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The Chiral Anomaly in Non-Leptonic Radiative Kaon Decays*

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Abstract

The weak radiative kaon decays $K \to \pi\pi\gamma(\gamma)$ and $K \to \pi\pi\pi\gamma(\gamma)$ are sensitive to the chiral anomaly. Chiral perturbation theory is the appropriate framework for an investigation of these processes. The phenomenology of the decays $K^+ \to \pi^+\pi^0\gamma$ and $K_L \to \pi^+\pi^-\gamma$ is discussed in some detail.

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1 Introduction

Anomalies constitute an essential ingredient in our present understanding of relativistic quantum field theories. Gauge symmetries should be free of anomalies in order to allow a consistent quantization procedure. Global symmetries, however, can be broken at the quantum level. A well known example is the chiral anomaly [1, 2], present in quantum field theories with chiral structure, such as the standard model. Its origin is an unavoidable clash between the chiral symmetry of the classical Lagrangian and the necessary regularization at small distances to obtain a well defined quantum field theory. Although the mathematical properties of the anomaly are well understood, experimental tests of this important aspect of modern particle physics are relatively rare.

The chiral anomaly manifests itself most directly in the low-energy interactions of the pseudoscalar mesons. The appropriate framework to study these effects is chiral perturbation theory (CHPT) [3, 4, 5], an effective field theory at the hadronic level. Its Lagrangian is constructed in terms of the asymptotically observed fields, using the symmetries of the standard model. The effective theory gives rise to a systematic low-energy expansion of the amplitudes, solving all constraints imposed by chiral symmetry and unitarity. The chiral anomaly is included in a natural way by the Wess-Zumino-Witten (WZW) functional [6]. For the strong, electromagnetic and semileptonic weak interactions, all anomalous Green functions can be obtained from this term. In contrast to most other aspects of the standard model in the hadronic sector, the translation from the fundamental level to the mesonic level is unambiguous and free of hadronization problems.

The chiral anomaly also appears in the non-leptonic weak interactions. It has been shown [7, 8, 9], that only radiative kaon decays are sensitive to the chiral anomaly in the non-leptonic sector. There are two different manifestations of the anomaly: the reducible amplitudes [7], which can again be derived directly from the WZW functional, and direct contributions [8, 9, 10], which are subject to some theoretical uncertainties.

In the following, I shall briefly review the construction of the effective chiral Lagrangian, including non-leptonic weak interactions. It will be explained how the chiral anomaly enters in the non-leptonic sector. Finally the phenomenology of the two most frequent "anomalous" decays $K^+ \to \pi^+\pi^0\gamma$ and $K_L \to \pi^+\pi^-\gamma$ will be discussed.

2 The Low-Energy Limit of QCD

The Green functions of the vector-, axialvector, scalar and pseudoscalar quark currents built out of the three light flavours u, d, s are generated by the vacuum-to-vacuum amplitude

$$e^{iZ[v,a,s,p]} = \langle 0\, out | 0\, in \rangle_{v,a,s,p} \tag{2.1}$$

associated with the Lagrangian

$$\mathcal{L} = \mathcal{L}^0_{QCD} + \bar{q}\gamma^\mu(v_\mu + a_\mu\gamma_5)q - \bar{q}(s - ip\gamma_5)q. \tag{2.2}$$

$\mathcal{L}^0_{QCD}$ is the QCD Lagrangian with the masses of the three light quarks $q = (q_u, q_d, q_s)^\top$ set to zero. The external fields v_μ, a_μ, s and p are Hermitean 3×3 matrices in flavour

space. To describe electromagnetic and semileptonic interactions the relevant fields of the standard model are

$$\begin{aligned} r_\mu &= v_\mu + a_\mu = eQA_\mu, \\ \ell_\mu &= v_\mu - a_\mu = eQA_\mu + \frac{e}{\sqrt{2}\,sin\theta_W}(W_\mu^+ T_+ + H.c.), \end{aligned} \tag{2.3}$$

with

$$Q = \text{ diag } (2/3, -1/3, -1/3), \quad T_+ = \begin{pmatrix} 0 & V_{ud} & V_{us} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{2.4}$$

