UNIVERSITÉ DE GENÈVE

SCHOLA GENEVENSIS MDLIX

HADRONIC TRANSITIONS AMONG QUARKONIUM STATES IN A SOFT-EXCHANGE-APPROXIMATION. CHIRAL BREAKING AND SPIN SYMMETRY BREAKING PROCESSES.*

R. Casalbuoni
Dipartimento di Fisica, Univ. di Firenze
I.N.F.N., Sezione di Firenze

A. Deandrea, N. Di Bartolomeo and R. Gatto
Département de Physique Théorique, Univ. de Genève

F. Feruglio
Dipartimento di Fisica, Univ. di Padova
I.N.F.N., Sezione di Padova

G. Nardulli
Dipartimento di Fisica, Univ. di Bari
I.N.F.N., Sezione di Bari

UGVA-DPT 12-801
December 1992

* Partially supported by the Swiss National Foundation

UGVA-DPT-12-801

Fermilab Library
0 1160 0029340 1

ABSTRACT

Although no asymptotic heavy quark spin symmetry, and even more no flavor symmetry, are expected for systems such as quarkonium, a numerical discussion shows that for some processes and in a preasymptotic region which may roughly include charmonium and bottomonium, the use of the spin-symmetry may be useful in conjunction with chiral symmetry for light hadrons (soft-exchange- approximation regime, SEA). We continue our discussion of hadronic transitions in the SEA-regime by studying in particular chiral breaking transitions such as ${}^3P' \to {}^3P\pi^0$, ${}^3P\eta$, level splittings and transitions which break both chiral and spin symmetry, such as $\psi' \to J/\psi\pi^0$, $J/\psi\eta$, and ${}^1P_1 \to J/\psi\pi^0$.

1 Introduction

The success of heavy quark symmetry [1], when applied to systems containing one heavy quark, does not unfortunately justify its extension to systems with more than one heavy quark, such as quarkonium, etc. A critical discussion [2] of such an issue for quarkonium states has lead us to recognize that no asymptotic symmetry of the heavy quark type (neither of the flavor-type nor of spin) is expected to hold for such systems. However at a numerical level, when limited to a class of processes, and in a preasymptotic quark mass region, application of a heavy quark formalism, only for the heavy quark spin symmetry and excluding the flavor symmetry, may be expected to be of some use. This class of processes excludes those which violate the Zweig rule and is limited to a kinematical domain which we have called the SEA (Soft-Exchange-Approximation) regime, for which it is essential that gluonic exchanges be predominantly of limited momenta.

The numerical examination of the possible preasymptotic range for the heavy quark mass suggests that it may be possible to use the formalism for charmonium and bottomonium, within the mentioned class of processes, and an effective lagrangian for quarkonia and light mesons was written down to be used within the SEA regime [2].

A number of applications [2] to transitions among charmonium states, for not too large momenta of the emitted hadrons, showed the usefulness of the formalism to derive results which would have otherwise required longer approximate QCD calculations. The heavy quark spin symmetry alone leads very simply to general relations for the differential decay rates in hadronic transitions among quarkonium states, which in the known cases reproduce the results of a multiple QCD multipole expansion [3] for gluonic emission. Further use of chiral symmetry leads to differential pion decay distributions valid in the soft regime. As shown in ref. [2] and [4] the general relations following from heavy quark spin symmetry alone relate the allowed transitions between two quarkonium multiplets, such as ${}^3S_1 \to {}^3S_1 + h$ and ${}^1S_0 \to {}^1S_0 + h$, ${}^3P_2 \to {}^3S_1 + h$, ${}^3P_1 \to {}^3S_1 + h$, ${}^3P_0 \to {}^3S_1 + h$, ${}^1P_1 \to {}^1S_0 + h$, all the transitions ${}^3P_2$, ${}^3P_1$, ${}^1P_1 \to {}^3P_2$, ${}^3P_1$, ${}^3P_0$, ${}^1P_1 + h$, those of the type ${}^3D_3$, ${}^3D_2$, ${}^3D_1$, ${}^1D_2 \to {}^3S_1$, ${}^1S_0 + h$, independently of the nature of the light final state h. Heavy quark spin symmetry, when supplemented with the lowest order chiral expansion for the emitted pseudoscalars leads to a general rule allowing only for even (odd) number of emitted pseudoscalars for transitions between quarkonium states of orbital angular momenta different by even (odd) units [4]. Such a rule can be violated by higher chiral terms, by chiral breaking, and by terms breaking the heavy quark spin symmetry. Specialization to a number of hadronic transitions reproduces by elementary tensor construction the known results from the cumbersome multiple expansion in gluon multipoles, providing for a simple explanation for the vanishing of certain coefficients which would otherwise be allowed in the chiral expansion. In certain cases, such as for instance ${}^3P_0 \to {}^3P_2\pi\pi$, ${}^3P_1 \to {}^3P_2\pi\pi$, or $D-S$ transitions via 2π, the final angular and mass distributions are uniquely predicted from heavy quark spin and lowest order chiral expansion. Other processes such as ${}^3S_1 \to {}^3S_1\pi\pi$ will depend on two chiral parameters, as in the case of ${}^3P_0 \to {}^3P_2\pi\pi$ and ${}^3P_1 \to {}^3P_2\pi\pi$, whereas ${}^3P_0 \to {}^3P_1\pi\pi$ receives no contributions, within the approximation. We shall not dwell here with the derivation and presentation of these results for which we refer to [4].

