TOPICS IN HEAVY FLAVOR DECAYS AND GRAND UNIFIED THEORIES

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ABSTRACT

This work consists of two parts. The first part deals with the study of decays of heavy flavor baryons (baryons containing one heavy quark). The form factors describing the decay of a heavy flavor baryon to a light baryon are studied in the heavy quark limit. The results for the form factors are used to calculate nonleptonic and semileptonic decays of the heavy flavor baryons.

The second part of this work concerns the effects of Planck scale corrections (from higher dimensional gravity induced operators) on the low energy predictions of $SO(10)$ and the supersymmetric $SU(5)$ grand unified theories. In particular we show that the Planck scale corrections can allow a low Left-Right symmetry scale $M_R \approx TeV$ within a $SO(10)$ grand unified theory which is consistent with precision measurements of the Standard model gauge coupling constants at LEP. In supersymmetric $SU(5)$ we show that the effects of gravity induced dimension six operators can destroy the prediction of $\alpha_s(M_Z)$ unless the color triplet higgs mass is constrained to be below $7 \times 10^{16} GeV$.

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INTRODUCTION

Heavy Quark Effective theory and grand unified theories are two major areas of research in particle theory. The discovery of additional symmetries in systems containing a heavy quark has considerably simplified the study of such systems. One of the several important applications of the heavy quark effective theory (HQET) is the determination of CKM matrix elements with small theoretical error. HQET has been used extensively to study heavy mesons but not much work has been done with heavy baryons. In this work we study the decays of heavy flavor baryons in HQET. In chapter 1 we discuss the general features of the Standard model relevant to weak decays of hadrons and the heavy quark effective theory.

Following the precision measurement of the Standard model gauge coupling constants at LEP it has been observed that when the gauge couplings are evolved from their measured values at the Z mass then they appear to meet at a single point (indicating unification of gauge couplings) in SUSY $SU(5)$ grand unified theory . This has generated a lot of interest in the area of grand unified theories (GUT). The second part of this work deals with the effects of higher dimensional operators, induced by gravity, on the low energy predictions of grand unified theories. In chapter 5 we give an introduction to grand unified theories.

The first part of this thesis is concerned with the the study of heavy baryon (containing one heavy quark) nonleptonic and semileptonic decays using the idea of Heavy Quark Effective Theory. In chapter 2 a heavy quark analysis of the heavy to light transitions in baryons is carried out up to order $1/m_Q$ where m_Q is the heavy quark mass. In the infinite heavy quark mass limit only two form factors describe the transition of a heavy Lambda type baryon to another light baryon. A Lambda

type baryon is a system where the heavy quark determines the spin and isospin of the heavy baryon as the light degrees of freedom in the heavy baryon are in a spin zero and isospin zero state. When the $1/m_Q$ corrections to the form factors are considered several new matrix elements have to be evaluated. These matrix elements are fixed, with the help of some simplifying assumptions, from the data on nonleptonic decays of charmed baryons. The calculated form factors are then used to study the nonleptonic (chapter 3) as well as the semileptonic decays (chapter 4) of the charmed and the bottom baryons. Of particular relevance is the semileptonic decay of the Λ_c as the CLEO experiment has data on this decay which can be used to test our calculation against other existing calculations. The semileptonic decay of the Λ_b is also important as this decay offers an independent extraction of the CKM matrix element $|V_{ub}|$.

The second part of this thesis involves the investigation of gravity induced higher dimensional operators in grand unified theories(GUT). In modern GUT's the unification scale is higher, typically of the order $10^{16}GeV$ which is not very far from the effective Planck scale $2.4 \times 10^{18}GeV$ and it is likely that quantum gravity effects will effect GUT predictions at low energies. This work considers two problems; the first (described in chapter 6) studies the effects of higher dimensional operators in a SO(10) GUT Lagrangian breaking down to the Standard model through the Pati-Salam and the Left-Right symmetric group. It is shown that in the presence of higher dimensional operators it is possible to satisfy the LEP constraints on the measured couplings of the Standard model and still have a Left-Right breaking scale of $\sim TeV$. This fact has several important phenomenological implications. The second problem (chapter 7) deals with the the effects of higher dimension operators in SU(5) SUSY(supersymmetric) GUT and considers for the first time effects of dimension six and higher operators on the prediction of the strong coupling constant at the Z mass

scale. It is shown that the the effects of higher dimensional operators can spoil the low energy prediction of α_s in SUSY SU(5) GUT unless the heavy color triplet higgs has a mass $\leq 7 \times 10^{16}$ GeV.

CHAPTER 1

GENERAL CONSIDERATIONS

In this chapter we discuss some general ideas about weak decays of hadrons. The subject of low-energy semileptonic and nonleptonic interactions is fairly old with its experimental foundations already well established by the 1960's. Since the 1960s, very profound changes have occurred in our understanding of weak interactions. New ideas based on the principle of local gauge invariance provide a unified description of the weak and electromagnetic interactions and inspite of striking differences in their observed characteristics, both the interactions have been shown to be two different manifestations of a single, more fundamental, electroweak interaction. The principle of local gauge invariance has also been applied to the strong interactions resulting in the construction of quantum chromodynamics(QCD). QCD is closely related to the electroweak theory and it is hardly possible to separate them. This fact is clearly evident in weak decays of hadrons where the electroweak and the strong interactions are inextricably linked. QCD along with the electroweak theory constitute the the very successful standard model of high energy physics. In the next section we give a brief description of the standard model.

1.1 Electroweak($SU(2)_L \times U(1)$) interactions

According to this model developed by Glashow- Salam - Weinberg [1] the chiral quarks transform under the operation of weak isospin as doublets if left-handed

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L \tag{1.1}$$

and singlets if right-handed

$$u_R, d_R, c_R, s_R, b_R, t_R \tag{1.2}$$

There is mixing among the left-handed quarks,

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.3}$$

where the matrix V is unitary. In the Kobayashi-Maskawa parameterization one can write [2]

$$V = \begin{pmatrix} c_1 & s_1 c_3 & s_1 s_3 \\ -s_1 c_2 & c_1 c_2 c_3 + s_2 s_3 e^{i\delta} & c_1 c_2 s_3 - s_2 c_3 e^{i\delta} \\ -s_1 s_2 & c_1 s_2 c_3 - c_2 s_3 e^{i\delta} & c_1 s_2 s_3 + c_2 c_3 e^{i\delta} \end{pmatrix} \tag{1.4}$$

where $c_i = \cos\theta_i (i = 1, 2, 3)$, $s_i = \sin\theta_i (i = 1, 2, 3)$ and a non-zero value for δ signals the presence of CP violation. For quarks the charged weak current, the neutral weak current and the electromagnetic current are respectively,

$$J^c_\mu = (\,\bar{u} \quad \bar{c} \quad \bar{t}\,)\,\Gamma_{L\mu} V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.5}$$

$$J^N_\mu = (\,\bar{u} \quad \bar{c} \quad \bar{t}\,)\,[\frac{1}{2}\Gamma_{L\mu} - \frac{4}{3}\sin^2\theta_W \gamma_\mu] \begin{pmatrix} u \\ c \\ t \end{pmatrix} + (\,\bar{d} \quad \bar{s} \quad \bar{b}\,)\,[-\frac{1}{2}\Gamma_{L\mu} + \frac{2}{3}\sin^2\theta_W \gamma_\mu] \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.6}$$

$$J^{EM}_\mu = \frac{2}{3}(\,\bar{u} \quad \bar{c} \quad \bar{t}\,)\,\gamma_\mu \begin{pmatrix} u \\ c \\ t \end{pmatrix} - \frac{1}{3}(\,\bar{d} \quad \bar{s} \quad \bar{b}\,)\,\gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.7}$$

where $\Gamma_{L,R\mu} = \gamma_\mu(1 \mp \gamma_5)$. These currents couple locally to the $W^\pm$ and Z^0 fields and the photon field A as

$$L_{int} = e\left[\frac{W^{\mu\dagger} J^c_\mu + h.c.}{2\sqrt{2}\sin\theta_W} + \frac{Z^\mu J^N_\mu}{\sin 2\theta_W} + A^\mu J^{EM}_\mu\right] \tag{1.8}$$

where e is the electric charge, normalized in terms of the fine structure constant α as $\alpha = e^2/4\pi$. For momentum transfer $Q^2 << M^2_{W,Z}$ it is useful to work with effective nonleptonic interaction Lagrangian.

$$L_{NL} = \frac{G_F}{\sqrt{2}}\left[J^\mu_c J^{c\dagger}_\mu + 2\rho J^\mu_N J^N_\mu\right] \tag{1.9}$$

where

$$\frac{G_F}{\sqrt{2}} = \frac{\pi\alpha}{2M^2_W \sin^2\theta_W}, \rho = (\frac{M_W}{M_Z\cos\theta})^2\, v \tag{1.10}$$

The effective nonleptonic interaction is modified in several important respects by QCD radiative corrections which will be discussed in the next section. The origin of quark masses and KM mixing angles as well as the mechanism of the breaking of electroweak symmetry are issues of great interest and will be addressed in the second part of the thesis. In the next section we discuss strong interactions and its effects on nonleptonic decays.

1.2 Strong Interactions

The strong interactions are believed to be described by a $SU(3)$ gauge theory called QCD. The degree of freedom that is gauged to construct this theory is called color. Each quark, it is assumed, can exist in three different colored states. The quarks interact with each other by the exchange of colored gluons and the gluons, because they carry color also, can interact with each other. The QCD Lagrangian is given by

$$L_{QCD} = \bar{q}(i\not{D} - M)q - \frac{1}{4}F^A_{\mu\nu}F^{\mu\nu}_A \tag{1.11}$$

where q represents the quark fields u,d,s,c,b,t and M is the quark mass matrix. The gluon field tensor $F^A_{\mu\nu}$ is related to the gluon field G^A_μ by

$$F^A_{\mu\nu} = \partial_\mu G^A_\nu - \partial_\nu G^A_\mu + f_{ABC} G^B_\mu G^C_\nu \tag{1.12}$$

and the covariant derivative is defined by

$$\not{D}q = \gamma^\mu(\partial_\mu - ig\frac{\lambda_A}{2}G^A_\mu)q \tag{1.13}$$

The matrices $\lambda_A/2$ are the generators of the $SU(3)_c$ in the triplet representation with $Tr(\lambda_A\lambda_B) = 2\delta_{AB}$. The f_{ABC}'s are the structure constants of the gauge group and

g is the gauge coupling constant with an associated strong fine structure constant $\alpha_s = g^2/4\pi$. In the quantum field theory, the Lagrangian written above has to be supplemented by additional terms viz. the gauge fixing terms and the ghost terms which are needed to avoid divergence in the generating functional of the gauge theory as well as to maintain unitarity. Moreover, the bare parameters and the fields of the theory have no direct physical significance. In perturbation theory the Green's functions of these bare quantities diverge. It is necessary to renormalize the bare parameters and fields before they can be related to observable quantities. In an intermediate step the theory has to be regularized. The most common and convenient regularization scheme is dimensional regularization [3], in which the dimension of space time is continued to $D = 4 - 2\epsilon$ with $\epsilon \to 0$. The Green's functions of bare fields in the regularized theory are finite but diverge as $\epsilon \to 0$. From the fact that the action $S = \int d^D x L(x)$ is dimensionless, one can derive the mass dimensions of the fields and parameters of the theory. In particular the bare coupling constant g^{bare} is no longer dimensionless if $D \neq 4$;

$$dim[g^{bare}] = \frac{4-d}{2} = \epsilon \tag{1.14}$$

In a renormalizable theory it is possible to rewrite the Lagrangian in terms of renormalized quantities such that the Green's functions of the renormalized fields are finite as $\epsilon \to 0$. For QCD we can write the relations between the bare and renormalized quantities as

$$q^{bare} = Z_q^{\frac{1}{2}} q^{ren} \tag{1.15}$$

$$A^{bare} = Z_A^{\frac{1}{2}} A^{ren} \tag{1.16}$$

$$g^{bare} = Z_g^{\frac{1}{2}} g^{ren} \tag{1.17}$$

$$m_q^{bare} = m_q^{ren} - \delta m_q \tag{1.18}$$

For simplicity we have not considered the renormalization of the gauge parameter(associated with the gauge fixing term) and the ghost fields.

Prior to the development of QCD the electroweak interactions, $SU(2)_L \times U(1)$ (or the (2,1) theory) was used to derive the consequent traditional four fermion weak interaction Hamiltonian by contracting out all W propagators. Particular difficulties were encountered in the nonleptonic sector as the quarks involved were strongly interacting and this badly distorted what was expected from the pure (2,1) theory. With the establishment of QCD as a theory of strong interactions it was now possible to calculate the effects of strong interactions in weak decays. In 1974 Gaillard and Lee [4] and Altarelli and Maiani [5] laid down the basic rules for these calculations. The essential techniques (along with the use of asymptotic freedom of QCD) were the renormalization group and the operator insertion technique to calculate the anomalous dimensions of four quark operators. In their calculations only the heavy W gauge boson was decoupled. In 1975 Vainshtein, Zakharov and Shifman [6] showed that if one decouples the c quark as well as the W boson substantially new structures are observed to be generated in $\Delta S = 1$ effective Hamiltonian which were referred to as penguin operators. The four quark operators existing before QCD correction have a left-left(L-L) chiral structure, penguin operators are four quark operators with L-R chiral structure. The work of SVZ was carried further by Gilman and Wise [7] who developed a systematic and rigorous method to decouple the heavy quarks. In this section we will consider briefly the construction of QCD corrected weak effective Hamiltonians. This subject is covered in details in Ref [8].

The basic framework for calculating the effective Hamiltonian for the weak decays of hadrons containing u, d, s, c, b quarks is the effective field theory relevant for scales $\mu << M_W, M_Z$. The central idea of effective field theory is essentially one of approx-

imation. Suppose we have a Lagrangian L_1 which reproduces all the physics up to a certain energy scale. There is no guarantee that this Lagrangian will reproduce the physics at higher energies, which may be described by the Lagrangian L_2. We can say the L_2 is effectively L_1 at low energies or that L_1 describes a low energy effective theory. L_2 might loosely be called the complete theory (L_2 could be an effective theory itself of L_3 at even higher energies; it need not be the complete or ultimate theory and so on). Fields appearing in L_2 whose corresponding particles are not produced in the low energy regime need not appear in L_1 but the heavy fields can appear as virtual particles in low energy processes giving rise to higher dimensional operators (constructed out of the light fields) in the Lagrangian which are suppressed by the heavy particle masses.

Since L_1 is an approximation we can not use it to generate L_2 but given L_2 we can obtain the approximation L_1 and the process for doing that is called decoupling. That decoupling works beyond tree level is not an obvious fact and it was Appelquist and Carrazone [9] who first demonstrated this in their decoupling theorem. The gist of the decoupling theorem may be stated thus: In a renormalizable field theory the heavy fields decouple from the light field low energy sector. The effective Lagrangian is obtained by disposing of terms with explicit reference to the heavy fields and replacing them with an infinite sum of purely light field terms inversely proportional to the powers of the heavy masses in the theory. The unsuppressed light field terms also undergo finite renormalizations. One advantage of working with an effective theory L_1 at low energies instead of working with the full theory L_2 is the considerable simplification of the calculation of low energy processes. If we are using the Lagrangian L_2 then we have to consider extra Feynman diagrams for each and every process considered. Now during the calculation of graphs containing heavy fields the

approximation $M^2 >> |p_{external}|^2$ will have to be made continually. It soon becomes clear that it would be beneficial if this approximation could be made once and for all at the beginning of all calculations rather than during each and every calculation. In an effective theory there are fewer particles to deal with and fewer Feynman diagrams to consider and so the final calculations are significantly simplified.

The first step in the construction of the effective Hamiltonian is called matching. In this step the top quark and the weak gauge bosons are integrated out and the full theory is matched to an effective five quark theory. If we are interested in charm decays then we have to decouple the b quark also to construct an effective four quark theory and if we are interested in strange quark decays then we have to decouple the charmed quark also. For scales well below 1 GeV the quantitative description of strong dynamics in terms of quarks and gluons ceases to be useful and a description in terms of mesons and baryons having the symmetries of the QCD Lagrangian appear to be more appropriate. The various stages in the construction of effective theories is shown in Fig. 1.1.

In general we can write the effective Hamiltonian as a linear combination of operators O_i

$$H_{eff} = \sum C_i(m_{heavy})O_i \tag{1.19}$$

where m_{heavy} is the mass scale where the decoupling takes place. To calculate the Wilson coefficients C_i we make a calculation (Green's function) with the full Hamiltonian and the effective Hamiltonian. We then match the results of the two theories to calculate $C_i(m_{heavy})$. The C_i's are scale dependent but we know that if we calculate the matrix element of H_{eff}(which is a physical observable) the result should be scale independent. This implies that the scale dependence of the Wilson's coefficient $C_i(\mu)$ is such that it cancels the μ dependence coming from the matrix elements of

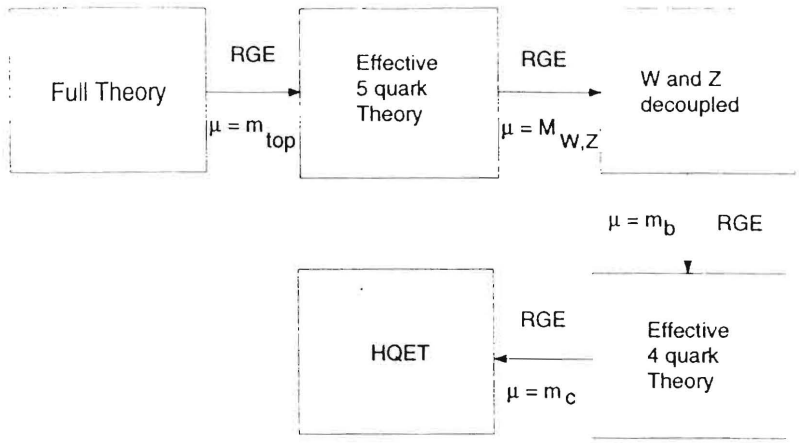

Figure 1.1: The various stages in the construction of effective theories.

the operators $< O_i(\mu) >$. This fact results in the $C_i(\mu)$'s satisfying a differential equation called the renormalization group equation(RGE), which depends on how the operators O_i renormalize i.e. on the anomalous dimensions of the operators O_i. This RGE can be used to relate $C_i(\mu)$ evaluated at different mass scales. In principle if we could write down the exact H_{eff} by performing calculations to all orders in perturbation theory then the matrix element of the effective Hamiltonian would be scale independent and it would not be necessary to worry about what scale to choose in our calculation. In real life, however, this is not possible and we always work with a truncated perturbation series. So if we are calculating the matrix element of the effective Hamiltonian with external state momentum $p_{ext} << \mu$ then there will be large logarithms $\ln(\frac{\mu^2}{p_{ext}^2})$ coming from higher order terms which we have neglected because we are working with a truncated series. To avoid this problem we start at

the high scale $\mu = m_{heavy}$. We calculate the $C_i(\mu)$'s by considering the matrix element(or Green's function) of H_{eff} with external state momentum $p_{ext} \gtrsim m_{heavy}$. This will prevent the appearance of large logarithms from higher order terms which have been omitted. Rigorously speaking the choice of the external momentum should be such that 1)$|p_{ext}|$ must be sufficiently small that terms of order $|p_{ext}|^2/m_{heavy}^2$ can be neglected. 2)$|p_{ext}|$ must be at the same time be sufficiently large such that $\ln(\frac{\mu^2}{p_{ext}^2})$ do not invalidate the perturbation theory. Next we use RGE to evolve the $C_i(\mu)$'s from the heavy scale $\mu = m_{heavy}$ to $\mu = m_{low}$ where m_{low} is the energy scale where the low energy processes we are interested in take place. The effective Hamiltonian with $C_i(\mu = m_{low})$ is the appropriate Hamiltonian to use when calculating matrix elements of external states with momentum $p_{ext} \sim m_{low}$. We do not have to worry about large logarithms any more because their effects have been taken into account(summed up) by the RGE analysis of the Wilson's coefficient $C_i(\mu)$.

To consider a specific example we will outline the process for writing the effective Hamiltonian for charmed decays. We start from the top mass scale ($m_{top} \sim 175$ GeV). For charged current interaction we can write the weak Hamiltonian as

$$H_{CC} = \frac{g_W}{\sqrt{2}}(\bar{U}^i\Gamma_L^\mu V D^i)W_\mu + h.c \tag{1.20}$$

where

$$U = \begin{pmatrix} u \\ c \\ t \end{pmatrix} \tag{1.21}$$

and

$$D = \begin{pmatrix} d \\ s \\ b \end{pmatrix} \tag{1.22}$$

where a is the flavor index and i is the color index and V is the CKM matrix. On

decoupling the top quark we get

$$\sum_{\alpha\beta} V_{\alpha\beta}(\bar{U}^i_\alpha \Gamma^\mu_L D^i_\beta)W_\mu \rightarrow \sum_{\alpha\beta} B_\alpha(m_{top})V_{\alpha\beta}(\bar{U}^i_\alpha \Gamma^\mu_L D^i_\beta)W_\mu \tag{1.23}$$

where the Wilson coefficient $B_\alpha(m_{top})$

$$B_\alpha(m_{top}) = (1 - \delta_{\alpha t}) \tag{1.24}$$

Now we have to use the RGE to evolve down to the scale of the weak gauge bosons for which we need the anomalous dimension of the operator in the effective Hamiltonian. Since the W does not couple to the gluons the anomalous dimension of the operator is just the anomalous dimension of the charged current γ_J which vanishes at the one loop level level in all gauges in MS($\bar{MS}$) scheme. This indicates that $B_\alpha(m_{top})$ is scale independent; its value remains constant from the top mass to the W mass scale. This is also true for all heavy flavors below the W mass threshold *i.e.* the effect of decoupling heavy flavor are of two loop order.

We next decouple the W boson by contracting the W propagator. The resulting Hamiltonian is the usual current × current Hamiltonian and so the structure of the effective Hamiltonian is of the form Eq. (1.19) where the O_i's are four quark operators. The RGE for the Wilsons coefficient is

$$[\mu\frac{d}{d\mu}\delta_{ij} - \gamma_{ji}]C_j = 0 \tag{1.25}$$

where

$$\gamma_{ik} = -Z^{-1}_{ij}\mu\frac{d}{d\mu}Z_{jk} \tag{1.26}$$

is the anomalous dimension matrix and the renormalization constants Z_{ij} are defined as

$$O^{bare}_i = Z_{ij}O^{renom}_j \tag{1.27}$$

To solve the differential equation for the Wilsons's coefficients we need to calculate the anomalous dimensions of the four quark operators. In general given a four quark operator it is found that on renormalization this operator generates a set of operators O_i. Furthermore it can be shown that four quark operators only generate four quark operators and therefore to calculate $\mathbf{Z} = [Z_{ij}]$ we calculate $G^{4\psi}_0(O_i)$ where

$$G^{4\psi}_0(O_i)(x_1, x_2, x_3, x_4, y) = < 0|T(\psi(x_1)\bar{\psi}(x_2)\psi(x_3)\bar{\psi}(x_4)O_i(y)|0 > \tag{1.28}$$

Working in the Landau gauge($Z_\psi = 1$) it is easily shown, in the minimal subtraction scheme,

$$G^{4\psi}_{0(1)}(O^0_i)_p = Z^2_{ij}G^{4\psi}_{(0)}(O_j) \tag{1.29}$$

where the subscripts (1) and p on the LHS of the equation refer to the one loop ϵ^{-1} term and the 0 subscript of the RHS side indicates it is the zero loop term. **Z** has been expanded perturbatively as

$$Z_{ij} = \delta_{ij} + Z^2_{ij} + O(g^4) \tag{1.30}$$

The various Feynman diagrams contributing to $G^{4\psi}_0(O_i)$ are shown in Fig. 1.2

From the diagram we see that the Green's function consists of 1PI(one particle irreducible diagrams) and 1PR (one particle reducible diagrams) which are the penguin diagrams. As a specific example we can consider the four quark operator responsible for Cabibbo favored charmed decays. The relevant four quark operator is

$$O_1 = (\bar{s}^i c^i)_L(\bar{u}^j d^j)_L \tag{1.31}$$

It can be shown that the renormalization of O_1 generates O_2 given by

$$O_2 = (\bar{s}^i c^j)_L(\bar{u}^i d^j)_L \tag{1.32}$$

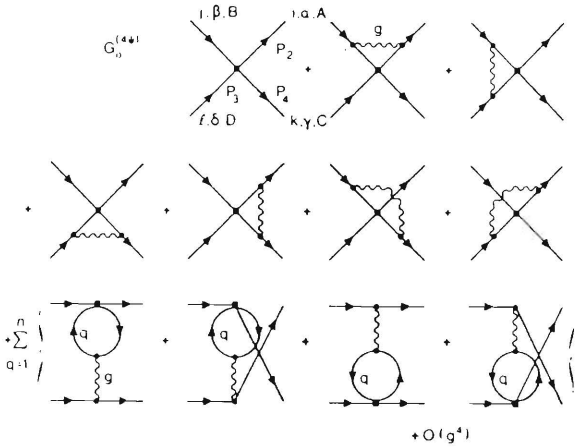

Figure 1.2: The expansion of $G_0^{4\psi}(O_i)$ in Feynman diagrams.

and the anomalous dimension matrix is given by

$$\gamma = \begin{pmatrix} -1 & 3 \\ 3 & -1 \end{pmatrix} \tag{1.33}$$

Finally we can now solve for the Wilson coefficient by solving the RGE. The solution at the one loop level can be written as

$$C_i(\mu) = V_{il}[\frac{\alpha(m)}{\alpha(\mu)}]^{\frac{\gamma_{0l}}{2\beta_0}}(V^{-1})_{lj}C_j(m) \tag{1.34}$$

where V is the matrix that diagonalizes γ and is defined as

$$V^{-1}\gamma V = \gamma^{diagonal} \tag{1.35}$$

and the running coupling constant α is given by

$$\alpha(\mu) = \frac{4\pi}{\beta_0 \ln(\mu^2/\Lambda_{QCD}^2)} \tag{1.36}$$

where

$$\beta_0 = (33 - 2n_f) \tag{1.37}$$

with n_f being the number of flavors in the effective theory.

After having constructed the effective Hamiltonian, the next step in the calculation of weak processes is the evaluation of the matrix elements of this Hamiltonian between hadronic states. These calculations are non-perturbative in nature and no known calculational tools are known to handle this problem. There are several methods available in the literature which can be used to perform this step: lattice gauge theory, $1/N$ (N = number of colors) approach, quark models, chiral perturbation theory, QCD sum rules and in the case of heavy (b and c) quarks the heavy quark effective theory (HQET). In the next section we will discuss HQET in some details as this technique has been used quite extensively in the work on heavy baryon decays.

1.3 Heavy Quark Effective Theory

The study of heavy quark system is simple compared to systems composed of light quarks because of a crucial property of QCD *viz.* asymptotic freedom. The strong coupling constant gets smaller at large momentum or small distances because the familiar screening of the charge is overcompensated by the antiscreening of the charge due to the self interaction of the colored gauge field, a phenomenon unique to non-abelian gauge theories. The domination of the antiscreening effect persists as long as the number of quark flavors does not exceed sixteen. The running of the strong coupling constant in QCD in the leading log approximation is given by by Eq. (1.36) where we set $\mu^2 = Q^2$, Q^2 being the typical momentum scale in the process. $\Lambda_{QCD} \sim$ 0.2 GeV is the the QCD scale and roughly represents the energy scale that separates

the region of large and small couplings. At large distances(small Q^2) the coupling gets strong, leading to nonperturbative phenomena such as confinement of quarks and gluons inside hadrons on a length scale $R_{Hadron} \sim 1/\Lambda_{QCD} \sim fm$. When the mass of the quark is very heavy i.e $m_Q >> \Lambda_{QCD}$ then Q is called a heavy quark. The quarks in the standard model fall in two groups; the u,d and s quarks are the light quarks while the c,b and t quarks fall in the heavy category. For heavy quarks $\alpha_s(m_Q)$ is small and so the strong interaction over the length scale of the Compton wavelength of the heavy quark $\Lambda_Q \sim 1/m_Q$ is perturbative. For a quarkonia system made of heavy quarks (e.g $Q\bar{Q}$), the typical length of which is of the order $\Lambda_Q/\alpha_s(m_Q) << R_{had}$, perturbative QCD is applicable and the spectra of such systems are very hydrogen like. In fact, after the discovery of asymptotic freedom the properties of the charmonium and bottomium system were predicted even before the discovery of these states.

For a system containing a heavy quark and a light quark the situation is a little more complicated. The size of such a system is determined by $1/\Lambda_{QCD}$ but the compton wavelength of the heavy quark is much smaller than the size of the hadron and this leads to simplification. The heavy quark in such a system is surrounded by a strongly interacting cloud of quarks and gluons and the only interaction this cloud (also called "brown muck") has with the heavy quark is through the emission of soft gluons with momentum much smaller that the heavy quark mass. This means that the light degrees of freedom cannot probe the quantum numbers of the heavy quark, for to do that would require a probe with momentum greater than or of the order of the heavy quark mass. The result of all this is that the light degrees of freedom in the hadron are blind to the mass and spin of the heavy quark and it can only see the long range color field produced by the heavy quark. In fact, in the rest frame of the heavy quark only the color electric field of the heavy quark is important as the color

magnetic field decouples when the mass of the heavy quark becomes large. In this heavy mass limit the dynamics of the light degrees of freedom are then completely determined by solving the QCD equations of motion in a static color field in the rest frame of the heavy quark (also the rest frame of the hadron) and the mass and flavor of the heavy quark plays no role at all. It therefore follows that hadronic systems, in the $m_Q \to \infty$ limit, (which differ only in the spin or flavor quantum number of the heavy quark) should have the same configuration for the light degrees of freedom. As a result even without being able to calculate the configuration of the light degrees of freedom it is possible to find relations between properties of such particles as the heavy mesons B, D, B* and D* or the heavy baryons Λ_b and Λ_c. Stated differently, in the limit that the mass of the heavy quark goes to infinity new symmetries, *viz.* a spin and a flavor symmetry arise in the effective theory of the strong interactions at low energies and this results, as observed by Isgur and Wise [10], in simple relations between the properties of heavy hadrons.

The heavy quark symmetry is an approximate symmetry, and corrections have to be computed because the quark mass is not infinite. These corrections can be written as a power series in Λ_{QCD}/m_Q. The condition $m_Q >> \Lambda_{QCD}$ is necessary and sufficient for the application of HQET and in that sense the situation is complementary to chiral symmetry which arises when the quark masses are much smaller than the QCD scale Λ_{QCD}. However it should be kept in mind that the heavy quark symmetry is not a symmetry of the full QCD Lagrangian but is a symmetry of an effective Lagrangian which is a good approximation to QCD in a certain kinematical regime, *viz.* where the heavy quark interacts with the light degrees of freedom through the exchange of soft gluons. In systems where the heavy quark symmetry is realized the heavy quark is almost on the mass shell with its momentum fluctuation around the mass

shell being of the order Λ_{QCD}. The corresponding change in the velocity of the heavy quark vanish in the infinite quark mass limit and so the velocity of the heavy quark is now no longer a dynamical degree of freedom but instead a conserved quantity. In the next section we briefly describe the construction of the effective theory in the infinite mass limit and the calculation of the $1/m_Q$ corrections.

