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Particle mass interpreted as a toroidal surface

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Abstract

A toroidal mass formula is introduced that economically reproduces the tau-, muon-, and neutron-electron mass ratios, as well as two mass ratios between quarks.

I. Introduction

Earlier, the author introduced a mass formula that reproduced the π^0 meson-, J/ψ meson-, muon-, and neutron-electron mass ratios with an accuracy at, or very near, the experimental limit [1], while also demonstrating, by using information theory, that this achievement is unlikely to be purely coincidental [2]. In addition, the author reproduced the masses of the quarks and leptons with a mass formula that exploited exponents that derived from a symmetry present in the initial terms of the Fibonacci sequence [3]. It is noteworthy that all of the above mass formulae succeeded by exploiting the constants 4.1 and 0.1, where these are equal to the beta coefficients $b_1 = 41/10$ and $\tilde{b}_1 = 1/10$ of the extra-dimensional, non-supersymmetric GUT described by Dienes, Dudas, and Gherghetta [4]. In this article, a toroidal mass formula is introduced that also employs 4.1 and 0.1, while reproducing closely the tau-, muon-, and neutron-electron mass ratios, as well as two quark mass ratios.

II. A toroidal mass formula

A torus is uniquely specified by its outer radius R, and its inner radius r, measured from the torus's center point. Its surface area is then equal to

$$S(R,r) = \pi^2\left(R^2 - r^2\right) \quad . \tag{1}$$

Now define the mass formula

$$R(L,M,N,K) = \frac{S\left(\sqrt{2}\times 4.1^M, \frac{4.1^M}{\sqrt{2}}\right) - S\left(\sqrt{2}\times 0.1^M, \frac{0.1^M}{\sqrt{2}}\right)\times L + K}{-S\left(\sqrt{2}\times 0.1^{\frac{5}{2}}, \frac{0.1^N}{\sqrt{2}}\right)} \quad , \tag{2}$$

and let L equal 1 for mass ratios that involve pairs of leptons, and 0 for those that do not. Now let $K = 0$, so that an approximation of the ratio of t-quark mass to c-quark may be produced by

$$. \quad \frac{M_t}{M_c} \approx R(0,\frac{1}{2},\frac{1}{2},0) = \frac{S\left(\sqrt{2}\times 4.1^{\frac{1}{2}}, \frac{4.1^{\frac{1}{2}}}{\sqrt{2}}\right)}{-S\left(\sqrt{2}\times 0.1^{\frac{5}{2}}, \frac{0.1^{\frac{1}{2}}}{\sqrt{2}}\right)} = 123.04... \quad . \tag{3a}$$

Similarly the ratio of b-quark mass to c-quark mass may be reproduced approximately by

$$\frac{M_b}{M_c} \approx R(0,0,0,0) = \frac{S\left(\sqrt{2}\times 4.1^0, \frac{4.1^0}{\sqrt{2}}\right)}{-S\left(\sqrt{2}\times 0.1^{\frac{5}{2}}, \frac{0.1^0}{\sqrt{2}}\right)} = 3.00012... \quad . \tag{3b}$$

And an approximation of the ratio of tau mass to electron mass may be produced by

$$\frac{M_\tau}{M_e} \approx R(1,\frac{5}{2},0,0) = 3\frac{S\left(4.1^{\frac{5}{2}}, 0.1^{\frac{5}{2}}\right)}{S\left(1,2\times 0.1^{\frac{5}{2}}\right)} = 3\frac{4.1^5 - 0.1^5}{0.99996} = 3475.82... \quad . \tag{3c}$$

Finally the ratio of muon mass to electron mass is closely reproduced by

$$\frac{M_\mu}{M_e} \approx R(1,\frac{3}{2},0,0) = 3\frac{S\left(4.1^{\frac{3}{2}}, 0.1^{\frac{3}{2}}\right)}{S\left(1,2\times 0.1^{\frac{5}{2}}\right)} = 3\frac{4.1^3 - 0.1^3}{0.99996} = 206.768\ 270\ 7 \quad . \tag{3d}$$

The above quark mass ratios, Eqs. (3a) and (3b), differ only slightly from the

corresponding experimental data. The t-quark's mass of 174,300 ± 5,100 MeV and the c-quark's mass of 1,150 to 1,350 MeV [5] suggest a possible t-quark / c-quark mass ratio of 169,200 / 1,350 = 125.33..., which is near its calculated value of 123.04... . The b-quark's mass of 4,100 to 4,400 MeV and the c-quark's mass of 1,150 to 1,350 MeV [5] suggest a possible b-quark / c-quark mass ratio of 4,100 / 1,350 = 3.037..., which is near its calculated value of 3.00012... .

Similarly, the calculated value of the tau mass relative to electron mass $\frac{M_\tau}{M_e}$ equals

$$0.510998918\,\text{MeV} \times 3\frac{4.1^5 - 0.1^5}{0.99996} = 1776.14...\ \text{MeV} \quad ,$$

which is within roughly 1 part in 2,000 of its experimental value of $1776.99^{+0.29}_{-0.25}$ MeV [5].

The experimental value of the muon-electron mass ratio $\frac{M_\mu}{M_e}$ equals 206.768 283 8 (54) [6], which differs by just 63 ppb, or roughly 1 part in 16,000,000, from the calculated muon-electron mass ratio yielded by Eq. (3d).