In the present note we shall concentrate on hadronic transitions among states of quarkonium which proceed either by breaking of the chiral symmetry, but consistently with heavy quark spin symmetry, or transitions which break the heavy quark spin symmetry. For instance the transitions among P-states, ${}^3P_{J'} \to {}^3P_J\pi_0$ or $\to {}^3P_J\eta$ proceed through chiral breaking but heavy spin conserving terms, whereas for instance $\psi' \to J/\psi\pi_0$ or $\to J/\psi\eta$ go through terms violating both symmetries. Apart from deriving general relations for the matrix elements of π^0 and η^0 emission in transitions among P-states, one can estimate the suppression factors entering in these transitions, related to the current quark masses. For transitions $\Upsilon(3P_{J'}) \to \Upsilon(1P_J)$ where in the final state also a η could be kinematically allowed, one can estimate the π^0 versus η emission ratio, roughly expected of the order 10^{-2} within conventional assumptions.

The first test for heavy quark spin breaking is of course in the structure of levels. The spin breaking in the formalism is expected to go by insertion of matrices $\sigma_{\mu\nu}$ multiplied by the relevant projectors at their left and right, and with a depression factor in front of them of the order of the inverse of the heavy quark mass. We have tried to reproduce the observed level patterns, as given by spin-spin, spin-orbit, and tensor splittings, in terms of $\sigma_{\mu\nu}$ insertions, and found a general consistency for charmonium and bottomonium, although the lack of flavor symmetry does not allow for a reliable quantitative comparison between the two systems.

We have studied the transitions $\psi' \to J/\psi\pi_0$ and $\psi' \to J/\psi\eta$, which go through spin breaking in our formalism, and which are of interest as the ratio of their partial widths is related to quark masses, apart from meson mixing terms. The transition ${}^1P_1 \to J/\psi\pi\pi$ goes through spin breaking chiral conserving terms, whereas ${}^1P_1 \to J/\psi\pi_0$ breaks both symmetries. In view of a recent upper limit by the E760 collaboration and of future accurate experiments we have studied both transitions, getting to a rough estimate of the ratio of their widths, which is in agreement with the present limits. Increase of experimental accuracies and availability of heavy meson factories [5] would make this whole field of experimentation of renewed interest.

2 Discussion of the approximation and formal description

The usual description of quarkonium states is based on a short-distance regime, coulomb-like apart radiative corrections, and on a long-distance regime closer to a string-like description.

A velocity heavy-quark description might make some approximate sense within the string-like regime, but will certainly fall down in the short distance regime. For large quark mass the coulombic regime will prevail, and in such a case one would have from the virial theorem $< T >= -E$, where T is the kinetic energy and $-E$ the binding energy. Also, from the Feynmann-Hellmann theorem, one would have a kinetic energy increasing linearly with the heavy quark mass, implying a corresponding increase of the relative momentum. The exchange of hard gluons of large momentum will become dominant. No spin symmetry is then expected to hold.

Even worse for what concerns a possible heavy-quark flavor symmetry. In general gluon radiation exchanged between static quarks brings about infrared divergences. In a bound state, potential and kinetic energy play a delicate balance against each other. The regularization of the infrared divergences then implies a large breaking of any flavour symmetry because of the explicit appearance of the heavy quark mass in the kinetic energy.

The conclusion is that, in total contrast to the situation for systems containing a single heavy quark, no heavy quark spin symmetry emerges asymptotically for infinite heavy quark mass, and even worse for a hypothetical heavy quark flavor symmetry.

For an assessment of the situation in some preasymptotic region one has to look at the existing quarkonium calculations. We can make use of calculations of Buchmueller and Tye [6] with a potential behaving like r^{-1} at short distance and like r at large distance to extract mass behaviours of the type: $< k > \approx 1.0\ m_Q^{0.66}$, $< v > \approx\ 0.5\ m_Q^{-0.34}$, $< T > \approx\ 0.25\ m_Q^{0.32}$ for the residual momentum, the relative velocity and the kinetic energy T within the $Q\bar{Q}$ bound state against the quark mass m_Q expressed in GeV. These formulae are expected to hold at least up to $m_Q \approx\ 80\ GeV$. From calculations by Quigg and Rosner [7] for a potential $C\ log(r/r_0)$, with $C = 3/4\ GeV$, we obtain, by using the virial theorem for this case: $< k > \approx 1.22\ m_Q^{0.5}$, $< v > \approx 0.65\ m_Q^{-0.5}$, $< T > \approx 0.375\ GeV$. By applying the virial theorem and the Feynmann-Hellmann theorem to more recent calculations by Grant and Rosner [8] we obtain: $< k > \approx 1.22\ m_Q^{0.54}$, $< v > \approx 0.61\ m_Q^{-0.46}$, $< T > \approx 0.37\ m_Q^{0.08}$. The conclusions seems to be that the kinetic energy and the residual momentum increase with increasing m_Q, while the relative velocity decreases. This seems empirically true (under all the assumptions for such calculations) in a preasymptotic region which contains both charmonium and bottomonium. On the other hand we know that the asymptotic behaviour for very heavy mass could well imply a linear growing of the kinetic energy, by naively applying the Feynmann-Hellmann theorem to a dominant Coulomb force.