1.4 The Effective HQET Lagrangian

We begin with a very simple derivation of the effective Lagrangian by looking at the Feynman rules for a heavy quark in QCD in the infinite mass limit. As already mentioned in the previous section, inside the hadron the heavy quark is almost on-shell and moving almost with the velocity of the hadron. It is interacting with the soft degrees of freedom surrounding it *via* the exchange of soft gluons carrying momentum $k \sim \Lambda_{QCD}$. We can therefore write the heavy quark momentum as

$$P_Q^\mu = m_Q v^\mu + k^\mu \tag{1.38}$$

Here v is the heavy quark or the heavy hadron velocity with $v^2 = 1$. It is quite straightforward to show that the heavy quark propagator can be expanded in $1/m_Q$ as

$$\frac{i}{\not{P}_Q - m_Q} = \frac{i}{v\cdot k}\frac{\not{v}+1}{2} + O(\frac{k}{m_Q}) \rightarrow \frac{i}{v\cdot k}P_+ \tag{1.39}$$

where the the positive energy projection operator $P_\pm$ is defined by

$$P_\pm = \frac{\not{v} \pm 1}{2} \tag{1.40}$$

has the following properties

$$P_\pm^2 = P_\pm \tag{1.41}$$

$$P_\pm P_\mp = 0 \tag{1.42}$$

$$P_+\gamma^\mu P_+ = P_+ v^\mu P_- \tag{1.43}$$

It is easy to see that the last property of the projection operator P_+ simplifies the QCD heavy quark gluon vertex

$$ig\gamma^\mu T^a \rightarrow igv^\mu T^a \tag{1.44}$$

where T^a are the generators of the color $SU(3)$ with the normalization $Tr(T^aT^b) = (1/2)\delta^{ab}$. In the figure below we show the Feynman graphs for the effective theory. One can easily reconstruct the Lagrangian from which the above Feynman rule follows

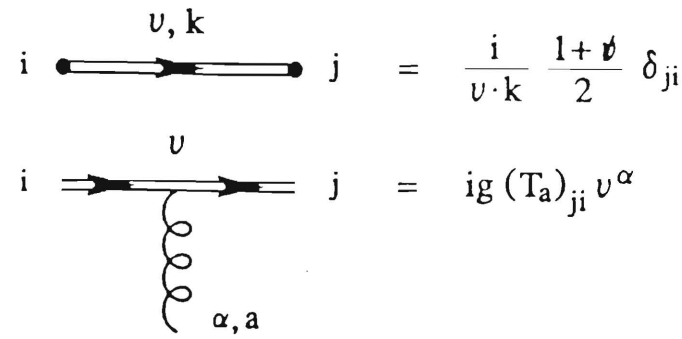

Figure 1.3: The Feynman rules for the propagator and the vertex in the heavy quark effective theory.

as

$$L_{eff} = \bar{h}_v iv\cdot Dh_v \tag{1.45}$$

where

$$D_\mu = \partial_\mu - igT^aA_{\mu a} \tag{1.46}$$

The effective quark field $h_v(x)$ is related to the actual quark field $Q(x)$ as

$$Q(x) \approx e^{-im_Q v\cdot x} h_v \tag{1.47}$$

Formally we can derive the HQET Lagrangian by introducing a large and a small component field h_v and H_v via the equations

$$h_v(x) = e^{im_Q v\cdot x} P_+ Q(x) \tag{1.48}$$

and

$$H_v(x) = e^{im_Q v\cdot x} P_- Q(x) \tag{1.49}$$

so that the full quark field $Q(x)$ can be written as

$$Q(x) = e^{-im_Q v\cdot x}[h_v + H_v] \tag{1.50}$$

It is easily verified that the large and the small components satisfy the following relations

$$\not{v} h_v = hv \tag{1.51}$$

$$\not{v} H_v = -H_v \tag{1.52}$$

In the rest frame of the heavy quark h_v corresponds to the upper two components of Q while H_v correspond to the lower two components. For an on-shell quark H_v is of course zero. In field theoretic language while h_v annihilates a heavy quark with velocity v, H_v creates a heavy anti-quark with velocity v. To describe a situation where the hadron contains a heavy anti-quark the sign of the momentum has to be changed and the small and the large components are switched. In terms of these new fields the QCD Lagrangian can be written as

$$L_Q = \bar{h}_v iv\cdot Dh_v - \bar{H}_v(iv\cdot D + 2m_Q)h_v + L_M \tag{1.53}$$

where

$$L_M = \bar{h}_v i\not{D}_p H_v + \bar{H}_v i\not{D}_p h_v \tag{1.54}$$

and

$$D_p^\mu = D^\mu - v^\mu v\cdot D \tag{1.55}$$

From an examination of the Lagrangian above we can interpret h_v as a massless degree of freedom, whereas the fluctuation H_v propagates with mass $2m_Q$. To obtain the effective Lagrangian we have to integrate out the heavy degrees of freedom H_v. This has been done by working with the generating functional for the QCD Green's function [11]. The same result can be derived at a classical level in the following manner. We first write down the equations of motion for the fields h_v and H_v that follow from the Lagrangian written above.

$$-iv\cdot Dh_v = i\not{D}_p H_v \tag{1.56}$$

$$(iv\cdot D + 2m_Q)h_v = i\not{D}_p h_v \tag{1.57}$$

One can now solve for H_v as

$$H_v = \frac{1}{(iv\cdot D + 2m_Q - i\epsilon)} i\not{D}_p h_v \tag{1.58}$$

This shows that the field H_v is of the order $1/m_Q$. Substituting the expression for H_v in the effective Lagrangian we can write an effective non local Lagrangian

$$L_{eff} = \bar{h}_v iv\cdot Dh_v + \bar{h}_v i\not{D}_p \frac{1}{(iv\cdot D + 2m_Q - i\epsilon)} i\not{D}_p h_v \tag{1.59}$$

Since the action of the D operator on the field h_v brings in a factor of k, the non local Lagrangian can be expanded in a power series in inverse powers of $1/m_Q$ and we can write the effective Lagrangian as

$$L_{eff} = \bar{h}_v iv\cdot Dh_v + 1/2m_Q \bar{h}_v (iD_p)^2 h_v + 1/2m_Q \bar{h}_v \sigma^{\alpha\beta} G_{\alpha\beta} h_v + O(1/(m_Q)^2) \tag{1.60}$$

The origin of the $1/2m_Q$ terms can be understood in terms of the diagram in Fig. 1.4 When the heavy degree of freedom in the figure is contracted we end up with an

Figure 1.4: The diagrams responsible for the origin of the $1/2m_Q$ terms in the effective theory. Time flows to the right and so the quark lines going in the negative direction of time represent the propagation of the H_v field.

effective interaction

$$L_1 = \bar{h}_v i\not{D}_p \frac{1}{2m_Q} i\not{D}_p h_v \tag{1.61}$$

It is straightforward to show that the term above generates the $1/2m_Q$ corrections in the Lagrangian. The first term in the Lagrangian represents the kinetic energy originating from the offshell residual motion of the heavy quark while the second term which goes as $\sim S \cdot B$ (where S is the spin operator and B is the color magnetic field) is just the hyperfine interaction. In the infinite mass limit it is now clear that the spin of the heavy quark decouples from the theory. The Lagrangian written above will be corrected by radiative corrections and we will consider this issue later. We next consider the problem of constructing operators, specifically currents,in the HQET at tree level. Consider the current $V^\mu = \bar{q}\gamma^\mu Q$. First, we write the full heavy quark field as (usinf Eqs. (1.52) and (1.60))

$$Q(x) = e^{-im_Q v\ cdotx}(1 + \frac{i\not{D}_p}{2m_Q} + ..)h_v(x) \tag{1.62}$$

and consequently,

$$V^\mu(x) = e^{-im_Q v\cdot x}\bar{q}\gamma^\mu(1 + \frac{i\not{D}_p}{2m_Q} + ..)h_v(x) \tag{1.63}$$

We can also expand the states in the full QCD in terms of the states in the HQET as

$$|M(v)\rangle = |M(v)\rangle_{HQET} + \frac{1}{2m_Q}\int dyT(L_1(y))|M(v)\rangle_{HQET} \tag{1.64}$$

and so finally we can write the matrix element

$$\begin{aligned}\langle A|V^\mu|M(v)\rangle &= \langle A|\bar{q}\gamma^\mu h_v|M(v)\rangle_{HQET} + \frac{1}{2m_Q}\langle A|\bar{q}\gamma^\mu i\not{D}_p h_v|M(v)\rangle_{HQET} \\ &+ \frac{1}{2m_Q}\int dyT(\bar{q}\gamma^\mu h_v(0)L_1(y)|M(v)\rangle_{HQET} + \ldots\end{aligned} \tag{1.65}$$

where $\langle A|$ represents any hadronic state. It is quite easy to continue this expansion to include higher orders in $1/m_Q$.

So far we have been dealing only at tree level and it is necessary to consider constructing operators in the effective theory which at the matching scale give a similar physical description obtained in terms of the operator of the full theory including loop effects.[12]. It can be shown that

$$\bar{q}\gamma^\mu Q = \sum_i C_i(\mu)J_i + \frac{1}{2m_Q}\sum_i B_i(\mu)O_i + O(1/m_Q^2) \tag{1.66}$$

where the equality sign in the above equation has the interpretation that the relation holds at the level of matrix elements. In HQET the operators J_i form a complete set of local dimension three current operators with the same quantum numbers as the vector current in the full theory. In HQET there are two such operators

$$J_1 = \bar{q}\gamma^\mu h_v \quad , \quad J_2 = \bar{q}v^\mu h_v \tag{1.67}$$

The set O_i also denotes a complete set of dimension four operators and they are listed in chapter 2.

1.5 Soft Pion Techniques

escribing some of the soft pion limit results which have been utilized in the study of the nonleptonic decays of the heavy baryons. We also describe the basis for q^2 dependence of the form factors that has been assumed in our analysis. Soft pion techniques involve combining PCAC and the algebra of the currents. In the limit that the pion momentum goes to zero it is possible to relate the weak process $\alpha \to \beta\pi$ to the simpler transition $\alpha \to \beta$ where α and β are two hadronic states. In the limit of vanishing quark masses (for u,d,s quarks) the QCD Lagrangian has a $SU(3)_L \times SU(3)_R$ symmetry which is spontaneously broken to the diagonal vector $SU(3)_V$. The pseudoscalar mesons arise as the goldstone bosons in the theory and they can acquire mass when the $SU(3)_L \times SU(3)_R$ is explicitly broken with a mass term for the quarks. It is possible to write down an effective Lagrangian consistent with the chiral symmetry of the theory. The Lagrangian can be written as an infinite series with the expansion parameter being the ratio of the momentum of the pseudoscalar meson to the chiral symmetry breaking scale ($\sim$ 1 GeV). In the soft meson limit we can truncate the series by including only a few terms and the PCAC relation can be derived from the resulting Lagrangian. If we define R as [13]

$$R_k = \langle \pi_k(q)\beta|H_W(0)|\alpha\rangle \tag{1.68}$$

where H_W is the weak Hamiltonian, then in the limit $q_\mu \to 0$ one can use the LSZ reduction technique to derive

$$\frac{f_\pi}{\sqrt{2}}R_k = -i\langle\beta|[Q_5^j(0), H_W(0)]|\alpha\rangle + iq_\mu M_k^\mu \tag{1.69}$$

where Q_5^j is the axial charge and the commutator in the equation above is easily calculated once the transformation property of H_W is known and

$$M_k^\mu = i\int d^4x e^{iq\cdot x}\theta(x_0)\langle p'|[j_k^\mu, H_W(0)]|p\rangle \tag{1.70}$$

One can calculate and show that $q_\mu M_k^\mu$ for $q_\mu \to 0$ is non zero when it receives contribution from intermediate states degenerate with the initial or the final states. The quantity $q_\mu M_k^\mu$ does not have a well defined limit in the soft meson and $SU(3)$ limit but the combination of this term and the pole term has an unambigous limit in the soft q limit. This issue will be taken up again when the nonleptonic decays of the heavy baryons are discussed in chapter 3.

The hadronic form factors are a crucial input in the calculation of the decay amplitude. A clear definition of these form factors can be found in the next chapter but here we briefly discuss the structure of the form factors in a qualitative way. Typically the form factors characterize matrix elements of the type $\langle\beta(p')|J_\mu|\alpha(p)\rangle$ where J_μ is some current. Assuming that the form factors in this matrix element are analytic in the whole q^2 plane ($q = p' - p$) except for cuts and poles on the real axis one can write a dispersion relation for the form factors of the form

$$F(t) = \frac{1}{\pi}\int \frac{ImF(t')}{t' - t} \tag{1.71}$$

where $t = q^2$. Now the imaginary part of the form factor is calculated by evaluating the absorptive part of $\langle\beta(p')|J_\mu|\alpha(p)\rangle$ and it can be shown [13] that this quantity goes as abs part $\sim \sum_n \langle\beta(p')|f_\alpha(0)|n\rangle\langle n|J_\mu|0\rangle\delta^4(p - p' + p_n)$ where $f_\alpha(0)$ is some operator constructed out of the field describing the hadron α. It is clear from this expression that only those states that have the quantum number of the current can contribute to the absorptive part. As a model for the form factors one can consider only one particle intermediate states in the sum above and then use the dispersion integral to calculate the form factors.

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CHAPTER 2

$1/m_Q$ CORRECTIONS TO FORM FACTORS

In this chapter I address the problem of calculating $1/m_Q$ corrections to the form factors that describe the transition of a heavy flavour baryon into a light baryon. For transitions of a charmed (bottom) baryon into an light baryon a total of six vector and axial vector form factors completely specify the hadronic part of the matrix element in semi-leptonic charmed(bottom) baryon decays and the factorizable part in the two body hadronic decays of charmed(bottom) baryons. There exist several calculations in the literature on charmed baryon decays and some on bottom baryon decays where these form factors are calculated in specific models [2]. However, it is interesting and useful to study these form factors in the limit that the charmed quark or the bottom quark is treated as heavy, because in this limit it is possible to systemetically expand the form factors in inverse powers of the heavy quark mass. The usefulness of such an expansion stems from the fact that the sources of the various coefficients in the power series expansion can be identified and one can either make reasonable estimates of these coefficients or fix them from experimental data. Not all the coefficients in the expansion are independent as the symmetries of the heavy quark effective theory can be used to find relations among the coefficients. In the heavy quark limit the spin symmetry of the heavy quark results in relations among the form factors. In particular, the spin symmetry is very effective in reducing the number of independent form factors for the lowest lying charmed(bottom) baryons because the light degrees of freedom are in a configuration of spin zero. For heavy to light transitions of the form $c(b) \to q$, where q is one of the light (u,d or s) quarks it is known that the six

form factors can be expressed in terms of only two independent form factors in the limit of $m_{c(b)} \to \infty$ [2]. In this chapter I estimate the $1/m_Q$ corrections to these form factors. In the next two chapters this result will be used to calculate the nonleptonic and semileptonic decays of the charmed and bottom baryons.

As already mentioned in the previous chapter in HQET the Q.C.D Lagrangian for the heavy quark is expanded in inverse powers of the heavy quark mass. In the limit $m_Q \to \infty$ the heavy quark field $Q(x)$ is replaced by $h_v(x)$ [3] :

$$h_v(x) = e^{im_Q v\cdot x} P_+ Q(x) \tag{2.1}$$

where $P_+ = \frac{1+\not{v}}{2}$ is the positive energy projection operator. The effective Lagrangian is written as

$$L_{HQET} = \bar{h}_v \, iv\cdot D \, h_v \tag{2.2}$$

where $D^\alpha = \partial^\alpha - ig_s t^a A^\alpha_a$ is the gauge covariant derivative. Corrections to the above Lagrangian come from higher dimensional operators suppressed by inverse powers of m_Q. Including $1/m_Q$ corrections we write [3, 4]

$$\begin{aligned} L &= L + \delta L/2m_Q \\ \delta L &= \bar{h}\,(iD)^2\, h + \frac{g_s}{2}\bar{h}\,\sigma_{\alpha\beta}G^{\alpha\beta}\, h \end{aligned} \tag{2.3}$$

where $G^{\alpha\beta} = [iD^\alpha, iD^\beta] = ig_s t^a G^{\alpha\beta}_a$ is the gluon field strength. The equation of motion for the heavy quark is

$$v\cdot Dh_v = 0 \tag{2.4}$$

One also has to expand the currents that mediate the weak decays of hadrons. In our case we are interested in currents of the form $\bar{q}\,\Gamma\,Q$. At the tree level the expansion of the current in the HQET gives

$$\bar{q}\,\Gamma\,Q \to \bar{q}\,\Gamma\,h + \frac{1}{2m_Q}\bar{q}\,\Gamma\,i\not{D}\,h + \cdots \tag{2.5}$$

where Γ is any arbitrary Dirac structure.

The vector and axial form factors in the weak decays of charmed baryons are conventionally parametrized in terms of 6 form factors, f_i and g_i, defined by (although here we are considering charmed baryons it is understood that the results are applicable to bottom baryons also)

$$\begin{aligned}
\langle B'(p',s') \mid \bar{q}\ \gamma^\mu\ Q \mid B_c(p,s)\rangle &= \bar{u}_{B'}(p',s')[f_1\gamma^\mu - i\frac{f_2}{m_{B_c}}\sigma^{\mu\nu}q_\nu \\
&+ \frac{f_3}{m_{B_c}}q^\mu]u_{Bc}(p,s) \\
\left\langle B'(p',s') \mid \bar{q}\ \gamma^\mu\gamma^5\ Q \mid B_c(p,s)\right\rangle &= \bar{u}_{B'}(p',s')[g_1\gamma^\mu - i\frac{g_2}{m_{B_c}}\sigma^{\mu\nu}q_\nu \\
&+ \frac{g_3}{m_{B_c}}q^\mu]\gamma^5 u_{Bc}(p,s) \qquad (2.6)
\end{aligned}$$

where $q^\mu = p^\mu - p'^\mu$ is the four momentum transfer. In our case it is convenient to write a different parametrization of form factors.

$$\langle B'(p',s') \mid \bar{q}\ \gamma^\mu\ Q \mid B_c(p,s)\rangle = \bar{u}_{B'}(p',s')\left[F_1\gamma^\mu + F_2 v^\mu + \frac{F_3}{m_{B'}}p'^\mu\right]u_{B_c}(p,s)$$

$$\begin{aligned}
\left\langle B'(p',s') \mid \bar{q}\ \gamma^\mu\gamma^5\ Q \mid B_c(p,s)\right\rangle &= \bar{u}_{B'}(p',s')[G_1\gamma^\mu + G_2 v^\mu \\
&+ \frac{G_3}{m_{B'}}p'^\mu]\gamma^5 u_{B_c}(v,s) \qquad (2.7)
\end{aligned}$$

where v^μ is the velocity of the heavy baryon. The two sets of form factors are related by

$$\begin{aligned}
f_1 &= F_1 + (m_{B_c} + m_{B'})\left[\frac{F_2}{2m_{B_c}} + \frac{F_3}{2m_{B'}}\right] \\
\frac{f_2}{m_{B_c}} &= -\frac{F_2}{2m_{B_c}} - \frac{F_3}{2m_{B'}} \\
\frac{f_3}{m_{B_c}} &= \frac{F_2}{2m_{B_c}} - \frac{F_3}{2m_{B'}} \\
g_1 &= G_1 - (m_{B_c} - m_{B'})\left[\frac{G_2}{2m_{B_c}} + \frac{G_3}{2m_{B'}}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{g_2}{m_{B_c}} &= -\frac{G_2}{2m_{B_c}} - \frac{G_3}{2m_{B'}} \\
\frac{g_3}{m_{B_c}} &= \frac{G_2}{2m_{B_c}} - \frac{G_3}{2m_{B'}} \qquad (2.8)
\end{aligned}$$

In the limit $m_Q \to \infty$, the spin symmetry allows one to write

$$\langle B' \mid \bar{q}\ \Gamma\ h_v \mid B_c\rangle = \bar{u}_{B'}\ [\theta_1 + \not{v}\ \theta_2]\,\Gamma\ U_{B_c} \qquad (2.9)$$

where θ_1, θ_2 are Lorentz-invariant and Γ is any arbitrary Dirac matrix, with

$$\not{v}U_{B_c}(v,s) = U_{B_c}(v,s) \qquad (2.10)$$

Note the normalization of U_{B_c}, the baryon spinor, is $\bar{U}_{B_c}U_{B_c} = 2M$ where M is the effective mass of the baryon in HQET and is given by

$$M = m_Q + \bar{\Lambda} \qquad (2.11)$$

The physical mass of the baryon is given by

$$m_{B_c} = M + O(1/m_Q) \qquad (2.12)$$

The spinors U_{B_c} are therefore related to the physical spinors u_{B_c} by

$$U_{B_c} = Z_M^{-1/2}u_{B_c} \qquad (2.13)$$

where

$$Z_M = \frac{m_{B_c}}{M} = 1 + O(1/m_Q^2) \qquad (2.14)$$

Since we are interested only in $1/m_Q$ corrections we can take $Z_M \approx 1$. Setting $\Gamma = \gamma^\mu, \gamma^\mu\gamma^5$ in Eq. (2.9) we get the following relations.

$$\begin{aligned}
F_1^0 &= \theta_1 - \theta_2 \\
F_2^0 &= 2\theta_2 \\
F_3^0 &= 0
\end{aligned}$$

$$
\begin{aligned}
G_1^0 &= \theta_1 + \theta_2 \\
G_2^0 &= 2\theta_2 \\
G_3^0 &= 0
\end{aligned} \tag{2.15}
$$

where $F^0{}_i, G_i^0$ are the zeroth order form factors in the $1/m_Q$ expansion. Eq. (2.15) lead to the following relations between the form factors [2].

$$
G_1^0 = F_1^0 + F_2^0 \quad ; \quad G_2^0 = F_2^0 \quad ; \quad G_3^0 = 0 \quad ; \quad F_3^0 = 0 \tag{2.16}
$$

or equivalently,

$$
g_1 = f_1 \quad ; \quad g_2 = f_2 \quad ; \quad g_3 = -f_2 \quad ; \quad f_3 = -f_2 \tag{2.17}
$$

We next turn to $1/m_c$ corrections to these relations. As we noted in chapter 1, the two sources of $1/m_c$ corrections are from the current and the effective Lagrangian. The first source of $1/m_c$ corrections comes from the expansion of the currents in powers of $1/m_c$. The matrix element of interest here is

$$
\langle B' \mid \bar{q}\, \Gamma\, i\not{D}\, h \mid B_c \rangle = \bar{u}_{B'} \phi_\nu \Gamma \gamma^\nu U_{B_c} \tag{2.18}
$$

where the form of the R.H.S of the above equation follow from spin symmetry. The general form of ϕ_ν is

$$
\phi_\nu = (\phi_{11} v_\nu + \phi_{12} v'_\nu + \phi_{13}\gamma_\nu) + \not{v}(\phi_{21} v_\nu + \phi_{22} v'_\nu + \phi_{23}\gamma_\nu) \tag{2.19}
$$

Using the equation of motion for the heavy quark we have the following conditions.

$$
\begin{aligned}
\langle B' \mid \bar{q}\, iv \cdot D\, h \mid B_c \rangle &= \bar{u}_{B'} v \cdot \phi U_{B_c} = 0 \\
\left\langle B' \mid \bar{q}\, \gamma^5\, iv \cdot D\, h \mid B_c \right\rangle &= \bar{u}_{B'} v \cdot \phi \gamma^5 U_{B_c} = 0
\end{aligned} \tag{2.20}
$$

which results in

$$
\begin{aligned}
\phi_{11} + \omega\, \phi_{12} &= -\phi_{23} = x \\
\phi_{21} + \omega \phi_{22} &= -\phi_{13} = y
\end{aligned} \tag{2.21}
$$

where $\omega = v \cdot v'$, v' being the velocity of the baryon B'. Note x and y are defined through Eq. (2.21). The corrections to the form factors due to the corrections to the current can now be easily written down from Eq. (2.19).

$$
\begin{aligned}
\delta F_1 &= \frac{\left[\left(\frac{\omega+1}{\omega}\right)(\phi_{11} + \phi_{21}) - \frac{y}{\omega} - \left(\frac{2\omega+1}{\omega}\right) x\right]}{2m_c} \\
\delta F_2 &= \frac{2\left[\phi_{21}\left(\frac{\omega-1}{\omega}\right) + 2x + \frac{y}{\omega}\right]}{2m_c} \\
\delta F_3 &= \frac{2\left[-\frac{\phi_{11}}{\omega} - \frac{\phi_{21}}{\omega} + \frac{(x+y)}{\omega}\right]}{2m_c} \\
\delta G_1 &= \frac{\left[\left(\frac{1-\omega}{\omega}\right)(\phi_{11} - \phi_{21}) + \frac{2\omega-1}{\omega} x + \frac{y}{\omega}\right]}{2m_c} \\
\delta G_2 &= \frac{2\left[-\phi_{21}\left(\frac{\omega+1}{\omega}\right) + 2x + \frac{y}{\omega}\right]}{2m_c} \\
\delta G_3 &= \frac{2\left[-\frac{\phi_{11}}{\omega} + \frac{\phi_{21}}{\omega} + \frac{(x-y)}{\omega}\right]}{2m_c}
\end{aligned} \tag{2.22}
$$

Using the equation of motion one can derive two relations between the ϕ's. We start with

$$
\begin{aligned}
\langle B' \mid i\partial_\mu(\bar{q}\, \Gamma\, h_v) \mid B_c \rangle &= \left\langle B' \mid [i(\partial_\mu + ig_s t_a A^a_\mu)\bar{q}]\, \Gamma\, h_v + \bar{q}\, \Gamma\, [i(\partial_\mu - ig_s t_a A^a_\mu) h_v] \mid B_c \right\rangle \\
&= \left[(M - m_Q) v_\mu - m_{B'} v'_\mu\right] \langle B' \mid \bar{q}\, \Gamma\, h_v \mid B_c \rangle
\end{aligned} \tag{2.23}
$$

where M is the heavy baryon mass in the $HQET$ limit. We can rewrite the above identity as

$$
\begin{aligned}
\left\langle B' \mid (iD^*_\mu q)\, \Gamma\, h_v + \bar{q}\, \Gamma\, (iD_\mu h_v) \mid B_c \right\rangle &= \left(\bar{\Lambda} v_\mu - m_{B'} v'_\mu\right) \\
&\quad \langle B' \mid \bar{q}\, \Gamma\, h_v \mid B_c \rangle
\end{aligned} \tag{2.24}
$$

where $\bar{\Lambda} = M - m_c$. With $\Gamma = \gamma^\mu \hat{\Gamma}$ and using the equation of motion for the light quark q one can write

$$
\begin{aligned}
\left\langle B' \mid \bar{q}\, \gamma^\mu \hat{\Gamma}\, iD_\mu\, h_v \mid B_c \right\rangle &= [\bar{\Lambda} v_\mu - (M_{B'} - m_q) v'_\mu] \\
&\quad \left\langle B' \mid \bar{q}\, \gamma^\mu \hat{\Gamma}\, h_v \mid B_c \right\rangle
\end{aligned} \tag{2.25}
$$

With $\hat{\Gamma}= 1$ one obtains,

$$(\phi_{11} - \phi_{21})\frac{\omega - 1}{\omega} - \left(\frac{2\omega + 1}{\omega}\right) y - \left(\frac{4\omega - 1}{\omega}\right) x$$
$$= F_1^0(\bar{\Lambda} - \hat{m}_{B'}) + F_2^0(\bar{\Lambda} - \hat{m}_{B'}\,\omega) \tag{2.26}$$

where F_1^0, F_2^0 are the vector form factors in the $m_c \to \infty$ limit and $\hat{m}_B = m_{B'} - m_q$, while with $\hat{\Gamma} = \gamma_5$ one obtains,

$$(\phi_{11} + \phi_{21})\frac{\omega + 1}{\omega} + \left(\frac{4\omega - 1}{\omega}\right) x + \left(\frac{2\omega - 1}{\omega}\right) y$$
$$= G_1^0(\bar{\Lambda} + \hat{m}_{B'}) - G_2^0\left(\bar{\Lambda} - \hat{m}_{B'}\,\omega\right) \tag{2.27}$$

where G_1^0, G_2^0 are the axial vector form factors in the $m_c \to \infty$ limit. One can now solve for ϕ_{11}, ϕ_{21} in terms of $F_1^0, F_2^0, G_1^0, G_2^0, x$ and y

We note that at $\omega = 1$

$$y = -\frac{G_1^0(\omega = 1)(\bar{\Lambda} - \hat{m'}_B)}{3} - x \tag{2.28}$$

We see therefore that at $\omega = 1$ the corrections to the form factor can be expressed in terms of the zeroth order form factors and x. If we further assume that $x \sim y$ with both x and y being small in the spirit the of $1/m_Q$ expansion, then

$$x \sim -\frac{G_1^0(\bar{\Lambda} - \hat{m}_{B'})}{6} \tag{2.29}$$

so all the corrections to the form factors can be expressed, at $\omega = 1$, in terms of the zeroth order form factors. (Note that for heavy-heavy transition $\bar{\Lambda} \simeq \hat{m}_{B'}$ up to order $1/m_Q$ and so $x \simeq 0$ which is what is expected.)

The second source of $1/m_Q$ corrections comes from $\delta L/2m_Q$ and one has to calculate

$$\frac{1}{2m_Q}\langle B' \mid i\int dy\; T\{\bar{q}\,\Gamma\, h_v(0),\; \delta L(y)\} \mid B_c\rangle >_{HQET}$$

Using spin symmetry we can write the corrections from the kinetic energy operator as

$$\frac{1}{2m_Q}\left\langle B' \mid i\int dy\; T\left\{\bar{q}\,\Gamma\, h_v(0),\; \bar{h}\,(iD)^2\, h(y)\right\} \mid B_c\right\rangle_{HQET} \tag{2.30}$$
$$= \frac{A(\omega)}{2m_Q}\langle B' \mid \bar{q}\,\Gamma\, h_v(0) \mid B_c\rangle \tag{2.31}$$

Clearly this correction only renormalizes the zeroth order form factors. We also note that we can replace the zeroth order form factors in Eqs. (2.22) and (2.23) by the renormalized form factors because the corrections are of the order $1/(2m_Q)^2$ which we neglect.

The corrections from the chromomagnetic operator can be written as, using spin symmetry,

$$\frac{1}{2m_Q}\left\langle B' \mid i\int dy\; T\left\{\bar{q}\,\Gamma\, h_v(0), \frac{g_s}{2}\,\bar{h}\,\sigma_{\alpha\beta}G^{\alpha\beta}\, h(y)\right\} \mid B_c\right\rangle_{HQET}$$
$$= \bar{u}_{B'}\,\chi_{\alpha\beta}\,\Gamma\left(\frac{1+\not{v}}{2}\right) s^{\alpha\beta}\, U_{B_c} \quad \text{where} \tag{2.32}$$

$$\chi_{\alpha\beta} = (\chi_{11} + \not{v}\,\chi_{12})\frac{(\gamma_\alpha\gamma_\beta - \gamma_\beta\gamma_\alpha)}{4} + (\chi_{21} + \not{v}\,\chi_{22})\frac{(\gamma_\alpha v'_\beta - \gamma_\beta v'_\alpha)}{4}$$
$$\text{and} \quad s^{\alpha\beta} = \frac{\gamma^\alpha\gamma^\beta - \gamma^\beta\gamma^\alpha}{4}$$

The corrections from the chromomagnetic operator to the form factors are then

$$\Delta F_1 = \frac{[(\chi_{11} - \chi_{12}]}{2}$$
$$\Delta F_2 = \left[-2\chi_{11} - \chi_{12} + \frac{\chi_{21}}{2} + (2\omega - 1)\frac{\chi_{22}}{2}\right]$$
$$\Delta F_3 = \frac{[-(\chi_{21} + \chi_{22})]}{2}$$
$$\Delta G_1 = \frac{[(\chi_{11} + \chi_{12})]}{2}$$

$$\Delta G_2 = \left[2\chi_{11} - \chi_{12} - \frac{\chi_{21}}{2} - (2\omega+1)\frac{\chi_{22}}{2}\right]$$
$$\Delta G_3 = \frac{[(\chi_{22} - \chi_{21})]}{2}$$

Therefore we see that for corrections from the chromagnetic term there are four new matrix elements that have to be calculated to estimate the $1/m_Q$ corrections.

It is straightforward to include radiative corrections in our analysis. For instance the relations between form factors in the $m_Q \to \infty$ limit get modified in the presence of radiative corrections to [5]

$$\begin{aligned} G_1 &= \left[\frac{C_1(\mu)}{C_1(\mu)+C_2(\mu)}\right][F_1+F_2] \\ G_2 &= \frac{C_1(\mu)}{C_1(\mu)+C_2(\mu)}F_2 - \frac{C_2(\mu)}{C_1(\mu)}F_1\left[1+\frac{C_1(\mu)}{C_1(\mu)+C_2(\mu)}\right] \\ G_3 &= F_3 = 0 \end{aligned} \tag{2.33}$$

where $C_1(\mu), C_2(\mu)$ are the Wilson's co-efficients that occur in the short-distance expansion of the currents. At the $1/m_Q$ level there are extra operators that arise namely [5, 6]

$$\begin{aligned} O_1 &= \bar{q}\, v^\mu\, i\not{D}\, h_v \quad ; \quad O_2 = \bar{q}\, iD^\mu\, h_v \quad ; \quad O_3 = \bar{q}\,(-iv\cdot \overleftarrow{D})\,\gamma^\mu\, h_v \\ O_4 &= \bar{q}\,(-iv\cdot \overleftarrow{D})\, v^\mu\, h_v \quad ; \quad O_5 = \bar{q}\,(-i\,\overleftarrow{D}^\mu)\, h_v \\ O_6 &= m_q\, \bar{q}\,\gamma^\mu\, h_v \quad ; \quad O_7 = m_q\, \bar{q}\, v^\mu\, h_v \end{aligned} \tag{2.34}$$

and the chromomagnetic operator also gets renormalised. It is straightforward to calculate the matrix elements of operators $O_1 - O_5$ using equations (2.15) and (2.20). The matrix elements of the operators O_6 and O_7 will just renormalize the zeroth order form factors. Radiative corrections are typically $\sim 5\%$ and we neglect these corrections in our calculations.