where V_{ij} are Kobayashi-Maskawa matrix elements. The mass matrix of the light quarks is contained in the scalar field $s(x)$. The Lagrangian (2.2) exhibits a local $SU(3)_L \times SU(3)_R$ symmetry

$$\begin{aligned} q_{R,L} &\to g_{R,L}\, q_{R,L}, \\ r_\mu &\to g_R\, r_\mu\, g_R^\dagger + i g_R\, \partial_\mu g_R^\dagger, \\ \ell_\mu &\to g_L\, \ell_\mu\, g_L^\dagger + i g_L\, \partial_\mu g_L^\dagger, \\ s + ip &\to g_R(s + ip) g_L^\dagger, \\ g_{R,L} &\in SU(3)_{R,L}. \end{aligned} \tag{2.5}$$

The generating functional Z admits an expansion in powers of the external momenta and of quark masses. Approximating Z by a given order in this expansion is called CHPT [3, 4, 5]. To lowest order in this chiral expansion, the effective Lagrangian describing pseudoscalar mesons is given by

$$\mathcal{L}_2 = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U \rangle \tag{2.6}$$

where

$$D_\mu U = \partial_\mu U - i r_\mu U + i U \ell_\mu, \quad \chi = 2B_0(s + ip), \tag{2.7}$$

and $\langle ... \rangle$ denotes the trace in three-dimensional flavour space. U is a unitary and unimodular 3×3 matrix which transforms as

$$U \to g_R\, U g_L^\dagger \tag{2.8}$$

under $SU(3)_L \times SU(3)_R$. The parameters F and B_0 are the only free constants of $\mathcal{L}_2$: F is the pion decay constant in the chiral limit,

$$F_\pi = F(1 + \mathcal{O}(m_{quark})). \tag{2.9}$$

whereas B_0 is related to the quark condensate,

$$\langle 0 \mid \bar{u}u \mid 0 \rangle = -F^2 B_0 (1 + \mathcal{O}(m_{quark})). \tag{2.10}$$

The matrix U incorporates the fields of the eight pseudoscalar mesons. It is convenient to use the parametrization

$$U = e^{i\sqrt{2}\Phi/F}. \tag{2.11}$$

In the limit $m_u = m_d$ the matrix Φ is related to the mass eigen-fields of the pseudoscalar octet by

$$\Phi = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}. \tag{2.12}$$

The Lagrangian (2.6) is referred to as the effective chiral Lagrangian of $\mathcal{O}(p^2)$. The chiral counting rules are the following: The field U is of $\mathcal{O}(p^0)$, the derivative ∂_μ and the external gauge fields v_μ, a_μ are terms of $\mathcal{O}(p)$ and the fields s, p count as $\mathcal{O}(p^2)$.

At $\mathcal{O}(p^4)$ the generating functional consists of three terms [4, 5]:

- The one-loop functional generated by the lowest order Lagrangian (2.6).
- The most general chiral-invariant local action of $\mathcal{O}(p^4)$ which respects also the discrete symmetries P and C.
- A contribution to account for the chiral anomaly.

Let me briefly discuss the last two points starting with the local action of $\mathcal{O}(p^4)$. It is generated by the Gasser-Leutwyler Lagrangian [5]

$$\mathcal{L}_4 = \sum_{i=1}^{12} L_i P_i, \tag{2.13}$$

where the P_i are operators of $\mathcal{O}(p^4)$ like $P_1 = \langle D_\mu U D^\mu U^\dagger \rangle^2$, etc. The twelve low-energy constants $L_1, ..., L_{12}$ arising here are in general divergent (except L_3, L_7). They absorb the divergences of the one-loop graphs. The finite parts of $L_1, ..., L_{10}$ can be determined [4, 5] by using experimental input. (P_{11} and P_{12} are contact terms which are not directly accessible to experiment.)