For higher waves one finds, using again the analysis of ref.[6], that for the $c - \bar{c}$ system the relative velocity increases of about 11-12 % in going from the s-wave to the d-wave, both for the radial states $n = 1$ and $n = 2$. In the case of the bottomonium, the velocity for the s, p and d states is almost the same for $n = 1$, whereas it increases of about 7% between the s-wave to the d-wave for $n = 2$. As a consequence we think that at least up to the d-waves, our approach is still consistent.

Once we accept the conjecture that approximate subasymptotic use of heavy quark symmetry, limited to the spin symmetry, may be useful in the case of bottomonium and charmonium, we can easily develop the formalism, following the notions and the notations of the heavy quark theory as developed for systems containing one heavy quark. The applications which we had considered in our previous note [2] showed no contradictions with existing knowledge and gave direct and transparent derivations of results which would have required a lengthy construction of QCD multipole expansion. We summarize here for completeness and for the notations the description of quarkonium states [2].

A heavy quark-antiquark bound state, characterized by radial number m, orbital angular momentum l, spin s and total angular momentum J, is denoted by:

$$m\,^{2s+1}l_J \tag{2.1}$$

In the limit of no spin-dependent interactions between the two quarks the singlet $m\,^1l_J$ and the spin triplet $m\,^3l_J$ form a single multiplet $J(m,l)$. For $l = 0$, when the triplet $s = 1$ collapses into a single state with total angular momentum $J = 1$, such a multiplet is described by:

$$J = \frac{(1+\not{v})}{2}[H_\mu\gamma^\mu - \eta\gamma_5]\frac{(1-\not{v})}{2} \tag{2.2}$$

Here v^μ denotes the four velocity associated to the multiplet J; H_μ and η are the spin 1 and spin 0 components respectively; the radial quantum number has been omitted.

For orbital angular momentum $l \neq 0$ the multiplet J generalizes to $J^{\mu_1\cdots\mu_l}$, with a decomposition

$$\begin{aligned} J^{\mu_1\cdots\mu_l} &= \frac{(1+\not{v})}{2}\Bigg[H_{l+1}^{\mu_1\cdots\mu_l\alpha}\gamma_\alpha + \frac{1}{\sqrt{l(l+1)}}\sum_{i=1}^{l}\epsilon^{\mu_i\alpha\beta\gamma}v_\alpha\gamma_\beta H_{l\gamma}^{\mu_1\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_l} \\ &+ \frac{1}{l}\sqrt{\frac{2l-1}{2l+1}}\sum_{i=1}^{l}(\gamma^{\mu_i} - v^{\mu_i})H_{l-1}^{\mu_1\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_l} \\ &- \frac{2}{l\sqrt{(2l-1)(2l+1)}}\sum_{i<j}(g^{\mu_i\mu_j} - v^{\mu_i}v^{\mu_j})\gamma_\alpha H_{l-1}^{\alpha\mu_1\cdots\mu_{i-1}\mu_{i+1}\cdots\mu_{j-1}\mu_{j+1}\cdots\mu_l} \\ &+ K_l^{\mu_1\cdots\mu_l}\gamma_5\Bigg]\frac{(1-\not{v})}{2} \end{aligned} \tag{2.3}$$

Here $K_l^{\mu_1\cdots\mu_l}$ represents the spin singlet ^1l_J, and the spin triplet ^3l_J is represented by $H_{l+1}^{\mu_1\cdots\mu_{l+1}}$ for $J = l+1$, $H_l^{\mu_1\cdots\mu_l}$ for $J = l$, and $H_{l-1}^{\mu_1\cdots\mu_{l-1}}$ for $J = l-1$. All these tensors are completely symmetric, traceless and satisfy transversality conditions

$$\begin{aligned} v_{\mu_1}K_l^{\mu_1\cdots\mu_l} &= 0 \\ v_{\mu_1}H_{l+1,l,l-1}^{\mu_1\cdots\mu_{l+1},l,l-1} &= 0 \end{aligned} \tag{2.4}$$

Moreover, to avoid orbital momenta other than l, we require that $J^{\mu_1\cdots\mu_l}$ itself is completely symmetric, traceless and orthogonal to the velocity. This allows to identify the states in 2.3 with the physical states. The normalisation for $J^{\mu_1\cdots\mu_l}$ has been chosen so that:

$$\begin{aligned} < J^{\mu_1\cdots\mu_l}\bar{J}_{\mu_1\cdots\mu_l} > = \ & 2\ \Big(H_{l+1}^{\mu_1\cdots\mu_{l+1}}H_{\mu_1\cdots\mu_{l+1}}^{\dagger l+1} - H_l^{\mu_1\cdots\mu_l}H_{\mu_1\cdots\mu_l}^{\dagger l} \\ & + H_{l-1}^{\mu_1\cdots\mu_{l-1}}H_{\mu_1\cdots\mu_{l-1}}^{\dagger l-1} - K_l^{\mu_1\cdots\mu_l}K_{\mu_1\cdots\mu_l}^{\dagger l}\Big) \end{aligned} \tag{2.5}$$