To use the above results in phenomenology one has to make estimates of the six new quantities x, y, χ_{11}, χ_{12}, χ_{21}, and χ_{22}. These quantities are uncalculable in

HQET and one has to use other techniques (QCD sum rules for example) to calculate them. In this work we do not address the problem of calculating these quantities but we try to get some estimates of the quantities using reasonable assumptions.

Starting with the corrections coming from the expansion of the current, a reasonable assumption is the one made in Eq. (2.29). Even for ω near 1 Eq. (2.29) can be used to estimate x provided x is a slowly varying function of ω.

For ω near 1 and making the approximation $x(\omega) \sim y(\omega) \sim x(\omega=1) \sim y(\omega=1)$ equations (2.26) and (2.27) reduce to

$$\begin{aligned} (\phi_{11} - \phi_{21})\frac{\omega-1}{\omega} &= F_1^0(\bar{\Lambda} - \hat{m}_{B'}) + F_2^0(\bar{\Lambda} - \hat{m}_{B'}\,\omega) - G_1^0(\bar{\Lambda} - \hat{m}_{B'}) \\ &= F_2^0(1-\omega)\,\hat{m}_{B'} \\ (\phi_{11} + \phi_{21})\frac{\omega+1}{\omega} &= G_1^0(\bar{\Lambda} + \hat{m}_{B'}) - G_2^0(\bar{\Lambda} - \hat{m}_{B'}\,\omega) \\ &+ \left[1 - \frac{1}{3\omega}\right] G_1^0(\bar{\Lambda} - \hat{m}_{B'}) \end{aligned}$$

For the factorized two body hadronic decays of charmed baryons taking, for example, $\Lambda_c \to \Lambda\pi$ and $\Lambda_c \to \Lambda\rho$ we find $\omega \sim 1.25$ and $\omega \sim 1.12$ respectively and so the above approximation is more reliable for nonleptonic decays involving a vector boson in the final state. We see that as far as corrections from expansion of currents are concerned the largest correction is $\sim G_1^0(\bar{\Lambda} + \hat{m}_{B'})/2m_c \sim 0.6G_1^0$ which is quite significant.

For the corrections from the chromomagnetic operator even at $\omega = 1$ there are four unknown functions $\chi_{11}, \chi_{12}, \chi_{21}$ and χ_{22}. We do not have estimates of these functions but the contribution of chromomagnetic operators have been calculated to be small in the case of mesons [9]. In the case of heavy-heavy transitions in baryons the matrix element of the chromomagnetic operator vanishes to first order in $1/m_Q$ [10]. In the case of heavy to heavy transitions at $\omega = 1$ there is a normalization condition which comes from the conservation of the flavour conserving vector current in the limit of

equal hadron masses [7, 8]. This absolute normalization relation involving the form factors can be used to find relations among matrix elements that arise as corrections to the form factors in the $1/m_Q$ expansion at $\omega = 1$. In the case of heavy-light transitions, however, we do not have such a normalization condition. There have been attempts to use HQET in $c \to s$ transitions treating both c and s quark as heavy [11, 12]. The basic assumption involved in such an analysis is the following ; in HQET on the scale of the heavy quark mass the light degrees of freedom have small momentum spread about their central equal velocity value. For strange baryons or mesons this is not true. In the limit of equal hadron masses we would then have the normalization condition at $\omega = 1$:

$$F_1 + F_2 + F_3 = 1 \quad \text{which implies} \tag{2.35}$$

$$F_1^0 + F_2^0 = 1. \tag{2.36}$$

We therefore get the condition that

$$\delta F_1 + \delta F_2 + \delta F_3 = 0 \quad \text{in the limit of equal hadron masses} \tag{2.37}$$

In this work we do not assume the validity of a $1/m_s$ expansion, but we make the assumption that that Eq. (2.37) is valid up to the order we are working in, even for unequal hadron masses or at most the R.H.S of Eq. (2.37) $\sim x/2m_c$ for unequal hadron masses. This is indeed the case in heavy to heavy transitons where, for example, both Eqs. (2.36) and (2.37) are true for $\Lambda_b \to \Lambda_c$ up to $1/m_Q^2$ for unequal hadron masses and is a consequence of Luke's theorem [7]. In our case it is unnecessary to use Eq. (2.36) to estimate the chromomagnetic corrections. Further making the assumption $\chi_{11} \sim \chi_{12} \sim \chi_{21} \sim \chi_{22}$ one obtains using $\delta F_1 + \delta F_2 + \delta F_3 = 0$

$$\chi_{11} = \chi_{12} = \chi_{21} = \chi_{22} = \frac{x}{m_c}$$

where we have set all the χ's equal to get an estimate of the chromomagnetic corrections. Then the corrections to the form factors at $\omega = 1$ can be written as

$$\begin{array}{ll} \Delta F_1 = 0 & \Delta G_1 = \frac{x}{m_c} \\ \Delta F_2 = -\frac{2x}{m_c} & \Delta G_2 = -\frac{x}{m_c} \\ \Delta F_3 = \frac{x}{m_c} & \Delta G_3 = 0 \end{array} \tag{2.38}$$

We see that the corrections from the chromomagnetic operator are rather small since $x/2m_c \sim$ few percent of G_1^0. Note that if we had used $\delta F_1 + \delta F_2 + \delta F_3 \sim x/2m_c$ the order of the corrections to the form factors would still be same as in the above equation. Therefore we have been able to express the $1/m_c$ corrections to the form factors at and near $\omega = 1$ in terms of the zeroth order form factors ${F_1}^0$ and ${F_2}^0$. Away from the zero recoil point, $\omega = 1$ or $q^2 = q^2_{max}$, one may chose a dipole form for the q^2 dependence and write a generic form factor F as [12]

$$F(q^2) = \frac{F(q^2_{max})}{(1 - \frac{q^2}{{m_{FF}}^2})^2}(1 - \frac{q^2_{max}}{{m_{FF}}^2})^2 \tag{2.39}$$

where m_{FF} is the appropriate pole mass. We see therefore that we can study heavy to light transitions in charmed baryons in terms of only two form factors evaluated at the zero recoil point. Note that in the limit $m_c \to \infty$ there are also two independent form factors. However because of $1/m_c$ corrections the relations in Eqs. (2.16) and(2.17) are no longer valid. As an application of our results we can study the various asymmetries in exclusive semi-leptonic decays of Λ_c. The asymmetries are expressed in terms of helicity amplitudes which in turn can be expressed in terms of the form factors [12]. Since all the form factors are expressible, including $1/m_c$ corrections, in terms of F_1^0 and F_2^0, all the asymmetries can be expressed as functions of F_2^0/F_1^0. We can write the form factors in Eq. (2.6) as

$$\begin{aligned} \frac{f_1}{F_1^0} &= 1 + a + (m_{B_c} + m_{B'})\left[\frac{r + b/3}{2m_{B_c}} - \frac{a + b/3}{2m_{B'}}\right] \\ \frac{f_2}{F_1^0 m_{B_c}} &= -\frac{r + b/3}{2m_{B_c}} + \frac{a + b/3}{2m'_B} \end{aligned}$$

$$
\begin{aligned}
\frac{f_3}{F_1^0 m_{B_c}} &= \frac{r+b/3}{2m_{B_c}} + \frac{a+b/3}{2m'_B} \\
\frac{g_1}{F_1^0} &= 1 + r + \frac{2b}{3} - (m_{B_c} - m'_B)\left[\frac{r-a-\rho b/3}{2m_{B_c}} + \frac{\rho b/3}{2m'_B}\right] \\
\frac{g_2}{F_1^0 m_{B_c}} &= -\frac{r-a-\rho b/3}{2m_{B_c}} - \frac{\rho b/3}{2m'_B} \\
\frac{g_3}{F_1^0 m_{B_c}} &= \frac{r-a-\rho b/3}{2m_{B_c}} - \frac{\rho b/3}{2m'_B}
\end{aligned}
\tag{2.40}
$$

where

$$
z_1 = (\bar{\Lambda} + \hat{m}_{B'}) \quad ; \quad z_2 = (\bar{\Lambda} - \hat{m}_{B'}) \quad ; \quad r = F_2^0/F_1^0 \quad ; \quad \rho = -\frac{6r}{1+r}\frac{\hat{m}_{B'}}{z_2}
$$

$$
a = \frac{(z_1 + \frac{4}{3}z_2) + r(z_1 + \frac{1}{3}z_2)}{2m_c} \quad ; \quad b = -\frac{(1+r)}{2m_c}z_2 \tag{2.41}
$$

In the above equation we have included corrections from the chromomagnetic operator although the maximum correction from this source is $2b/3 \sim -0.04G_1^0$ and so may be neglected

A fit to the data on the semi-leptonic decay of Λ_c performed by taking into account the $1/m_c$ corrections would result in a value for the ratio F_2^0/F_1^0. To calculate the absolute decay rates one needs F_1^0 and F_2^0 separately. This may be fixed from the measurement of absolute decay rates in semileptonic or hadronic decays of the charmed baryons, though for hadronic decays one has to contend with extra theoretical uncertainties. These issues will be discussed in the next two chapters.

Summarizing, in this chapter we have calculated the $1/m_Q$ corrections to the weak hadronic form factors which characterize charmed and bottom baryon decays. Using certain assumptions about matrix elements we can estimate these $1/m_Q$ corrections at or near $\omega \sim 1$ in terms of two form factors F_1^0, F_2^0. With a dipole form for the form factors one can extrapolate these form factors to arbitrary q^2. One can now use these results to study the weak decays of charmed baryons involving the transition of a heavy quark into a light quark. In the next chapter it will be shown that some of

the assumptions made in this chapter about the matrix elements that represent the $1/m_Q$ corrections have to be relaxed to be consistent with experimental data.

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CHAPTER 3

NONLEPTONIC DECAYS OF HEAVY FLAVOR BARYONS

3.1 Introduction

There is now a fair amount of experimental data available on charmed baryon decays while more data on bottom baryon decays will be available in the future. There are already several calculations of these decays in the literature. A crucial input in the calculation of the semileptonic as well as the nonleptonic decays of charmed and bottom baryons are the hadronic form factors. These form factors can be calculated in specific models like the quark model or the MIT bag model [1, 2] . Another approach, as discussed in the previous chapter, is to use HQET to find relations among form factors for baryons containing a heavy quark. For instance we saw in the previous chapter that in the heavy-to-heavy transition of the type $\Lambda_b \to \Lambda_c$ all form factors are expressible in terms of one Isgur-Wise function and a HQET mass parameter $\bar{\Lambda}$ up to order $1/m_Q$ where m_Q is the c or b quark mass. For a heavy to light transition, for example of the type $\Lambda_c \to \Lambda$, the use of HQET in the limit $m_Q \to \infty$ allows one to express all the form factors in terms of only two form factors [3].

As we showed in the previous chapter, in heavy to light transitions $1/m_Q$ corrections can be important, especially for the charm sector. Pure HQET analysis of these $1/m_Q$ corrections in the heavy to light transitions does not lead to interesting phenomenology as there are too many form factors and there is hardly any predictive power left [6, 7]. However, as shown in the previous chapter, using a combination

of HQET and some reasonable assumptions, all the form factors up to $1/m_Q$ corrections can be expressed in terms of only two form factors evaluated at maximum momentum transfer. A specific choice for the q^2 dependence of the form factors(*e.g.*, a monopole,dipole etc) can be used to extrapolate to arbitrary values of the four momentum transfer q^2. In this limit, therefore, there are two inputs, the zeroth order form factors F_1^0 and F_2^0 at maximum q^2 or $\omega = v.v' = 1$, where v and v' are the initial and final baryon velocities. In this chapter we will use a slightly modified version of the model for the form factors developed in the previous chapter to study the nonleptonic decays of charmed and bottom baryons. To proceed with our calculations we need the zeroth order form factors F_1^0 and F_2^0 at maximum q^2 or alternately F_1^0 and $r = F_2^0/F_1^0$ at maximum q^2. The best way to fix these inputs would be from measurements of semi-leptonic decays. For instance the asymmetry measurement in $\Lambda_c \to \Lambda l \nu_l$ could be used to fix r. There are measurements of $\Lambda_c \to \Lambda l^+ \nu_l$ form factors by the CLEO collaboration [1] but the fit to data in these studies assumes the KK(Körner Kramer) model [2] for the form factors and hence is not general enough for our use. The KK model, described in the next chapter, treats both the charm quark and the strange quark as heavy.

We next examine the data on non-leptonic decays of charmed baryons. The theoretical description of these processes is model dependent and to that extent an extraction of $F_1^0(\omega = 1)$ and $r = F_2^0(\omega = 1)/F_1^0(\omega = 1)$ using non-leptonic data would also be model dependent. Using current algebra we can use the value of the decay rate of $\Lambda_c \to \Sigma\pi^0$ to fix the non-factorizable contribution to the Cabibbo favored charmed baryon decays. Next, we can use the values of the decay rate and asymmetry of $\Lambda_c \to \Lambda\pi^+$ to fix F_1^0 and $r = F_2^0/F_1^0$ at $\omega = 1$. We calculate the decay rates and asymmetries of the Λ_c and the Ξ_c charmed baryons decaying into an uncharmed

baryon and a pseudoscalar or a vector meson. In our calculations we use SU(3) symmetry to relate the form factors in the $c \to s$ transition to $c \to u$ transitions. Using the flavor symmetry of HQET one can use the same inputs $F_1^0(\omega = 1)$ and τ, extracted from the charm sector, in the bottom sector to study the decays of the bottom baryon. Below we describe the basic features of the current algebra model that we employ in the calculation of the non-leptonic decays of the charmed and bottom baryons.

The starting point of non-leptonic decay calculations is the QCD corrected weak Hamiltonian. This effective current×current Hamiltonian gives rise to the following quark diagrams [9] : the internal and external W-emission diagrams, which result in the factorizable contribution, and the W-exchange diagrams which gives rise to the non-factorizable contribution. The W-annihilation diagram is absent in baryon decay and the W-loop diagram does not contribute to Cabibbo allowed decays. In the large N_c limit the non-factorizable contribution is no longer color suppressed because of N_c W-exchange diagrams . This combinatorial factor of N_c cancels a similar factor in the denominator.

The factorizable part of the decay amplitude is expressed in terms of six form factors. For the decay of the charmed baryon into an uncharmed baryon and the light pseudoscalar, to a very good approximation, only two form factors contribute for a pion in the final state . When the pseudoscalar is replaced by a vector meson four of these form factors contribute. We use the pole model to calculate the non-factorizable part. This model assumes that the non-factorizable decay amplitude receives contributions primarily from one particle intermediate states and these contributions then show up as simple poles in the decay amplitude. The various intermediate single particle states are the ground state positive parity baryons which contribute only to

the parity conserving amplitude, the parity violating amplitude being small[10] .The parity violating amplitude may receive contributions from excited negative parity baryons. In the limit that the momentum of the pseudoscalar $q \to 0$, the parity violating piece of the amplitude reduces to the usual current commutator term of current algebra. Even though in charmed baryon decay the final state pseudoscalar meson is not soft, we will still work in the soft-meson limit and represent the parity violating piece of the amplitude by the current commutator term. It is important to note that using SU(3) symmetry all the weak matrix elements between the positive parity baryon states can be expressed in terms of only one matrix element and therefore in this model the non-factorizable contribution is completely determined by one weak matrix element between positive parity ground state baryons. Hence the prediction for the asymmetry parameter for decays, which have no factorizable contribution (eg, $\Lambda_c \to \Sigma^+\pi^0$), is independent of the baryon-baryon weak matrix element and depends only on the baryon masses.

It is relevant to compare our model with some of the recent models employed in the calculation of Cabibbo favored charmed baryon decays. In our model we use a completely different model for the form factors than has been used in other models to calculate the factorizable piece of the decay amplitude. Regarding the non-factorizable contributions, we have assumed that the current commutator term represents the parity violating non-factorizable amplitude even in the case of charmed baryon decays where the pseudoscalar momentum q is far from zero. Large corrections to this current algebra result have been calculated in Ref. [11] and Ref. [12]. These corrections depend on the model used to estimate the baryon to baryon weak matrix element and the corrections calculated in Ref. [11] and Ref. [12] are quite different. Phenomenologically both these calculations fail in their prediction of the asymmetry

measured in the decay $\Lambda_c \to \Sigma^0\pi^+$. This is also true for another recent calculation on non-leptonic charmed baryon decays using a spectator quark model by Körner and Kramer [13]. However, the central value of the measured asymmetry for the decay $\Lambda_c \to \Sigma^0\pi^+$ compares very well with the current algebra prediction. This seems to indicate that, at least for the decay, $\Lambda_c \to \Sigma^0\pi^+$, the correction to the current algebra result is small. In the light of the experimental results we have therefore adopted the position that the major contribution to the non-factorizable parity violating part of the amplitude comes from the current algebra commutator term. The advantage of such a scenario is that the only parameter needed to specify the non-factorizable contribution is a single baryon-baryon matrix element which can be fixed from the decay rate of a process like $\Lambda_c \to \Sigma^+\pi^0$ (which has no factorizable contribution) and we do not have to rely on model dependent calculation of the weak matrix element.

The decay $\Lambda_c \to p\phi$ is Cabibbo suppressed and has only a factorizable contribution. The same form factors that characterize the $c \to u$ transition in Cabibbo favored decays can also be used for this decay.

For the Λ_b decay we ignore the non-factorizable contribution. For the form factors in this decay we have used the same value of $F_1^0(\omega = 1)$ and $F_2^0(\omega = 1)$ used in charmed baryon decays as these are the form factors for $m_Q \to \infty$ at $\omega = 1$ and so by heavy flavor symmetry they are the same for the charm and bottom sector.

The chapter is organized in the following way. In the next section we outline our model for the calculation of the various charmed and bottom baryon decays while in the third section we present our results.

3.2 Model

Non-Leptonic Decays: Here we develop the formalism for the Cabibbo favored decay of a charmed baryon into an uncharmed baryon and either a pseudoscalar or a vector meson. This formalism will also be used in the decay $\Lambda_b \to J/\psi \quad \Lambda$. We start with the decay of a charmed baryon into a baryon and a pseudoscalar. The amplitude for such a decay can be written as

$$M(B_i \to B_f P) = i\bar{u}_{B_f}(A + B\gamma_5)u_{B_i} \tag{3.1}$$

In the rest frame of the parent baryon the decay amplitude reduces to

$$M(B_i \to B_f P) = i\chi^\dagger_{B_f}(S + P\sigma.\mathbf{q})\chi_{B_i} \tag{3.2}$$

where $\mathbf{q}$ is the unit vector along the direction of the daughter baryon momentum and $S = \sqrt{(2m_c(E_f + m_f)}A$ and $P = \sqrt{(2m_c(E_f - m_f)}B$ with E_f and m_f referring to the final baryon energy and mass. The decay rates and various asymmetries are given by

$$\Gamma = \frac{Q}{8\pi m_c{}^2}(|S|^2+|P|^2); \alpha = \frac{2Re(S^*P)}{|S|^2 + |P|^2} \quad ; \quad \beta = \frac{2Im(S^*P)}{|S|^2 + |P|^2} \quad and \quad \gamma = \frac{|S|^2 - |P|^2}{|S|^2 + |P|^2} \tag{3.3}$$

where Q is the magnitude of the three momentum of the decay products. The starting point of our dynamical analysis is the QCD corrected effective weak Hamiltonian for Cabibbo favored decays

$$H_W = \frac{G_F}{2\sqrt{2}}V_{cs}V_{ud}(c_+O_+ + c_-O_-) \tag{3.4}$$

with $O_\pm = (\bar{s}c)(\bar{u}d) \pm (\bar{s}d)(\bar{u}c)$ where we have omitted the Dirac structure $\gamma_\mu(1 - \gamma_5)$ between the quark fields inside each parentheses. V_{cs} and V_{ud} are the usual CKM matrix elements while $c_\pm$ are the Wilson coefficients evaluated at the charm quark mass scale. In our model we write the decay amplitude as

$$M(B_i \to B_f P) = M(B_i \to B_f P)_{fac} + M(B_i \to B_f P)_{nonfac} \tag{3.5}$$

From the structure of the Hamiltonian, factorization occurs with a π^+ and $\bar{K}^0$ in the final state. The factorizable contribution is given by

$$M(B_i \to B_f \pi^+) = \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud} [c_1 + \frac{c_2}{N_c}] < \pi^+ | \bar{u} d | 0 >< B_f | \bar{s} c | B_i > \quad (3.6)$$

$$M(B_i \to B_f \bar{K}^0) = \frac{G_F}{2\sqrt{2}} V_{cs} V_{ud} [c_2 + \frac{c_1}{N_c}] < \bar{K}^0 | \bar{s} d | 0 >< B_f | \bar{u} c | B_i > \quad (3.7)$$

where $c_1 = \frac{1}{2}(c_+ + c_-)$; $c_2 = \frac{1}{2}(c_+ - c_-)$ with N_c being the number of colors. The N_c suppressed terms come from the Fierz reordering of the operators $O_\pm$. For a satisfactory description of non-leptonic decays of mesons it was found that the Fierz ordered contribution should be omitted [14]. This can be justified in the $1/N_c$ expansion method with $N_c \to \infty$ [15]. We shall therefore also work in the large N_c limit. The matrix elements of the current between baryonic states that appear in the equation above are parameterized in terms of form factors. We define the six vector and axial vector form factors through the following equations

$$\begin{aligned} \langle B'(p',s') \mid \bar{q}\, \gamma^\mu\, Q \mid B_Q(p,s) \rangle &= \bar{u}_{B'}(p',s')[f_1 \gamma^\mu - i \frac{f_2}{m_{B_c}} \sigma^{\mu\nu} q_\nu \\ &\quad + \frac{f_3}{m_{B_c}} q^\mu] u_{BQ}(p,s) \\ \left\langle B'(p',s') \mid \bar{q}\, \gamma^\mu \gamma^5\, Q \mid B_Q(p,s) \right\rangle &= \bar{u}_{B'}(p',s')[g_1 \gamma^\mu - i \frac{g_2}{m_{B_c}} \sigma^{\mu\nu} q_\nu \\ &\quad + \frac{g_3}{m_{B_c}} q^\mu] \gamma^5 u_{BQ}(p,s) \end{aligned} \quad (3.8)$$

where $q^\mu = p^\mu - p'^\mu$ is the four momentum transfer, B_Q is the baryon with a heavy quark and B' is the light baryon. In chapter 2 we studied the form factors for heavy to light transitions involving baryons in HQET including corrections up to $1/m_Q$. We found that at $\omega = 1$, in addition to the two zeroth order form factors form factors F_1^0 and F_2^0, there were five other unknown matrix elements, four of which represent corrections from the chromomagnetic operator. In chapter 2 we made some

assumptions about these unknown matrix elements and we were able to express all the form factors in terms of F_1^0 and F_2^0. Without making any assumptions about the corrections coming from the chromomagnetic operator we write the form factors as

$$\begin{aligned} \frac{f_1}{F_1^0} &= 1 + a + (m_{B_Q} + m_{B'}) \left[\frac{r + b/3}{2m_{B_Q}} - \frac{a' + b/3}{2m_{B'}} \right] \\ \frac{f_2}{F_1^0 m_{B_Q}} &= -\frac{r + b/3}{2m_{B_Q}} + \frac{a'' + b/3}{2m_{B'}} \\ \frac{f_3}{F_1^0 m_{B_Q}} &= \frac{r + b/3}{2m_{B_Q}} + \frac{a''' + b/3}{2m_{B'}} \\ \frac{g_1}{F_1^0} &= 1 + r + \frac{2b}{3} - (m_{B_Q} - m_{B'}) \left[\frac{r - a - \rho b/3}{2m_{B_Q}} + \frac{\rho b'/3}{2m_{B'}} \right] \\ \frac{g_2}{F_1^0 m_{B_Q}} &= -\frac{r - a - \rho b/3}{2m_{B_Q}} - \frac{\rho b''/3}{2m_{B'}} \\ \frac{g_3}{F_1^0 m_{B_Q}} &= \frac{r - a - \rho b/3}{2m_{B_Q}} - \frac{\rho b'''/3}{2m_{B'}} \end{aligned} \quad (3.9)$$

where

$$\begin{aligned} \bar{\Lambda} &= m_{B_Q} - m_Q \\ \hat{m}_{B'} &= m_{B'} - m_q \\ z_1 &= (\bar{\Lambda} + \hat{m}_{B'}) \\ z_2 &= (\bar{\Lambda} - \hat{m}_{B'}) \\ r &= F_2^0 / F_1^0 \\ a &= \frac{(z_1 + \frac{4}{3} z_2) + r(z_1 + \frac{1}{3} z_2)}{2m_Q} \\ \rho &= -\frac{6r}{1+r} \frac{\hat{m}_{B'}}{z_2} \\ b &= -\frac{(1+r)}{2m_Q} z_2 \end{aligned} \quad (3.10)$$

and m_{B_Q} and $m_{B'}$ are the heavy and the light baryon masses while m_Q and m_q are the masees of the heavy and the light quark respectively. The model used here corresponds to $a' = a'' = 0$, $\rho b' = \rho b(1 + 2m_{B'}/m_{B_Q})$ and $\rho b'' = \rho b$. The quantities

a''' and $\rho b'''$ are now calculable since the four matrix elements that represent the chromomagnetic corrections are determined by our choice of a',a'',$\rho b'$ and $\rho b''$. The expressions for a''' and $\rho b'''$ are

$$\begin{aligned} a''' &= a + a[1 - \frac{2m_{B_Q}}{m_{B_Q} - m_{B'}} + \frac{\rho b}{a}\frac{m_{B'}}{m_{B_Q}}] \\ \frac{\rho''' b}{3} &= \frac{\rho b}{3} + [(\frac{2m_{B'}}{3m_{B_Q}} + \frac{4{m_{B'}}^2}{(m_{B_Q} - m_{B'})m_{B_Q}})\rho b - \frac{4m_{B'}m_{B_Q}}{(m_{B_Q} - m_{B'})^2}a] \end{aligned}$$

The choice of the model described above is dictated by the fact that it works well phenomenologically and the fact that an expansion in $1/m_Q$ is valid. The condition for the validity of the $1/m_Q$ expansion is that $|r| \leq 1$. To connect these assumptions with the ones made in Ref. [7], we review the assumptions made about the corrections coming from the chromomagnetic operator in chapter 2. We consider $m_{B'}/m_{B_Q}$ to be small and we relax some of the assumptions about the chromomagnetic corrections in chapter 2. While we retain $\delta F_1 + \delta F_2 + \delta F_3 = 0$ (Eq. (2.33) of chapter. 1) we only assume (at $\omega = 1$) $\chi_{11} \sim \chi_{12} \sim \chi_1$ but do not constrain χ_{21} and χ_{22}. The above assumptions lead to $\chi_1(\omega = 1) = x(\omega = 1)/m_c$ [7]. The model for the form factors used in this work corresponds to $\chi_{21} = \chi_{22} = \chi_2 = -a$ in the limit $m_{B'}/m_{B_Q}$ is small. So we see that the model employed here is almost identical to the model in chapter 2(except for χ_1 not equal to χ_2) in the limit $m_{B'}/m_{B_Q}$ is small. For the decays of charmed baryons considered in this paper the difference between the two models can be significant given the fact that $m_{B'}/m_{B_Q}$ is no longer small. For bottom baryon decays we expect the two models to yield essentially identical results. Imposing the constraint on r we find that we can fix f and g from the measured asymmetry and decay rate of $\Lambda_c \to \Lambda\pi^+$. Taking into account the experimental errors, the form factors f and g are such that $(g - f)/g \leq 0.35$ if $f < g$ and $(f - g)/f \leq 0.35$ if $g < f$. Note that in the $m_c \to \infty$ limit the form factors f and g are equal. The inclusion of

$1/m_c$ corrections destroys this equality, and so the inequalities above represent the size of the $1/m_c$ corrections. We also assume $F_1^0(\omega = 1) > 0$ in our analysis. The factorizable contributions to the decay amplitude can now be written as

$$\begin{aligned} A_{fac} &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} f_P c_k [(m_f - m_i) f_1({m_P}^2) + f_3({m_P}^2)\frac{{m_P}^2}{m_i}] \\ B_{fac} &= \frac{G_F}{\sqrt{2}} V_{cs} V_{ud} f_P c_k [(m_f + m_i) g_1({m_P}^2) + g_3({m_P}^2)\frac{{m_P}^2}{m_i}] \end{aligned} \tag{3.11}$$

where $c_1(c_2)$ refer to π^+ ($\bar{K}^0$) emission, f_P is the pseudoscalar decay constant and $f_{1,3}$ and $g_{1,3}$ are the form factors defined in Eq. (3.8) and ${m_P}^2$ is the pseudoscalar mass . In our analysis we have used the SU(3) results

$$\begin{aligned} {f_1}^{\Lambda_c \Lambda} &= \sqrt{\frac{2}{3}} {f_1}^{\Xi_c{}^0 \Lambda \Xi^0} = -\sqrt{\frac{2}{3}} {f_1}^{\Xi_c{}^+ \Lambda \Xi^0} = -\sqrt{\frac{2}{3}} {f_1}^{\Lambda_c p} \\ &= -\sqrt{\frac{2}{3}} {f_1}^{\Xi_c{}^+ \Lambda \Sigma^+} = \sqrt{\frac{4}{3}} {f_1}^{\Xi_c{}^0 \Lambda \Sigma^0} = 2{f_1}^{\Xi_c{}^0 \Lambda \Lambda} \end{aligned} \tag{3.12}$$

It is important to note that strictly we should use the SU(3) relations for the zeroth order form factors since the $1/m_Q$ corrections involve the baryon masses and hence break SU(3), but this effect is small and is therefore neglected in our analysis.

For the non-factorizable term we will use the pole model and current algebra as outlined in the introduction. Following Ref. [16] we write the non-factorizable amplitude $R(q)$ as

$$R(q) = R_{Born}(q) + \bar{R}(q) \tag{3.13}$$

The usual approximation is to assume

$$R(q, q^2 = m_P^2) \simeq R_{Born}(q, q^2 = m_P^2) + \bar{R}(0) \tag{3.14}$$

Finally using reduction techniques for the amplitude one obtains

$$R(q, q^2 = m_P^2) \simeq \frac{-\sqrt{2}}{f_P} < B|[Q_5, H^{PV}]|B_c > + R_{Born}(q, q^2 = m_P^2) - \bar{R}_c(0) \tag{3.15}$$

where Q_5 is the axial charge, f_P is the pseudoscalar decay constant and

$$\bar{R}_c(0) = \lim_{q\to 0}[-\frac{\sqrt{2}}{f_\pi} i q^\mu M_\mu + R_{Born}(q)] \tag{3.16}$$

with M_μ defined in Eq. (1.72). Clearly the first term in the amplitude above contributes to the parity violating amplitude while the remaining terms contribute to the parity conserving amplitude as the parity violating amplitude is small [10]. (In the case of non-leptonic hyperon decays $< B_f|H^{PV}|B_i >= 0$ in the SU(3) limit [16]). Note in the case of charmed baryon decays, as opposed to the hyperon decay case, the contribution from $\bar{R}_c(0)$ is no longer small compared to $R_{Born}(q, q^2 = m_P^2)$. Hence in our model we have

$$A_{nonfac} = \frac{-\sqrt{2}}{f_P} < B|[Q_5, H^{PV}]|B_c > \tag{3.17}$$

$$\begin{aligned} B_{nonfac} &= -[g_{B''B'P}\frac{< B''|H^{PC}|B >}{m_B - m_{B''}}\frac{m_B + m_{B'}}{m_{B''} + m_{B'}} \\ &+ g_{BB'''P}\frac{< B'|H^{PC}|B''' >}{m_{B'} - m_{B'''}}\frac{m_B + m_{B'}}{m_B + m_{B'''}}] \end{aligned} \tag{3.18}$$

The first term in the expression for B is the s-channel pole contribution while the next term is the u-channel pole contribution. The strong pseudoscalar meson-baryon coupling $g_{B_i,B_j,P}$ can be related via the Goldberger-Treiman relation to the axial vector form factors $g^A{}_{B',B''}$ as

$$g_{B',B''} = \frac{1}{f_P}(m_{B'} + m_{B''})g^A{}_{B',B''} \tag{3.19}$$

The axial form factors $g^A{}_{B_i,B_j}$ are of two types, those between non-charmed baryons and those between charmed baryons. For the first type we use SU(3) parameterization with

$$D + F = 1.25 \quad ; \quad D/F \approx 1.8 \tag{3.20}$$

where the D/F ratio is taken from a fit to hyperon semileptonic decay [17]. The second type of form factors are between charmed baryons and it is reasonable to use SU(4) symmetry and use the same D and F in this case also. The justification for this lies in the fact the the transitions are $\Delta C = 0$ and so the baryon wavefunction mismatch in the overlap integral is small[12]. For the weak matrix element between the positive parity baryons we will use the following SU(3) relation

$$a_{\Sigma^+\Lambda_c^+} = a_{\Sigma_c^0\Lambda^0} = a_{\Xi^0\Lambda\Xi^0} = \sqrt{\frac{1}{3}}a_{\Xi^0S\Xi^0} = \sqrt{\frac{1}{3}}a_{\Sigma_c^+\Sigma^+} = -\sqrt{\frac{1}{3}}a_{\Sigma_c^0\Sigma^0} \tag{3.21}$$

where $a_{B_i,B_f} =< B_f|H^{PC}|B_i >$. Using the above SU(3) relations the non-factorizable term is completely specified in terms of one weak matrix element which we choose to be $a_{\Xi^0\Lambda\Xi^0}$, and which we fix from the measured decay rate of $\Lambda_c \to \Sigma^+\pi^0$.