Let us now turn to the anomaly. Under a chiral transformation

$$g_{R,L} = \mathbf{1} + i\alpha \mp i\beta + ... \tag{2.14}$$

the generating functional of QCD transforms in a non-trivial way:

$$\begin{aligned} \delta_\alpha Z[v, a, s, p] &= 0, \\ \delta_\beta Z[v, a, s, p] &= \beta \cdot G[v, a], \end{aligned} \tag{2.15}$$

where the explicit form of $G[v, a]$ has been given by Bardeen [2]. A functional $S[U, \ell, r]_{WZW}$ which reproduces the chiral anomaly has been constructed by Wess and Zumino and Witten [6]. Instead of writing down the complete expression, let me emphasize a crucial property of S_{WZW}. It has no free parameters at all and is given entirely in terms of the matrix field U and the gauge fields ℓ, r. This is indeed a remarkable feature. In general the transition from the fundamental level of quarks and gluons to the mesonic level involves a host of undetermined parameters (e.g., the twelve parameters in $\mathcal{L}_4$).

3 Non-Leptonic Weak Interactions

The effective Lagrangian constructed so far is describing strong, electromagnetic, and semileptonic weak interactions. One could naively think that non-leptonic weak processes can now be treated by simply contracting the W boson field between two hadronic Green functions. A moment's thought reveals that this cannot be a successful strategy. The reason are the strong interactions connecting the strongly interacting fields on both sides of the W.

The correct approach in non-leptonic weak interactions is the following: One first has to integrate out the W boson and the heavy quarks in the fundamental theory to arrive at an effective $|\Delta S| = 1$ Hamiltonian [11]

$$\mathcal{H}_{\text{eff}}^{|\Delta S|=1} = \frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i Q_i + \text{H.c.} \tag{3.1}$$

Neither the explicit form of the local four-quark operators Q_i nor the Wilson coefficients C_i are needed in the following. For the effective chiral realization of (3.1) at the hadronic level it is important that (3.1) transforms as

$$(8_L, 1_R) + (27_L, 1_R) \tag{3.2}$$

under the chiral group. The effective chiral Lagrangian for (3.1) to lowest order in the chiral expansion can be written as

$$\mathcal{L}_2^{|\Delta S|=1} = G_8 F^4 \langle \lambda L_\mu L^\mu \rangle + G_{27} F^4 \left(L_{\mu 23} L_{11}^\mu + \frac{2}{3} L_{\mu 21} L_{13}^\mu \right) + \text{H.c.}, \tag{3.3}$$

$$\lambda = \frac{1}{2}(\lambda_6 - i\lambda_7), \qquad L_\mu = iU^\dagger D_\mu U.$$

Neglecting the small $|\Delta I| = 1/2$ part of the 27-plet, the Lagrangian (3.3) produces the tree-level amplitudes

$$\begin{aligned} A(K_1^0 \to \pi^+\pi^-) &= 2iF(G_8 + G_{27}^{(3/2)})(M_\pi^2 - M_K^2), \\ A(K^+ \to \pi^+\pi^0) &= 3iFG_{27}^{(3/2)}(M_\pi^2 - M_K^2), \\ G_{27}^{(3/2)} &= \frac{5}{9} G_{27}. \end{aligned} \tag{3.4}$$

Up to radiative and higher-order chiral corrections [12, 13], the ratio

$$\frac{G_{27}^{(3/2)}}{G_8} = \frac{1}{32} \tag{3.5}$$

is small (and positive), expressing the $\Delta I = 1/2$ rule in $K \to 2\pi$ decays, and $|G_8| \simeq 9 \cdot 10^{-6}\ \text{GeV}^{-2}$.

At next-to-leading order in CHPT, the chiral Lagrangian $\mathcal{L}_4^{|\Delta S|=1}$ is already quite involved [14]. I shall only need the octet Lagrangian of $\mathcal{O}(p^4)$. Following the classification of Ecker, Kambor and Wyler [15], I write

$$\mathcal{L}_4^{|\Delta S|=1} = G_8 F^2 \sum_{i=1}^{37} N_i W_i + \text{H.c.} \tag{3.6}$$

with dimensionless coupling constants N_i and octet operators W_i. The operators $W_{19}, ..., W_{27}$; $W_{32}, ..., W_{35}$ can contribute only to processes with external W bosons (in addition to the non-leptonic weak transition) and W_{36}, W_{37} are contact terms, needed for renormalization. So, only 22 of the coupling constants N_i contribute to observable physical processes. Referring to Ref. [15] for the complete Lagrangian, I list here only a few terms that will be needed in the following:

$$\begin{aligned} W_{28} &= i\varepsilon_{\mu\nu\rho\sigma}\langle\lambda L^\mu\rangle\langle L^\nu L^\rho L^\sigma\rangle, \\ W_{29} &= 2\langle\lambda[U^\dagger \tilde{F}_R^{\mu\nu} U, L_\mu L_\nu]\rangle, \\ W_{30} &= \langle\lambda L_\mu\rangle\langle(\tilde{F}_L^{\mu\nu} + U^\dagger \tilde{F}_R^{\mu\nu} U)L_\nu\rangle, \\ W_{31} &= \langle\lambda L_\mu\rangle\langle\tilde{F}_L^{\mu\nu} - U^\dagger \tilde{F}_R^{\mu\nu} U)L_\nu\rangle, \end{aligned} \tag{3.7}$$

where $F_L^{\mu\nu}$, $F_R^{\mu\nu}$ are the field strength tensors associated to the external gauge fields ℓ_μ, r_μ and $\tilde{F}_{L,R\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} F_{L,R}^{\rho\sigma}$ are their duals.

4 The Chiral Anomaly in the Non-Leptonic Weak Sector

The chiral anomaly also contributes to non-leptonic weak amplitudes starting at $\mathcal{O}(p^4)$. Two different manifestations of the anomaly can be distinguished:

4.1 Reducible Anomalous Amplitudes

These amplitudes arise from the contraction of meson lines between a weak $|\Delta S| = 1$ Green function and the WZW functional. At $\mathcal{O}(p^4)$, there can only be one such contraction and the weak vertex must be due to the lowest-order non-leptonic Lagrangian $\mathcal{L}_2^{|\Delta S|=1}$ in Eq. (3.3).

Since $\mathcal{L}_2^{|\Delta S|=1}$ contains bilinear terms in the meson fields, the so-called pole contributions to anomalous non-leptonic amplitudes can be given in closed form by a simultaneous diagonalization [16] of the kinetic parts of the Lagrangians $\mathcal{L}_2$ and $\mathcal{L}_2^{|\Delta S|=1}$. The corresponding local Lagrangian (octet part only) is [7]:

$$\mathcal{L}_{\text{an}}^{|\Delta S|=1} = -\frac{ieG_8}{8\pi^2 F}\tilde{F}^{\mu\nu}\partial_\mu\pi^0 K^+ \overleftrightarrow{D}_\nu \pi^- + \frac{\alpha G_8}{6\pi F}\tilde{F}^{\mu\nu}F_{\mu\nu}\left(K^+\pi^-\pi^0 - \frac{1}{\sqrt{2}}K^0\pi^+\pi^-\right) + \text{H.c.} \tag{4.1}$$

Here $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, $\tilde{F}_{\mu\nu} = \varepsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}$ its dual and $D_\mu \varphi^\pm = (\partial_\mu \mp ieA_\mu)\varphi^\pm$ denotes the covariant derivative with respect to electromagnetism. In the limit of CP conservation, the anomalous Lagrangian (4.1) contributes only to the decays $K^+ \to \pi^+\pi^0\gamma, \pi^+\pi^0\gamma\gamma$ and $K_L \to \pi^+\pi^-\gamma\gamma$ with real or virtual photons.

There are of course other reducible anomalous amplitudes. A generic example is provided by a non-leptonic Green function where an external π^0 or η makes an anomalous transition to two photons. All reducible anomalous amplitudes of $\mathcal{O}(p^4)$ are proportional to G_8 in the octet limit. No other unknown parameters are involved.

4.2 Direct Weak Anomaly Functional

The second manifestation of the anomaly in non-leptonic weak amplitudes arises diagrammatically from the contraction of the W boson field between a strong Green function on one side and the WZW functional on the other side. As already explained in Sect. 3 such diagrams cannot be taken literally at a typical hadronic scale, because of the presence of strongly interacting fields on both sides of the W. Instead, one must first integrate out the W together with the heavy quark fields. The operators appearing in the operator product expansion must then be realized at the bosonic level in the presence of the anomaly.