where $\bar{J} = \gamma^0 J^\dagger\gamma^0$ and $< \ldots >$ means the trace over the Dirac matrices. The following applications will concern only s and p states, given respectively by 2.2 and

$$\begin{aligned} J^\mu = \frac{1+\not{v}}{2}\Bigg[\ & H_2^{\mu\alpha}\gamma_\alpha + \frac{1}{\sqrt{2}}\epsilon^{\mu\alpha\beta\gamma}v_\alpha\gamma_\beta H_{1\gamma} \\ & + \frac{1}{\sqrt{3}}(\gamma^\mu - v^\mu)H_0 + K_1^\mu\gamma_5\Bigg]\frac{(1-\not{v})}{2} \end{aligned} \tag{2.6}$$

Under a Lorentz transformation Λ we have:

$$J^{\mu_1\cdots\mu_l} \to \Lambda^{\mu_1}_{\ \nu_1}\ldots\Lambda^{\mu_l}_{\ \nu_l}D(\Lambda)J^{\nu_1\cdots\nu_l}D(\Lambda)^{-1} \tag{2.7}$$

where $D(\Lambda)$ is the usual spinor representation of Λ.

Parity and charge conjugation have the following action:

$$J^{\mu_1\cdots\mu_l} \xrightarrow{P} \gamma^0 J_{\mu_1\cdots\mu_l}\gamma^0 \tag{2.8}$$

$$J^{\mu_1\cdots\mu_l} \xrightarrow{C} (-1)^{l+1} C J^{\mu_1\cdots\mu_l T} C \tag{2.9}$$

where $C = i\gamma^2\gamma^0$ is the charge conjugation matrix.

Under heavy quark spin transformation one has

$$J^{\mu_1\cdots\mu_l} \to S J^{\mu_1\cdots\mu_l} S'^{\dagger} \tag{2.10}$$

with S, $S' \in SU(2)$ and $[S, \not{v}] = [S', \not{v}] = 0$. As long as one can neglect spin dependent effects, one will require invariance of the allowed interaction terms under the transformation 2.10.

3 Chiral breaking hadronic transitions

In this note we restrict ourselves to hadronic transitions with emission of light pseudoscalar mesons. Such a light sector, in the limit of vanishing quark masses, has a spontaneously broken $SU(3) \times SU(3)$ chiral symmetry. The light pseudoscalar octet is described in terms of pseudo-Goldstone bosons, assembled in the matrix

$$\Sigma = \exp \frac{2iM}{f_\pi} \tag{3.1}$$

where f_π is the pion decay constant, $f_\pi \simeq 132\, MeV$, and

$$M = \begin{pmatrix} \sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\sqrt{\frac{1}{2}}\pi^0 + \sqrt{\frac{1}{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \tag{3.2}$$

Frequently occurring quantities are the 1-forms $\mathcal{A}_\mu$ and $\mathcal{V}_\mu$, given by:

$$\begin{aligned} \mathcal{A}_\mu &= \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger\right) \\ \mathcal{V}_\mu &= \frac{1}{2}\left(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger\right) \end{aligned} \tag{3.3}$$

with $\xi^2 = \Sigma$.

Under the chiral symmetry the fields ξ and Σ transform as follows:

$$\begin{aligned} \xi &\to g_L \xi U^\dagger = U \xi g_R^\dagger \\ \Sigma &\to g_L \Sigma g_R^\dagger \end{aligned} \tag{3.4}$$

where g_L, g_R are global $SU(3)$ transformations and U is a function of x, of the fields, and of g_L, g_R. The forms $\mathcal{A}_\mu$ and $\mathcal{V}_\mu$ transform as:

$$\begin{aligned} \mathcal{A}_\mu &\to U \mathcal{A}_\mu U^\dagger \\ \mathcal{V}_\mu &\to U \mathcal{V}_\mu U^\dagger + U \partial_\mu U^\dagger \end{aligned} \tag{3.5}$$

Under parity we have:

$$\begin{aligned} \Sigma &\xrightarrow{P} \Sigma^\dagger \\ \mathcal{A}_\mu &\xrightarrow{P} -\mathcal{A}^\mu \\ \mathcal{V}_\mu &\xrightarrow{P} \mathcal{V}^\mu \end{aligned} \tag{3.6}$$

Under charge conjugation:

$$\begin{aligned} \Sigma &\xrightarrow{C} \Sigma^T \\ \mathcal{A}_\mu &\xrightarrow{C} \mathcal{A}_\mu^T \\ \mathcal{V}_\mu &\xrightarrow{C} -\mathcal{V}_\mu^T \end{aligned} \tag{3.7}$$

In this section we will discuss possible chiral breaking but spin conserving terms, which are important for transitions forbidden in the $SU(3) \times SU(3)$ symmetry limit. Examples of such kind of transitions are

$$^3P_{J'} \to {}^3P_J \pi_0,\ {}^3P_J \eta \tag{3.8}$$

The transitions

$$\psi' \to J/\psi \pi_0,\ J/\psi \eta \tag{3.9}$$

need terms which in addition violate the spin symmetry and will be discussed in the next section.