For the decay where the meson in the final state is a vector meson we can write the decay amplitude as

$$M(B_c \to B_f V) = i u_{B_f}\epsilon^{*\mu}[\gamma_\mu(a + b\gamma_5) + 2(x + y\gamma_5)P_{1\mu}]u_{B_c} \tag{3.22}$$

where $P_{1\mu}$ is the four-momentum of the parent baryon and $\epsilon^{*\mu}$ is the polarization of the vector meson. The kinematics for this decay have been worked out in details in Ref. [18]. We can write down the factorizable contribution as

$$\begin{aligned} a_{fac} &= \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}f_V m_V c_k[f_1(m_V{}^2) + \frac{m_f + m_i}{m_i}f_2(m_V{}^2)] \\ b_{fac} &= \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}f_V m_V c_k[g_1(m_V{}^2) + \frac{m_f - m_i}{m_i}g_2(m_V{}^2)] \\ x_{fac} &= \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}f_V m_V c_k[\frac{f_2(m_V{}^2)}{m_i}] \\ y_{fac} &= \frac{G_F}{\sqrt{2}}V_{cs}V_{ud}f_V m_V c_k[\frac{g_2(m_V{}^2)}{m_i}] \end{aligned} \tag{3.23}$$

where $c_1(c_2)$ refer to ρ^+ ($\bar{K}^{*0}$) emmision, f_V is the vector meson decay constant, m_V is the vector meson mass and f_1, f_2 and g_1, g_2 are the form factors. For the pole term

we will work in the approximation that ρ generates isospin and so the couplings g_{BBV} are pure F-type. Similar results apply to the decays $\Lambda_c \to p\phi$ and $\Lambda_b \to J/\psi\Lambda$ with the appropriate changes in the QCD correction factor and the CKM matrix elements.

Before we present our results in the next section we list the various inputs used in the calculations. We begin with the calculations of the non-leptonic decays of the charmed baryons. As outlined in the introduction a fit to the decay rate and the asymmetry for the decay $\Lambda_c \to \Lambda\pi^+$ is used to extract $F_1^0(\omega = 1)$ and r. The extracted values are $F_1^0(\omega = 1) = 0.46$ and $r = -0.47$. The values for the Wilson coefficients c_1 and c_2 were taken $\approx$ 1.32 and -0.59 respectively [11] and we have used $m_c = 1.4$ GeV and $m_s = 0.2$ GeV[19]. We found that the non-factorizable contribution could be expressed in terms of the single matrix element $a_{\Xi^{0A}\Xi^0}$. The measured decay rate of $\Lambda_c \to \Sigma^+\pi^0$ is used to extract $a_{\Xi^{0A}\Xi^0} = -5.48 \times 10^{-8}$ GeV. For the vector meson decays we use, following Ref. [11], $f_\rho = f_{K^*} = 0.221$ GeV. For the mode $\Lambda_c \to p\phi$ we have used $f_\phi = 0.23$ GeV for the ϕ decay constant. For the Λ_b decay we have used $|V_{cb}| = 0.040$ [22], $c_2 \approx 0.23$, $f_{J/\psi} = 395$ MeV, and pole masses $m_V \cong 5.42$ GeV, $m_A \cong 5.86$ GeV[1] . The quark masses were taken as $m_b = 4.74$ GeV and $m_s = 0.20$ GeV [19].

3.3 Results

Starting with the results on the non-leptonic decays of the charmed baryons, in Table 3.1 and 3.2 we give the predictions for the decay rates and asymmetry for the non-leptonic decays $B_i \to B_f P$ and $B_i \to B_f V$. In Table 3.3 we show the predictions for the mode $\Lambda_c \to p\phi$ and in Table 3.4 we show the predictions for $\Lambda_b \to J/\psi\Lambda$. In Table 3.5 and Table 3.6 we show the form factors for the $\Lambda_c \to \Lambda$ transition while in Table 3.7 and Table 3.8 we show the form factors for the $\Lambda_b \to \Lambda$ transition.

Table 3.1: Decay rates ($\times 10^{11} s^{-1}$), branching ratios ($\times 10^{-3}$) and asymmetry predictions for Cabibbo favored $B_i \to B_f P$ decays. The asterisks indicate the input values.

Process	Γ_{Th}	BR_{Th}	Γ_{Expt}	α_{Th}, α_{Expt}
$\Lambda_c \to \Lambda\pi^+$	0.40	7.9*	0.40 ± 0.11	-0.94^* , $-0.94^{+22}_{-0.06}$[20]
$\Lambda_c \to \Sigma^0\pi^+$	0.44	8.7*	0.44 ± 0.10	-0.47 , $--$
$\Lambda_c \to \Sigma^+\pi^0$	0.44	8.7	0.44 ± 0.12[20]	-0.47 , -0.45 ± 0.31[20]
$\Lambda_c \to pK^0$	0.68	13.4	1.05 ± 0.20	-0.91, $--$
$\Lambda_c \to \Xi^0K^+$	0.25	4.9	0.17 ± 0.05	0 , $--$
$\Xi_c^{0A} \to \Xi^-\pi^+$	0.17	1.6	$--$	0.06 , $--$
$\Xi_c^{+A} \to \Xi^0\pi^+$	0.88	31	0.03 , $--$	
$\Xi_c^{0A} \to \Xi^0\pi^0$	0.62	6.1	$--$	-0.89 , $--$
$\Xi_c^{+A} \to \Sigma^+K^0$	0.31	3.1	$--$	-0.005 , $--$
$\Xi_c^{0A} \to \Lambda K^0$	0.42	4.1	$--$	-0.76 , $--$
$\Xi_c^{0A} \to \Sigma^0K^0$	0.23	2.2	$--$	0.006 , $--$
$\Xi_c^{0A} \to \Sigma^+K^-$	0.24	2.3	$--$	0 , $--$

Table 3.2: Decay rates ($\times 10^{11} s^{-1}$), branching ratios ($\times 10^{-3}$) and asymmetry predictions for Cabbibo favored $B_i \to B_f V$ decays

Process	Γ_{Th}	BR_{Th}	Γ_{Expt}	BR_{Expt}	α_{Th}	α_{Expt}
$\Lambda_c \to \Lambda\rho^+$	0.55	11	< 2.1	< 42[23]	0.46	$--$
$\Lambda_c \to \Sigma^0\rho^+$	0.15	3	$--$	$--$	0.0	$--$
$\Lambda_c \to \Sigma^+\rho^0$	0.15	3	< 0.6	< 12[21]	0	$--$
$\Lambda_c \to pK^{*0}$	0.57	11.3	$--$	$--$	0.45	$--$
$\Lambda_c \to \Xi^0K^{*+}$	0.002	0.8	$--$	$--$	0	$--$
$\Xi_c^{0A} \to \Xi^-\rho^+$	1.3	12.8	$--$	$--$	0.54	$--$
$\Xi_c^{+A} \to \Xi^0\rho^+$	0.88	31	$--$	$--$	0.46	$--$
$\Xi_c^{0A} \to \Xi^0\rho^0$	0.11	1.1	$--$	$--$	0	$--$
$\Xi_c^{+A} \to \Sigma^+K^{*0}$	0.36	12.8	$--$	$--$	0.47	$--$
$\Xi_c^{0A} \to \Lambda K^{*0}$	0.10	1	$--$	$--$	-0.56	$--$
$\Xi_c^{0A} \to \Sigma^0K^{*0}$	0.17	1.7	$--$	$--$	0.37	$--$
$\Xi_c^{0A} \to \Sigma^+K^{*-}$	0.016	0.15	$--$	$--$	0	$--$

Table 3.3: Decay rate ($\times 10^{11} s^{-1}$), branching ratio relative to $pK^-\pi^+$ mode and asymmetry predictions for $\Lambda_c \to p\phi$

Process	Γ_{Th}, BR	Γ_{Expt}	α_{Th}	α_{Expt}
$\Lambda_c \to p\phi$	$0.02, BR \approx 0.01$	$0.023 \pm 0.006 \pm 0.003$[24]	0.31	--

Table 3.4: Decay rate ($\times 10^{11} s^{-1}$), branching ratio relative to the total decay width and asymmetry predictions for $\Lambda_b \to J/\psi \quad \Lambda$ decays

Process	Γ_{Th}	Γ_{Expt}	α_{Th}	α_{Expt}
$\Lambda_b \to J/\psi\Lambda$	$0.4 \times 10^{-3}, BR \approx 0.4 \times 10^{-4}$	--	0.25	--

Table 3.5: Form factors at the point q^2_{max} for $\Lambda_c \to \Lambda$

$f_1(q^2_{max})$	$f_2(q^2_{max})$	$f_3(q^2_{max})$	$g_1(q^2_{max})$	$g_2(q^2_{max})$	$g_3(q^2_{max})$
0.46	0.11	−0.84	0.46	0.26	−1.88

Table 3.6: Form factors at the point $q^2 = 0$ for $\Lambda_c \to \Lambda$

$f_1(q^2=0)$	$f_2(q^2=0)$	$f_3(q^2=0)$	$g_1(q^2=0)$	$g_2(q^2=0)$	$g_3(q^2=0)$
0.22	0.05	−0.40	0.28	0.16	−1.15

Table 3.7: Form factors at the point q^2_{max} for $\Lambda_b \to \Lambda$

$f_1(q^2_{max})$	$f_2(q^2_{max})$	$f_3(q^2_{max})$	$g_1(q^2_{max})$	$g_2(q^2_{max})$	$g_3(q^2_{max})$
0.38	0.11	−0.29	0.46	0.22	−0.0175

Table 3.8: Form factors at the point $q^2 = 0$ for $\Lambda_b \to \Lambda$

$f_1(q^2=0)$	$f_2(q^2=0)$	$f_3(q^2=0)$	$g_1(q^2=0)$	$g_2(q^2=0)$	$g_3(q^2=0)$
0.040	0.012	−0.03	0.08	0.04	−0.003

In conclusion in this chapter we have studied the non-leptonic two body decays of charmed and bottom baryons involving transition of a heavy to light quark based on a model for form factors that includes order $1/m_Q$ corrections.

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CHAPTER 4

SEMILEPTONIC DECAYS OF Λ_c AND Λ_b BARYONS

4.1 Introduction

Recently there has been a measurement of the form factors in the semi-leptonic decay $\Lambda_c \to \Lambda e^+ \nu_e$ [1], where a fit to the data was done using the model of Körner and Kramer [2]. This model, as we mentioned in the previous chapters, is constructed within the framework of HQET, but treats both the charm quark and the strange quark as heavy and considers only the $1/m_s$ expansion, keeping $m_c \to \infty$. Even though the model has only one input parameter, use of $1/m_s$ expansion is questionable and the omission of $1/m_c$ corrections is also not justified. For a heavy to light transition, for example of the type $\Lambda_c \to \Lambda$, the use of HQET in the limit $m_Q \to \infty$ allows one to express all the form factors in terms of only two form factors [3]. Semileptonic decays of Λ_c have been studied in this limit [2, 4]. In the previous two chapters we constructed a model for the form-factors which takes into account $1/m_Q$ corrections for heavy-to-light transitions of the Λ type baryon. In this chapter we use the form factors to study the decays $\Lambda_c \to \Lambda l \nu$ and $\Lambda_b \to p l \nu$.

This chapter is organized in the following way. In the next section we present the formulae for the calculation of asymmetries and decay rates for the semi-leptonic decays $\Lambda_c \to \Lambda l \nu$ and $\Lambda_b \to p l \nu$ while in the third section we present our results.

4.2 Formalism

Here we present the formalism for the semi-leptonic decays of $\Lambda_c \to \Lambda l \nu$ and $\Lambda_b \to p l \nu$. The hadronic part of the amplitude is the matrix element of the weak quark current between baryonic states that is parameterized in terms of form factors which have already been defined in the previous chapters.

We have shown in the previous chapters that we can write the form factors in the above equation in terms of two form factors F_1 and F_2 provided some assumptions are made about the $1/m_Q$ corrections. For both Λ_c and Λ_b semi-leptonic decays we will work in the approximation that the lepton mass is negligible. To make the kinematics of the decay $\Lambda_c \to \Lambda_s l \nu_l$ transparent we can imagine that this decay proceeds via $\Lambda_c \to \Lambda_s W^*$ followed by $W^* \to l \nu_l$. Following [2] we define the helicity amplitudes which are given by

$$\begin{aligned} H_{\lambda_2,\lambda_W} &= H^V_{\lambda_2,\lambda_W} + H^A_{\lambda_2,\lambda_W} \\ H^{V,(A)}_{-\lambda_2,-\lambda_W} &= +(-) H^{V,(A)}_{\lambda_2,\lambda_W} \end{aligned} \tag{4.1}$$

where λ_2, λ_W are the polarizations of the daughter baryon and the W-boson respectively. In terms of the form factors the helicity amplitudes are given by

$$\begin{aligned} H^V_{1/2,0} &= a_- \left[(M_1 + M_2) f_1 + \frac{q^2}{m_{B_c}} f_2 \right] V(q^2) \\ H^A_{1/2,0} &= a_+ \left[-(M_1 - M_2) g_1 + \frac{q^2}{m_{B_c}} g_2 \right] A(q^2) \\ H^V_{1/2,1} &= \sqrt{2Q_-} \left[-f_1 - \frac{(M_1 + M_2)}{m_{B_c}} f_2 \right] V(q^2) \\ H^A_{1/2,1} &= \sqrt{2Q_+} \left[g_1 - \frac{(M_1 - M_2)}{m_{B_c}} g_2 \right] A(q^2) \end{aligned} \tag{4.2}$$

where

$$Q_\pm = (M_1 \pm M_2)^2 - q^2 \quad ; \quad a_\pm = \sqrt{\frac{Q_\pm}{q^2}}$$

$$V(q^2) = \frac{(1-\frac{q^2_{max}}{m^V_{FF}{}^2})^2}{(1-\frac{q^2}{m^V_{FF}{}^2})^2} \quad ; \quad A(q^2) = \frac{(1-\frac{q^2_{max}}{m^A_{FF}{}^2})^2}{(1-\frac{q^2}{m^A_{FF}{}^2})^2} \tag{4.3}$$

where M_1, M_2 are the parent and daughter baryon masses and $m^{V,A}_{FF}$ are the appropriate pole masses and q is the momentum transfer. The decay $\Lambda_c \to \Lambda_s l\nu_l$ is analyzed by looking at the two sided cascade decay $\Lambda_c \to \Lambda_s[\to p\pi^-] + W[\to l\nu_l]$. The normalized four-fold joint angular distribution for the decay $\Lambda_1 \to \Lambda_2[\to a(\frac{1}{2}^+) + b(0^-] + W[\to l\nu_l]$ is given, following the notation in Ref. [2], as

$$\begin{aligned}\frac{d\Gamma}{dq^2 d\chi d\cos\Theta d\cos\Theta_\Lambda} &= B(\Lambda_2 \to a+b)\frac{1}{2}\frac{G^2}{(2\pi)^4}\frac{q^2 p}{24M_1}|V_{Q_1,Q_2}|^2[T_1 + T_2 \\ &\quad + T_3] \end{aligned} \tag{4.4}$$

where

$$\begin{aligned} T_1 &= [\frac{3}{8}(1 \pm \cos\Theta)^2|H_{1/2\,1}|^2(1+\alpha_\Lambda\cos\Theta_\Lambda) + \frac{3}{8}(1 \mp \cos\Theta)^2|H_{-1/2\,-1}|^2 \\ &\quad (1-\alpha_\Lambda\cos\Theta_\Lambda)] \\ T_2 &= \frac{3}{4}\sin^2\Theta[|H_{1/2\,0}|^2(1+\alpha_\Lambda\cos\Theta_\Lambda) + |H_{-1/2\,0}|^2(1-\alpha_\Lambda\cos\Theta_\Lambda)] \\ T_3 &= \mp\frac{3}{2\sqrt{2}}\alpha_\Lambda\cos\chi\sin\Theta\sin\Theta_\Lambda[(1\pm\cos\Theta)Re(H_{-1/2\,0}H^*_{1/2\,1}) \\ &\quad +(1\mp\cos\Theta)Re(H_{-1/2\,0}H^*_{-1/2\,-1})] \end{aligned}$$

where $p = \sqrt{Q_+Q_-}/2M_1$, the upper and lower signs in the above equation hold for the $l^-\nu_l$ and $l^+\nu_l$ leptonic final states respectively and V_{Q_1,Q_2} is the CKM element for the $Q_1 \to Q_2$ transition. The polar angles defined in Ref. [2] are shown in Fig. 4.1.

By integrating over two of the angles one can look at the following distributions

$$\begin{aligned} \frac{d\Gamma}{dq^2 d\cos\Theta_\Lambda} &\propto 1 + \alpha\alpha_\Lambda\cos\Theta_\Lambda \\ \frac{d\Gamma}{dq^2 d\cos\Theta} &\propto 1 \pm 2\alpha'\cos\Theta + \alpha''\cos\Theta^2 \\ \frac{d\Gamma}{dq^2 d\chi} &\propto 1 \mp \frac{3\pi^2}{32\sqrt{2}}\gamma\alpha_\Lambda\cos\chi \end{aligned} \tag{4.5}$$

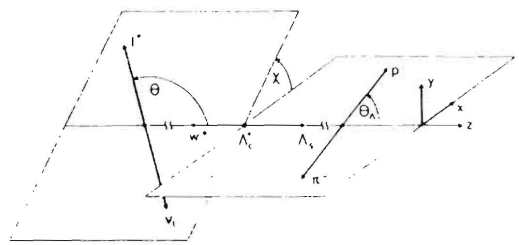

Figure 4.1: The decay process $\Lambda_1 \to \Lambda_2[\to a(\frac{1}{2}^+) + b(0^-] + W[\to l\nu_l]$. The polar angles Θ_Λ, Θ and χ are shown in this figure.

The first distribution in the above equation gives the polar angle distribution for the cascade decay $\Lambda_s \to p\pi^-$. The second distribution is the polar angle distribution for the decay $W \to l\nu_l$, while the third distribution is the azimuthal angle distribution. The asymmetries are given by the expressions below [2] and will depend only on the ratio $F_2^0(\omega=1)/F_1^0(\omega=1)$ in our model for the form factors

$$\begin{aligned} \alpha &= \frac{|H_{1/2\,1}|^2 - |H_{-1/2\,-1}|^2 + |H_{1/2\,0}|^2 - |H_{-1/2\,0}|^2}{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 + |H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2} \\ \alpha' &= \frac{|H_{1/2\,1}|^2 - |H_{-1/2\,-1}|^2}{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 + 2\,(|H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2)} \\ \alpha'' &= \frac{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 - 2\,(|H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2)}{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 + 2\,(|H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2)} \\ \gamma &= \frac{2\,Re(H_{-1/2\,0}H^*_{1/2\,1} + H_{1/2\,0}H^*_{-1/2\,-1})}{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 + |H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2} \end{aligned} \tag{4.6}$$

for unpolarized Λ_c. For polarized Λ_c one has similar decay distributions and we refer the reader to Ref. [2] for the relevant details. We give here the expression for the asymmetries

$$\begin{aligned} \alpha_P &= \frac{|H_{1/2\,1}|^2 - |H_{-1/2\,-1}|^2 - |H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2}{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 + |H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2} \\ \gamma_P &= \frac{2\,Re(H_{1/2\,0}H^*_{-1/2\,0})}{|H_{1/2\,1}|^2 + |H_{-1/2\,-1}|^2 + |H_{1/2\,0}|^2 + |H_{-1/2\,0}|^2} \end{aligned} \tag{4.7}$$

As $q^2 \to q^2{}_{max}$ all the polar asymmetries vanish while the azimuthal asymmetries $\gamma \to 2\sqrt{2}/3$ and $\gamma_P \to -1/3$. At the $q^2 = 0$ point however the limiting value of the asymmetries α, α_P and γ_P depend on the dynamics as shown below while the asymmetries α', γ and α'' go to zero 0,0 and -1 respectively. These asymmetries were studied for different representative values of the ratio $r_{KK} = f_2/f_1$ [2] which is related to the ratio $r = F_2^0(\omega = 1)/F_1^0(\omega = 1)$ via $r_{KK} = r/(2 + r)$. The fit, performed by Ref. [1], to the semileptonic decay data on $\Lambda_c \to \Lambda_s e^+ \nu_e$ uses the Korner Kramer(KK) model to extract r_{KK} and the asymmetry α [1]. Since, as already mentioned in the introduction, the KK model does not include $1/m_c$ corrections and the use of $1/m_s$ expansion is questionable, a more correct approach would be therefore to do a fit to the semi-leptonic data including $1/m_c$ corrections to extract the ratio $r(r_{KK})$ and hence the asymmetries.

4.3 Results

In our analysis we have studied the semi-leptonic decays for $|r| \leq 1$ and $0 \leq F_1^0(\omega = 1) \leq 1$. In chapter 3 we found the $1/m_c$ expansion to be valid for $|r| \leq 1$ and from the study of non-leptonic two body charmed baryon decays $F_1^0(\omega = 1) \leq 1$ was obtained (we assumed $F_1^0(\omega = 1) > 0$). In our analysis we found that the $1/m_Q$ corrections depend on the value of r. As far as the $1/m_Q$ corrections to the form factors are concerned for the Λ_c decay the $1/m_c$ corrections to the form factors are $> 50\%$ for $r \leq -0.62$. For Λ_b decays the $1/m_b$ corrections are small and are greater than 30% for $r \leq -0.5$. Fig. 4.2 shows $\alpha(q^2)$ versus q^2 for different r values.

We note that for $-0.5 \leq r \leq 1$ the asymmetry α is not very sensitive to r. This feature is common to most of the calculated integrated asymmetries. In Fig. 4.3 we show $\alpha(q^2 = 0)$ versus r.

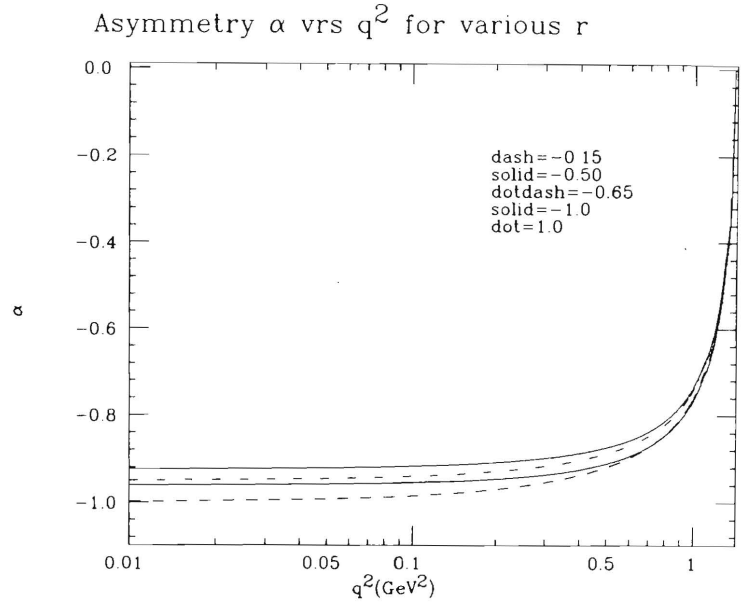

Figure 4.2: Asymmetry α versus q^2 for various r.

In the KK model or in general with $m_c \to \infty$, $\alpha(q^2 = 0) = -1$ for all r. Fig. 4.3 shows the effect of including $1/m_c$ corrections. Fig. 4.4 and Fig. 4.5 show the predictions for the integrated asymmetries for different r for unpolarized and polarized Λ_c and except for γ we see that the integrated asymmetries are insensitive to r in the range $-0.5 \leq r \leq 1$.

The $1/m_c$ corrections to the integrated asymmetries again depend on the value of r and can be as large as 40%.

The decay rates depend on both r and $F_1^0(\omega = 1)$. Instead of calculating the

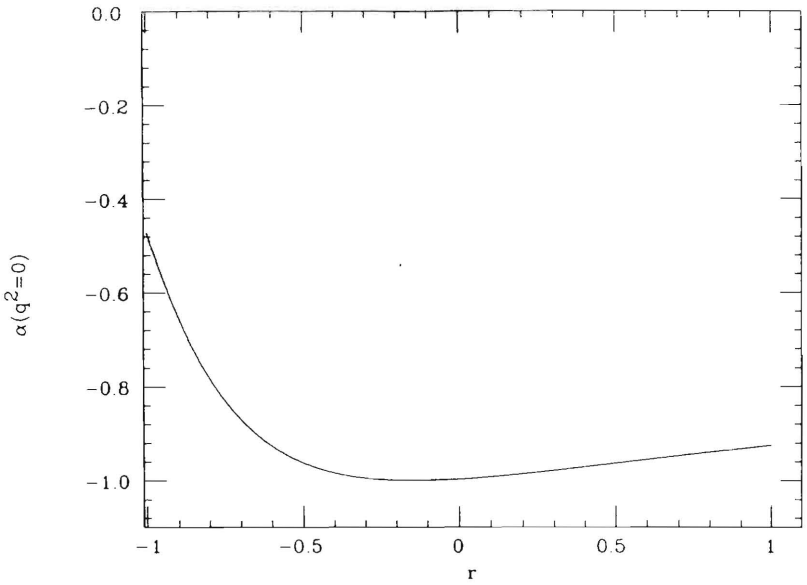

Figure 4.3: Asymmetry $\alpha(q^2 = 0)$ versus r.

individual decay rates for $\Lambda_c \to \Lambda l\nu$ and $\Lambda_b \to pl\nu$, to reduce the uncertainties from these sources we calculate the ratio of the decay rates which will depend only on r as the factor $F_1^0(\omega = 1)$ cancels. Fig. 4.6 shows this ratio as a function of r with and without $1/m_Q$ corrections with a monopole and dipole form for the form factors.

We see that the $1/m_Q$ corrections are small for the dipole form factors. The ratio above will receive corrections of the order $\sim 1/m_Q^2$ and higher where m_Q is the c or b quark. An estimate of these corrections will depend on the value of r which can be extracted by performing a fit to the $\Lambda_c \to \Lambda e^+\nu_e$ data. The final result can be

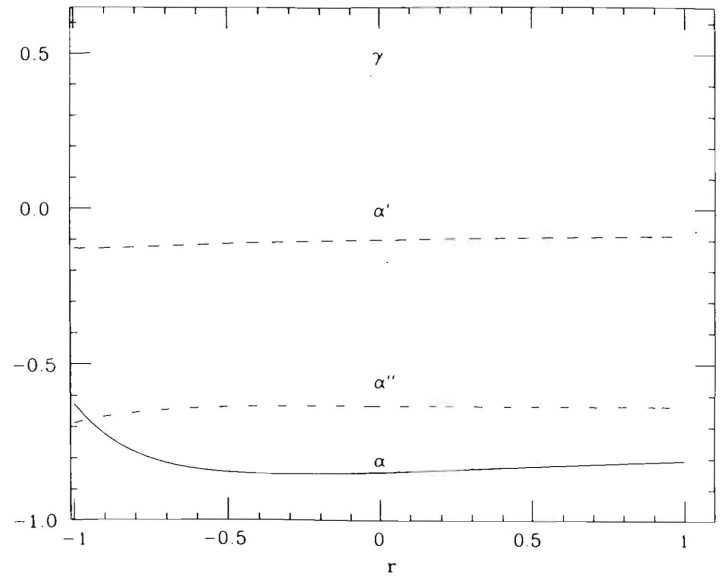

Figure 4.4: Average asymmetries (unpolarized case) versus r.

written as

$$\frac{\Gamma(\Lambda_b \to pl\nu)}{\Gamma(\Lambda_c \to \Lambda l\nu)} = R_0(r)[1 + \Delta_1(r) + \Delta_2(r) + ...]|\frac{V_{ub}}{V_{cs}}|^2 \tag{4.8}$$

where R_0 is the ratio with $m_Q \to \infty$ and $\Delta_j(r)$ represents the corrections to the ratio due to $1/m_Q^j$ corrections to the form factors. Since we do not know r we make an estimate of the ratio above by using the value of r extracted from nonleptonic charmed baryon decays with dipole form factors [6] i.e $r = r_0 = -0.47(r_{KK} = -0.31)$. We write

$$r = r_0 \pm \Delta r$$

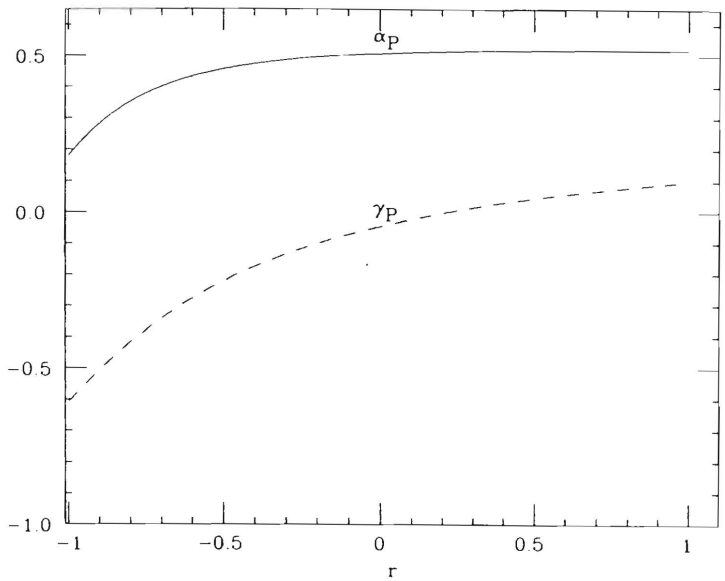

Figure 4.5: Average asymmetries (polarized case) versus r.

$$R_0(r) \approx R_0(r_0) \pm \Delta r R_0'(r_0) \tag{4.9}$$

where $R_0'(r_0)$ is the derivative of $R_0(r)$ at $r = r_0 = -0.47$, and where Δr may also include experimental uncertainties in extracting r. So an estimate of the ratio defined in Eq. (4.8) can be made through the following approximation

$$\frac{\Gamma(\Lambda_b \to pl\nu)}{\Gamma(\Lambda_c \to \Lambda l\nu)} \approx R_0(r_0)[1 \pm \Delta r R_0'(r_0)/R_0(r_0) + \Delta_1(r_0) + \Delta_2(r_0) + ...]|\frac{V_{ub}}{V_{cs}}|^2 \tag{4.10}$$

For monopole and dipole form factors we then have

$$\frac{\Gamma(\Lambda_b \to pl\nu)}{\Gamma(\Lambda_c \to \Lambda l\nu)} \approx 196.65[1 \pm 0.78\Delta r + 0.02 + \Delta_2(r_0) + ...]|\frac{V_{ub}}{V_{cs}}|^2 \tag{4.11}$$

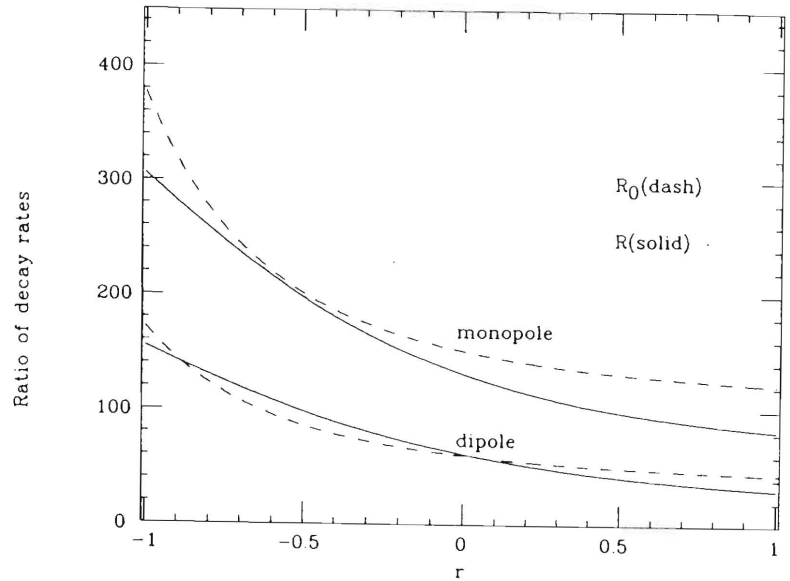

Figure 4.6: Ratio of $\Lambda_b \to pl\nu$ and $\Lambda_c \to \Lambda l\nu$ decay rates versus r.

and

$$\frac{\Gamma(\Lambda_b \to pl\nu)}{\Gamma(\Lambda_c \to \Lambda l\nu)} \approx 83.41[1 \pm 0.94\Delta r - 0.15 + \Delta_2(r_0) + ...]|\frac{V_{ub}}{V_{cs}}|^2 \tag{4.12}$$

Naively we can set $\Delta_2(r_0) \approx \Delta_1(r_0)\bar{\Lambda}/2m_Q$ and so we see that $\Delta_2(r_0) \gtrsim .05$ for the dipole form factors and $\Delta_2(r_0)$ is negligible for the monopole form factors. Measurement of the ratio R can be used to extract $|V_{ub}|/|V_{cs}|$. Clearly the largest uncertainty in R comes from the assumed q^2 dependence of the form factors and to reliably extract $|V_{ub}|/|V_{cs}|$ using R the q^2 dependence of the form factors must be measured. The assumptions made in the calculation of the form factors at $\omega = 1$ can be tested

by studying the decay distribution in $\Lambda_c \to \Lambda l\nu$. With enough statistical power such a study could determine the q^2 dependence of the form factors. Once the assumptions for the form factors at $\omega = 1$ is tested successfully by doing a fit to the $\Lambda_c \to \Lambda l\nu$ decay distribution, and r and information on the q^2 dependence of form factors extracted, we can use the ratio in Eq. (4.8) to determine $|V_{ub}|/|V_{cs}|$ with a small error.

