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The $\Delta^+ \to P + \gamma$ Transverse One-Half Helicity Transition Form Factor–An Algebraic Approach

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Abstract

The $\Delta^+ \to P + \gamma$ transverse one-half helicity transition form factor $h_3(q^2) \propto (G_M^*(q^2) - 3G_E^*(q^2))$ is examined using equal-time commutation relations (ETCRs) and the dynamical concept of asymptotic $SU_F(2)$ symmetry and realization. Utilizing as input only the well-known isovector part of the proton magnetic moment $G_M^V(q^2)$, a direct calculation of the closely related, tabulated $A_{1/2}$ $\Delta^+ P\gamma$ photon decay amplitude is made with results in good agreement with experiment. The ratio of the electric quadrupole moment to the magnetic dipole moment $(E_{1+}/M_{1+})_{q^2=0} \equiv$ electromagnetic ratio (EMR) is calculated as a function of $G_M^*(0)$. We also predict that $G_M^*(0) - 3G_E^*(0) \cong 2.97$ and confirm that $h_3(q^2)$ decreases more rapidly than the nucleon dipole form factor as a function of q^2.

1 INTRODUCTION

The main importance of the $\Delta^+ P\gamma$ transition form factors in elementary particle physics is to provide a basis for testing theories of effective quark forces and production models[1]. Specifically they are important in:

- QCD: One gluon exchange mechanism and tensor interactions.
- Skyrme[2] Models: Can some version be made to work or not?
- Enhanced Quark Models: They should be capable of predicting accurately the Δ–N transition form factors as a function of q^2.
- Electroproduction and Photoproduction: A good area where theoretical ideas may be tested.
- Symmetry Schemes: The Δ always plays an important role in models involving $SU(6)$, $U(6,6)$, *etc.*, and Melosh transformations.
- Dispersion relations: The Δ always plays an important role.
- Baryon Sum Rules: The Δ always plays an important role.

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2 The $\Delta^+ \to P + \gamma$ Transition Form Factors

In general[3] one may write for the $\Delta^+ \to N+\gamma$ transition amplitude the following expression:

$$\langle P(\vec{p},\lambda_P)|j_\mu(0)|\Delta^+(\vec{p}^*,\lambda_\Delta)\rangle = \frac{e}{(2\pi)^3}\sqrt{\frac{mm^*}{E_P E_\Delta}}\bar{u}_P(\vec{p},\lambda_P)[\Gamma_{\mu\beta}]u^\beta_\Delta(\vec{p}^*,\lambda_\Delta) \tag{1}$$

where

$$\begin{aligned}\Gamma_{\mu\beta} &= -ih_3\Delta^{-1}\gamma\cdot p^* q_\beta \epsilon_\mu(qp\gamma) \\ &+ h_2\Delta^{-1}[2\epsilon_{\beta\sigma}(p^*p)\epsilon_\mu{}^\sigma(p^*p)\gamma_5 - i\gamma\cdot p^* q_\beta\epsilon_\mu(qp\gamma)] \\ &+ h_1\Delta^{-1}q_\beta[p\cdot q q_\mu - q^2 p_\mu]\gamma_5 \,.\end{aligned} \tag{2}$$

Here, the electromagnetic current is denoted by j^μ, $\gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3$, $q \equiv (p^* - p)$, $\Delta^{-1} \equiv [((m^*+m)^2 - q^2)\,((m^*-m)^2 - q^2)]^{-1}$ is a kinematic factor which depends on q^2, m^* (the Δ^+ mass), and m (the nucleon mass); λ_P and λ_Δ are the helicities of the proton and Δ^+ respectively; and h_1, h_2, h_3 induce scalar, transverse $\frac{3}{2}$, and $\frac{1}{2}$ transitions respectively in the rest frame of the isobar (Δ^+). Also h_1, h_2, and h_3 are related to the more familiar form factors G^*_M, G^*_E, and G^*_C by the relations: $h_2 = -\frac{3(m^*+m)}{2m}(G^*_M + G^*_E)$, $h_3 = -\frac{3(m^*+m)}{2m}(G^*_M - 3G^*_E)$, and $h_1 = \frac{3(m^*+m)}{m}G^*_C$. G^*_M, G^*_E, and G^*_C induce magnetic, electric, and coulombic multipole transitions. For the process $P \to P+\gamma$, we have $\langle P(\vec{\tilde{p}},\lambda)|j_\mu(0)|P(\vec{\tilde{p}}^*,\lambda^*)\rangle = \frac{e}{(2\pi)^3}\sqrt{\frac{m^2}{E_{\tilde{p}}E_{\tilde{p}^*}}}\bar{u}_P(\vec{\tilde{p}},\lambda)[\Gamma_\mu]u_P(\vec{\tilde{p}}^*,\lambda^*)$, where $\Gamma_\mu = \left[1 - \frac{\tilde{q}^2}{4m^2}\right]^{-1}[\frac{i}{4m^2}G_M\epsilon_\mu(\tilde{P}\tilde{q}\gamma)\gamma_5 + \frac{1}{2m}G_E\tilde{P}_\mu]$, and $\tilde{P} \equiv \tilde{p} + \tilde{p}^*$. G_M and G_E are the familiar Sach's form factors.