We first discuss the masses and mixings of the octet and singlet η' pseudoscalar light meson states. The term which give mass to the pseudoscalar octet, massless in the chiral limit, is

$$\mathcal{L}_m = \frac{\mu f_\pi^2}{4} < M(\Sigma + \Sigma^\dagger) > \tag{3.10}$$

Here M is the current mass matrix:

$$M = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{3.11}$$

and μ is a scale parameter with dimensions of a mass. The lagrangian 3.10 gives in addition a mixing $\pi_0 - \eta$: the physical states $\tilde{\pi}_0$, $\tilde{\eta}$ turn out to be:

$$\begin{aligned} \tilde{\pi}_0 &= \pi_0 + \epsilon\eta \\ \tilde{\eta} &= \eta - \epsilon\pi_0 \end{aligned} \tag{3.12}$$

where the mixing angle ϵ is

$$\epsilon = \frac{(m_d - m_u)\sqrt{3}}{4(m_s - \frac{m_u + m_d}{2})} \tag{3.13}$$

The η', which is a chiral singlet, mixes with π_0, η. Such a mixing can be described by the term

$$\mathcal{L}_{\eta\eta'} = \frac{if_\pi}{4}\lambda < M(\Sigma - \Sigma^\dagger) > \eta' \tag{3.14}$$

where λ is a parameter with dimension of a mass. At first order in the mixing angles the physical states are:

$$\begin{aligned} \tilde{\pi}_0 &= \pi_0 + \epsilon\eta + \epsilon'\eta' \\ \tilde{\eta} &= \eta - \epsilon\pi_0 + \theta\eta' \\ \tilde{\eta}' &= \eta' - \theta\eta - \epsilon'\pi_0 \end{aligned} \tag{3.15}$$

where

$$\begin{aligned} \epsilon' &= \frac{\lambda(m_d - m_u)}{\sqrt{2}(m_{\eta'}^2 - m_{\pi_0}^2)} \\ \theta &= \sqrt{\frac{2}{3}}\ \frac{\lambda\left(m_s - \frac{m_u + m_d}{2}\right)}{m_{\eta'}^2 - m_\eta^2} \end{aligned} \tag{3.16}$$

and ϵ as given in 3.13.

We will consider chiral violating, spin-conserving hadronic transitions between charmonium states at first order in the chiral breaking mass matrix M. We are thus lead to consider the quantities:

$$\begin{aligned} &< M(\Sigma + \Sigma^\dagger) > \\ &< M(\Sigma - \Sigma^\dagger) > \end{aligned} \tag{3.17}$$

The first one is even under parity, the second odd, and both have $C = +1$.

The only term spin-conserving and of leading order in the current quark masses contributing to the transition 3.8 is

$$< J_\mu \bar{J}_\nu > v_\rho \epsilon^{\mu\nu\rho\sigma}\partial_\sigma \left[\alpha\frac{if_\pi}{4} < M(\Sigma - \Sigma^\dagger) > + \beta f_\pi \eta'\right] \tag{3.18}$$

where α and β are coupling constants of dimensions $(mass)^{-2}$. The direct coupling to η' contributes through the mixing 3.15. The spin symmetry of the heavy sector gives relations among the modulus square matrix elements of the transitions between the two p-wave states. In particular we find that

$$|\mathcal{M}|^2({}^3P_0 \to {}^3P_0\pi) = |\mathcal{M}|^2({}^3P_2 \to {}^3P_0\pi) = 0 \tag{3.19}$$

and that all non-vanishing matrix elements can be expressed in terms of ${}^3P_0 \to {}^3P_1\pi$:

$$\begin{aligned} |\mathcal{M}|^2({}^3P_1 \to {}^3P_1\pi) &= \frac{1}{4}|\mathcal{M}|^2({}^3P_0 \to {}^3P_1\pi) \\ |\mathcal{M}|^2({}^3P_1 \to {}^3P_2\pi) &= \frac{5}{12}|\mathcal{M}|^2({}^3P_0 \to {}^3P_1\pi) \\ |\mathcal{M}|^2({}^3P_2 \to {}^3P_2\pi) &= \frac{3}{4}|\mathcal{M}|^2({}^3P_0 \to {}^3P_1\pi) \\ |\mathcal{M}|^2({}^1P_1 \to {}^1P_1\pi) &= |\mathcal{M}|^2({}^3P_0 \to {}^3P_1\pi) \end{aligned} \tag{3.20}$$

where π stays for π_0 or η. The relations 3.20 can be generalized for any spin conserving transition between $l = 1$ multiplets, leading to the same results of a QCD double multipole expansion [3]