It is interesting to compare this method of extracting $|V_{ub}|/|V_{cs}|$ and hence $|V_{ub}|$ with other methods of extracting $|V_{ub}|$. One method is to use semileptonic inclusive B decays and study the decay distribution near the end point of the spectrum in the narrow window $2.3 \leq E_l \leq 2.6$ GeV [11]. However, in this region the $b \to u$ transition is subject to large QCD radiative corrections, as well as large non-perturbative corrections which are entangled with each other, and theoretical calculations are model dependent. Another way to extract $|V_{ub}|$ is through the study of exclusive B decays [12] and this approach is similar to our approach with the difference that we are working with baryons instead of mesons. The calculations of form factors in exclusive decays of B mesons are done in specific models, like the quark model, or with a chiral Lagrangian with a heavy meson. In Ref. [13] it is suggested that $|V_{ub}|$ may be extracted from the ratio of the lepton distribution in $B \to \pi l\nu$ and $D \to \pi l\nu$ with a theoretical uncertainty of 10-20 %. The relevant form factors are calculated up to order $1/m_Q$ in the soft pion limit and they (the form factors) are expressed in terms of unknown matrix elements. Certain assumptions are made about these matrix elements to estimate the uncertainty in their calculations. In Ref. [14] the ratio of the lepton distribution in $B \to \rho l\nu$ and $D \to \rho l\nu$ at maximum q^2 is used for an extraction of $|V_{ub}|$. The $1/m_Q$ corrections are calculated in the quark model and the theoretical uncertainty in the measurement of $|V_{ub}|$ is small. However, at maximum q^2 the decay rates vanish. Actual measurements are performed away from

maximum q^2 where additional form factors contribute adding to the theoretical uncertainty in the measurement of $|V_{ub}|$. Another approach to measuring $|V_{ub}|$, which is the mesonic counterpart of our method, is given in Ref. ([15]) where experimental data on $D \to \bar{K}$ semi-leptonic decay are used to calculate the $B \to \pi$ semileptonic decay. The q^2 dependence of the form factors is assumed to be of the monopole type, no $1/m_Q$ corrections are considered and the extraction of $|V_{ub}|$ is found to depend on the value of f_{D_s} and f_B whose values are not known very accurately. Since most approaches to extracting $|V_{ub}|$ from B meson decays are model dependent and can involve significant theoretical uncertainties, extraction of $|V_{ub}|$ using heavy baryon decays is interesting as it is possible that theoretical uncertainties in heavy baryon decays might be small.

The predictions for the decay rates for $\Gamma(\Lambda_c \to \Lambda l\nu)$ and $\Gamma(\Lambda_b \to pl\nu)$ are subject to significant $1/m_Q$ corrections and may not be reliable. Nevertheless we give the predictions for the decay rates for $r = -0.47$ and $F^0(\omega = 1) = 0.46$[6] obtained in chapter 3.

$$\Gamma(\Lambda_c \to \Lambda l\nu) = 5.36 \times 10^{10} s^{-1} \tag{4.13}$$

$$\Gamma(\Lambda_b \to pl\nu) = |V_{ub}|^2 6.48 \times 10^{12} s^{-1} \tag{4.14}$$

The Particle Data Group gives $\Gamma(\Lambda_c \to \Lambda e^+ X) = (7.0 \pm 2.5) \times 10^{10} s^{-1}$. A measurement of $\Gamma(\Lambda_b \to pl\nu)$ is likely to be made at LEP in the future. The numbers for the individual decay rates are calculated using $|V_{cs}| = 0.9745$ [9], the pole masses $m_V \cong 5.37$ GeV and $m_A \cong 5.80$ GeV[10] and the quark masses $m_b = 4.74$ GeV and $m_u = 0.005$ GeV[8]

In conclusion in this chapter we have studied the semileptonic decays of charmed and bottom baryons involving heavy to light quark transition based on a model for form factors that includes $1/m_Q$ corrections. We also suggest a method for determin-

ing $|V_{ub}|/|V_{cs}|$.

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CHAPTER 5

GENERAL INTRODUCTION TO GRAND UNIFIED THEORIES

The Standard Model of particle physics has now a strong experimental basis. The Standard Model consists of QCD, which provides a consistent picture of both the dynamics and symmetries of the strong interactions, and the electroweak interaction which represents the unification of the electromagnetic and the weak interactions. All the interactions are described by gauge theories and mediated by vector bosons. The Standard Model represents an impressive unification in our ideas of the strong, electromagnetic and weak interactions. There are, however, some unanswered questions. Although the model combines the weak and the electromagnetic interactions in the group $SU(2) \times U(1)$ there is an arbitrary parameter that describes the coupling of the two groups. Furthermore there is no understanding of why the electric charges come in discrete units of $e/3$. With the addition of the strong interactions the group is enlarged to $SU(3) \times SU(2) \times U(1)$ with no connection between the strong coupling and the electroweak couplings. The Standard Model also contains sixteen other parameters besides the three gauge couplings *viz.* the six quark and three lepton masses, three mixing angles and a phase parameterizing CP violation, the two boson masses M_W and M_{Higgs} and finally θ_{QCD} which describes potential violation of CP in the QCD Lagrangian. There is no understanding of the matter structure in the Standard Model with no explanation of the mass spectrum or the existence of three families. These questions suggest that the Standard Model is not a fundamental theory but is at best an effective theory derived from an underlying fundamental theory. One can

then consider two scenarios. The first is that some or all the fields of the Standard Model may be composite and there is some more fundamental level of structure. The second possibility is that the fields of the Standard model are fundamental but are related by additional symmetries broken at some high energy scale. The second approach leads to grand unified theories. Some of the questions posed above about the Standard Model can be answered if a simple group can accommodate the strong and the electroweak interactions , because then there is only one coupling constant and all internal quantum numbers are quantized. To achieve this $SU(3) \times SU(2) \times U(1)$ must be embedded in a larger simple group, the so-called grand unified group. The Standard Model then emerges as an effective theory of the original grand unified theory at the low energy scale $\sim$ 100 GeV.

The first attempt at lepton-quark unification was made by Pati and Salam [1] who unified the quarks and leptons within the group $SU(2)_L \times SU(2)_R \times SU(4)_c$ by extending the color gauge group to include the leptons. While this model explained charge quantization there were still three coupling constants g_{2L}, g_{2R} and g_{4c} in the model. The first GUT based on a simple group, the group SU(5), was proposed by Georgi and Glashow [2]. There has been a lot of activity in grand unified theories (GUT) following the precise measurement of the Standard model coupling constants at the Z mass scale at LEP. It was shown by Amaldi *et. al.*[3] that if one extrapolated the measured Standard Model couplings to the GUT scale in supersymmetric $SU(5)$ grand unified theory (SUSY $SU(5)$ GUT) then they meet at a single point, signifying unification. The minimal nonsupersymmetric $SU(5)$ cannot be made consistent with the LEP measurements and the proton decay results.

The basic rules to be followed for the construction of a GUT are the following:

1) Choose a suitable gauge group G. The requirement of gauge invariance then

specifies the gauge bosons of the theory.

2) Choose the fermion representation so that the standard $SU(3) \times SU(2) \times U(1)$ low energy structure emerges. The couplings of the fermions to the gauge bosons are then fixed by local gauge invariance.

3) Assign the scalars of the theory in the appropriate representations and choose the proper couplings so that a phenomenologically acceptable pattern of symmetry breaking of G to $SU(3) \times SU(2) \times U(1)$ is achieved. We choose the minimum number of scalar representations necessary to provide the necessary symmetry breaking described above.

4) Specify the Yukawa couplings of the theory so that an acceptable pattern of fermion masses and mixings emerge after spontaneous symmetry breaking.

Grand unified theories solve some of the problems of the Standard Model. Charge quantization is explained and in GUT all the coupling constants of the Standard Model are related. However there is still no good explanation of the family structure. There is no prediction for the KM angles unless extra discrete symmetries are assumed. There is also no improvement in the scalar sector as more parameters are needed in the scalar potential than are present in the Standard Model. The appearance of a large mass scale (M_X) where unification takes place explains why proton decay is slow but in GUT there is no explanation of why this scale should be so much greater than the weak scale. In nonsupersymmetric guts the existence of two very different mass scales leads to the hierarchy problem as it becomes necessary to fine tune parameters in the scalar potential to one part in $10^{13} - 10^{14}$. Finally grand unified theories do not include gravity even though the unification scale is close to the Planck mass.

5.1 Choosing The Gauge Group

The Standard Model ($SU(3)\times SU(2)\times U(1)$) has four diagonal generators corresponding to T_3 and T_8 of color, T_3 of weak isospin and Y the hypercharge generator. The observed particles carry definite values of these quantum numbers. Any group that contains the Standard Model should therefore have at least four diagonal generators or in other words it must have a rank of at least four. Furthermore the group should have complex representations necessary to accommodate the SU(3) complex triplet and the complex doublet fermion representation. The possible rank four groups are $[SU(2)]^4$, $[O(5)]^2$, $[G_2]^2$, $O(8)$, $O(9)$, $Sp(8)$, F_4, $[SU(3)]^2$ and $SU(5)$. The first two do not contain $SU(3)$ and the next five do not have complex representations, and so, are ruled out as candidates for the GUT group. For the case of $[SU(3)]^2$ the $SU(2)\times U(1)$ would come from an $SU(3)$ and so the charge operator would be a traceless generator of $SU(3)$. This would mean that the sum of quark charges would be zero which is inconsistent with the charges of the observed quarks. Hence $SU(5)$ emerges as the unique rank four candidate for a grand unified group.

Of course we can also consider groups with rank greater than four and there are certain shortcomings of the $SU(5)$ group to motivate such considerations. If we insist on the GUT group being simple then we have limited classes of simple groups at our disposal. They are $SU(n+1)$, $SO(2n)$, $SO(2n+1)$ and $Sp(2n)$ where n is a positive integer and then there are the exceptional groups namely G_2, F_4, E_6, E_7 and E_8 (the subscript represents the rank of the group). If we further insist that the group have complex representation then the only possibilities are $SU(n+1)$, $SO(4n+2 = 2(2n+1))$ and E_6. It should be pointed out that it is possible to construct GUT's with fermions in the real representation provided we allow extra mirror fermions in the theory. This would then require the explanation of why the

mirror fermions are heavy and the conventional fermions light. The necessity of mirror fermions in real representations can be seen very easily by the following argument. In GUT it is convenient to express all fields in terms of a definite chirality. So instead of working with ψ_L and ψ_R one works with ψ_L and $\psi_L^c = \sigma_2\psi_R^*$ or with ψ_R and $\psi_R^c = \sigma_2\psi_L^*$. In terms of left handed fields the fermions in the Standard Model have the following transformation under the $SU(3)\times SU(2)\times U(1)$

$$f_L = N_g \times [(3,2,1/3) + (\bar{3},1,-4/3) + (\bar{3},1,2/3) + (1,2,-1) + (1,1,1)] \quad (5.1)$$

where N_g is the number of generations. In terms of the right handed fields one can write

$$f_R^c = N_g \times [(\bar{3},2,-1/3) + (\bar{3},1,4/3) + (\bar{3},1,-2/3) + (1,2,1) + (1,1,1)] \quad (5.2)$$

In a real representation f_L and its conjugate representation f_L^* transform in the same way (their transformations are equivalent). This also means that the transformation of f_L and $\sigma_2 f_L^* = f_R^c$ are similar (note σ_2 acts on the Dirac space and not on the symmetry group space). Clearly the fermionic representation written above does not satisfy the condition that f_L and f_R^c are equivalent. To remedy the situation we have to add mirror fermions F_L to f_L such that F_L transforms as $f_R^c(F_R^C \equiv f_L)$ so that the representation $f_L + F_L$ is now real.

5.2 Phenomenology Of General GUT

Two crucial predictions from GUT are gauge coupling unification and the prediction for proton decay. In the following figure we show the unification of coupling constants observed recently after the precision measurement of the gauge couplings at LEP. Once the gauge couplings are known at a certain energy scale then one can use the

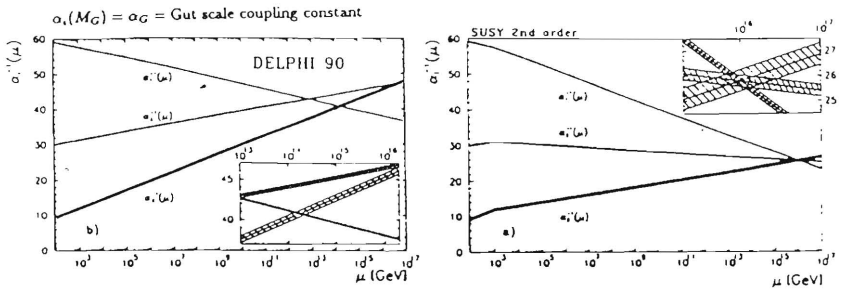

Figure 5.1: Evolution of the coupling constants in minimal and supersymmetric $SU(5)$ GUT.

renormalization group evolution (RGE) to evolve the couplings up to the unification scale. If we define

$$\vec{\alpha}^{-1}(M_Z) = (\alpha_1(M_Z), \alpha_2(M_Z), \alpha_3(M_Z)) \tag{5.3}$$

with

$$\alpha_1^{-1}(M_Z) = \frac{3}{5}\frac{\cos^2\theta_W}{\alpha_{em}(M_Z)} \tag{5.4}$$

$$\alpha_2^{-1}(M_Z) = \frac{\sin^2\theta_W}{\alpha_{em}(M_Z)} \tag{5.5}$$

$$\alpha_3(M_Z) = \alpha_{strong}(M_Z) \tag{5.6}$$

and

$$\vec{\alpha}^{-1}(M_X) = \alpha_X^{-1}(1,1,1) \tag{5.7}$$

where α_X is the gauge coupling at the unification scale then we can write the evolution equation (at one loop) as

$$\vec{\alpha}^{-1}(M_Z) = \vec{\alpha}^{-1}(M_X) + \vec{\beta}\ln\frac{M_X}{M_Z} \tag{5.8}$$

where $\vec{\beta}$ contains the beta functions for the three gauge couplings. From the above equation if one is given any two of the coupling constants (generally α_1 and α_2) then the third (generally α_s) can be predicted and one can also predict the unification scale. If one uses the beta functions for the suspersymmetric $SU(5)$ GUT then the gauge couplings have been observed to meet at $M_X \sim 10^{16}$ GeV. Minimal nonsupersymmetric $SU(5)$ seems to be ruled out by this kind of analysis. One can also obtain gauge unification with other GUT groups which have additional intermediate scales and extra particles, for example, like the group $SO(10)$.

The next crucial prediction of GUT's is that of nucleon decay. The origin of this process in general GUT can be understood in the following manner. A general GUT contains several heavy fields besides the particles in the Standard Model. These heavy particles have mass $\sim M_X$ which is the unification scale. When the heavy fields are integrated out to construct the low energy effective theory we end up with the Lagrangian of the Standard model as well as higher dimension terms suppressed by the heavy mass scale. We can write the effective Lagrangian as

$$L_{eff} = \sum_{i,d} C_i\left(\frac{1}{M_{X_i}}\right)^{d-4} O_i^d \tag{5.9}$$

where O_i^d are $SU(3) \times SU(2) \times U(1)$ and Lorentz invariant operators and d is the dimension of the operator. These operators are constructed out of the light fields *viz.* the light fermions, the gauge bosons and the light Higgs. M_{X_i} is generic for the exchange particle mass that generates the operator O_i^d. To construct all possible operators with the correct invariance properties it is convenient to define a F parity

[4]. Let (a, b) denote the representation of a given state under the two $SU(2)$ groups that define the Lorentz transformation. Lorentz invariance requires $(-1)^{2a}$ and $(-1)^{2b}$ to be multiplicatively conserved. Similarly invariance under weak isospin requires $(-1)^{2T}$ to be multiplicatively conserved. One can define F parity by

$$F = (-1)^{2T+2a} \tag{5.10}$$

All allowed operators should have $F = 1$. The F quantum numbers of the fields in the Standard Model are the following. The fermions ψ_L and ψ_R have $F = 1$ while the charge conjugate states have $F = -1$. All the bosons, the vector bosons and the Higgs, have $F = -1$. The allowed operators must also be invariant under the color $SU(3)$. In non supersymmetric GUT the lowest operator having baryon number 1 and having the correct invariance property is the dimension six operator of the form $qqql$ and this operator generates nucleon decay. This is not true in supersymmetric GUT where nucleon decay can proceed through the exchange of a heavy Higgsino and the corresponding operator has dimension five. Some proton decay diagrams are shown in Fig. 5.2. Other interesting effects in general GUT's which can be generated by higher dimensional operators are neutrino masses and $n - \bar{n}$ oscillations. The next two sections will be devoted to the study of $SU(5)$ and the $SO(10)$ groups, which are of interest to us here.

5.3 SU(5)

$SU(5)$ is the group of 5×5 complex unitary matrices with determinant $= 1$. The unitarity and unimodularity conditions lead to 24 independent matrices. The defining relation for $SU(5)$ is

$$\psi' = U\psi \tag{5.11}$$

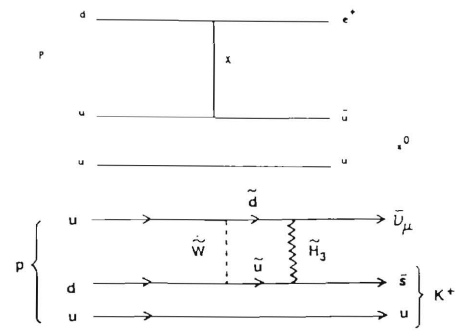

Figure 5.2: Proton decay diagrams in Minimal $SU(5)$ via vector boson(X) exchange and SUSY $SU(5)$ via higgisino($\tilde{H}_3$) exchange. The Wino ($\tilde{W}$) "dressing" converts quarks to squarks. There are additional diagrams with $\bar{\nu}_e$, $\bar{\nu}_\tau$ in the final states. The CKM matrices appear at the Wino vertex allowing all three generations to enter in the loop.

such that

$$\psi'^\dagger \psi' = \psi^\dagger \psi \tag{5.12}$$

A general matrix of $SU(5)$ can then be written as

$$U = \exp(-i \sum_{i=1,24} \beta_i L_i) \tag{5.13}$$

where L_i are the 24 hermitian, traceless generators of the group and the β_i's are the parameters of the transformation. Further, the generators are normalized to

$$Tr(L^a L^b) = 2\delta^{ab} \tag{5.14}$$

A choice of the generators is given as

$$L^a = \begin{pmatrix} \lambda^a & 0 \\ 0 & 0 \end{pmatrix} \tag{5.15}$$

where λ^a (a=1 to 8) are the usual Gell-Mann matrices. and

$$L^{9,10,11} = \begin{pmatrix} 0 & 0 \\ 0 & \sigma^{1,2,3} \end{pmatrix} \tag{5.16}$$

The diagonal generators of the group are L^3, L^8 L^{11} and L^{12} where

$$L^{11} = \mathrm{Diag}(0,0,0,1,-1) \tag{5.17}$$

and

$$L^{12} = \frac{1}{\sqrt{15}}\mathrm{Diag}(-2,-2,-2,3,3) \tag{5.18}$$

The gauge bosons associated with the generators above(see eqn (5.23) are the gauge bosons of the Standard Model. The generators associated with the extra heavy fields of the theory have the form

$$L^{13,15\ldots 23} = \begin{pmatrix} 0 & X_i \\ X_i^\dagger & 0 \end{pmatrix} \tag{5.19}$$

and

$$L^{14,16\ldots 24} = \begin{pmatrix} 0 & Y_i \\ Y_i^\dagger & 0 \end{pmatrix} \tag{5.20}$$

where the X_i's are 3×2 matrices with all elements zero except one which has the value 1. For example

$$X_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{5.21}$$

and so on. The Y_i's are obtained from X_i's via

$$Y_i = (-1)^{i+1} X_i \tag{5.22}$$

The vector boson matrix V_μ is defined through

$$\frac{V_\mu}{\sqrt{2}} = \frac{1}{2}\sum_a V_\mu^a L^a \tag{5.23}$$

Explicitly the V_μ matrix can be written as

$$V_\mu = \begin{pmatrix} g_{11} - \frac{2B}{\sqrt{30}} & g_{12} & g_{13} & \bar{X}_1 & \bar{Y}_1 \\ g_{21} & g_{22} - \frac{2B}{\sqrt{30}} & g_{23} & \bar{X}_2 & \bar{Y}_2 \\ g_{31} & g_{32} & g_{33} - \frac{2B}{\sqrt{30}} & \bar{X}_3 & \bar{Y}_3 \\ X_1 & X_2 & X_3 & \frac{W_3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} & W^+ \\ Y_1 & Y_2 & Y_3 & W^- & -\frac{W_3}{\sqrt{2}} + \frac{3B}{\sqrt{30}} \end{pmatrix} \tag{5.24}$$

where the g's and the W are the gluons and the weak gauge bosons of the Standard Model. The fermions of the Standard Model are placed in two separate representation of $SU(5)$ *viz.* in the $\bar{5}$ and in the 10. The fundamental 5 representation decomposes under $SU(3) \times SU(2)$ as

$$5 = (3,1) \oplus (1,2) \tag{5.25}$$

Because the charge operator can be written as a linear combination of the diagonal generators, the trace of the charge operator Q is zero

$$\sum_a \langle a|Q|a\rangle = 0 \tag{5.26}$$

This leads to the identification of the fields in the $\bar{5}$ representation as

$$\bar{5} = \begin{pmatrix} d_1 \\ d_2 \\ d_3 \\ e^c \\ -\nu^c \end{pmatrix}_R \tag{5.27}$$

The interaction of this representation can be written as

$$L \sim i\bar{\psi}_{5\alpha}\gamma^\mu[\partial_\mu \delta^{\alpha\beta} - ig(\frac{V_\mu}{\sqrt{2}})^{\alpha\beta}]\psi_{5\beta} \tag{5.28}$$

The remaining fermions are put in the 10 representation of $SU(5)$. The 10 can be obtained as an irreducible representation in the decomposition of the reducible representation constructed by multiplying two fundamental representations

$$5 \otimes 5 = 15 \oplus 10 \tag{5.29}$$

This representation is antisymmetric under the group indices and its transformation can be written as

$$\chi'_{ij} = [\delta_{ik}\delta_{jl} - i\frac{g}{2}\alpha^a(\Lambda^a)_{ij;kl}]\chi_{kl} \tag{5.30}$$

where

$$(\Lambda^a)_{ij;kl} = \frac{1}{2}[\delta_{ik}(L^a)_{jl} + \delta_{jl}(L^a)_{ik} - \delta_{jk}(L^a)_{il} - \delta_{il}(L^a)_{jk}] \tag{5.31}$$

Explicitly we can write the 10 representation as

$$\chi = \begin{pmatrix} 0 & u_3^c & u_2^c & u_1^c & d_1^c \\ u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & d_3 \\ u_1 & u_2 & u_3 & 0 & -e^c \\ d_1 & d_2 & d_3 & e^c & 0 \end{pmatrix}_L \tag{5.32}$$

The interaction Lagrangian for the 10 is given by

$$L \sim i\frac{1}{2}\mathrm{Tr}[\bar{\chi}_{10}\gamma^\mu[\partial_\mu - 2ig(\frac{V_\mu}{\sqrt{2}})]\chi_{10}] \tag{5.33}$$

The interaction of the fermions with the X and Y bosons are shown in the following figure.

We now turn to the question of the spontaneous breaking of the $SU(5)$. There are two symmetry breakings that we have to consider. The first is the breaking of $SU(5)$ to the Standard model group. This is achieved by introducing Higgs fields Σ in the adjoint representation which acquires a *vev* to produce the correct breaking of $SU(5)$. A Higgs field in the 5 representation (which contains the Higgs doublet of the Standard Model) is included to provide the second breaking of the electroweak interactions to electromagnetism. We start by considering the Σ field and its interactions. The covariant derivative for the Σ field is given by

$$D_\mu\Sigma = \partial_\mu\Sigma - \frac{ig}{\sqrt{2}}[V_\mu, \Sigma] \tag{5.34}$$

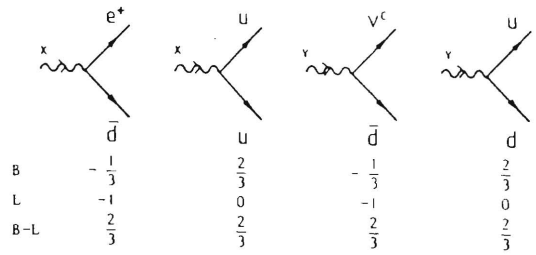

Figure 5.3: X and Y boson couplings to the fermions.

where, like the vector boson field V_μ, the Σ field can be written in terms of its components as

$$\frac{\Sigma}{\sqrt{2}} = \frac{1}{2}\sum_a \Sigma^a L^a \tag{5.35}$$

The kinetic part of the gauge invariant Lagrangian can be written as

$$L_\Sigma = \frac{1}{2}\mathrm{Tr}[(D_\mu\Sigma)^\dagger(D^\mu\Sigma)] \tag{5.36}$$

The most general potentials containing up to quartic terms for the Σ field is (imposing the discrete symmetry $\Sigma \to -\Sigma$)

$$V(\Sigma) = -\mu^2\mathrm{Tr}(\Sigma^2) + \frac{1}{4}a[\mathrm{Tr}(\Sigma^2)]^2 + \frac{1}{2}bTr(\Sigma^4) \tag{5.37}$$

Note that the discrete symmetry simplifies the potential and the subsequent analysis for minima, but is not essential in constructing the potential. The unique minimum of this potential is obtained for the choice

$$\langle 0|\Sigma|0\rangle = \sqrt{2}v\mathrm{Diag}(1, 1, 1, -3/2, -3/2) \tag{5.38}$$

for $\mu^2 > 0$, $b > 0$, and $a \geq (-7/15)b$.

The kinetic part of the Lagrangian then generates masses for the heavy X and Y bosons

$$m_{X_i}^2 = m_{Y_i}^2 = \frac{25}{8} g^2 v^2 \tag{5.39}$$

The electroweak symmetry is broken by introducing a 5 of Higgs

$$H = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \\ h^+ \\ -h_0 \end{pmatrix} \tag{5.40}$$

and we introduce the potential

$$V(H) = -\frac{1}{2}\mu'^2 |H|^2 + \frac{\lambda}{4}(|H|^2)^2 \tag{5.41}$$

Minimizing the potential one ends up with

$$\langle 0| - h_0 |0\rangle = v_0 \tag{5.42}$$

$$\mu'^2 = \lambda v_0^2 \tag{5.43}$$

and the W,Z acquire masses

$$M_W^2 = M_Z^2 \cos^2\theta_W = \frac{1}{4} g_2^2 v_0^2 \tag{5.44}$$

At tree level the color triplet Higgs $\vec{h} = (h_1, h_2, h_3)$ is massless and this is a problem because these Higgs can cause proton decay. These colored Higgs can obtain mass through the coupling of the Σ and the H field. Even if such coupling terms are absent at the tree level they can be generated radiatively through the following graphs shown in Fig. 5.4

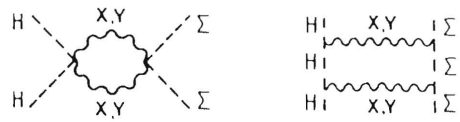

Figure 5.4: Examples of Feynman graphs that can lead to Σ H couplings.

Moreover, these diagrams are divergent and so one must have bare coupling terms at tree level to provide the necessary counterterms to cancel these divergences. We therefore add an extra piece to the potential

$$V(\Sigma, H) = \alpha |H|^2 Tr(\Sigma^2) + \beta H^\dagger \Sigma^2 H \tag{5.45}$$

The minimization of the full potential then leads to to the following two conditions

$$\mu^2 = \frac{15}{2} a v^2 + \frac{7}{2} b v^2 + \alpha v_0^2 + + \frac{9}{30} \beta v_0^2 \tag{5.46}$$

$$\mu'^2 = \frac{1}{2}\lambda v_0^2 + +15\alpha v^2 + +(\frac{9}{2} - 3\epsilon)\beta v^2 \tag{5.47}$$

where

$$\epsilon \approx \frac{3}{20} \frac{\beta v_0^2}{6 v^2} \tag{5.48}$$

and the vev of the Σ field is given by

$$\langle 0|\Sigma|0\rangle = \sqrt{2} v \mathrm{Diag}(1, 1, 1, -3/2 - (1/2)\epsilon, -3/2) + (1/2)\epsilon) \tag{5.49}$$

The formulae for the W and Z masses are changed by the presence of the ϵ term but because such terms are small these corrections can be neglected. However we see from the equation for μ'^2 that there must be delicate cancellation, about one part in

$10^{13} - 10^{14}$, between this term and the $v^2 \sim m_X^2$ terms to obtain the v_0^2 terms which is of the order of the weak scale. This is the fine tuning problem and can be resolved if we introduce supersymmetry.