Lastly, the neutron-electron mass ratio $\frac{M_n}{M_e}$ may be reproduced precisely by letting

$L = 0$, and $K = S\left(\frac{1}{0.1^{\frac{4}{2}}}, \frac{1}{0.1^{\frac{3}{2}}}\right) + S\left(\frac{1}{0.1^{\frac{2}{2}}}, \frac{1}{0.1^{\frac{1}{2}}}\right)$, so that

$$\frac{M_n}{M_e} \approx R\left(0, \frac{3}{2}, -\frac{1}{2}, S\left(\frac{1}{0.1^{\frac{4}{2}}}, \frac{1}{0.1^{\frac{3}{2}}}\right) + S\left(\frac{1}{0.1^{\frac{2}{2}}}, \frac{1}{0.1^{\frac{1}{2}}}\right)\right)$$

$$\approx \frac{3 \times S\left(4.1^{\frac{3}{2}}, 0\right) + 2\left[S\left(\frac{1}{0.1^{\frac{4}{2}}}, \frac{1}{0.1^{\frac{3}{2}}}\right) + S\left(\frac{1}{0.1^{\frac{2}{2}}}, \frac{1}{0.1^{\frac{1}{2}}}\right)\right]}{S\left(\frac{1}{0.1^{\frac{1}{2}}}, 2 \times 0.1^{\frac{5}{2}}\right)}$$

$$\approx 3\frac{4.1^3 + 6060}{9.99996} = 1838.683\,654\,7\ldots \quad . \tag{3e}$$

This mass ratio is within 2.8 parts per billion (or roughly 1 part in 360,000,000) of its experimental value of 1838.683 659 8 (13) [6]. Although this is unexpected, it is not clear that any law of physics disallows a simple but precise equation for the neutron-electron mass ratio.

It is important to recognize how few differences there are between the parameters used in Eq. (3c), which produce the tau-electron mass ratio; and those of Eq. (3d), which produce the muon-electron mass ratio; and those of Eq. (3e), which produce the neutron-electron mass ratio,

as is seen in the following parameter table.

Mass ratio	*L*	*M*	*N*	*K*
t-quark / c-quark	0	1/2	1/2	0
b-quark / c-quark	0	0	0	0
tau / electron	1	5/2	0	0
muon / electron	1	3/2	0	0
neutron / electron	0	3/2	−1/2	$S\left(\frac{1}{0.1^{\frac{4}{2}}},\frac{1}{0.1^{\frac{3}{2}}}\right)+S\left(\frac{1}{0.1^{\frac{2}{2}}},\frac{1}{0.1^{\frac{1}{2}}}\right)$

In particular, it appears that the remarkable accuracy of Eq. (3e), the equation that yields the neutron-electron mass ratio, may in some way non-coincidentally relate to the success of Eq. (3d), the equation that yields the muon-electron mass ratio. This specific point was assessed in detail by the author using information theory and number theory, where it was calculated that these mass equations jointly compress the data they summarize to such a large degree, that it is unlikely they do so purely by coincidence [2].

Also note that in Eq. (3e) the expression $S\left(\frac{1}{0.1^{\frac{4}{2}}},\frac{1}{0.1^{\frac{3}{2}}}\right)+S\left(\frac{1}{0.1^{\frac{2}{2}}},\frac{1}{0.1^{\frac{1}{2}}}\right)$ does not "fine-tune" the neutron-electron mass ratio to fit the experimental mass data, but instead is by far the major contributor to the neutron's mass (it contributes approximately 99% of the mass). Still, despite the expression's obvious simplicity, it combines with the other terms to produce precisely the right amount of mass to fit the experimental data to within 2.8 parts per billion.

Lastly, note that the parameters that produce $\frac{M_t}{M_c}$ need only be changed slightly to produce instead $\frac{M_n}{M_e}$: the value for M must be incremented by 1, the value for N decremented by 1, and K, which equals 0, must be altered to equal $S\left(\frac{1}{0.1^{\frac{4}{2}}},\frac{1}{0.1^{\frac{3}{2}}}\right)+S\left(\frac{1}{0.1^{\frac{2}{2}}},\frac{1}{0.1^{\frac{1}{2}}}\right)$.

III. Conclusion

One may speculate as to why Eqs. (3a)-(3e) prove so effective. In particular, the toroidal Eqs. (3c)-(3e) economically reproduce the tau-, muon-, and neutron-electron mass ratios with unexpected precision: to 1 part in 2,000, 1 part in 16,000,000, and 1 part in 360,000,000, respectively. One should be careful not to assume automatically that the toroidal mass formula, Eq. (2), compactly summarizes so much data purely for coincidental reasons. There are limits to the compressibility of data, and when such limits are surpassed this suggests that something is at work other than coincidence.

This issue, particularly as it relates to the muon- and neutron-electron mass ratios, has already been investigated in detail by the author [2]. It was shown that, even when the muon- and neutron-electron mass ratio equations are considered separately, coincidental success is improbable. But these equations exploit 4.1 and 0.1, which the author has already shown to compactly summarize the neutral meson masses [1], as well as the quark and lepton mass ratios [3]. These additional results make the possibility of coincidence still less plausible.

It is interesting to conjecture whether ultimately a physical explanation of the efficiency of the above equations may be reached by in some way establishing a relationship between the values 4.1 and 0.1 and the beta coefficients b_1 and $\tilde{b}_1$ of the extra-dimensional non-supersymmetric GUT described by Dienes, Dudas, and Gherghetta [4]. Finally, it should be noted that in string theory toroidal equations play a role in relation to particle mass.

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