, for instance, $t = t_0 < 0$, and then decreased some time afterward. Moreover, a uniform gravity must be confined in a finite region; otherwise the light speed as the maximum velocity would be violated. In other words, uniform gravity like an electromagnetic plane wave, also does not really exist in nature. Thus, the equivalence of acceleration and uniform gravity is best described, as Einstein did, in terms of an accelerated chest [14]. (In Einstein's derivation of gravitational red shifts [12], the obvious, though crucial, step of replacing the infinitesimal energy γh by $\Delta\Phi$ is omitted.) In practice, the uniform gravity is essentially a local idealization of a non-uniform gravity.

Consider a system K' accelerated with an acceleration a relative to an inertial system K_0. Then, if the coordinates of the origin of K' in system K_0 is $(X_{0'}(t), 0, 0, t)$, we have

$$\frac{d^2X_{0'}}{dt^2} = a, \tag{15a}$$

and

$$X_{0'}(t) = X_{0'}(0) + at^2/2; \quad y = y'; \quad z = z', \quad if \quad v(0) = 0. \tag{15b}$$

where $X_{0'}(0)$ is arbitrary. Thus, to obtain a transformation compatible with form (14), we may assume similar to (11) that

$$dx' = dx - v(x,t)dt, \quad x = x' - X_{0'}(0) + at^2/2 \tag{16a}$$

and

$$cdt' = [1 - F(x,t)]^{-1}[cdt - f(x,t)c^{-1}dx], \tag{16b}$$

where $v(x,t)$ is a relative velocity of the two systems, and $f(x,t)$ is an unknown function. Note that (15a) is equivalent to

$$dx' \equiv 0; \quad and \quad \frac{d^2x}{dt^2} = a \quad (\frac{dx}{dt} = 0 \quad and \quad \frac{dv}{dt} = a). \tag{16c}$$

Substituting (16) to metric form (14), a comparison with the flat metric leads to three relations as follows:

$$-(1 - 2U/c^2)^{-1} + (1 - F)^{-2}(1 - 2U/c^2)(f/c)^2 = -1; \tag{17a}$$

$$-(1 - 2U/c^2)^{-1}(v/c)^2 + (1 - F)^{-2}(1 - 2U/c^2) = 1; \tag{17b}$$

$$(1 - 2U/c^2)^{-1}v = (1 - F)^{-2}(1 - 2U/c^2)f. \tag{17c}$$

It follows that

$$v = (1 - F)^{-2}(1 - 2U/c^2)^2 f; \quad 1 - vf/c^2 = (1 - 2U/c^2); \quad and \quad v = f \tag{18a}$$

Thus

$$U = (v)^2/2 \geq 0; \quad and \quad F = 2U/c^2 = (v/c)^2. \tag{18b}$$

Since v is the relative velocity of the two systems of coordinates K' and K_0, as expected, metric (14) is in the form of (13).

Also, in terms of physics, (16) is in complete agreement with (11) as shown be the following relations:

$$dx' = dx - vdt;. \tag{19a}$$

and

$$cdt' = [1 - (v/c)^2]^{-1}[cdt - vc^{-1}dx]; \tag{19b}$$

or

$$dx = [1 - (v/c)^2]^{-1}dx' + vc^{-1}cdt'. \tag{19c}$$

and

$$cdt = cdt' + [1 - (v/c)^2]^{-1}v^{c-1}dx'. \tag{19d}$$

The limitation on the velocity of light and the definite sign of ds^2 for the time line element, also requires

$$c^2/2 > U(x', t') \geq 0, \quad and \quad \frac{d^2x'}{ds^2} = \frac{1}{c^2}\frac{\partial U}{\partial x'}, \tag{20}$$

from the geodesic equation for the case of $dx'/ds = 0$.