5.4 Supersymmetric GUT

The chief motivation for introducing supersymmetry in particle physics is to solve the fine tuning problem problem. We already encountered this problem in the previous section and here we look at the same problem again in the context of the Standard Model. The loop corrections to the Standard Model Higgs are quadratically divergent leading to a mass

$$m_H^2 \approx m_{bare}^2 + cg^2\Lambda^2 \tag{5.50}$$

where Λ is the cutoff. If we assume the Standard Model to describe physics up to the GUT scale then $\Lambda = M_G \sim 10^{16}$ GeV. Therefore, to obtain the correct order for the mass of the Higgs, which is of the weak scale, we require delicate cancellations (one part in 10^{14}) between the bare mass term and the cutoff term. Alternatively one can interpret the above relation to indicate that the Standard Model is not the right theory up to the GUT scale and we should expect new physics beyond a cutoff of $\Lambda \sim O(TeV)$. Supersymmetry(SUSY) is one of the several extensions of the Standard Model which can solve this hierarchy problem. In SUSY theories there is a symmetry between the fermionic and the bosonic degrees of freedom. The SUSY algebra is generated by spinorial charges Q_α which relate fermionic and bosonic states

$$Q|\text{Fermion}\rangle = |\text{Boson}\rangle \tag{5.51}$$

The Q's satisfy

$$[Q_\alpha, Q_\beta^\dagger]_+ = -2[(\frac{(1-\gamma_5)}{2}\gamma^\mu\gamma_0]P_\mu \tag{5.52}$$

$$[Q_\alpha, Q_\beta]_+ = [Q_\alpha, P^\mu] = 0 \tag{5.53}$$

$P^2 = M^2$ is a Casimir operator of the algebra and so all states in a SUSY multiplet have the same mass. Further taking the trace of the first anticommutator relation written above one obtains

$$P^0 = H = \frac{1}{4}(QQ^\dagger + Q^\dagger Q) \geq 0 \tag{5.54}$$

The inequality in the above eqn is to be understood at the level of matrix elements. Hence the Hamiltonian of SUSY theories is positive definite. The simplest SUSY multiplets that are required for constructing models are the chiral multiplets and the vector multiplets. Ignoring the auxiliary fields the left(right) chiral multiplet contains a complex scalar and a Weyl left(right) spinor. The vector multiplet contains a real vector field V_μ and a Majorana spinor λ. The Minimal Supersymmetric Standard Model(MSSM) is the simplest extension of the Standard Model which is constructed by promoting each particle in the Standard Model to either a chiral or a vector multiplet. In following Fig. 5.5 we show the particles in MSSM [5]

The fine tuning problem now can be solved because the Higgs has interactions not only with the quarks and gauge bosons but also with the scalar partners of the quarks, the squarks, and the fermionic partner of the gauge bosons, the gauginos. The diagrams contributing to the Higgs mass in the SUSY case is shown in the next Fig. 5.6

The cancellation of all divergences depends on the couplings of the theory (dictated by SUSY) and the equality of the masses within a given supermultiplet. However the equality of the masses of the particles and their superpartners is obviously wrong as no SUSY particles have been observed and hence SUSY has to be broken. Spontaneous symmetry breaking(SSB) of SUSY is difficult because if the vacuum is

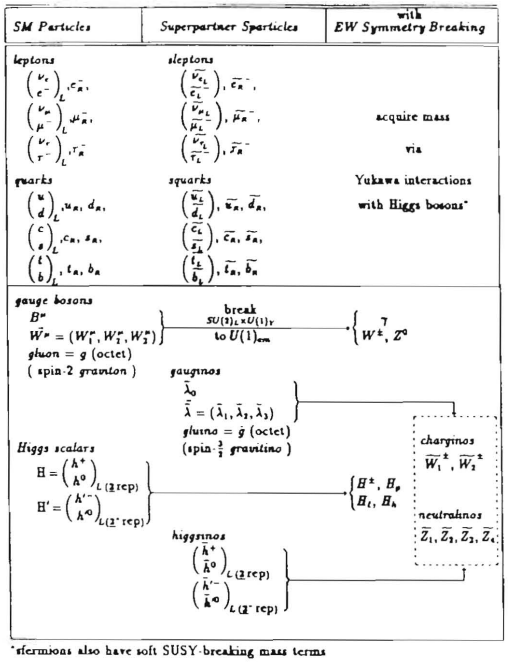

Figure 5.5: Particle contents of MSSM.

supersymmetric ($Q_\alpha|0\rangle = 0$) then the energy of the vacuum is 0 and is a global minimum because we already saw that the energy is positive definite in SUSY theories. So if the vacuum state is not supersymmetric($Q_\alpha|0\rangle \neq 0$) then the energy is greater than zero and the broken vacuum will have a higher energy than the symmetric state. Hence to have SSB of SUSY one has arrange it so that the symmetric vacuum does not exist. However most models of SSB of SUSY do not lead to realistic mass relations. Another way is to break SUSY explicitly by soft breaking terms which do not destroy the solution of the hierarchy problem. A list of all possible soft breaking terms with this property were classified by Girardello and Grisaru [6]. With the addition of the

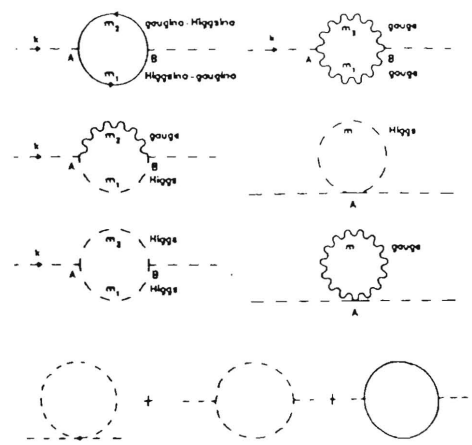

Figure 5.6: One loop contributions to the Higgs mass in SUSY models. The quarks and squarks contributions (which dominate the radiative corrections to the Higgs mass) are shown on the bottom of the figure with the dashed lines in the loop representing the squarks and the solid lines in the loop representing the quarks.

soft breaking terms the cutoff in the equation for the h Higgs mass gets replaced by

$$\Lambda^2 \rightarrow (m_{\tilde{q}}^2 - m_q^2) \ln \frac{\Lambda^2}{m_{\tilde{q}}^2} \tag{5.55}$$

Hence to avoid the fine tuning problem the squark masses should be around a TeV. Thus SUSY particles are likely to be accessible in the next generation of accelerators [7]. The origin of the soft breaking terms can be understood in terms of supergravity (SUGRA) models where SUSY is broken at a high scale when a gauge singlet fields, which couples to the observable physical fields(particles and their superpartners) only through gravity, acquires a *vev*. When the heavy fields in this theory are integrated out then one ends up with the required soft breaking terms which can be expressed in terms of a few parameters. The masses of the superparticles can now be expressed

in terms of these parameters. The masses of the superparticles are important inputs in the study of SUSY GUTS. In the next section we discuss briefly SUSY GUT's and SUGRA models.

5.5 SUSY GUT And SUGRA Models

The construction of the SUSY SU(5) GUT is done by adding the super partners of the Σ, X ,Y and the H fields .However we need two Higgs, H_1 and H_2 to give masses to the fermions as the superpotential has to be analytic. Also, anomaly cancellation from the fermionic Higgsino requires the existence of two Higgs doublets. After electroweak symmetry breaking one has now 5 physical higgs boson; a pair of charged Higgs, two neutral CP even states and one CP odd neutral state. The fine tuning of the Higgs mass is solved in SUSY GUT provided again that the supersymmetric particles have masses around a TeV. We now give a short discussion of SUGRA models.

To construct a SUGRA model one has to promote supersymmetry from a global symmetry to a local symmetry. If we start from a noninteracting SUSY Lagrangian

$$L = -\partial_\mu z^\dagger \partial^\mu z + \bar{\chi}(i)\partial\!\!\!/\chi \tag{5.56}$$

Then the Lagrangian is invariant under global SUSY transformations but not when we consider local SUSY transformations

$$\delta z(x) = \bar{\epsilon}\chi(x) \quad ; \quad \delta\chi(x) = (-i)[\partial\!\!\!/ z]\epsilon(x) \tag{5.57}$$

Invariance under the local transformation can be obtained by adding a spin 3/2 field and a spin 2 field (ψ^μ_α and $g^{\mu\nu}$) transforming as

$$\delta\psi^\mu = \kappa^{-1}\partial^\mu\epsilon(x) \tag{5.58}$$

$$\delta g^{\mu\nu} = \kappa\bar{\psi}^\mu\gamma^\nu\epsilon(x) \tag{5.59}$$

where κ has the dimension of inverse mass and this mass is identified as the Planck mass. The Lagrangian has to be augmented by interaction terms between the chiral multiplet and the supergravity multiplet as

$$L_{int} = -\kappa\bar{\psi}_\nu\gamma^\nu[\partial\!\!\!/ z]\chi(x) - g^{\mu\nu}T_{\mu\nu} \tag{5.60}$$

where $T_{\mu\nu}$ is the stress tensor for the scalar field. Now we have to consider Lagrangians which are not only invariant under local supersymmetry but also invariant under general co-ordinate transformations. Starting with the supergravity multiplet($g_{\mu\nu}, \psi_\mu$) we can write the SUGRA Lagrangian as [8].

$$L_{SG} = -\frac{1}{2}\kappa^{-2}eR(e^m_\mu, w^{mn}_\mu) - \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\rho\psi_\sigma \tag{5.61}$$

where the vierbein e^m_μ are introduced through

$$g_{\mu\nu} = e^m_\mu(x)\eta_{mn}e^n_\mu(x) \tag{5.62}$$

Here η is the flat metric in the local inertial frame at the point x and $g_{\mu\nu}$ is the metric tensor. R is the curvature scalar and

$$e = \det[e^m_\mu(x)] \tag{5.63}$$

D_ρ is defined by

$$D_\rho = \partial_\rho + \frac{1}{2}w^{mn}_\rho\sigma_{mn} \tag{5.64}$$

where w^{mn}_ρ are the spin connections. The Lagrangian is invariant under local SUSY transformations

$$\delta\psi^\mu = \kappa^{-1}D^\mu\epsilon(x) \tag{5.65}$$

and

$$\delta e^m_\mu(x) = \frac{1}{2}\kappa\bar{\epsilon}(x)\gamma^m\psi_\mu \tag{5.66}$$

The problem of coupling gravity to matter and maintaining supergravity invariance is generally quite complicated [9, 10]. The general couplings of $N = 1$ SUGRA depend on three functions, the superpotential $\tilde{W}$, the Kahler potential $d(z, z^\dagger)$ and the gauge kinetic function $f_{\alpha\beta}$ where α, β are adjoint representation group indices. The full super potential in the SUGRA model is not positive definite although in the limit $\kappa \to 0$ the effective potential does reduce to the form for a potential with global supersymmetry. However Planck scale effects, are crucial because they allow SUSY to be broken in a natural way. Because the potential is not positive definite it can easily accommodate SUSY breaking and this happens when the "super Higgs" fields z acquire a $vev \sim O(M_{Pl})$. In SUGRA GUT models the super Higgs particle is a gauge singlet and couples to the regular fields of MSSM through gravity. This coupling is suppressed by the Planck scale. This prevents large Planck scale masses to be communicated to the physical fields due to the SSB of SUSY around the Planck scale. When one integrates out the super Higgs fields as well as the heavy fields at the GUT scale then at lower energies one ends up with an effective potential given by [12]

$$V = \sum_a |\frac{\partial W}{\partial z_a}|^2 + V_D + [m_0^2 z_a z_a^\dagger + (A_0 W^3 + B_0 W^2) + h.c] \tag{5.67}$$

where z_a are the physical scalar fields and

$$V_D = \frac{1}{2}\sum_{i,a} g_i^2 D_i^a D_i^a \tag{5.68}$$

$$D_i^a = \sum_m z_m^\dagger T_i^a z_m \tag{5.69}$$

the superpotential W is given by

$$W = \mu_0 H_1 H_2 + [\lambda^{(u)} q_i H_2 u_j^C + \lambda^{(d)} q_i H_1 d_j^C + \lambda^{(e)} l_i H_2 e_j^C] \tag{5.70}$$

This is the usual Yukawa interaction(we have assumed R-parity invariance) and W^2 and W^3 in the potential above are just the quadratic and the trilinear term in W but with the superfields replaced by the scalar components. There arises also a gaugino mass term in the Lagrangian

$$L_{gaugino} = -m_{1/2}\bar{\lambda}^\alpha \lambda_\alpha \tag{5.71}$$

Using RGE evolution [11] one can now predict the SUSY spectrum in terms of the parameters $m_0, m_{1/2}, A_0, B_0$ and μ_0. An attractive feature of such models is that for large top quark Yukawa coupling one of the Higgs mass can be driven negative by the RGE evolution from the GUT scale to the weak scale thus signalling electroweak symmetry breaking. The requirement of electroweak symmetry breaking fixes μ_0 up to a sign and so the low energy physics can be described in terms of $m_0, m_{1/2}, A_0, \tan\beta = v_2/v_1 (< H_i >= v_i)$ where the parameter B_0 has been eliminated in terms of $\tan\beta$. We turn to the study of the $SO(10)$ group in the next section.

5.6 SO(10) Group

$SO(n)$ is the group of real rotations in n-dimensions. It is the set of $n \times n$ orthogonal matrices with $\det R = 1$ (R is the group element). The defining relation for $SO(n)$ is

$$\phi' = R\phi \tag{5.72}$$

where ϕ is a real column vector such that $\phi^T\phi$ is invariant which leads to

$$R^T R = 1 \tag{5.73}$$

One can write R as

$$R = \exp[\frac{1}{2}\omega_{ab} M^{ab}] \tag{5.74}$$

where $[w_{ab}]$, the matrix of the parameters of the transformation, is antisymmetric. A choice for the generators M is

$$(M_{ab})_{ij} = -i(\delta_{ai}\delta_{bj} - \delta_{aj}\delta_{bi}) \tag{5.75}$$

and the M's satisfy the commutation relation

$$[M_{ab}, M_{cd}] = -i[\delta_{bc}M_{ad} - \delta_{ac}M_{bd} - \delta_{bd}M_{ac} - \delta_{ad}M_{bc}] \tag{5.76}$$

For the group $SO(2r)$ and $SO(2r+1)$ there are r commuting generators which can be taken as $M_{12}, M_{34} \ldots M_{2r-1,2r}$ and so the rank of both these groups is r.

We will now specialize the discussion to the $SO(10)$ group. This group has as its maximal subgroups $SU(5) \times U(1)$ and $SU(4) \times SU(2)_L \times SU(2)_R$(Pati-Salam group) and $SO(10)$ can break into these two groups. This work is concerned with the breaking of $SO(10)$ to the Pati-Salam group and so all discussions are confined to this particular pattern of symmetry breaking. The $SO(10)$ group. The $SO(10)$ group admits spinor representation [4]. The spinor representation in $SO(2n)$ can be constructed by taking the tensor product of the spinors of $SO(3)(\psi)$

$$\chi = \psi_1 \otimes \psi_2 \otimes \psi_3 \ldots \psi_n \tag{5.77}$$

The spinor χ is however reducible and can be split up in to two pieces χ^+ and χ^-. For n even these states are self conjugate (under C) while for n odd the C transforms one into another. Since we want a complex representation for the GUT we want n to be odd. Combined with the fact that the group should have at least rank 4 we see that $SO(10)$ is the smallest of such groups. The generators for the spinor representation are of the form

$$\sigma_{ab} = \frac{1}{2i}[\gamma^a, \gamma^b] \tag{5.78}$$

where γ^a are the generalized Pauli matrices given in the Appendix. For $SO(10)$ the spinor representation has dimension 16 and this decomposes under $SU(3) \times SU(2)_L \times SU(2)_R$(Left-Right Symmetric Group) as

$$16 = \begin{pmatrix} u \\ d \end{pmatrix}_L (3,2,1) \oplus \begin{pmatrix} \nu \\ e \end{pmatrix}_L (1,2,1) \oplus \begin{pmatrix} d^C \\ -u^C \end{pmatrix}_L (\bar{3},1,2) \oplus \begin{pmatrix} e^+ \\ -\nu^C \end{pmatrix}_L (1,1,2) \tag{5.79}$$

We have an extra right handed component for the neutrino and in principle we can construct a Dirac mass term for the neutrino. We see that since all the fermions are put in a single representation it is easy to understand anomaly cancellation in this model ($SO(10)$ is anomaly free) rather than in $SU(5)$ where the fermions are in two different representations and one has to check for anomaly cancellation between the two chiral representations which are not anomaly free on their own. The gauge bosons are in the 45 dimensional adjoint representation and they decompose under the Left-Right Symmetric group as

$$45_1 = (8,1,1)(g) \oplus (1,3,1)\vec{W}_L \oplus (1,1,3)\vec{W}_R \oplus (1,1,1)(B) \tag{5.80}$$

and

$$45_2 = \begin{pmatrix} X_\alpha & \bar{Y}'_\alpha \\ Y_\alpha & \bar{X}'_\alpha \end{pmatrix} (\bar{3},2,2) \oplus \begin{pmatrix} X'^\alpha & \bar{Y}^\alpha \\ Y'^\alpha & \bar{X}^\alpha \end{pmatrix} (3,2,2) \oplus (3,1,1)X_s \oplus (\bar{3},1,1)\bar{X}_s \tag{5.81}$$

As in the case the $SU(5)$ GUT one can consider the symmetry breaking of the $SO(10)$ group to the Standard Model group. Generally one can consider various chains of descent from this GUT group to the Standard Model group. Some of these chains will be considered in the next chapter. One can also consider constructing fermion masses and mixing in this model and finally it is possible to construct supersymmetric version of the $SO(10)$ GUT model.

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CHAPTER 6

EFFECTS OF HIGHER DIMENSIONAL OPERATOR IN $SO(10)$

The precision measurements [1] at LEP have put constraints [2, 8, 9, 10] on many extensions of the Standard Model. Grand Unified Theories are also constrained [9, 10]. The measurements of the Z mass and width, the jet cross-sections, and the various asymmetries provide very accurate values for $\sin^2\theta_w$ and α_s at the scale M_Z. Using these experimental values[1, 9],

$$\begin{aligned} \sin^2\theta_W &= 0.2333 \pm 0.0008 \\ \alpha_s &= 0.118 \pm 0.005 \end{aligned} \tag{6.1}$$

and taking the fine structure constant at the electroweak scale to be, $\alpha_{em}(M_z) = 1/127.9$,[9] one can write down the values [9] of the three coupling constants at the electroweak scale (M_z). Then using the evolution of these coupling constants with energy it is possible to see if the coupling constants meet at a point giving rise to grand unification of all the three forces [9]. It was found that any GUT without supersymmetry with no intermediate mass scale is ruled out by this analysis, whereas theories with intermediate an mass scale [10] get constraints on the allowed values for the mass scale. We point out that that the present values of $\sin^2\theta_w$ and α_s at the scale M_Z are (from Particle Data Group,1994)

$$\begin{aligned} \sin^2\theta_W &= 0.2319 \pm 0.0005 \\ \alpha_s &= 0.120 \pm 0.007 \pm 0.002 \end{aligned} \tag{6.2}$$

These new values for $\sin^2\theta_w$ and α_s do not change the results of our analysis in this chapter which uses the older values.

An important class of GUTs with intermediate symmetry breaking scales is the one containing those with left-right symmetry [3]. In these theories the standard electroweak model $SU(2)_L \otimes U(1)_Y$ emerge at low energy as a result of symmetry breaking of a larger left-right symmetric $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ group. In the Standard Model the (V–A) nature of the theory is put in by hand, whereas in an left-right symmetric extension [3] of the Standard Model this comes about through spontaneous symmetry breaking (See Appendix for a brief discussion of the left-right Symmetric model) when appropriate Higgs fields acquire vacuum expectation values (vev). In this model there will then be new scalar and gauge particles of mass of the order of this symmetry breaking scale M_R. The mixing of these gauge bosons with the Standard Model gauge bosons puts a lower bound on this scale. The $K_L - K_S$ mass difference gives a lower bound [4] of about 1.6 TeV on M_R from the box diagram with both W_L and W_R exchanges. However, this constraint is subject to the assumption of manifest left-right symmetry, which is to assume that the Kobayashi-Maskawa matrices of the left- and the right- handed sectors are same. In the absence of this artificial symmetry (which does not have any natural explanation) the bound [5] on M_R is relaxed to 300 GeV. From the direct search [6] at CDF the lower bound on M_{W_R} is 520 GeV. This bound is not applicable to left-right symmetric models where the W_R couples only to the heavy neutrinos. The strongest bound on M_R comes from an analysis [7, 8] of the precision measurement of the Z–pole from the CERN e^+e^- collider LEP [1]. From a fit of the 1992 data and for the commonly chosen Higgs triplet fields for the left-right symmetry breaking, the lower bound on M_R is of the order of TeV. However, if this theory is embedded in a GUT, then the constraint

becomes [10],

$$M_R \geq 10^9 GeV. \tag{6.3}$$

In this chapter we address the question : if the right handed gauge bosons are seen in the near future accelerators then will that rule out the possibility of a GUT ?

Several attempts have been made to understand this problem [12, 13]. It was found that if one breaks the left-right D parity spontaneously, then one can have $g_L \neq g_R$. In this case the Higgs sector is left-right asymmetric and one can have low M_R. But for $M_R \sim$ TeV, it is required to break the $SU(2)_R \rightarrow U(1)_R$ at a high scale and as a result one can naturally have a light Z_R but not light right handed charged gauge bosons [12]. To get all the right handed gauge bosons light one requires large numbers of artificial Higgs scalars, which are left-right asymmetric. In the supersymmetric $SO(10)$ theory it is possible to have low M_R for a very specific choice of Higgs scalars [13], which do not give masses to the fermions. Addition of any choice of Higgs scalars to create a realistic fermion mass spectrum destroys this result.

Since the GUT scale is very close to the Planck scale, the effects of gravity may not be negligible. Higher dimensional operators were considered originally [14] to help solve some problems in fermion masses. The idea is to find out if the low energy physics contains some signatures of gravity effects. In all these analyses the coupling constants in these nonrenormalizable terms are free parameters. Someday we may learn if such coupling constants may arise naturally from gravity. In the case of minimal $SU(5)$ GUT it has been argued that the addition of nonrenormalizable terms induced by quantum gravity or by the compactification of the higher dimensions (in some theories like Kaluza-Klein theories) can allow a range of parameters [14, 15, 16] for which the correct values for the $\sin^2\theta_w$ and α_s are reproduced and which remain

consistent with proton decay. In our study we consider higher dimensional operators in an $SO(10)$ version of the GUT and see if for some range of values for the coupling constants of the higher dimensional operators we can have low value of M_R consistent with $\sin^2\theta_w$, α_s and proton decay.

In the case of $SU(5)$ it was found that by inclusion of the dimension five operators [14] alone it is not possible to get the correct value of $\sin^2\theta_w$ and α_s, but by including the dimension six operators [15] along with the dimension five operators it is possible to solve the problem. In the case of the $SO(10)$ GUT, if we have only two stages of symmetry breaking then we can not have low energy left-right symmetry breaking [19], but if one breaks $SO(10)$ in two stages near the Planck scale to the left-right symmetric model then we show that it is possible to have low energy left-right symmetry breaking with a wide range of values of the coupling constants [20].

To begin with let us consider the symmetry breaking chain to be,

$$\begin{aligned}
SO(10) \xrightarrow{M_U} & \quad SU(4) \times SU(2)_L \times SU(2)_R & [\equiv G_{PS}] \\
\xrightarrow{M_1} & \quad SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{(B-L)} & [\equiv G_{LR}] \\
\xrightarrow{M_R} & \quad SU(3)_c \times SU(2)_L \times U(1)_Y & [\equiv G_{std}] \\
\xrightarrow{M_W} & \quad SU(3)_c \times U(1)_{em}.
\end{aligned}$$

The $SO(10)$ invariant Lagrangian, which allows the above symmetry breaking chain, is given in the domain of energies near the Planck scale, as a combination of the usual four dimensional terms and the new induced higher dimensional nonrenormalizable terms. These higher dimensional terms will be suppressed by the Planck scale (M_{Pl})

(or by the compactification scale which can be as much as two orders of magnitude below the Planck scale [16]). The Lagrangian can then be written as,

$$L = -\frac{1}{2}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) + \sum_{n=1} L^{(n)} \tag{6.4}$$

and the sum in Eq. (6.4) runs over all possible higher dimensional operators. We write down the five- and six-dimensional operators explicitly as

$$L^{(1)} = -\frac{1}{2}\frac{\eta^{(1)}}{M_{Pl}}\mathrm{Tr}(F_{\mu\nu}\phi F^{\mu\nu}) \tag{6.5}$$

$$\begin{aligned} L^{(2)} &= -\frac{1}{2}\frac{1}{M_{Pl}^2}\Big[\eta_a^{(2)}\mathrm{Tr}(F_{\mu\nu}\phi^2 F^{\mu\nu}) + \eta_b^{(2)}\mathrm{Tr}(\phi^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \\ &\quad +\eta_c^{(2)}\mathrm{Tr}(F^{\mu\nu}\phi)\mathrm{Tr}(F_{\mu\nu}\phi)\Big] \end{aligned} \tag{6.6}$$

In the above equations $\eta^{(n)}$ specify the couplings of the higher dimensional operators. In our analysis we consider dimension five operator and dimension six operators when the contribution from dimension five operator vanish. We note that the effect of all operators higher than dimension six can be absorbed in the couplings of the dimension six operators and hence their inclusion does not increase the number of parameters.We therefore consider only dimension five and dimension six operators in our analysis.

Let us first consider the case when only the scale M_U is large and M_I is some intermediate symmetry breaking scale which does not receive any contribution from the higher dimensional operators. Suppose the $SO(10)$ is broken by a 54-plet of Higgs fields Σ. Σ is a traceless symmetric field of the $SO(10)$ and the *vevs* of Σ which can mediate this symmetry breaking to the group G_{PS}, are given by,

$$\langle\Sigma\rangle = \frac{1}{\sqrt{30}}\ \Sigma_0\ \mathrm{diag}(1,1,1,1,1,1,-\frac{3}{2},-\frac{3}{2},-\frac{3}{2},-\frac{3}{2}). \tag{6.7}$$

where, $\Sigma_0 = \sqrt{\frac{6}{5\pi\alpha_G}}M_U$ and $\alpha_G = g_0^2/4\pi$ is the GUT coupling. The G_{PS} invariant Lagrangian will be given by,

$$-\frac{1}{2}(1+\epsilon_4)\ \mathrm{Tr}(F_{\mu\nu}^{(4)}\ F^{(4)\mu\nu}) - \frac{1}{2}(1+\epsilon_2)\ \mathrm{Tr}(F_{\mu\nu}^{(2L)}\ F^{(2L)\mu\nu})$$

$$-\frac{1}{2}(1+\epsilon_2)\ \mathrm{Tr}(F_{\mu\nu}^{(2R)}\ F^{(2R)\mu\nu}) \tag{6.8}$$

Now defining the physical gauge fields below the unification scale to be

$$A_i' = A_i\sqrt{1+\epsilon_i}$$

, we recover the usual G_{PS} Lagrangian with modified coupling constants

$$\begin{aligned} g_4^2(M_U) &= \bar{g}_4^2(M_U)(1+\epsilon_4)^{-1} \\ g_{2L}^2(M_U) &= \bar{g}_{2L}^2(M_U)(1+\epsilon_2)^{-1} \\ g_{2R}^2(M_U) &= \bar{g}_{2R}^2(M_U)(1+\epsilon_2)^{-1} \end{aligned} \tag{6.9}$$

The couplings $\bar{g}_i$ are the couplings that would have appeared in the absence of the higher dimensional operators, whereas the g_i are the physical couplings which are evolved down to lower scales.

We introduce the parameter $\epsilon^{(n)}$ associated with a given operator of dimension $n+4$ in the following way:

$$\epsilon_{a,b,c}^{(n)} = \left[\frac{1}{\sqrt{30}}\frac{\phi_0}{M_{Pl}}\right]^n \eta^{(n)} \tag{6.10}$$

We then have

$$\epsilon_{a,b,c}^{(n)} = \left[\left\{\frac{2}{25\pi\alpha_G}\right\}^{\frac{1}{2}}\frac{M_U}{M_{Pl}}\right]^n \eta^{(n)} \tag{6.11}$$

The changes in the coupling constants are,

$$\epsilon_4 = \epsilon^{(1)} + \epsilon_a^{(2)} + \frac{1}{2}\epsilon_b^{(2)}$$

and

$$\epsilon_2 = -\frac{3}{2}\epsilon^{(1)} + \frac{9}{4}\epsilon_a^{(2)} + \frac{1}{2}\epsilon_b^{(2)}.$$

It is to be noted that the $SU(2)_L$ and $SU(2)_R$ always receive equal contributions. As a result, the effect of the higher dimensional terms will be to shift the relative

contributions between the $SU(2)$ and the $SU(4)$ gauge coupling constants (overall shifts of all the coupling constants do not contribute to the predictions of $\sin^2\theta_w$ and α_s). For this reason no matter how many higher dimensional terms we consider, what contributes to the low energy predictions of $\sin^2\theta_w$ and α_s is only the combination,

$$\epsilon = \epsilon_4 - \epsilon_2. \tag{6.12}$$

If we now assume that the dimension 6 terms $\epsilon_i^{(2)}$ are negligible compared to the dimension 5 terms $\epsilon^{(1)}$, then we further get,

$$\epsilon_4 = \frac{2}{5}\epsilon \quad \text{and} \quad \epsilon_2 = -\frac{3}{5}\epsilon.$$

As we argued earlier, this does not reduce the number of parameters in the theory. If we include the higher dimensional terms, then the allowed region in ϵ will be shared by the other $\epsilon^{(n)}$. This relative shift can be parameterized by one parameter, which is related to $\epsilon^{(1)}$. As discussed above, the effect of higher dimensional terms will change the boundary conditions and at the unification scale M_U we have,

$$\bar{g}_4^2 = \bar{g}_{2L}^2 = \bar{g}_{2R}^2 = g_0^2$$

or equivalently,

$$g_4^2(M_U)(1+\epsilon_4) = g_{2L}^2(M_U)(1+\epsilon_2) = g_{2R}^2(M_U)(1+\epsilon_2) = g_0^2. \tag{6.13}$$

Using the matching conditions at the scale M_I,

$$g_{3c}^{-2}(M_I) = g_{1(B-L)}^{-2}(M_I) = g_4^{-2}(M_I) \tag{6.14}$$

and M_R,

$$\begin{aligned} g_{1Y}^{-2}(M_R) &= \frac{3}{5}g_{2R}^{-2}(M_R) + \frac{2}{5}g_{1(B-L)}^{-2}(M_R) \\ g_{2L}^{-2}(M_R) &= g_{2R}^{-2}(M_R) \end{aligned} \tag{6.15}$$

and the evolution of the coupling constants[17],

$$\mu\frac{d\alpha_i(\mu)}{d\mu} = 2\beta_i\alpha_i^2(\mu) \tag{6.16}$$

where $\alpha_i = \frac{g_i^2}{4\pi}$, and $\beta_i = -\frac{1}{(4\pi)}\left(\frac{11}{3}T_g[i] - \frac{4}{3}T_f[i] - \frac{1}{6}T_s[i]\right)$, we can now write the following relations between the Standard Model coupling constants and the unification coupling constant α_G :

$$\begin{aligned} \alpha_Y^{-1}(M_Z) &= \alpha_G^{-1}(1+\frac{3}{5}\epsilon_2+\frac{2}{5}\epsilon_4) + (\frac{6}{5}b_{2R}+\frac{4}{5}b_4)M_{UI} \\ &\quad +(\frac{6}{5}b_{2R}+\frac{4}{5}b_{1(B-L)})M_{IR} + 2b_{1Y}M_{Rw} \\ \alpha_{2L}^{-1}(M_Z) &= (1+\epsilon_2)\alpha_G^{-1} + 2b_{2L}(M_{UI}+M_{IR}+M_{Rw}) \\ \alpha_{3c}^{-1}(M_Z) &= (1+\epsilon_4)\alpha_G^{-1} + 2b_4M_{UI} + 2b_{3c}(M_{IR}+M_{Rw}) \end{aligned} \tag{6.17}$$

where $M_{ij} = \ln\frac{M_i}{M_j}$.