The width for the emission of a π_0 follows from 3.18:

$$\Gamma({}^3P_0 \to {}^3P_1\pi_0) = \frac{3}{8\pi}|\vec{p}_\pi|^3(m_d - m_u)^2\left[\alpha + \frac{2}{3}\beta\frac{\lambda f_\pi}{(m_{\eta'}^2 - m_{\pi_0}^2)}\right]^2 \tag{3.21}$$

where $\vec{p}_\pi$ is the momentum of the emitted pion in the rest frame of the decaying particle. The width is suppressed approximately by a factor $(m_u - m_d)^2/\Lambda^2$ where $\Lambda = \Lambda_{QCD}$. For most of the transitions between P-states there is not enough phase space for the emission of an η. However a η could be observed for $\Upsilon(3P_J)$ going to $\Upsilon(1P_J)$. For such transitions the ratio of the partial widths turns out to be:

$$\frac{\Gamma({}^3P_J \to {}^3P'_{J'}\pi_0)}{\Gamma({}^3P_J \to {}^3P'_{J'}\eta)} = \frac{27}{16}\left|\frac{\vec{p}_\pi}{\vec{p}_\eta}\right|^3\left[\frac{m_d - m_u}{m_s - \frac{m_u+m_d}{2}}\right]^2\left[\frac{1 + \frac{2}{3}\frac{\beta}{\alpha}\frac{\lambda f_\pi}{(m_{\eta'}^2 - m_{\pi_0}^2)}}{1 + \frac{\beta}{\alpha}\frac{\lambda f_\pi}{(m_{\eta'}^2 - m_\eta^2)}}\right]^2 \tag{3.22}$$

By assuming a small direct coupling of the η' ($\beta \ll \alpha$), or by neglecting the mixing $\pi_0 - \eta'$ and $\eta - \eta'$ (small λ), we can estimate the previous ratio. Taking $m_d - m_u = 5\,MeV$, $m_s = 150\,MeV$, and the mass of $\Upsilon(3P_J)$ equal to $10.53\ GeV$, as predicted in potential models [6], one has for the ratio 3.22 the value:

$$R = 1.3 \times 10^{-2} \tag{3.23}$$

4 Spin breaking

For heavy mesons there are only two types of operators that can break spin symmetry. The simple reason is that on the quark (antiquark) indices of the quarkonium wave function act projection operators $(1 + \not{v})/2$ and $(1 - \not{v})/2$ which reduce the original 4×4-dimensional space to a 2×2-dimensional one. Obviously, in the rest frame, the most general spin symmetry breaking term is of the form $\vec{a} \cdot \vec{\sigma}$, where $\vec{\sigma}$ are the Pauli matrices. In an arbitrary frame one observes that any Γ-matrix sandwiched between two projectors $(1 + \not{v})/2$, or $(1 - \not{v})/2$, can be reexpressed in terms of $\sigma_{\mu\nu}$ sandwiched between the same projectors:

$$\frac{1+\not{v}}{2}1\frac{1+\not{v}}{2} = \frac{1+\not{v}}{2} \tag{4.24}$$

$$\frac{1+\not{v}}{2}\gamma_5\frac{1+\not{v}}{2} = 0 \tag{4.25}$$

$$\frac{1+\not{v}}{2}\gamma_\mu\frac{1+\not{v}}{2} = v_\mu\frac{1+\not{v}}{2} \tag{4.26}$$

$$\frac{1+\not{v}}{2}\gamma_\mu\gamma_5\frac{1+\not{v}}{2} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}v^\nu\frac{1+\not{v}}{2}\sigma^{\alpha\beta}\frac{1+\not{v}}{2} \tag{4.27}$$

$$\frac{1+\not{v}}{2}\gamma_5\sigma_{\mu\nu}\frac{1+\not{v}}{2} = -\frac{i}{2}\epsilon_{\mu\nu\alpha\beta}\frac{1+\not{v}}{2}\sigma^{\alpha\beta}\frac{1+\not{v}}{2} \tag{4.28}$$

and analogous relations with $(1+\not{v})/2 \to (1-\not{v})/2$. We use here $\epsilon_{0123} = +1$. Let us define

$$\sigma^{(\pm)}_{\mu\nu} = \frac{1 \pm \not{v}}{2}\sigma_{\mu\nu}\frac{1 \pm \not{v}}{2} \tag{4.29}$$

In the rest frame, $\sigma^{(\pm)}_{\mu\nu}$ reduce to Pauli matrices. From the previous identities it follows that the most general spin symmetry breaking terms in the quarkonium space are of the form $G_1^{\mu\nu}\sigma^{(+)}_{\mu\nu}$, or $G_2^{\mu\nu}\sigma^{(-)}_{\mu\nu}$, with $G_i^{\mu\nu}$ two arbitrary antisymmetric tensors. There is another convenient way to express this result by means of the Pauli-Lubanski four-vector

$$\Sigma_\mu = \frac{1}{4}\epsilon_{\mu\nu\alpha\beta}v^\nu\sigma^{\alpha\beta} = \frac{i}{2}\gamma_5\sigma_{\mu\nu}v^\nu \tag{4.30}$$

In fact, due to the following identity, $\sigma^{(+)}_{\mu\nu}$ can be evaluated in terms of Σ_μ

$$\sigma^{(+)}_{\mu\nu} = 2\epsilon_{\mu\nu\alpha\beta}\frac{1+\not{v}}{2}\Sigma^\alpha v^\beta\frac{1+\not{v}}{2} \tag{4.31}$$

Σ_μ is orthogonal to v_μ, and in the rest frame we have $\Sigma^\mu = (0, \vec{\sigma}/2)$.