On the other hand, the gravitational acceleration of a particle in K' is equivalent to

$$dx \equiv 0; \tag{21a}$$

and

$$ds = cdt, \quad dx' = -vdt, \quad and \quad dt' = [1 - (v/c)^2]^{-1}dt. \tag{21b}$$

Thus

$$\frac{d^2x'}{ds^2} = -\frac{dv}{c^2dt} \quad and \quad -a = \frac{\partial U}{\partial x'}. \tag{21c}$$

where a is the constant acceleration by assumption. Thus, if U is independent of the time t',

$$U(x') = -ax' + C, \quad or \quad U(x') = -a(x'x_0) \tag{22}$$

where C is a constant. Since $U(x')$ is bounded according to (20), we must also have a range for x'. From (16a), we have

$$U = -a(x' - x - X_{0'}(0)). \tag{23}$$

According to (21a) $dx \equiv 0$, $x + X_{0'}(0)$ should be considered as just an arbitrary constant. Thus, (23) agrees with (22).

To verify time dilation and spatial contraction, one should consider a system in relative rest at the beginning of a free fall. Let us consider the following Lorentz coordinate transformation,

$$dx = [1 - (v/c)^2]^{-1/2}[dx'' + vdt''], \tag{24a}$$

and

$$cdt = [1 - (v/c)^2]^{-1/2}[cdt'' + (v/c)dx'']. \tag{24b}$$

where $v = [2U(x')]^{1/2}$. Then, we have the expected relation

$$dx = [1 - (v/c)^2]^{1/2}dx'', \quad and \quad cdt' = [1 - (v/c)^2]^{-1/2}cdt'' \tag{24c}$$

and

$$ds^2 = c^2dt''^2 - dx''^2 - (dy'^2 + dz'^2). \tag{24d}$$

Thus, the initial form of Einstein's equivalence principle is indeed compatible with Einstein's notion of Riemannian space.

There is an arbitrary constant in the potential U of metric (14) for uniform gravity. This uncertainty is actually a necessary feature that a uniform gravity can be considered as a local idealization of a non-uniform gravity. Without this arbitrary constant, one cannot adjust the metric for uniform gravity to have the same local value of a changeable non-uniform gravity. Note also that metric (14) for a uniform gravity has the form of the Schwarzschild exterior solution far from the source. Just as the plane wave is a local idealization of an electromagnetic wave, the uniform gravity can be considered as a local idealization of the Schwarzschild solution. As a local idealization, one may not expect that metric (14) satisfies the normal boundary condition at infinite, just as an electromagnetic plane-wave has the same amplitude everywhere, and appears as having no source.

To calculate the space-time metric of uniform gravity, it is crucial to recognize the dual characteristics of the spatial subspace of Einstein's Riemannian space-time. Thus, for this case, the equivalence of gravitational force and acceleration is accurately valid. In other words, the criticisms of Fock [10] and his followers [31] on Einstein's general relativity are baseless. This further strengthens the status of Einstein's equivalence principle that has been further established due to its role in the derivation of the Maxwell-Newton Approximation [18,19]. Weinberg [4, p.3] declared, "In my view, it is much more useful to regard general relativity above all as a theory of gravitation, whose connection with geometry arises from the peculiar empirical properties of gravitation, properties summarized by Einstein's Principle of the Equivalence of Gravitation and Inertia."

An interesting result of this calculation is that it confirms Einstein's 1911 calculation on the gravitational red shifts is valid in terms of a Euclidean-like structure instead of a

curved space. The notion of Einstein space clarifies that it is valid to consider the frame of reference as if a three-dimensional Euclidean-like structure. Since a Euclidean-like structure is independent of gravity, it makes sense to define the Euclidean-like structure first to obtain the Schwarzschild solution and/or the other metric solutions.