We define two quantities A and B by the following relations,

$$\begin{aligned} A &= \alpha_Y^{-1} - \alpha_{2L}^{-1} \\ B &= \alpha_{2L}^{-1} + \frac{5}{3}\alpha_Y^{-1} - \frac{8}{3}\alpha_{3c}^{-1} \end{aligned} \tag{6.18}$$

A and B are related to $\sin^2\theta_w$, α, and α_s by

$$\begin{aligned} \sin^2\theta_w &= \frac{3}{8} - \frac{5}{8}\alpha A \\ 1 - \frac{8}{3}\frac{\alpha}{\alpha_s} &= \alpha B \end{aligned} \tag{6.19}$$

Equations (6.17) may be solved for M_{UI} and M_{IR} in terms of A and B to obtain

$$\begin{aligned} M_{UI} &= -\frac{X}{2} + \frac{M_{Rw}}{2} - \epsilon\frac{\alpha_G^{-1}}{2b_2} \\ M_{IR} &= \frac{Y}{2} - 2M_{Rw} \end{aligned} \tag{6.20}$$

where, $X = -\frac{5A-B}{4b_2}$; $Y = -\frac{5A+B}{4b_2}$; $b_2 = -\frac{11}{6\pi}$ and $\epsilon = \epsilon_4 - \epsilon_2$. Using the measured values of $\sin^2\theta_w$ and α_s from Eq. (6.1) one gets $M_I \sim 2\times10^{22}M_R$ for $M_R \sim 10M_w$.

Thus we see that when the $SO(10)$ GUT breaks to the G_{PS} at the unification scale it is impossible to have low M_R. Let us now consider the symmetry breaking chain in which $SO(10)$ breaks down to G_{LR} at the unification scale M_U directly, when the component of a 45-plet of Higgs H which is invariant under G_{LR} acquires *vev*. This can be written as,

$$\langle H \rangle = \frac{1}{\sqrt{12}\, i} \; H_0 \begin{pmatrix} 0_{33} & 1_{33} & 0_{34} \\ -1_{33} & 0_{33} & 0_{34} \\ 0_{43} & 0_{43} & 0_{44} \end{pmatrix} \tag{6.21}$$

where, 0_{mn} is a $m \times n$ null matrix and 1_{mm} is a $m \times m$ unit matrix. The antisymmetry of the matrix will imply that there is no contribution to the dimension five operator from this Higgs. The lowest order contribution comes from the dimension six operators. To order six the G_{LR} invariant Lagrangian is written as,

$$\begin{aligned} &-\frac{1}{2}(1+\epsilon_3)\,\mathrm{Tr}(F^{(3)}_{\mu\nu}\,F^{(3)\mu\nu}) - \frac{1}{2}(1+\epsilon_2)\,\mathrm{Tr}(F^{(2L)}_{\mu\nu}\,F^{(2L)\mu\nu}) \\ &-\frac{1}{2}(1+\epsilon_2)\,\mathrm{Tr}(F^{(2R)}_{\mu\nu}\,F^{(2R)\mu\nu}) - \frac{1}{2}(1+\epsilon_1)\,\mathrm{Tr}(F^{(1)}_{\mu\nu}\,F^{(1)\mu\nu}) \end{aligned} \tag{6.22}$$

where, $\epsilon_3 = \epsilon_1$ and $\epsilon_2 = 0$. As argued earlier, in this case there is also no relative shift in the boundary condition between the $SU(3)$ and the $U(1)$ coupling constants and as a result it will not be possible to have low M_R.

In all the analysis done above we have assumed that the intermediate scale M_I does not receive any contribution from the higher dimension operators. In the following analysis we change this assumption and consider effects of the higher dimensional operators at the intermediate scale M_I also. For these effects to be significant it is obvious that the scale M_I has to be close to the GUT unification scale M_U. In the following analysis we also include threshold corrections due to the heavy masses in the theory and we use two loop RGE for the running of the coupling constants. Let us again consider the symmetry breaking chain shown at the beginning of our analysis at

the scale M_U. As we have already mentioned, symmetry breaking is mediated by the *vev* of a 54-plet of Higgs. In this case the left-right parity is broken at M_R only when $SU(2)_R$ is broken and the gauge coupling constants g_L and g_R corresponding to the groups $SU(2)_L$ and $SU(2)_R$ respectively evolve similarly between M_U and M_R so that $g_L(M_R) = g_R(M_R)$. We will now also consider the second case where the symmetry breaking at the scale M_U is by a 210-plet of Higgs. This breaks the discrete left-right parity symmetry [11] D, so that g_L and g_R evolve in a different way below M_U and as a result one obtains $g_L(M_R) \neq g_R(M_R)$.

As we have aready observed then *vev* of a 45-plet field H can break the symmetry group G_{PS} to G_{LR}, but the antisymmetry of the matrix H will imply that to dimension five operators there is no contribution from this Higgs. The lowest order contribution comes from the dimension six operators,

$$\begin{aligned} L'^{(2)} \;=\; & -\frac{1}{2}\frac{1}{M_{Pl}^2}\Big[\eta_a'^{(2)}\mathrm{Tr}(F_{\mu\nu}H^2F^{\mu\nu}) + \eta_b'^{(2)}\mathrm{Tr}(H^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \\ & +\eta_c'^{(2)}\mathrm{Tr}(F^{\mu\nu}H)\mathrm{Tr}(F_{\mu\nu}H)\Big] \end{aligned} \tag{6.23}$$

The *vev* of H does not modify the $SU(2)$ couplings. The $SU(4)$ invariant effective Lagrangian will only contain a new contribution,

$$\begin{aligned} L''^{(2)} \;=\; & -\frac{1}{2}\frac{1}{M_{Pl}^2}\Big[\eta_a'^{(2)}\mathrm{Tr}(F_{\mu\nu}\phi_{15}^2F^{\mu\nu}) + \eta_b'^{(2)}\mathrm{Tr}(\phi_{15}^2)\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \\ & +\eta_c'^{(2)}\mathrm{Tr}(F^{\mu\nu}\phi_{15})\mathrm{Tr}(F_{\mu\nu}\phi_{15})\Big] \end{aligned} \tag{6.24}$$

where ϕ_{15} transforms as $(15,1,1)$ of G_{PS}. At M_I the symmetry group $SU(4)_c$ breaks down to $SU(3)_c \otimes U(1)_{B-L}$ when the field ϕ_{15} acquires a *vev*,

$$\phi_{15} = \frac{1}{\sqrt{24}}\phi_0 \mathrm{diag}[1,1,1,-3]. \tag{6.25}$$

with $\phi_0 = \sqrt{6/5\pi\alpha_4}M_I$. We now define

$$\epsilon_i'^{(2)} = \frac{\eta_i'^{(2)}\phi_0^2}{24M_{Pl}^2} = \left[\frac{1}{20\pi\alpha_4}\left[\frac{M_I}{M_{Pl}}\right]^2\right]\eta_i'^{(2)} \tag{6.26}$$

where $i = a, b, c$. The $SU(3)_c \otimes U(1)_{B-L}$ invariant kinetic energy term for the gauge bosons will then be given by

$$-\frac{1}{2}(1+\epsilon_3')\,\mathrm{Tr}(F^{(3)}_{\mu\nu}\,F^{(3)\mu\nu}) - \frac{1}{2}(1+\epsilon_1')\,\mathrm{Tr}(F^{(1)}_{\mu\nu}\,F^{(1)\mu\nu}) \tag{6.27}$$

where

$$\epsilon_3' = \epsilon_a'^{(2)} + 12\epsilon_b'^{(2)}$$

and

$$\epsilon_1' = 7\epsilon_a'^{(2)} + 12\epsilon_b'^{(2)} + 12\epsilon_c'^{(2)}.$$

In general, ϵ_1' and ϵ_3' may be treated as two free parameters. But we shall assume $\epsilon_b'^{(2)} = \epsilon_a'^{(2)} = \epsilon_c'^{(2)}$ and hence,

$$\epsilon_3' = 0.42\epsilon' \quad \epsilon_1' = \epsilon'(\text{say}). \tag{6.28}$$

Thus the parameter space in ϵ' and ϵ we consider here may be somewhat relaxed. However, the number of parameters in the $\sin^2\theta_w$ and α_s is not changed and we cannot expect any change in low energy predictions. In our analysis we will consider the parameter space of ϵ' and ϵ which allows low M_R. For the D-nonconserved case a 210-plet of Higgs is used to break $SO(10)$ to the group G_{PS} without D-parity conservation.The *vev* of the Higgs is given by

$$\langle H_{210}\rangle = \frac{1}{\sqrt{32}}\,H_0\ \mathrm{diag}(1_{44}, 1_{44}, -1_{44}, -1_{44}). \tag{6.29}$$

where H_0 is related to the vector boson mass M_X by $\sqrt{\frac{2}{\pi\alpha_G}}M_X = H_0$. Keeping only the dim-5 operator we get

$$\epsilon_4 = 0$$

$$\epsilon_{2L} = -\epsilon_{2R} = 8\epsilon^{(1)} = \epsilon$$

where

$$\epsilon^{(1)} = \sqrt{\frac{2}{32\pi\alpha_G}}\left[\frac{M_X}{M_{Pl}}\right]$$

For the evolution of the coupling constants we use the two loop renormalization group equations [9, 18, 21],

$$\mu\frac{\partial}{\partial\mu}\alpha_i(\mu) = \frac{2}{4\pi}\left[b_i + \sum_j \frac{b_{ij}}{4\pi}\alpha_j(\mu)\right]\alpha_i^2(\mu) + \frac{2b_{ij}}{(4\pi)^2}\alpha_i^3 \tag{6.30}$$

where the i, j indices represents the different subgroups at the energy scale μ and $\alpha_i = \frac{1}{4\pi}g_i^2$. The formulae for the β-functions with SUSY and without SUSY are given in Ref [22]. We define $C(R)$ as the Casimir invariant for the representation R, m_R as the multiplicity of the representation(*e.g.* number of flavors,number of flavors or left and right components) and $T(R)$ is 1/2 Dynkin index where the Dynkin index l is defined as

$$l = \frac{d(R)}{d(adj)}C(R) \tag{6.31}$$

where $d(R)$ and $d(adj)$ are the dimensions of R and the adjoint representation. The one loop β functions are then given by

$$b_i = -\frac{11}{3}T(g) + \frac{4}{3}T(f) + \frac{1}{3}T(s) \tag{6.32}$$

where g,f and s refer to the gauge, fermion and scalar representation of the theory. The two loop β functions are

$$b_{ii} = [\frac{10}{3}C(G_i) + 2C(R_i)]T(R_i)m_{R_i} + [\frac{2}{3}C(G_i) + 4C(S_i)]T(S_i)m_{S_i} - \frac{34}{3}[C(G_i)]^2 \tag{6.33}$$

and

$$b_{ij} = [2C(R_j)d(R_j)T(R_i)N_{families}] + [4C(S_j)d(S_j)T(S_i)] \tag{6.34}$$

where in the formulae above we have to sum over all the representations in the theory.

For the two cases considered in our work the explicit form of the β functions are given in the Appendix.

Table 6.1: Higgs spectrum at various mass scales for the D-conserved and the D-nonconseved chain

Group G_i	*Higgs content*
$(2_L 2_R 4_C P)$	$(2,2,1)_{10}$ $(1,3,\bar{10})_{126}$ $(3,1,10)_{126}$ $(1,1,15)_{45}$
$(2_L 2_R 4_C)$	$(2,2,1)_{10}$ $(1,3,\bar{10})_{126}$ $(1,1,15)_{45}$
$(2_L 2_R 1_{B-L} 3_C P)$	$(2,2,0,1)_{10}$ $(1,3,2,1)_{126}$ $((3,1,2,1)_{126}$
$(2_L 2_R 1_{B-L} 3_C)$	$(2,2,0,1)_{10}$ $(1,3,2,1)_{126}$

We use the hypothesis of minimal fine tuning[23] to find the Higgs content at the various mass scales for any given chain. This hypothesis assumes that only a minimum number of fine tunings are done to ensure the hierarchy of the gauge boson masses. This leads to the following symmetry breaking scenario. Consider

$$G_0 \overset{M_U}{\longrightarrow} G_1 \overset{M_I}{\longrightarrow} G_2$$

Let ϕ_0 be some representation of G_0 where

$$\phi_0 = \sum_{\alpha=1,m_i} \oplus \phi_i^\alpha \tag{6.35}$$

where the ϕ_i^α are irreducible representations of G_1. Now if some of the ϕ_i^α acquire *vev* to break G_1 to G_2 then according to the survival hypothesis those fields acquire masses of the order of M_I while the other ϕ fields get masses of the order M_U. In Table 6.9 we show the Higgs bosons that live at different mass scales.

An approximate solution of the evolution equation can be written as

$$\alpha_i^{-1}(\mu') = \beta_0 \ln \frac{\mu'}{\mu} + \frac{1}{\alpha_i(\mu)} + \frac{\beta_1}{\beta_0} \ln \left[\frac{\alpha^{-1}(\mu') + \frac{\beta_1}{\beta_0}}{\alpha^{-1}(\mu) + \frac{\beta_1}{\beta_0}} \right]$$

Table 6.2: The Higgs bosons at M_U

SO(10) multiplet	$G_{2,2,4}$ *multiplet*
54	$S_1(1,1,1)$,$S_2(1,1,20')$,$S_3(3,3,1)$
45	$\phi_1(3,11)$,$\phi_2(2,2,6)$,$\phi_3(1,3,1)$
126	$\Sigma_1(2,2,15)$,$\Sigma_2(1,1,6)$

$$\begin{aligned} \beta_0 &= \frac{1}{2\pi}\left[b_i + \sum b_{ij}\alpha_j(\mu)\right] \\ \beta_1 &= -\frac{2b_{ii}}{(4\pi)^2} \end{aligned} \tag{6.36}$$

At each symmetry breaking threshold we use the following matching conditions for the couplings when the group G breaks to the group G_i [24],

$$\alpha_i^{-1}(\mu) = \alpha_G^{-1} - \frac{\lambda_i}{12\pi} \tag{6.37}$$

where

$$\lambda_i = C_G - C_{G_i} + \mathrm{Tr}(\theta_i^H)^2 \ln \frac{M_H}{\mu}$$

The θ_i^H are the generators of G_i for the representation in which the Higgs, M_H, appear.C_G and C_{G_i} are the quadratic Casimir invariants for the group G and the group G_i while μ is the symmetry breaking scale. Gravity induced corrections change $\alpha_i^{-1}(\mu)$ to $\alpha_i^{-1}(\mu)(1+\epsilon_i)^{-1}$ as in Eq. (6.9). In our analysis we identify μ, the unification scale with the vector boson mass. Threshold corrections will occur due to the non-degeneracy of the Higgs masses with the vector boson mass. Using the D-conserved non-SUSY case as an example we have calculated the effect of threshold corrections at the heavy scales M_I and M_U. The Higgs masses are assumed to vary between $\frac{1}{5}$ and 5 times the vector boson mass.The threshold corrections enter through the factors λ_i appearing in equation above. The Higgs that live around the mass scale M_U and M_I are given in Table 6.2 and Table 6.3.

Table 6.3: The Higgs bosons at M_I

$SO(10)$ multiplet	$G_{2_L,2_R,1_{B-L},3_C}$ multiplet
45	$Y_1(1,1,1,0)$,$Y_2(1,1,0,8)$,$X(1,1,\frac{4}{3},\bar{3})$,$X(1,1,-\frac{4}{3},3)$
126	$\xi_1(3,1,-\frac{2}{3},6)$,$\xi_2(1,3,\frac{2}{3},\bar{6})$, $\xi_3(3,1,\frac{2}{3},3)$,$\xi_4(1,3,-\frac{2}{3},\bar{3})$

Defining $\eta_H = \ln \frac{M_H}{M_X}$ one can write at M_U,

$$\begin{aligned} \lambda_4 &= (4 + 16\eta_{S_2} + 8\eta_{\phi_2} + 32\eta_{\Sigma_1} + 2\eta_{\Sigma_2} + 2\eta_{H'}) \\ \lambda_2 = &= (6 + 12\eta_{S_3} + 12\eta_{\phi_2} + 30\eta_{\Sigma_1} + 4\eta_{\phi_1}) \end{aligned} \tag{6.38}$$

At the scale M_I one has,

$$\begin{aligned} \lambda_3 &= (1 + 6\eta_{Y_2} + 15\eta_{\xi_1} + 15\eta_{\xi_2} + 3\eta_{\xi_3} + 3\eta_{\xi_4}) \\ \lambda_{B-L} &= (4 + 6\eta_{\xi_1} + 6\eta_{\xi_2} + 3\eta_{\xi_3} + 3\eta_{\xi_4}) \\ \lambda_{2L} &= (24\eta_{\xi_1} + 12\eta_{\xi_3}) \\ \lambda_{2R} &= (24\eta_{\xi_2} + 12\eta_{\xi_4}) \end{aligned} \tag{6.39}$$

The quantities $\lambda_4 - \lambda_2$ and $\lambda_3 - \lambda_{B-L}$ appear in the solution for M_U and M_I respectively.We consider two cases where the Higgs masses are chosen such that the above quantities are at their extreme values.We further make the assumption that the Higgs at a given scale coming from the same SO(10) multiplet have the same masses. In the first case we choose M_Σ,M_S,$M_{H'}$ to be $(\frac{1}{5})M_U$ while M_ϕ to be $5M_U$. At M_I we choose M_Y and M_ξ to be $\frac{1}{5}M_I$.For the second case we just flip the Higgs bosons around at the two scales. We refer to these two cases as case(a) and case(b).We have only considered the cases when M_I is not equal to M_U.

6.0.1 Results

Using the values of the Standard Model couplings at M_Z (Eq. 6.1),the evolution equations and the matching conditions we find regions in the ϵ , ϵ' space which allow a low M_R for various values of the intermediate scale M_I and the unification scale M_U. In Fig. 6.1 the allowed regions for D-conserved non-SUSY case are shown.

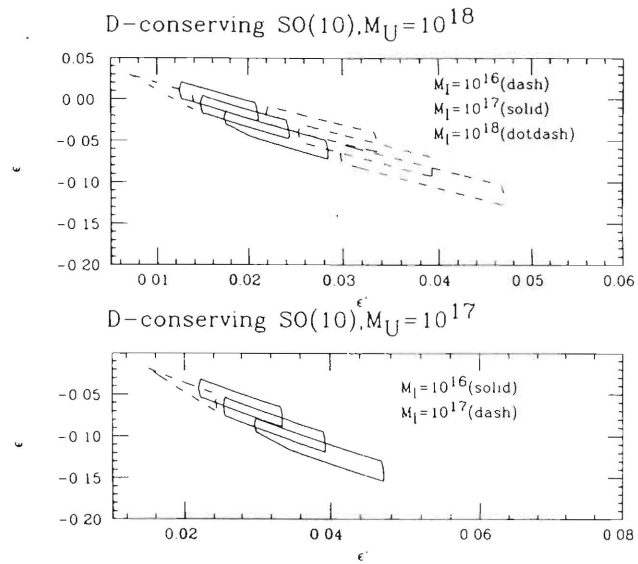

Figure 6.1: Allowed regions in the ϵ , ϵ' space which allow a low $M_R(\approx 1 \quad TeV)$ for various values of the intermediate scale M_I and the unification scale M_U for the D-conserved non-SUSY case.

For M_U not equal to M_I the effects of threshold corrections have been included. In Fig. 6.2 and Fig. 6.3 the allowed regions for D-conserved and D-non-conserved

SUSY case are shown.

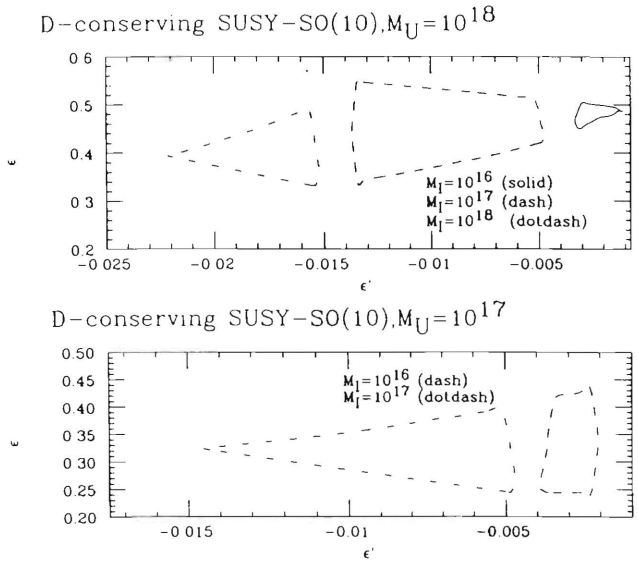

Figure 6.2: Allowed regions in the ϵ , ϵ' space which allow a low $M_R(\approx 1 \quad TeV)$ for various values of the intermediate scale M_I and the unification scale M_U for the D-conserved SUSY case.

For the D-broken non-SUSY case the widths of the allowed regions are too small to be shown graphically and therefore we present the results for this case in Table 6.4. For the supersymmetric version the allowed regions are larger but no solution was found for the case $M_I = 10^{16}$, $M_U = 10^{18}$. In our analysis for the SUSY case we have taken the SUSY scale to be at the Z scale. Even though we have not carried out a full analysis of the threshold effects, from the examples considered, we do not expect moderate threshold effects to alter the regions in the parameter space drastically. In

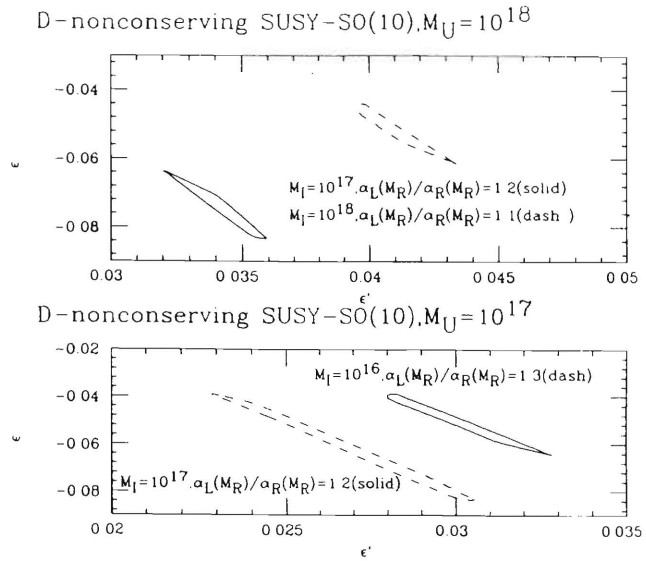

Figure 6.3: Allowed regions in the ϵ , ϵ' space which allow a low $M_R(\approx 1 \quad TeV)$ for various values of the intermediate scale M_I and the unification scale M_U for the D-nonconserved SUSY case.

conclusion we have shown that both for the D-conserved and D-nonconserved case it is possible in $SO(10)$ GUT to have the low energy left-right symmetry scale M_R in the TeV range by including Planck scale corrections. We have also shown that there are fairly wide regions in the parameter space of gravity induced couplings that allow M_R in the TeV range and this can be contrasted with the other known way of achieving low M_R(as already mentioned) i.e to introduce very specific Higgs representations in SUSY $SO(10)$. We have also shown that our analysis works for both SUSY and non-SUSY $SO(10)$.

Table 6.4: Allowed ranges for ϵ and ϵ' for D-broken non-SUSY case

ϵ' (10^{-3})	ϵ (10^{-3})	M_I	M_U	$\frac{\alpha_{2L}(M_R)}{\alpha_{2R}(M_R)}$
4.92 – 10.38	-10 – -5.43	10^{16}	10^{17}	1.4
11.69 – 12.46	-9.72 – -1.032	10^{16}	10^{18}	1.3
6.46 – 8.46	-7.09 – -5.32	10^{17}	10^{17}	1.3
11.08 – 13.23	-9.59 – -7.91	10^{17}	10^{18}	1.2
9.23 – 10.92	-9.18 – -7.75	10^{17}	10^{18}	1.3
8.23 – 10.4	-7.33 – -5.50	10^{18}	10^{18}	1.2

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CHAPTER 7

EFFECTS OF HIGHER DIMENSIONAL OPERATORS IN SUSY SU(5) GUT

In this chapter we consider the effects of higher dimensional operators in SUSY SU(5) GUT. Again we consider these effects on the gauge coupling unification condition only. As we mentioned in the previous chapter, in the case of non-supersymmetric GUTs it was shown[1] that by considering dimension five operators alone it is not possible to make minimal $SU(5)$ GUT consistent with the LEP data and proton decay limit. Where as by considering both dimension five and dimension six operators one can make the minimal $SU(5)$ GUT consistent with LEP data and stable proton.[2]

The effects of higher dimension operators in SUSY GUT's have been studied and recently, Hall and Sarid,[3] and Langacker and Polonsky[4] have shown that the prediction of the strong coupling constant α_s in the minimal supersymmetric $SU(5)$ grand unified theory is smeared out when dimension five non-renormalizable operators arising from gravity is included (Recently Planck scale effects have also been considered by A.Vayonakis[2]). In this chapter we point out that for high GUT scale higher dimensional operators (with dimensions ≥ 6) can be as significant as dimension five operators. In particular we show that these operators can wash out the prediction for α_s completely.

As we observed in Chapter 5, the particle spectrum is a crucial input in studying the evolution of the coupling constants. In this analysis we employ the SUSY mass spectrum used in Ref.1. The particle content of the model is the following: all the Standard Model particles are already present at the initial scale m_Z(including the

top quark and the Higgs). We then include the second Higgs doublet at m_H, the squarks at an average mass $m_{\tilde{q}}$, the sleptons at their average mass $m_{\tilde{l}}$, the Winos at $m_{\tilde{W}}$ the gluinos at $m_{\tilde{g}}$, the Higgsinos at $m_{\tilde{H}}$, the color triplet component of the 5 of the Higgs boson at M_{tr}, the non-Goldstone boson members of the 24 Higgs at M_Σ and finally the superheavy gauge bosons and their superpartners at M_X. Using a supergravity inspired model all the SUSY masses are written in terms of four mass parameters chosen as m_0 (common scalar mass), $m_{1/2}$ (common gaugino mass at the GUT scale), μ(the coupling of the two Higgs in the superpotential) and m_H. A simplified spectrum used here is given as

$$m_{\tilde{q}} = \sqrt{m_0^2 + 6m_{1/2}^2} \tag{7.1}$$

$$m_{\tilde{l}} = \sqrt{m_0^2 + 0.4m_{1/2}^2} \tag{7.2}$$

$$m_{\tilde{g}} = 2.7m_{1/2} \tag{7.3}$$

$$m_{\tilde{W}} = 0.8m_{1/2} \tag{7.4}$$

Note that up to one loop the gaugino masses are given by

$$m_{\tilde{B},\tilde{W},\tilde{g}} = \frac{\alpha_{1,2,3}(M_Z)}{\alpha_G} m_{1/2} \tag{7.5}$$

The Bino mass does not appear in our formulae. Finally,

$$m_{\tilde{H}} = \mu \tag{7.6}$$

In chapter 5 we had mentioned that the SUSY spectrum can be expressed in terms of the parameters $m_0, m_{1/2}, A_0, \tan\beta$. In our analysis we have eliminated $A_0, \tan\beta$ in terms of the second Higgs doublet mass m_H. Note that the Z mass can be expressed as

$$\frac{1}{2}M_Z^2 = \frac{m_1^2 - m_2^2\tan^2\beta}{\tan^2\beta - 1} \tag{7.7}$$

$$m_i^2(t) = m_{H_i}^2(t) + \mu^2(t) \tag{7.8}$$

for $i = 1, 2$ and where $t = \ln\frac{M_G^2}{Q^2}$. Further

$$m_3^2(t) = -B(t)\mu(t) \tag{7.9}$$

In the formulae above we have neglected the loop corrections to the Higgs potential. The two Higgs masses m_{H_1}, m_{H_2} start off with the same mass m_0 at the GUT scale but evolve differently under the RGE and one of the Higgs mass m_{H_1} turns negative (due to large top quark Yukawa coupling) at the weak scale signaling electroweak symmetry breaking. The RGE of m_{H_2} does not depend on A_0 but only on $m_0, m_{1/2}, \alpha_G$, while that of m_{H_1} depends on A_0. After symmetry breaking there are 5 physical Higgs $h, H, H^\pm, A$. In this calculation m_h is taken to be light at around the Z scale while all the other Higgs are considered degenerate with

$$m_H^2 = m_A^2 = m_H^2 = m_{H^\pm}^2 = m_1^2 + m_2^2 \tag{7.10}$$

and so we see that $m_{H_2}^2$ can be expressed in terms of $m_H, m_{H_1}^2(m_0, m_{1/2}, \alpha_G)$ It is clear now that from Eq. (7.7) that we can now express $\tan\beta$ in terms of $\mu, m_0, m_{1/2}, m_H$. In passing we note that the relation

$$\sin 2\beta = \frac{2m_3^2}{m_1^2 + m_2^2} \tag{7.11}$$

can be used to eliminate B_0. We would also like to mention that in general after electroweak symmetry breaking the charged winos mix with the charged Higgsinos to form mass diagonal states called the charginos while the neutral higgsinos can mix with the neutral gauginos($\tilde{W}^3, \tilde{B}$) to form neutralinos. In this discussion the mixings are neglected.