The bosonization of four-quark operators in the odd-intrinsic parity sector was investigated in Ref. [8]. As in the even-intrinsic parity sector, the bosonized four-quark operators contain factorizable (leading in $1/N_C$, where N_C is the number of colours) and non-factorizable parts (non-leading in $1/N_C$). The factorizable contribution of $\mathcal{O}(p^4)$ can be calculated exactly [8]. Specializing to the dominant octet operator in $\mathcal{H}_{\text{eff}}^{|\Delta S|=1}$, one obtains the following bosonized form of $\mathcal{O}(p^4)$ in the factorizable approximation for the odd-parity part [8, 10]:

$$\frac{F^2}{16\pi^2}\Big(2i\varepsilon^{\mu\nu\alpha\beta}\langle\lambda L_\mu\rangle\langle L_\nu L_\alpha L_\beta\rangle + \Big\langle\lambda[U^\dagger\tilde{F}_R^{\mu\nu}U, L_\mu L_\nu]\Big\rangle$$
$$+ 3\langle\lambda L_\mu\rangle\Big\langle(\tilde{F}_L^{\mu\nu} + U^\dagger\tilde{F}_R^{\mu\nu}U)L_\nu\Big\rangle + \langle\lambda L_\mu\rangle\Big\langle(\tilde{F}_L^{\mu\nu} - U^\dagger\tilde{F}_R^{\mu\nu}U)L_\nu\Big\rangle\Big). \tag{4.2}$$

Comparison with the general weak Lagrangian $\mathcal{L}_4^{|\Delta S|=1}$ of $\mathcal{O}(p^4)$ in (3.6), (3.7) shows that all the possible octet operators proportional to the ε tensor (W_{28}, W_{29}, W_{30} and W_{31}) appear in (4.2). Thus the chiral anomaly contributes to all the coefficients $N_{28}, \ldots, N_{31}$ of normal octet operators. Moreover, the non-factorizable parts must be of the same form (4.2). The corresponding coefficients will differ from those in Eq. (4.2). In fact, they must depend on the QCD scale μ to cancel the μ-dependence of the Wilson coefficients in the $|\Delta S| = 1$ effective Hamiltonian [17].

Since all octet operators in $\mathcal{H}_{\text{eff}}^{|\Delta S|=1}$ produce the same structure (4.2), the $|\Delta S| = 1$ effective Lagrangian in the anomalous parity sector of $\mathcal{O}(p^4)$ can be characterized by the coefficients [8]

$$N_{28}^{\text{an}} = \frac{a_1}{8\pi^2}, \qquad N_{29}^{\text{an}} = \frac{a_2}{32\pi^2}, \qquad N_{30}^{\text{an}} = \frac{3a_3}{16\pi^2}, \qquad N_{31}^{\text{an}} = \frac{a_4}{16\pi^2}, \tag{4.3}$$

Table 1: A complete list of local anomalous non-leptonic weak K decay amplitudes of $\mathcal{O}(p^4)$ in the limit of CP conservation.

Transition	$\mathcal{L}_{\rm an}^{\lvert\Delta S\rvert=1}$	W_{28}	W_{29}	W_{30}	W_{31}	expt.
$K^+ \to \pi^+\pi^0\gamma$	x		x	x		x
$K^+ \to \pi^+\pi^0\gamma\gamma$	x		x	x		
$K_L \to \pi^+\pi^-\gamma$			x		x	x
$K_L \to \pi^+\pi^-\gamma\gamma$	x		x		x	
$K^+ \to \pi^+\pi^0\pi^0\gamma$			x	x		x
$K^+ \to \pi^+\pi^0\pi^0\gamma\gamma$			x	x		
$K^+ \to \pi^+\pi^+\pi^-\gamma$			x		x	x
$K^+ \to \pi^+\pi^+\pi^-\gamma\gamma$			x		x	
$K_L \to \pi^+\pi^-\pi^0\gamma$		x	x	x		
$K_S \to \pi^+\pi^-\pi^0\gamma(\gamma)$			x	x	x	

where $a_i = 1$ corresponds to the factorizable approximation (4.2). There are some theoretical arguments that the actual values of the a_i are slightly smaller than one [8, 9].