5. THE SCHWARZSCHILD SOLUTION AND EINSTEIN'S EQUIVALENCE PRINCIPLE

The Schwarzschild solution, though the first, is not the only solution. In fact, depending on the gauge condition used, one can obtain a new diffeomorphic solution almost at will. Other well-known metric solutions for the gravity of a spherical distribution of mass are respectively the isotropic solution, and the harmonic solution as follows [4]:

$$ds^2 = [(1 - M\kappa/2r)^2/(1 + M\kappa/2r)^2]dt^2 - (1 + M\kappa/2r)^4(dx^2 + dy^2 + dz^2) \qquad (25)$$

and

$$ds^2 = [(1-M\kappa/r)/(1+M\kappa/r)]dt^2-[(1+M\kappa/r)/(1-M\kappa/r)]dr^2-(1+M\kappa/r)^2r^2(d\theta^2+sin^2\theta d\phi^2) \qquad (26)$$

where $\kappa = K/8\pi c^2$, M is the total mass of a spherical mass distribution with the center at the origin of the frame of reference $K(x, y, z)$, $r = [x^2 + y^2 + z^2]^{1/2}$, κ is a coupling constant. Since the metrics are functions of r, this illustrates again that the Euclidean-like structure is necessarily included in a Riemannian space-time of Einstein. In fact, Einstein [12] had made clear that the light velocity is "defined in the sense of Euclidean geometry".

These three different solutions of the space-time metric for the same frame of reference produced the same (first order) deflection of a light ray. Einstein [1] even remarked, "It should be noted that this result, also, of the theory is not influenced by our arbitrary choice of a system of coordinates." Subsequently, the first order results of the gravitational red shift, the perihelion of Mercury, and the retardation of light are also the same from these solutions. These inconclusive evidences were sufficient to convince many that general relativity is gauge invariant. This also led many to believe that Einsteins equivalence principle is non-essential for Einstein's predictions, and Pauli's version were sufficient.

However, if one ventures to the second order, the gauge invariance cannot be maintained. For instance, the second order effects of gravitational red shifts are different since these three

metrics have three distinct time-time components of the space-time metrics in terms of the second order. Thus to decide which of the metrics (if the correct one is among them) give the correct gravitational red shifts, one must consider Einsteins equivalence principle.

According to Einsteins equivalence principle, local space contractions and time dilations are measurable [1,12]. However, since these different solutions have the same frame of reference, only one of them can be valid. Unfortunately, the current theory does not favor any of these solutions. Moreover, their differences in the well-known four tests are in the second order approximations, which remain beyond the accuracy of current experimental capability. Therefore, to have a first order approximation, it is necessary to measure the local light speeds (for instance) of the earth such that the realistic gauge would be identified [27]. The isotropic and the harmonic solutions are indistinguishable in term of the first order Maxwell-Newton approximation,

$$ds^2 = (1 - 2M\kappa/r)dt^2 - (1 + 2M\kappa/r)(dx^2 + dy^2 + dz^2), \tag{27}$$

which is incompatible with the Schwarzschild solution. From metric (27), the local light speeds of first order is

$$\frac{dr}{dt} = (1 - \frac{2M\kappa}{r})\ c. \tag{28a}$$

But

$$\frac{d\rho}{dt} = (1 - \frac{2M\kappa}{\rho})\ c \quad and \quad \frac{\rho d\theta}{dt} \simeq (1 - \frac{2M\kappa}{\rho})\ c. \tag{28b}$$

where c is the light velocity at a free fall in vacuum, would be the local vertical and horizontal light speeds of the Schwarzschild solution for the earth. Thus, it is possible to distinguish experimentally between the Schwarzschild solution and the Maxwell-Newton approximation. An experimental difficulty for not reaching the required high accuracy is the gravitational stretch and compression effects on the vertical arm [40]. However, this difficult can be circumvented by measuring horizontal light speeds at different heights and at free fall [26].

In view of this, the Schwarzschild solution is in an unfavorable position for having no specific experimental support. Although the calculated results show that gravity can be anisotropic, this gives little theoretical support to the Schwarzschild solution since different situations give different space contractions. On the other hand, the Maxwell-Newton approximation is the basis of a very accurate Einstein-Infeld-Hoffman equation for tracking planets spacecraft [5]. Moreover, the accurate experiments on binary pulsars also support

the Maxwell-Newton approximation [18-20,44,45]. Therefore, it is expected that the experiments on local light speeds will directly support the Maxwell-Newton approximation, but reject the Schwarzschild solution.