In the gauge part of the GUT Lagrangian we include both dimension 5 and dimension 6 operators, which might originate from non-renormalizable quantum gravity

effect, and write

$$\delta\mathcal{L} = \frac{c}{2\hat{M}_P}\mathrm{tr}(GG\Sigma) + \frac{1}{2\hat{M}_P^2}[d_{11}\frac{1}{2}\mathrm{tr}(G\Sigma^2G) + d_{12}\frac{1}{2}\mathrm{tr}(G\Sigma G\Sigma) + d_2\mathrm{tr}(G^2)\mathrm{tr}(\Sigma^2) + d_3\mathrm{tr}(G\Sigma)\mathrm{tr}(G\Sigma)] \quad (7.12)$$

Then these terms will modify the kinetic energy terms of the Standard Model gauge bosons to,

$$\begin{aligned}\mathcal{L}_{gauge} &= -\frac{1}{4}(FF)_{U(1)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(-\frac{1}{2\sqrt{15}}\right)+\frac{v^2}{2\hat{M}_P^2}\left(\frac{1}{15}\right)\left(d_1\frac{7}{4}+d_2\frac{15}{2}+d_3\frac{15}{2}\right)\right] \\ &-\frac{1}{4}(FF)_{SU(2)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(-\frac{3}{2\sqrt{15}}\right)+\frac{v^2}{2\hat{M}_P^2}\left(\frac{1}{15}\right)\left(d_1\frac{9}{4}+d_2\frac{15}{2}\right)\right] \\ &-\frac{1}{4}(FF)_{SU(3)}\left[1+\frac{c}{2}\frac{v}{\hat{M}_P}\left(\frac{1}{\sqrt{15}}\right)+\frac{v^2}{2\hat{M}_P^2}\left(\frac{1}{15}\right)\left(d_1+d_2\frac{15}{2}\right)\right] \quad (7.13)\end{aligned}$$

where we have defined $d_1 = \frac{d_{11}+d_{12}}{2}$ as the the first two operators in Eq. (7.12) always contribute equally. Note that in principle one can also include operators of dimensions higher than six in our analysis but their contributions to $\vec{\epsilon}$, where $\vec{\epsilon}\alpha_G{}^{-1}$ is the amount by which $\vec{\alpha_G}{}^{-1}$ gets modified in the evolution equations for the coupling constants, can be included by absorbing them in the co-efficients d_1, d_2 and d_3. Since we are interested only in gauge coupling evolutions it is thus sufficient to confine our analysis to just dimension five and dimension six operators for minimal supersymmetric $SU(5)$ GUT and see how they can affect the predictions of α_s. At the one loop level the gauge coupling, evaluated at the Z mass $\vec{\alpha}^{-1} \equiv \vec{\alpha}^{-1}(m_Z) \equiv (\alpha_1{}^{-1}, \alpha_2{}^{-1}, \alpha_3{}^{-1})$ will be related to the GUT scale (M_G) gauge coupling constant,

$$\vec{\alpha}^{-1} = \alpha_G^{-1}\left(\vec{1}+\vec{\epsilon}_5+\vec{\epsilon}_6\right) - \sum_a \vec{\beta}_a \ln\left(\frac{M_a}{M_G}\right)$$

where,

$$\vec{\epsilon}_5 \equiv \frac{c}{2}\frac{v}{\hat{M}_P}\left(\frac{1}{\sqrt{15}}\right)\left(-\frac{1}{2}, -\frac{3}{2}, 1\right)$$

and

$$\vec{\epsilon}_6 \equiv \frac{d}{30}\frac{v^2}{\hat{M}_P^2}\left(d_1\frac{7}{4}+d_2\frac{15}{2}+d_3\frac{15}{2}, d_1\frac{9}{4}+d_2\frac{15}{2}, d_1+d_2\frac{15}{2}\right)$$

Then, the modified unification equations are given by,

$$\begin{aligned}\frac{2}{\alpha_s}+\frac{6}{5\pi}\ln\frac{M_{tr}}{m_Z}-\sqrt{\frac{12}{5}}\frac{c}{2}\frac{v}{\hat{M}_P}\frac{1}{\alpha_G}+[\frac{1}{5}d_1-\frac{1}{2}d_3]\frac{v^2}{2\hat{M}_P^2}\frac{1}{\alpha_G} &= f_1(s^2, m_0, m_{\frac{1}{2}}, \mu, m_H) \\ \frac{2}{\alpha_s}+\frac{9}{\pi}\ln\frac{5}{12}+\frac{12}{\pi}\ln g_5+\frac{6}{\pi}\ln\lambda_{24}+\frac{18}{\pi}\ln\frac{v}{m_Z}+\frac{5}{2}\frac{d_3}{2}\frac{v^2}{\hat{M}_P^2}\frac{1}{\alpha_G} &= f_2(s^2, m_0, m_{\frac{1}{2}}). \quad (7.14)\end{aligned}$$

where

$$f_1 = \frac{3(6s^2-1)}{5\alpha} - \frac{3}{20\pi}\ln\frac{m_{\tilde{q}}^2}{m_{\tilde{l}}^2} - \frac{2}{\pi}\ln\frac{2.7}{0.8} + \frac{4}{5\pi}\ln\frac{\mu}{m_Z} + \frac{1}{5\pi}\ln\frac{m_H}{m_Z} - 1.52 \quad (7.15)$$

$$f_2 = \frac{3(1-2s^2)}{\alpha} - \frac{3}{4\pi}\ln\frac{m_{\tilde{q}}^2}{m_{\tilde{l}}^2} - \frac{2}{\pi}\ln 2.70.8 - \frac{4}{5\pi}\ln\frac{m_{1/2}}{m_Z} + \frac{1}{\pi} + 1.13 \quad (7.16)$$

Subtracting one of the equations in Eq. (7.14) from the other we obtain an equation for M_{tr} which can be written as,

$$\frac{-84}{5\pi}\ln t = w_1t^2 + w_2t + b \quad (7.17)$$

where $t = \frac{M_{tr}}{M_P}$, $w_1 = \frac{1}{\alpha_G\lambda_5{}^2}[\frac{18}{5}d_3 - \frac{6}{25}d_1]$, $w_2 = \frac{6}{5\alpha_G\lambda_5}c$ and $b = f_1 - f_2 + \frac{6}{\pi}\ln\frac{\lambda_{24}}{\lambda_5{}^4} + \frac{6}{\pi}\ln 4\pi\alpha_G + \frac{84}{5\pi}\ln\frac{M_P}{m_Z}$. Defining, $x = \frac{M_{tr}}{\lambda_5 M_P}$ we can rewrite the first equation in Eq. (7.14) as,

$$\frac{2}{\alpha_s} = f_1(s^2, m_0, m_{\frac{1}{2}}, \mu, m_H) - \frac{6}{5\pi}\ln\frac{M_{tr}}{m_Z} + \frac{6}{5}\frac{c}{\alpha_G}x + [\frac{3}{5}d_3 - \frac{6}{25}d_1]\frac{x^2}{\alpha_G} \quad (7.18)$$

We now numerically solve Eq. (7.17) for t and then use Eq. (7.18) to calculate α_s. We use the same mass spectrum and ranges of parameters (s^2, m_0, $m_{\frac{1}{2}}$, μ, m_{H_2},λ_5,λ_{24},c) as in Ref.1. In other words we vary the light superpartner masses and the second Higgs doublet mass between 100 GeV and 1 TeV, s^2 between 0.2314 and 0.2324 [5], λ_5 and λ_{24} between .1 and 3 while we constrain $|c| < 1$. The co-effecients d_1, d_2 and d_3 are unknown, but we see from Eq. (7.17) and Eq. (7.18) that only d_1

and d_3 contribute to the equations for M_{tr} and α_s. We also observe from Eq. (7.17) that d_3 has a much larger co-efficient . We can now consider two scenarios, one with $|d_1| < 1; d_3 = 0$ and $|d_1| = 0; |d_3| < 1$. There may be multiple solutions to Eq. (7.17) and we have chosen the lowest solution in our analysis. To select the lowest solution we define two critical solutions t_1 and t_2 which are given by,

$$t_1 = \frac{t_{ex}}{2}[1 + \sqrt{(1-2y)}] \tag{7.19}$$

$$t_2 = \frac{t_{ex}}{2}[1 - \sqrt{(1-2y)}] \tag{7.20}$$

where $t_{ex} = \frac{-w_2}{2w_1}$ and $y = \frac{a}{w_1 t_{ex}^2}$ For $w_2 = 0$ we have one critical solution t_{cr} given by

$$t_{cr} = \sqrt{\frac{-a}{2w_1}} \tag{7.21}$$

The critical solutions correspond to points where the tangent to the logarathmic function on the left hand side of Eq. (7.17) equals the tangent to the parabola on the right hand side of Eq. (7.17). When t_1 and t_2 are both real and positive and distinct from one another we can have at most three solutions, one below t_1, one between t_1 and t_2 and one above t_2. If instead the critical solutions are real and positive, but equal then we can have at most two solutions. For $w_1 < 0$ there is always one real positive critical solution and so there can be up to two solutions one on either side of the critical solution. When there is no real, positive critical solution there can be up to one solution to Eq. (7.17). For $w_1 = 0$, as observed in Ref.1, there can be only one solution for w_2 greater than 0 while for w_2 less than 0 there can be upto two solutions lying on either side of the critical solution $t_{critical} = \frac{-a}{w_2}$.

7.1 Results

For the case where $|d_1| < 1; d_3 = 0$, the effect of dimension 6 operators are found negligible. However for the case where $|d_1| = 0; |d_3| < 1$, the effect of dimension six operator can be significant. In Fig. 7.1(a) we show a plot of the solutions in the $\alpha_s - M_{tr}$ plane. Although we cut off the figure at $\alpha_s = 1$, we mention that there are solutions for larger values of α_s [1]. In Table 7.1 we show the ranges of α_s for different M_{tr}. Fig. 7.1(b) is a blow up of Fig. 7.1(a) for $\alpha_s \leq 0.12$. Here, we have used a much smaller grid size for λ_5 in our numerical computation; as a result, some solutions that do not show up in Fig. 7.1(a) now appear in Fig. 7.1(b) .

We observe that for $M_{tr} \geq 7 \times 10^{16}$ GeV the range of the solutions for α_s is greatly increased. We also note that with dimension 6 operators it is now possible to get values of α_s below 0.11 which was not possible with pure dimension 5 term. This could be of interest if in the future the central value of $\alpha_s = 0.120 \pm 0.007 \pm 0.002$ shifts down by $\sim 1.5\sigma$. (It is interesting to note that such a low value of α_s (0.108 ± 0.004) is indeed obtained in an analysis of LEP data by Maxwell et al. [6] where it is claimed that the standard perturbative QCD analyses used to extract α_s from LEP data do not correctly take into account higher order NNLO corrections which can be sizeable for some of the LEP observables used in the determination of α_s.) We found that solutions with large values of α_s and small values of α_s (less than 0.11) correspond to small values of λ_5 in the range 0.1 to 0.3 indicating a high value for M_X (or x) and consequently large gravitational corrections. When the unification scale is close to the Planck scale the magnitude of the terms induced by the higher dimensional operators in Eq. (7.18) can become comparable to the combination of the first two terms, resulting in a much wider range for α_s. In our calculations we have constrained the

[1] Of course, the equations themselves cease to be valid if α_s is too large.

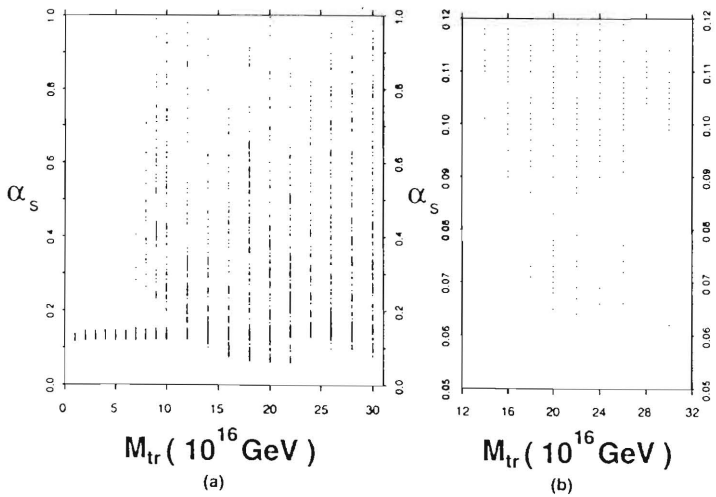

Figure 7.1: Plot of the solutions in the $\alpha_s - M_{tr}$ plane. Fig. (b) is a blow up of Fig. (a) for $\alpha_s \leq 0.12$. For Fig. (b) a much smaller grid size for λ_5 was used in the numerical computation; as a result, some solutions that do not show up in Fig. (a) now appear in Fig. (b).

Table 7.1: Allowed ranges of α_s for various M_{tr}

$M_{tr} \times 10^{16}$ GeV	α_s(max)	α_s(min)
1	0.144	0.115
2	0.146	0.118
3	0.146	0.119
4	0.147	0.120
5	0.147	0.119
6	0.147	0.120
7	0.404	0.124
8	0.705	0.121
9	1.408	0.121
10	2.95	0.121
14	2.82	0.101
18	1.742	0.0660
22	3.89	0.0590
26	3.82	0.0570
30	3.36	0.0560

heavy masses to be less than $\tilde{M}_P$. To compare to the results with only the dimension five operator included, we note that in that case, the parameter x always is of the order of 10^{-2}. However the inclusion of the dimension six operators allows x to be an order of magnitude higher indicating a higher unification scale close to $\tilde{M}_P$. (Note $\frac{M_X}{\tilde{M}_P} = \sqrt{8\pi\alpha_G}x \sim x$ for $\alpha_G = \frac{1}{25}$; where M_X is the vector boson mass) so that it should not be surprising that the higher dimensional operators are significant.

In summary, we have shown that the inclusion of dimension 6 operators may totally wash out the predictions for the strong coupling constant and further,that the correlation between α_s and M_{tr} is also destroyed unless we constrain the triplet Higgs mass $M_{tr} < 7 \times 10^{16}$ GeV. Turning this around, if we require that SUSY-GUT make calculable predictions at the electroweak scale in the presence of gravity induced non-renormalizable operators we may infer more restrictive bounds on the triplet Higgs mass than are available in the literature [7].

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Appendix A

SOME $SO(10)$ RELATED RESULTS

A.1 D Parity

One of the maximal subgroups of $SO(10)$ is the group $SU(2)_L \times SU(2)_R \times SU(4) \times D$ where D is called the D-parity. The D-parity operator is similar to the charge conjugation operator for the fermions and this operator can be expressed as a product of the $SO(10)$ generators

$$D = M_{23} M_{67} \tag{A.1}$$

and for the fermion f_L

$$f_L \xrightarrow{D} f_L^C \tag{A.2}$$

Note that in $SO(10)$ all the fermions (f_L and f_L^C) are in the same representation and the group generators just allow you to go from a state in a given representation to another state in the same representation. The transformation of the scalars $\Delta_L \equiv (3,1,10)$ is given under D-parity as

$$\Delta_L \equiv (3,1,10) \xrightarrow{D} \Delta_R^* \equiv (1,3,\bar{1}0) \tag{A.3}$$

So we see that D-parity is not the charge conjugation operator and neither can it be identified with the CP operator. In fact one can have Lagrangians with complex Yukawa couplings that violate CP but conserve D-parity. The importance of the concept of D-parity comes from the fact that there are Higgs multiplet in $SO(10)$ which contain submultiplets that are singlets under $SU(2)_L \times SU(2)_R \times G$ but are odd or even under D-parity. For example the 54 representation contains the D-even singlet $(1,1,1)$ under $SU(2)_L \times SU(2)_R \times SU(4)$ and so when it acquires a *vev* to break $SO(10)$, because of D-parity conservation, the resulting group is $SU(2)_L \times SU(2)_R \times SU(4) \times D$. The 210 representation on the other hand contains a D-odd singlet under $SU(2)_L \times SU(2)_R \times SU(4)$ and so when it is used for symmetry breaking we end up with $SU(2)_L \times SU(2)_R \times SU(4)$ without D-parity. In the D-broken case there are asymmetric Higgs bosons contribution to the RGE for g_L and g_R and so even if we start with both of these couplings equal at a certain scale at some different scale this equality will no longer be maintained.

A.2 One And Two Loop β Functions

For the region between $M_Z - M_R$

$$\begin{pmatrix} b_Y \\ b_{2LW} \\ b_{3c} \end{pmatrix} = \begin{pmatrix} 41/10 \\ -19/6 \\ -7 \end{pmatrix} \tag{A.4}$$

and the two loop functions are

$$\begin{pmatrix} 199/50 = b_{YY} & 27/10 & 44/5 \\ 9/10 & 35/6 = b_{2LW2LW} & 12 \\ 11/10 & 9/2 & -26 = b_{3c3c} \end{pmatrix} \tag{A.5}$$

For the region between $M_R - M_I$(D-conserving)

$$\begin{pmatrix} b_{2L} \\ b_{2R} \\ b_{B-L} \\ b_{3c} \end{pmatrix} = \begin{pmatrix} -7/3 \\ -7/3 \\ 7 \\ -7 \end{pmatrix} \tag{A.6}$$

and the two loop functions are

$$\begin{pmatrix} 80/3 = b_{2L2L} & 3 & 27/2 & 12 \\ 3 & 80/3 = b_{2R2R} & 27/2 & 12 \\ 81/2 & 81/2 & 115/2 = b_{B-LB-L} & 4 \\ 9/2 & 9/2 & 1/2 & -26 = b_{3c3c} \end{pmatrix} \tag{A.7}$$

For the region between $M_R - M_I$(D-nonconserving)

$$\begin{pmatrix} b_{2L} \\ b_{2R} \\ b_{B-L} \\ b_{3c} \end{pmatrix} = \begin{pmatrix} -3 \\ -7/3 \\ 11/2 \\ -7 \end{pmatrix} \tag{A.8}$$

and the two loop functions are

$$\begin{pmatrix} 8 = b_{2L2L} & 3 & 3/2 & 12 \\ 3 & 80/3 = b_{2R2R} & 27/2 & 12 \\ 9/2 & 243/6 & 61/2 = b_{B-LB-L} & 4 \\ 9/2 & 9/2 & 1/2 & -26 = b_{3c3c} \end{pmatrix} \tag{A.9}$$

For the region between $M_I - M_U$(D-conserving)

$$\begin{pmatrix} b_{2L} \\ b_{2R} \\ b_{4c} \end{pmatrix} = \begin{pmatrix} 11/3 \\ 11/3 \\ -10/3 \end{pmatrix} \tag{A.10}$$

and the two loop functions are

$$\begin{pmatrix} 584/3 = b_{2L2L} & 3 & 765/2 \\ 3 & 584/3 = b_{2R2R} & 765/2 \\ 153/2 & 153/2 & 2207/6 = b_{4c4c} \end{pmatrix} \tag{A.11}$$

For the region between $M_I - M_U$(D-nonconserving)

$$\begin{pmatrix} b_{2L} \\ b_{2R} \\ b_{4c} \end{pmatrix} = \begin{pmatrix} -3 \\ 11/3 \\ -19/3 \end{pmatrix} \tag{A.12}$$

and the two loop functions are

$$\begin{pmatrix} 8 = b_{2L2L} & 3 & 45/2 \\ 3 & 584/3 = b_{2R2R} & 765/2 \\ 9/2 & 153/2 & 1091/6 = b_{4c4c} \end{pmatrix} \tag{A.13}$$

For the region between $M_Z - M_R$(SUSY)

$$\begin{pmatrix} b_Y \\ b_{2LW} \\ b_{3c} \end{pmatrix} = \begin{pmatrix} 66/10 \\ 1 \\ -3 \end{pmatrix} \tag{A.14}$$

and the two loop functions are

$$\begin{pmatrix} 199/25 = b_{YY} & 27/5 & 88/5 \\ 9/5 & 25 = b_{2LW2LW} & 31 \\ 11/5 & 9 & 14 = b_{3c3c} \end{pmatrix} \tag{A.15}$$

For the region between $M_R - M_I$(D-conserving SUSY)

$$\begin{pmatrix} b_{2L} \\ b_{2R} \\ b_{B-L} \\ b_{3c} \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 15 \\ -3 \end{pmatrix} \tag{A.16}$$

and the two loop functions are

$$\begin{pmatrix} 49 = b_{2L2L} & 3 & 9 & 24 \\ 3 & 49 = b_{2R2R} & 9 & 24 \\ 45 & 45 & 61 = b_{B-LB-L} & 8 \\ 9 & 9 & 1 & 14 = b_{3c3c} \end{pmatrix} \tag{A.17}$$

For the region between $M_R - M_I$(D-nonconserving SUSY)

$$\begin{pmatrix} b_{2L} \\ b_{2R} \\ b_{B-L} \\ b_{3c} \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \\ 21/2 \\ -3 \end{pmatrix} \tag{A.18}$$

and the two loop functions are

$$\begin{pmatrix} 25 = b_{2L2L} & 3 & 3 & 24 \\ 3 & 49 = b_{2R2R} & 9 & 24 \\ 9 & 45 & 34 = b_{B-LB-L} & 8 \\ 9 & 9 & 1 & 14 = b_{3c3c} \end{pmatrix} \tag{A.19}$$

For the region between $M_I - M_U$(D-conserving SUSY)

$$\begin{pmatrix} b_{2L} \\ b_{2R} \\ b_{4c} \end{pmatrix} = \begin{pmatrix} 21 \\ 21 \\ 16 \end{pmatrix} \tag{A.20}$$

and the two loop functions are

$$\begin{pmatrix} 265 = b_{2L2L} & 3 & 405 \\ 3 & 265 = b_{2R2R} & 405 \\ 81 & 81 & 538.5 = b_{4c4c} \end{pmatrix} \tag{A.21}$$

For the region between $M_I - M_U$(D-nonconserving SUSY)

$$\begin{pmatrix} b_{2L} \\ b_{2R} \\ b_{4c} \end{pmatrix} = \begin{pmatrix} 1 \\ 21 \\ 7 \end{pmatrix} \tag{A.22}$$

and the two loop functions are

$$\begin{pmatrix} 25 = b_{2L2L} & 3 & 45 \\ 3 & 265 = b_{2R2R} & 405 \\ 9 & 81 & 304.5 = b_{4c4c} \end{pmatrix} \tag{A.23}$$

A.3 Generalized Pauli Matrices

For the Group $SO(2n+2)$ the generalized $2^n \times 2^n$ Pauli matrices are defined iteratively as

$$\gamma_i^{n+1} = \begin{pmatrix} \gamma_i^n & 0 \\ 0 & -\gamma_i^n \end{pmatrix} \qquad i = 1, 2, 3.....2n \tag{A.24}$$

$$\gamma_{2n+1}^{n+1} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{A.25}$$

$$\gamma_{2n+2}^{n+1} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{A.26}$$

A.4 Left-Right Symmetric Model

One of the motivation for the Left-Right symmetric model is to understand parity violation in low energy physics. The Left-Right symmetric model assumes that the interaction Lagrangian is intrinsically left-right symmetric and the asymmetry observed in nature (eg β decay, μ decay etc) arises from the vacuum being noninvariant

under parity symmetry. The interesting feature of this model is that at low energies, it reproduces all the features of the electroweak theory but as one goes to higher energies new effects associated with the parity invariance of the model appear such as the second neutral Z-boson, right-handed charged currents, right handed neutrinos etc.

The Left-Right symmetric models are based on the gauge group

$$SU(2)_L \times SU(2)_R \times U_{B-L}$$

. Considering only a single generation, the fermions have the assignments

$$Q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L = (1/2, 0, 1/3) \quad ; \quad Q_R = \begin{pmatrix} u \\ d \end{pmatrix}_R = (0, 1/2, 1/3)$$

$$\psi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L = (1/2, 0, 1/3) \quad ; \quad \psi_R = \begin{pmatrix} \nu \\ e \end{pmatrix}_R = (0, 1/2, 1/3)$$

. The electric charge is given by the formula

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2} \tag{A.27}$$

If we assume discrete parity symmetry then the group that we are considering is $SU(2)_L \times SU(2)_R \times U_{B-L} \times P$. This model therefore has only two coupling constants $g_2 = g_{2L} = g_{2R}$ and g'(associated with $U(1)_{B-L}$). The kinetic energy part of the interaction for the fermions can be written as

$$\begin{aligned} L_{kin} &= -\bar{Q}_L \gamma^\mu D_{L\mu} Q_L - \bar{\psi}_L \gamma^\mu D_{L\mu} \psi_L - \bar{Q}_R \gamma^\mu D_{R\mu} Q_R \\ &\quad -\bar{\psi}_R \gamma^\mu D_{R\mu} \psi_R \end{aligned} \tag{A.28}$$

where

$$D_{L\mu} = (\partial_\mu - \frac{ig}{2}\vec{\tau}.\vec{W}_{L\mu} - \frac{ig'}{6}B_\mu) \tag{A.29}$$

and

$$D_{R\mu} = (\partial_\mu - \frac{ig}{2}\vec{\tau}.\vec{W}_{R\mu} - \frac{ig'}{6}B_\mu) \tag{A.30}$$

Now one can consider two stage breaking of this model to the electroweak group where first at the scale M_P the parity symmetry is broken and at a lower scale the gauge symmetry $SU(2)_R$ is broken. Alternately, one can consider the breaking of the parity and the gauge symmetry at the same scale and in our discussions we consider this scenario only. First we consider the breaking of the Left-right Symmetry to the electroweak symmetry which can be achieved by introducing scalars

$$\Delta_L(1,0,2) + \Delta_R(0,1,2)$$

The most general parity invariant and gauge invariant potential involving these two fields is

$$\begin{aligned} V(\Delta_L, \Delta_R) &= -\mu^2 Tr(\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R) \\ &\quad +\rho_1[Tr((\Delta_L^\dagger \Delta_L))^2 + Tr((\Delta_R^\dagger \Delta_R))^2] \\ &\quad +\rho_2[Tr(\Delta_L^\dagger \Delta_L \Delta_L^\dagger \Delta_L) + Tr(\Delta_R^\dagger \Delta_R \Delta_R^\dagger \Delta_R)] \\ &\quad +\rho_3[Tr(\Delta_L^\dagger \Delta_L) Tr(\Delta_R^\dagger \Delta_R)] \\ &\quad +\rho_4[Tr(\Delta_L^\dagger \Delta_L^\dagger) Tr(\Delta_L \Delta_L) + Tr(\Delta_R^\dagger \Delta_R^\dagger) Tr(\Delta_R \Delta_R)] \end{aligned} \tag{A.31}$$

where we write

$$\Delta_{LR} = \vec{\tau}.\vec{\delta}_{LR} = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \delta^0 & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}_{LR} \tag{A.32}$$

By suitable symmetry transformation the vev of the Δ_{LR} can be written as

$$\langle \Delta_{LR} \rangle = \begin{pmatrix} 0 & 0 \\ v_{LR} & 0 \end{pmatrix} \tag{A.33}$$

The potential V then reduces to

$$V(v_L, v_R) = -\mu^2(v_L^2 + v_R^2) + (\rho_1 + \rho_2)(v_L^4 + v_R^4) + \rho_3 v_L^2 v_R^2 \tag{A.34}$$

The minimum for this potential occurs for

$$\rho_3 > 2(\rho_1 + \rho_2) \quad ; \quad \alpha = 0, \pi/2 \tag{A.35}$$

where

$$v_L = v \sin\alpha \quad ; \quad v_R = v \cos\alpha \tag{A.36}$$

To break the electroweak symmetry we introduce the Higgs $\phi(1/2, 1/2, 0)$ and the full potential is now written as

$$\begin{aligned} V(\Delta_L, \Delta_R, \phi) &= V(\Delta_L, \Delta_R) - \sum_{ij} \mu_{ij}^2 Tr(\phi_i^\dagger \phi_j) \\ &+ \sum_{ijkl} \lambda_{ijkl}[Tr(\phi_i^\dagger \phi_j) Tr(\phi_k^\dagger \phi_l)] \\ &+ \sum_{ijkl} \lambda'_{ijkl}[Tr(\phi_i^\dagger \phi_j \phi_k^\dagger \phi_l)] \\ &+ \sum_{ij} \alpha_{ij}[Tr(\Delta_L^\dagger \Delta_L + \Delta_R^\dagger \Delta_R) Tr(\phi_i^\dagger \phi_j)] \\ &+ \sum_{ij} \beta_{ij}[Tr(\Delta_L^\dagger \Delta_L (\phi_i \phi_j^\dagger) + \Delta_R^\dagger \Delta_R (\phi_i^\dagger \phi_j)] \\ &+ \sum_{ij} \gamma_{ij}[Tr(\Delta_L^\dagger \phi_i \Delta_R \phi_j^\dagger)] + h.c \end{aligned} \tag{A.37}$$

where

$$\phi_1 = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} \tag{A.38}$$

and

$$\phi_2 (\equiv \tau_2 \phi_1^* \tau_2) = \begin{pmatrix} \phi_2^{0*} & -\phi_2^+ \\ -\phi_1^- & \phi_1^{0*} \end{pmatrix} \tag{A.39}$$

For a wide range of parameters the minimum for the potential is given by

$$\langle \Delta_{LR} \rangle = \begin{pmatrix} 0 & 0 \\ v_{LR} & 0 \end{pmatrix} \tag{A.40}$$

$$\langle \phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} e^{i\alpha} \tag{A.41}$$

if $\kappa' << \kappa$ then

$$v_L = [\frac{\gamma_{12}}{2(\rho_1 + \rho_2) - \rho_3}] \frac{\kappa^2}{v_R} \tag{A.42}$$

In the limit that κ / v_R is small the spontaneous CP phase α vanishes. Also from the formula above we obtain $v_L << \kappa$. The mass matrix for the charged gauge boson has the following form

$$\begin{pmatrix} \frac{1}{2} g^2(\kappa^2 + \kappa'^2 + 2v_L^2) & g^2 \kappa \kappa' \\ g^2 \kappa \kappa' & \frac{1}{2} g^2(\kappa^2 + \kappa'^2 + 2v_R^2) \end{pmatrix} \tag{A.43}$$

The eigenstates of the matrix are the physical bosons $W_{1,2}$

$$W_1 = W_L \cos\zeta + W_R \sin\zeta \tag{A.44}$$

$$W_2 = -W_L \sin\zeta + W_R \cos\zeta \tag{A.45}$$

with

$$\tan 2\zeta = \frac{2\kappa\kappa'}{v_R^2 - v_L^2} \tag{A.46}$$

The effective four fermion interaction at low energy($q^2 << M_{W_{1,2}}^2$) has the form

$$\begin{aligned} L_{cc} = \frac{G_F}{\sqrt{2}} &[(\cos^2\zeta + \eta \sin^2\zeta) J_{\mu L}^\dagger J_L^\mu + (\eta \cos^2\zeta + \sin^2\zeta) J_{\mu R}^\dagger J_R^\mu \\ &-(\eta - 1)\cos\zeta \sin\zeta)(J_{\mu L}^\dagger J_R^\mu + J_{\mu R}^\dagger J_L^\mu)] \end{aligned} \tag{A.47}$$

with

$$J_{\mu LR} = \bar{u}_{LR} \gamma_\mu d_{LR} + \bar{\nu}_{LR} \gamma_\mu e_{LR} \tag{A.48}$$

$$\eta = \frac{M_{W_1}^2 \approx \frac{1}{2} g^2(\kappa^2 + \kappa'^2)}{M_{W_2}^2 \approx \frac{1}{2} g^2(\kappa^2 + \kappa'^2 + 2v_R^2)} \tag{A.49}$$

The neutral current interaction is given as

$$L_{NC} = ig(J_{\mu L}W^{3\mu L} + J_{\mu R}W^{3\mu R} + J_{\mu B-L}B^{\mu}) \tag{A.50}$$

The mass matrix for the neutral bosons W_{3L}, W_{3R}, B is

$$\begin{pmatrix} \frac{g^2}{2}(\kappa^2+\kappa'^2+4v_L^2) & -\frac{g^2}{2}(\kappa^2+\kappa'^2) & -2gg'v_R^2 \\ \frac{g^2}{2}(\kappa^2+\kappa'^2) & -\frac{g^2}{2}(\kappa^2+\kappa'^2+4v_R^2) & -2gg'v_R^2 \\ -2gg'v_L^2 & -2gg'v_R^2 & g'^2(v_L^2+v_R^2) \end{pmatrix} \tag{A.51}$$

The mass eigenstates are the following fields

$$A = \sin\theta_W(W_{3L}+W_{3R}) + \sqrt{\cos 2\theta_W}B \tag{A.52}$$

$$Z_1 = Z_L\cos\xi + Z_R\sin\xi \tag{A.53}$$

$$Z_2 = -Z_L\sin\xi + Z_R\cos\xi \tag{A.54}$$

where

$$Z_L = \cos\theta_W W_{3L} - \sin\theta_W\tan\theta_W W_{3R} + \tan\theta_W\sqrt{\cos 2\theta_W}B \tag{A.55}$$

$$Z_R = \frac{\sqrt{\cos 2\theta_W}}{\cos\theta_W}W_{3R} + \tan\theta_W B \tag{A.56}$$

$$\tan 2\xi = 2\sqrt{\cos 2\theta_W}(M_{Z_L}/M_{Z_R})^2 \tag{A.57}$$

and

$$M_{Z_L}^2 \approx \frac{g^2}{2\cos^2\theta_W}(\kappa^2+\kappa'^2+4v_L^2) \tag{A.58}$$

$$M_{Z_R}^2 \approx \frac{g^2}{2\cos^2\theta_W\cos 2\theta_W}[(\kappa^2+\kappa'^2)\cos^2\theta_W + 4v_L^2\sin^2\theta_W + 4v_R^2\cos^4\theta_W] \tag{A.59}$$

Finally, the neutral current interactions can be written in terms of the physical A, Z_1 and Z_2 fields.

We now turn to the question of fermion masses and mixing. The most general gauge invariant interaction can be written as

$$\begin{aligned} L_Y = & \sum_{ij} h_{ij}^q\bar{Q}_{Li}\phi Q_{Rj} + \tilde{h}_{ij}^q\bar{Q}_{Li}\tilde{\phi}Q_{Rj} \\ & + \sum_{ij} h_{ij}^l\bar{\psi}_{Li}\phi\psi_{Rj} + \tilde{h}_{ij}^l\bar{\psi}_{Li}\tilde{\phi}\psi_{Rj} \\ & + f_{ij}(\psi_{Li}^T C^{-1}\tau_2\Delta_L\psi_{Lj} + \psi_{Ri}^T C^{-1}\tau_2\Delta_R\psi_{Rj}) + h.c \end{aligned} \tag{A.60}$$

The last two terms can generate Majorana masses for the neutrinos after SSB. The quark and charged lepton masses are given by

$$M_{ij}^u = h_{ij}^q\kappa e^{i\alpha} + \tilde{h}_{ij}^q\kappa' e^{-i\alpha} \tag{A.61}$$

$$M_{ij}^d = h_{ij}^q\kappa' e^{-i\alpha} + \tilde{h}_{ij}^q\kappa e^{i\alpha} \tag{A.62}$$

$$M_{ij}^e = h_{ij}^l\kappa' e^{-i\alpha} + \tilde{h}_{ij}^l\kappa e^{i\alpha} \tag{A.63}$$

One can consider a couple of interesting scenarios. If we demand that the Lagrangian is invariant under parity($\phi \leftrightarrow \phi^\dagger, Q_L \leftrightarrow Q_R, \psi_L \leftrightarrow \psi_R$) then the matrices $[h_{ij}]$ and $[\tilde{h}_{ij}]$ are hermitian. If $\alpha = 0$ then the mass matrices are hermitian which means they can be diagonalized by unitary matrices and consequently the mixing angles and phases in the left and the right handed quark sectors are the same (manifest left -right symmetry). Another case is when the h's are real (CP conserved Lagrangian) but $\alpha \neq 0$(CP broken spontaneously). The mass matrices are then complex and symmetric. In this case the mixing angles in the left and right handed quark sectors are the same but the phases are different(pseudo left-right symmetry). If the Lagrangian is not invariant under parity then in general the left and the right mixing matrices are different. Also if we introduce an additional ϕ field then a parity invariant Lagrangian can lead to different left and right mixing matrices in the quark sector.