3 The Tabulated $A_{\frac{1}{2}}$ and $A_{\frac{3}{2}}$ Helicity Amplitudes

The total Δ^+ radiative width $\equiv \Gamma^T_\gamma$, for decay into $P+\gamma$ is given by[4] :

$$\Gamma^T_\gamma = \frac{mq_c^2}{2m^*\pi}\sum_{\lambda=\frac{1}{2},\frac{3}{2}} A^2_\lambda \tag{3}$$

where[5,6,7,8,9]

$$A_{\frac{1}{2}} = -e\left(\frac{\sqrt{3}}{12}\right)\left[\frac{3}{2}\frac{m^{*2}-m^2}{m^3}\right]^{\frac{1}{2}}[G^*_M(0) - 3G^*_E(0)], \tag{4}$$

q_c = magnitude of the CM three-momentum, and

$$A_{\frac{3}{2}} = -e\left(\frac{1}{4}\right)\left[\frac{3}{2}\frac{m^{*2}-m^2}{m^3}\right]^{\frac{1}{2}}[G^*_M(0) + G^*_E(0)] \tag{5}$$

Experimentally[4] $A_{\frac{1}{2}} \cong (-141 \pm 5)\times 10^{-3}\,GeV^{-1/2}$ and $A_{\frac{3}{2}} \cong (-258 \pm 11)\times 10^{-3}\,GeV^{-1/2}$. In the naïve quark model, it can be shown that the quantity $(E_{1+}/M_{1+})_{q^2=0}$ =

$-G_E^*(0)/G_M^*(0)=$ ratio of the electric quadrupole moment to the magnetic dipole moment $\equiv EMR = 0$, whereas in the Skyrme model[2], the EMR is large and of the order of -5%. Experimentally[4], however, $EMR \approx (-1.3 \pm 0.5)\%$. Clearly, the viability of any particular theoretical model (including ours) to yield non-zero EMR values of the right sign and magnitude will be greatly enhanced by a successful calculation. As has been noted, the EMR ratio is especially effective for testing effective quark forces such as occur in QCD one-gluon exchange tensor interactions, various types of enhanced quark models, symmetry schemes such as $SU(6)$, $U(6,6)$, Melosh transformations, dispersion relations and sum rules (where the Δ always plays an important role). If one uses[10] the following commutation relations judiciously, namely $[A_{\pi^+}, A_{\pi^-}] = 2V_{\pi^0}$, $[A^\mu_{\pi^+}, A_{\pi^-}] = 2V^\mu_{\pi^0} = 2j^\mu_V$, $[A_{\pi^0}, A^\mu_{\pi^+}] = V^\mu_{\pi^+}$, then one may obtain[11] the following important relation:

$$\langle P, \Downarrow, \vec{k}|j_\mu|\Delta^+, \Uparrow, \vec{t}\rangle = \epsilon\langle P, \Downarrow, \vec{k}|j^V_\mu|P \Uparrow, \vec{t}\rangle \ , \tag{6}$$

where $|\vec{t}|$, $|\vec{k}| \to \infty$, $\epsilon =$ calculable, fixed, pure number, $\Uparrow \sim$ helicity $= +\frac{1}{2}$, $\Downarrow \sim$ helicity $= -\frac{1}{2}$, and j^V_μ is the vector part of the electromagnetic current. We also find two solutions[12] (up to a sign) for ϵ: $\epsilon = \frac{5}{8}\sqrt{2}$ or $\frac{2}{5}\sqrt{2}$. It turns out that $\epsilon = \frac{5}{8}\sqrt{2}$ yields good agreement with experiment when one uses the transverse ($\mu = 1,2$) components of the electromagnetic current.