In Table 1 all local contributions from either the Lagrangian $\mathcal{L}_{\rm an}^{\lvert\Delta S\rvert=1}$ in (4.1) or the direct terms of $\mathcal{O}(p^4)$ to all kinematically allowed non-leptonic K decays are listed. A separate column indicates whether the corresponding decay has been observed experimentally. Let me note that the transitions with either three pions and/or two photons in the final state are in general also subject to non-local reducible anomalous contributions. Finally, one observes that in the non-leptonic weak sector the chiral anomaly contributes only to radiative K decays.

5 The Decays $K^+ \to \pi^+\pi^0\gamma$ and $K_L \to \pi^+\pi^-\gamma$

In view of the experimental observation of anomalous contributions, the decays $K^+ \to \pi^+\pi^0\gamma$ and $K_L \to \pi^+\pi^-\gamma$ are the most interesting ones. The amplitude for $K(P) \to \pi_1(p_1) + \pi_2(p_2) + \gamma(q)$ is decomposed into an electric amplitude $E(x_i)$ and a magnetic amplitude $M(x_i)$:

$$A(K \to \pi\pi\gamma) = \varepsilon^\mu(q)^*[E(x_i)(p_1 q\, p_{2\mu} - p_2 q\, p_{1\mu}) + M(x_i)\varepsilon_{\mu\nu\rho\sigma} p_1^\nu p_2^\rho q^\sigma]/M_K^3 \tag{5.1}$$

$$x_i = \frac{Pp_i}{M_K^2} \qquad (i = 1, 2), \qquad x_3 = \frac{Pq}{M_K^2}, \qquad x_1 + x_2 + x_3 = 1.$$

There is no interference between E and M as long as the photon helicity is not measured.

5.1 $K^+ \to \pi^+\pi^0\gamma$

The electric amplitude for $K^+ \to \pi^+\pi^0\gamma$ at $\mathcal{O}(p^2)$ is pure bremsstrahlung [7, 9],

$$E_B(x_i) = \frac{eA(K^+ \to \pi^+\pi^0)}{M_K x_3(\frac{1}{2} - x_0)}, \qquad p_1 = p_+, \quad p_2 = p_0. \tag{5.2}$$

According to (3.4) it is suppressed by the $\Delta I = 1/2$ rule. This facilitates the detection of the magnetic amplitude which occurs at $\mathcal{O}(p^4)$ and consists of both a reducible and a direct amplitude [7, 8, 9]

$$M_4 = \frac{eG_8 M_K^3}{2\pi^2 F}\left(-1 + \frac{3}{2}a_2 - 3a_3\right), \tag{5.3}$$

in terms of the couplings a_i defined in (4.3).

At $\mathcal{O}(p^4)$ there is also a local scale-independent contribution to the electric amplitude E [7, 9]:

$$E_4^{\text{local}} = \frac{2ieG_8 M_K^3}{F}(N_{14} - N_{15} - N_{16} - N_{17}). \tag{5.4}$$

By measuring the energy spectrum of the photon, the counterterm amplitude (5.4) can in principle be isolated through its interference with the bremsstrahlung amplitude (5.2). The size of this interference can be estimated by appealing to the so-called factorization model (see [17] and references quoted therein) which predicts [15]

$$N_{14} - N_{15} - N_{16} - N_{17} = -k_f \frac{F_\pi^2}{2M_V^2} = -7 \cdot 10^{-3} k_f. \tag{5.5}$$

The constant k_f is a fudge factor which the naive factorization model puts equal to one. For $k_f > 0$, the interference is predicted to be positive [7, 9]:

$$\frac{E_4^{\text{local}}}{E_B} \simeq 2.3x_3(1 - 2x_0)(-N_{14} + N_{15} + N_{16} + N_{17})/7 \cdot 10^{-3}. \tag{5.6}$$

Except for small photon energies ($x_3 \to 0$, $2x_0 \to 1$) where bremsstrahlung is bound to dominate, the amplitude E_4^{local} should be detectable. In fact, the experiment of Abrams et al. [18] is consistent with constructive interference between E_B and E_4^{local}, but the available data [18] are not precise enough to separate the amplitudes $E - E_B$ and M experimentally.

As the counterterm contributions (5.4) are scale-independent, the loop amplitude is necessarily finite. It turns out to be rather small [9]:

$$\left|\frac{E_4^{\text{loop}}}{E_B}\right| \leq 4 \cdot 10^{-2}. \tag{5.7}$$

At least in the foreseeable future, the loop amplitude can safely be neglected in comparison with the bremsstrahlung amplitude (5.2). On the other hand, the counterterm amplitude (5.4) should be within reach of facilities with intense K^+ beams.