Since the current theory of back hole depends on the presumed validity of the Schwarzschild solution [30,31], this theory would have to go through a revision unless the experimental result of local light speeds turns out to be unexpected. The current dependency on this solution is at least in two aspects: 1) It is crucial to interpret dt/dr (instead of normally dr/dt) as velocity inside the "event horizon" [31]. This disagrees with Einstein [1] who believes in a fixed signature for any space-time coordinate. However, such an exchange of signatures between a space and a time coordinate does not happen in an isotropic solution. 2) For an isotropic solution, the radius of the event of horizon would be only 1/4 of that for the Schwarzschild solution. Thus, it is not surprising that theorists such as Wald [25], E. J. Weinberg [30], and Landau and Lifshitz [32], who believed in the current notion of black hole, actually rejected Einstein's equivalence principle but accepted Pauli's version.

6. DISCUSSIONS AND CONCLUSIONS

Some theorists [35] believed "a homogeneous field is characterized by that any part of it is representative of the whole", and thus turn against Einstein's equivalence principle. Experimentally [4,5], it is known that the gravitational redshifts are related to the gravitational potential, but not the gravitational field. Moreover, this belief is also not true in electrodynamics since experiments show that the electromagnetic potential actually has physical influence [36] just as Aharonov and Bohm predicted [37]. It should be noted also that both metrics (6) and (14) do not satisfy the harmonic gauge condition [4], and thus seem to against the proposal of Zhou [38] that the harmonic gauge condition should be an additional physical condition. However, Peng [39] clarified that such a condition is meant to apply for the case when the metric is asymptotically flat.

Based on Pauli's version, Yu [7, p. 58] misleadingly claimed that all measurable quantities must be scalars, and Liu [42] defined a light speed such that the local light speeds are always isotropic for any othorgonal coordinate system. According to these theories, Zhou's experiment [38,40] of local light speeds would appear to be irrelevant. However, Liu's light speeds give only one half of the observed deflection angle [1,2], and it is well known that there

are non-scalars such as vectors in physics. Moreover, as illustrated by the local distance formula of Landau and Lifshitz [32], Pauli's version leads to incorrect acceptance of any Lorentz manifolds as valid in physics [22]. Since Pauli's version leads to disagreements with observation and theoretical inconsistency [23,24,33], it is necessary to re-establish Einstein's equivalence principle on a firm theoretical ground.

In Einstein's theory [1], the reality is modeled with a physical space (-time) that has a frame of reference. In such a physical space, all physical requirements are sufficiently satisfied. In particular, in a physically valid space-time coordinate system, Einstein [1,12] has shown that time is compatible with the rate of a local clock.

Thus, a different coordinate system can fail as a space-time coordinate system in physics although some calculations can be carried out with an arbitrary mathematical coordinate system (see Sections 2 and 3) Moreover, it has been illustrated that the local Minkowski space at a point is obtained by means of choosing the appropriate acceleration. This requires that the coordinates of a physical space must have physical meanings. As illustrated in Sections 3, Einsteins equivalence principle is applicable only in a physical space. Otherwise, the so-calculated local time rate and local spatial contraction would be incorrect in physics.

Now, it is clear that Einstein's equivalence principle cannot be replaced with merely the existence of local Minkowski spaces since, as Logunov et al [23] showed, Pauli's "equivalence principle" has been proven as inadequate in physics. (There is a class of Lorentz manifolds any of which is not diffeomorphic to a physical space [19].) Otherwise, not only this will surely end up in theoretical disagreement with Einstein, but also against the weighty fact that there are non-scalars in physics. Moreover, the existence of definitive gravitational red shifts testifies that a valid space-time coordinate system cannot be arbitrary [27].

Historically, Eddington [2] is the first who recognized that a gauge couldn't be arbitrary. Zhou [38,40] is probably the first who did an experiment[8)] in order to show that a gauge is unique for a physical coordinate system although his reason for uniqueness was essentially an assumption. In this paper, it is shown that the uniqueness of a gauge for a given frame of reference is actually built in the theoretical framework of general relativity. Thus, this paper supports the essential part of Zhou's proposal.