From equation (6), we can extract $h_3'(q^2) \equiv -\frac{2m}{3(m+m^*)}h_3(q^2)$ as a function of the vector part of $G_M(\tilde{q}^2) \equiv G^V_M(\tilde{q}^2)$ by considering matrix elements of $\frac{1}{\sqrt{2}}(j_1 - ij_2)$ (corresponding to a helicity $+1$ photon moving in the $+z$ direction) and Lorentz transformations ("Z-boosts") G and $\tilde{G}$ such that $G : \{p^{*\mu} = (p^{*0}, 0, 0, k) \to p^{*'\mu} = (m^*, 0, 0, 0),\ p^\mu = (p^0, 0, 0, rk) \to p'^\mu = (\frac{r^2m^{*2}+m^2}{2rm^*}, 0, 0, \frac{r^2m^{*2}-m^2}{2rm^*})\}$ and $\tilde{G} : \{\tilde{p}^{*\mu} = (\tilde{p}^{*0}, 0, 0, k) \to \tilde{p}^{*'\mu} = (m, 0, 0, 0),\ \tilde{p}^\mu = (\tilde{p}^0, 0, 0, rk) \to \tilde{p}'^\mu = (\frac{m(r^2+1)}{2r}, 0, 0, -\frac{(1-r^2)m}{2r})\}$, where $k \to \infty$. We find that $q^2 = \frac{(1-r)}{r}(m^{*2}r - m^2)$, $\tilde{q}^2 = -\frac{(1-r)^2}{r}m^2$,and $0 < r \leq \frac{m}{m^*}$. When $r = r_0 = m^2/m^{*2}$, $q^2 = 0$ and $\tilde{q}^2 = \tilde{q}_0^2 \approx -0.2342\ GeV^2/c^2$, where the pole mass value $m^* = 1.211\ GeV/c^2$ was used. After some algebra, we obtain:

$$h_3'(q^2) \equiv G^*_M(q^2) - 3G^*_E(q^2) = \left[\frac{5\sqrt{3}m\sqrt{-\tilde{q}^2}}{3(m^*+m)\left[(m^*-m)^2 - q^2\right]^{1/2}}\right] G^V_M(\tilde{q}^2) \tag{7}$$

Using $G^V_M(\tilde{q}^2) \approx 2.35\left[1 - \frac{\tilde{q}_0^2}{.71}\right]^{-2}$, we calculate numerically that:

$$h_3'(0) \equiv G^*_M(0) - 3G^*_E(0) \approx 2.9718 \tag{8}$$

This then implies that:

$$A_{\frac{1}{2}} \approx -0.1340\, GeV^{-1/2}\ , \tag{9}$$

From equation (7), we can now calculate the EMR as a function of $G^*_M(0)$, and find that:

$$EMR \approx -\frac{1}{3}\left[1 - \frac{2.9718}{G^*_M(0)}\right] \tag{10}$$

If we take $G^*_M(0) \approx 3.0$, we obtain $EMR \approx -0.3\%$.

Note that while the EMR is a very sensitive function of $G^*_M(0)$ and the Δ^+ mass, $A_{\frac{1}{2}}$ is not. In addition, we point out that Eq.(7) predicts a faster than dipole fall-off behavior for $G^*_M(q^2)$ consistent with experimental data in the region where G^*_E is known to be small.

4 Summary

- The transverse 1/2 helicity form factor $h_3(q^2)$ or the tabulated $A_{\frac{1}{2}}$ photon decay amplitude can be calculated and is in good agreement with experiment. A *more rapid fall-off* in $h_3(q^2)$ (and G_M^* if G_E^* remains small compared to G_M^*) than the nucleon dipole form factor as a function of q^2 is confirmed;
- $G_M^*(0) - 3G_E^*(0) \cong 2.97 \Rightarrow G_M^*(0) \approx 3$ if $|G_E^*(0)/G_M^*(0)| \ll 0$;
- Our treatment is *completely relativistic.* Current conservation is guaranteed. Additionally, the correct transition operator is used in all calculations;
- Our treatment is also non-perturbative and performed in a broken symmetry hadronic world. We do not require the use of "mean" mass approximations. Physical masses are used at all times. Thus, G_E^* is not forced to equal zero as in the Skyrme model and in the naïve quark model;
- The EMR is calculated as a function of $G_M^*(0)$. Its value is very sensitive to the Δ mass;
- $A_{\frac{1}{2}}$ is computed in good agreement with experiment.

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