One can also try to estimate the dominant effects of $\mathcal{O}(p^6)$ due to vector meson exchange. For a detailed discussion the reader is refered to Ref. [9]. Here, I just give the result:

$$M_6 = \frac{eG_8 M_K^3}{2\pi^2 F}\left\{ r_V(k_f - \frac{1}{2}) + \frac{L_9 M_K^2}{F^2} k_f(3 - 8x_+ - 2x_0)\right\}, \tag{5.8}$$

where

$$r_V = \frac{64\sqrt{2}\,\pi^2 g_V h_V M_K^2}{3M_V^2} \simeq 0.4 \simeq \frac{2L_9^r(M_\rho)M_K^2}{F_\pi^2}$$

with the vector couplings g_V, h_V [19] and the coupling constant L_9 [5] of(2.13):

$$g_V \simeq \frac{F_\pi}{\sqrt{2}\,M_V}, \qquad |h_V| \simeq 3.7\cdot 10^{-2}, \qquad L_9^r(M_\rho) \simeq 6.9\cdot 10^{-3}. \tag{5.9}$$

Under the assumption that direct emission is entirely due to the magnetic part, experiments [18] find a branching ratio

$$BR(55 < T_{\pi^+}(\mathrm{MeV}) < 90) = (1.8 \pm 0.4)\cdot 10^{-5} \tag{5.10}$$

for the given cuts in the kinetic energy of the charged pion. This experimental result can be used to determine the quantity $A_4 = -2 + 3a_2 - 6a_3$. For the values of r_V and L_9 listed in (5.9), A_4 is found to be

$$A_4 = -4.1 - 0.3k_f \pm 0.5, \qquad 0 \le k_f \le 1. \tag{5.11}$$

being very much consistent with the theoretical expectation based on $a_i \lesssim 1$.

5.2 $K_L \to \pi^+\pi^-\gamma$

The bremsstrahlung amplitude of $\mathcal{O}(p^2)$ [7, 9]

$$E_B(x_i) = \frac{\varepsilon e A(K_1^0 \to \pi^+\pi^-)}{M_K(\frac{1}{2} - x_+)(\frac{1}{2} - x_-)}, \qquad p_1 = p_+, \quad p_2 = p_-, \tag{5.12}$$

is again suppressed because it violates CP. Here ε is the standard CP violation parameter in $K \to \pi\pi$ decays. From $\mathcal{O}(p^4)$ on I assume CP conservation implying

$$\begin{aligned} E(x_-, x_+) &= -E(x_+, x_-), \\ M(x_-, x_+) &= M(x_+, x_-). \end{aligned} \tag{5.13}$$

The dominant contribution of $\mathcal{O}(p^4)$ occurs in the magnetic amplitude and it is due to the anomaly. As can be seen from (4.1), there is no reducible anomalous amplitude of $\mathcal{O}(p^4)$. The direct weak anomaly functional gives rise to [8, 10]

$$M_4 = \frac{eG_8 M_K^3}{2\pi^2 F}(a_2 + 2a_4). \tag{5.14}$$

Because of (5.13) there is no local contribution to E at $\mathcal{O}(p^4)$. The one-loop amplitude is again finite and small:

$$\left|\frac{E_4^{\rm loop}(K^\pm\eta)}{E_B}\right| \leq 1.2\cdot 10^{-2}. \tag{5.15}$$

There are several contributions of $\mathcal{O}(p^6)$. First of all, there is a reducible amplitude due to the anomaly of the form [7, 9]

$$M_6^{\rm anom} = -\frac{eG_8M_K^3}{2\pi^2F}F_1, \tag{5.16}$$

$$F_1 = \frac{1}{1-r_\pi^2} - \frac{(c-\sqrt{2}\,s)(c+2\sqrt{2}\,\rho s)}{3(r_\eta^2-1)} + \frac{(\sqrt{2}\,c+s)(2\sqrt{2}\,\rho c-s)}{3(r_{\eta'}^2-1)},$$

$$r_i = M_i/M_K, \qquad c=\cos\Theta, \qquad s=\sin\Theta.$$

Θ denotes the η–η' mixing angle and $\rho\neq 1$ takes into account possible deviations from nonet symmetry for the non-leptonic weak vertices (nonet symmetry is assumed for the strong WZW vertices). At $O(p^4)$ ($\Theta=0$, $M_{\eta'}\to\infty$), F_1 vanishes because of the Gell-Mann–Okubo mass formula. In the real world, the η and η' contributions interfere destructively for $0\leq\rho<1$ and $\Theta\simeq -20°$ as in the similar case of the $K_L\to 2\gamma$ amplitude.