Note that the uniqueness of a realistic gauge for a given frame of reference is compatible with the fact that the same deflection angle of light was obtained by two different metrics, namely, the Schwarzschild and the isotropic metrics. First, these two gauges are related

to the same frame of reference. Moreover, both metrics are asymptotically flat and have the same first order gravitational red shift. Thus, these gauges are restricted. Second, the deflection angle is an integrated effect over a very long range, and thus would not be subjected to the same rule for local effects such as gravitational red shift and local light speeds. In view of that the realistic gauge is not yet known, such a restricted gauge invariance in mathematics may be needed to explain the observational confirmation of light bending. Besides, the second order approximation is no longer invariant.

In conclusion, a uniform acceleration, as an idealization, is valid for an illustration of Einstein's equivalence principle as a foundation of general relativity. Since the physical meaning of space-time coordinates is included in the theoretical framework of general relativity, there is no longer any reason to substitute Einstein's equivalence principle with Pauli's version [3]. Another source of difficulty was a misconception that the curved space-time is expected to be embedding in a higher dimensional flat space [41]. It goes without saying that Whitehead's objection [21] is no longer valid. It is hope that this paper would help rectifying the damages done by Bergmann's misinterpretation [13] and Fock's misconceptions [10]. On this basis, one may expect that general relativity would be infused with new life and be even more fruitful in the future.

However, since the application of Einstein's equivalence principle still cannot decide the realistic gauge, one might ask what is its difference from Pauli's version in physics. The main difference is that as illustrated by the local distance formula of Landau and Lifshitz [32], Pauli's version leads to incorrect acceptance of any Lorentz manifolds as valid in physics [22]. On the other hand, Einstein's equivalence principle, which enables us to have a clear physical meaning of coordinates, would reject some unphysical Lorentz manifolds [26]. Logically, Einstein's equivalence principle demands the burden of proof from current advocates of black holes for choosing the Schwarzschild solution instead of the possibility of others since the gauge is unique for a given frame. Moreover, both Whitehead [21] and Fock [10] rejected general relativity because they believe uniformity of space-time owing to their philosophies. However, their objection turns out to be irrelevant since their required uniformity is actually implied by Einstein's equivalence principle [26].

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ENDNOTES

1. In fact, Einstein [12, p. 144] wrote, "For it is clear that, e.g., the gravitational field generated by a material point in its environment certainly cannot be transformed away by any choice of the system of coordinates, i.e. it cannot be transformed to the case of constant $g_{\mu\nu}$". Thus, some theorists misrepresented Einstein's theory simply because of carelessness in reading.

2. To be more precise, the Riemannian space is actually a pseudo-Riemannian space because the metric signature is indefinite as that of a Minkowski space.

3. The Maxwell-Newton approximation is derived independent of the Einstein equation, which was guessed by Einstein.

4. In Euclidean geometry, the measurements in terms of the coordinates or in terms of the metric are the same. For a curved space, as shown by Weinberg [4], these two kinds of measurements would be different unless the curved space is embedded in a Euclidean space of higher dimension.

5. The principle of causality [17] assumes that the causes of phenomena are identifiable. This principle is commonly used in symmetry consideration in electrodynamics. In general relativity, Einstein and subsequent theorists have used this principle implicitly on symmetry considerations [1-10]. Thus, in practice, the physical meaning of space coordinates has been used not only subsequently but also right at the beginning.

6. Landau and Lifshitz [32] erroneously claimed that their local distance formula always produces "actually (local) distances".

7. Einstein declared, "The general laws of nature are to be expressed by equations which hold good for all systems of coordinates, that is, are covariant with respect to any substitutions whatever (generally covariant)." However, Einstein's equivalence principle and the principle of causality are not just tensor equations.

8. Although Yilmaz [43] proposed a similar experiment in 1979 earlier than Zhou's, the purpose of Yilmaz's proposal was to compare his own theory with Einstein's theory.

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