In addition, there are $\mathcal{O}(p^6)$ contributions induced by vector meson exchange [9]:

$$M_6^V = \frac{eG_8M_K^3}{2\pi^2F}\left\{r_V[1+x_3(2k_f-3)] + \frac{2L_9M_K^2}{F^2}k_f(2-5x_3)\right\}. \tag{5.17}$$

There are indeed strong experimental indications for the presence of a sizeable magnetic amplitude beyond $O(p^4)$. A recent analysis of $K_L\to\pi^+\pi^-\gamma$ at FNAL [20] confirms an earlier result from Brookhaven [21] finding evidence for a dependence of the direct emission amplitude on the photon energy consistent with (5.17).

6 Summary

- The chiral anomaly appears also in non-leptonic weak interactions.
- Reducible and direct anomalous amplitudes can be distinguished.
- Weak radiative kaon decays receive contributions from the chiral anomaly.
- Chiral perturbation theory is the appropriate tool to study these processes.
- Present experiments are already sensitive to effects of $\mathcal{O}(p^6)$ in the amplitudes.

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References

[1] S.L. Adler, Phys. Rev. 177 (1969) 2426;
J.S. Bell and R. Jackiw, Nuovo Cim. 60A (1969) 47.

[2] W.A. Bardeen, Phys. Rev. 184 (1969) 1848.

[3] S. Weinberg, Physica 96A (1979) 327.

[4] J. Gasser and H. Leutwyler, Ann. Phys. 158 (1984) 142.

[5] J. Gasser and H. Leutwyler, Nucl. Phys. B250 (1985) 465.

[6] J. Wess and B. Zumino, Phys. Lett. 37B (1971) 95;
E. Witten, Nucl. Phys. B223 (1983) 422.

[7] G. Ecker, H. Neufeld and A. Pich, Phys. Lett. B278 (1992) 337.

[8] J. Bijnens, G. Ecker and A. Pich, Phys. Lett. B286 (1992) 341.

[9] G. Ecker, H. Neufeld and A. Pich, Non-Leptonic Kaon Decays and the Chiral Anomaly, CERN-TH.6920/93.

[10] H.-Y. Cheng, Phys. Rev. D42 (1990) 72.

[11] F.J. Gilman and M.B. Wise, Phys. Rev. D20 (1979) 2392.

[12] E. de Rafael, Nucl. Phys. B (Proc. Suppl.) 7A (1989) 1.

[13] J. Kambor, J. Missimer and D. Wyler, Phys. Lett. B261 (1991) 496.

[14] J. Kambor, J. Missimer and D. Wyler, Nucl. Phys. B346 (1990) 17.

[15] G. Ecker, J. Kambor and D. Wyler, Nucl. Phys. B394 (1993) 101.

[16] G. Ecker, A. Pich and E. de Rafael, Nucl. Phys. B303 (1988) 665.

[17] A. Pich and E. de Rafael, Nucl. Phys. B358 (1991) 311.

[18] R.J. Abrams et al., Phys. Rev. Lett. 29 (1972) 1118;
V.N. Bolotov et al., Sov. J. Nucl. Phys. 45 (1987) 1023.

[19] G. Ecker, A. Pich and E. de Rafael, Phys. Lett. B237 (1990) 481;
G. Ecker, J. Gasser, H. Leutwyler, A. Pich and E. de Rafael, Phys. Lett. B223 (1989) 425.

[20] E.J. Ramberg et al. (E731-FNAL), Phys. Rev. Lett. 70 (1993) 2525.

[21] A.S. Carroll et al., Phys. Lett. 96B (1980) 407.