We have shown that the most general operators which break spin symmetry are $\sigma^{(\pm)}_{\mu\nu}$. We can try to get some more insight at the problem by looking at the underlying QCD theory. Following [9], we analyze the QCD equations of motion of a heavy quark

$$(i\not{D} - M)\psi(x) = 0 \tag{4.32}$$

where $D = \partial + ig_sG$, and G the gluon field. Introducing the velocity dependent fields

$$\psi(x) = e^{-iMv\cdot x}Q_v \tag{4.33}$$

we get for the projections

$$Q_v^{(\pm)} = \frac{1 \pm \not{v}}{2}Q_v \tag{4.34}$$

the following equations of motion:

$$iv\cdot DQ_v^{(+)} = -\frac{1+\not{v}}{2}i\not{D}Q_v^{(-)}, \qquad \left(1+\frac{iv\cdot D}{2M}\right)Q_v^{(-)} = \frac{1}{2M}\frac{1-\not{v}}{2}i\not{D}Q_v^{(+)} \tag{4.35}$$

We can solve formally the equation for $Q_v^{(-)}$

$$Q_v^{(-)} = \frac{1}{2M}\frac{1}{1+\frac{iv\cdot D}{2M}}\frac{1-\not{v}}{2}i\not{D}Q_v^{(+)} \tag{4.36}$$

and substitute the result into the first equation, obtaining

$$iv\cdot DQ_v^{(+)} = -\frac{1}{2M}\frac{1+\not{v}}{2}i\not{D}\frac{1-\not{v}}{2}\frac{1}{1+\frac{iv\cdot D}{2M}}i\not{D}\frac{1+\not{v}}{2}Q_v^{(+)} \tag{4.37}$$

The price for eliminating $Q_v^{(-)}$ is a non-local equation for $Q_v^{(+)}$. However, the usefulness of the previous equation is in the expansion in $1/M$. By using the identity

$$\frac{1+\not{v}}{2}\gamma_\mu\frac{1-\not{v}}{2}\gamma_\nu\frac{1+\not{v}}{2} = \frac{1+\not{v}}{2}\left(g_{\mu\nu} - v_\mu v_\nu - i\sigma_{\mu\nu}\right)\frac{1+\not{v}}{2} \tag{4.38}$$

we get our final result

$$iv\cdot DQ_v^{(+)} = \frac{1}{2M}\frac{1+\not{v}}{2}\left[g_{\mu\nu} - v_\mu v_\nu - i\sigma_{\mu\nu}\right]D^\mu\frac{1}{1+\frac{iv\cdot D}{2M}}D^\nu Q_v^{(+)} \tag{4.39}$$

In particular, at the first order in $1/M$ we have

$$iv\cdot DQ_v^{(+)} = \frac{1}{2M}\frac{1+\not{v}}{2}\left[D^2 - (v\cdot D)^2 + \frac{g_s}{2}G_{\mu\nu}\sigma^{\mu\nu}\right]Q_v^{(+)} \tag{4.40}$$

where

$$G_{\mu\nu} = \frac{1}{ig_s}\left[D_\mu, D_\nu\right] \tag{4.41}$$

In the rest frame, this equation is nothing but the Pauli-Schrödinger equation. We could also use the relation between $Q_v^{(-)}$ and $Q_v^{(+)}$ in the equation of motion for the gluons, to obtain that the only spin symmetry breaking term is proportional to $\sigma^{(+)}_{\mu\nu}$, with the further information that the coefficient of this operator starts with $1/M$. From this argument we expect that any insertion of the operator $\sigma^{(+)}_{\mu\nu}$ gives a suppression factor $1/M$. Analogous conclusions can be reached for $\sigma^{(-)}_{\mu\nu}$ by considering the heavy anti-quark field.

The first example of spin breaking within the formalism will concern the fine structure of $J^{\mu_1\cdots\mu_\ell}$ levels in a few interesting cases. The general expression for the fine structure in terms of spin and angular momentum consists of a linear combination of

$$a = \mathbf{S_1}\cdot\mathbf{S_2} \tag{4.42}$$

$$b = \mathbf{L}\cdot\mathbf{S} \tag{4.43}$$

$$c = -\frac{1}{(2\ell-1)(2\ell+3)}\left[12(\mathbf{L}\cdot\mathbf{S})^2 + 6\mathbf{L}\cdot\mathbf{S} - 4\mathbf{S}^2\mathbf{L}^2\right] \tag{4.44}$$

where $\mathbf{S_1}$ and $\mathbf{S_2}$ are the quark spins, $\mathbf{S}$ is the total spin of the system, and $\mathbf{L}$ its orbital angular momentum. The first term gives the hyperfine splitting, the second the spin-orbit splitting and the third comes from the tensor term. The corresponding matrix elements for S, P, D states of quarkonium are given in table I. Within our formalism, for the S-wave, the hyperfine splittings arise from the following term:

$$A(S) = < \sigma^{\mu\nu}J\sigma_{\mu\nu}\overline{J} > \tag{4.45}$$

The values of table I, column a, are reproduced with an appropriate numerical coefficient in front of this term and recalling our normalization for the S-wave of eq.2.5. In the case of the P-wave, the spin-spin, spin-orbit, and tensor terms, are given respectively by:

$$A(P) = < J^\mu\sigma_{\nu\rho}\overline{J}_\mu\sigma^{\nu\rho} > \tag{4.46}$$

$$B(P) = i < J^\mu\sigma_{\mu\nu}\overline{J}^\nu > -i < \overline{J}^\nu\sigma_{\mu\nu}J^\mu > \tag{4.47}$$

$$C(P) = < J^\mu\sigma_{\mu\nu}\overline{J}_\rho\sigma^{\rho\nu} > + < J^\mu\sigma^{\rho\nu}\overline{J}_\rho\sigma_{\mu\nu} > \tag{4.48}$$

where the last term is in effect a combination of the usual tensor and spin-spin terms. The analogous terms for D-waves are:

$$A(D) = < J^{\mu\nu}\sigma_{\rho\lambda}\overline{J}_{\mu\nu}\sigma^{\rho\lambda} > \tag{4.49}$$

for the spin-spin term,

$$B(D) = i < J^{\mu\nu}\sigma_{\mu\rho}\overline{J}^{\rho}_{\nu} > -i < \overline{J}^{\rho}_{\nu}\sigma_{\mu\rho}J^{\mu\nu} > \tag{4.50}$$

for the spin-orbit term, and

$$C(D) =< J^{\mu\nu}\sigma_{\mu\rho}\overline{J}_{\lambda\nu}\sigma^{\rho\lambda} > + < J^{\mu\nu}\sigma^{\rho\lambda}\overline{J}_{\lambda\nu}\sigma_{\mu\rho} > \tag{4.51}$$

$$C'(D) =< J^{\mu\nu}\sigma^{\mu\rho}\overline{J}_{\rho\lambda}\sigma_{\nu\lambda} > \tag{4.52}$$

which are both combinations of the usual tensor and spin-spin terms. It is easy to build a linear combination of the two which gives the tensor splittings of table I.

From the mass values of table II one can extract the physical splittings and perform a fit to arrive at the numerical coefficients in front of our lagrangian terms. As the coefficients have the dimensions of mass, we choose them to be in MeV and obtain for the S-wave spin-spin splitting $-7.3A(S)$ and $-2.5A(S)$ for $c\bar{c}$ and $b\bar{b}$ respectively, in front of an unperturbed mass levels of $1534 < J\overline{J} >$ and $4730 < J\overline{J} >$ for the two cases.

For P-waves the corresponding coefficients are $2A(P)$ and $0.8A(P)$, to be compared with the unsplitted common mass term of $1762.6 < J^{\mu}\overline{J}_{\mu} >$ and $4950.1 < J^{\mu}\overline{J}_{\mu} >$ for charmonium and bottomonium respectively. For the splittings within the triplets, the mass spectra are reproduced by the following combinations:

$$8.75B(P) - 2[A(P) - 1.5C(P)] \tag{4.53}$$

in the case of $c\bar{c}$ and

$$3.5B(P) - 0.6[A(P) - 1.5C(P)] \tag{4.54}$$

in the case of $b\bar{b}$. We see that, in agreement with our previous considerations, the spin-spin and the tensor terms are depressed with respect to the spin-orbit coupling, which contains a single $\sigma_{\mu\nu}$ insertion. Also these terms are more depressed for bottomonium than for charmonium. In the previous computation we have assumed that possible mixing terms among different waves are negligible. It is however possible, within the formalism, to include such mixing terms in the lagrangian. For example in the case of a $S - D$ wave mixing, such a term is given by

$$< J\sigma_{\mu\rho}\overline{J}^{\mu\nu}\sigma^{\rho}_{\nu} > + < \overline{J}\sigma_{\mu\rho}J^{\mu\nu}\sigma^{\rho}_{\nu} > \tag{4.55}$$

mixing $^3S_1$ and $^3D_1$ states.

5 Spin breaking hadronic transitions

We apply now our formalism to the transitions $\psi' \to J/\psi\pi_0$ and $\psi' \to J/\psi\eta$. Of particular interest is the ratio

$$R = \frac{\Gamma(\psi' \to J/\psi\pi_0)}{\Gamma(\psi' \to J/\psi\eta)} \tag{5.1}$$

which provides for a measure of the light-quark mass ratio

$$r = \frac{m_d - m_u}{m_s - \dfrac{m_u + m_d}{2}} \tag{5.2}$$

Using partial conservation of axial-vector current Ioffe and Shifman [10] give the prediction

$$R = \frac{27}{16}\left[\frac{\vec{p_\pi}}{\vec{p_\eta}}\right]^3 r^2 \tag{5.3}$$

The calculation of R is straightforward with the heavy quark formalism. Eq. 5.3 will be recovered when neglecting the mixings $\pi_0 - \eta$ and $\eta - \eta'$ (or a possible direct coupling of η').

The most general spin breaking lagrangian for the processes $\psi' \to J/\psi\pi_0, \eta$ is

$$\begin{aligned}\mathcal{L} = \; & i\epsilon_{\mu\nu\rho\lambda}\left[< J'\sigma^{\mu\nu}\bar{J} > - < \bar{J}\sigma^{\mu\nu}J' >\right] v^{\rho} \times \\ & \partial^{\lambda}\left[\frac{iA}{4} < M(\Sigma - \Sigma^{\dagger}) > + B\eta'\right] + h.c.\end{aligned} \tag{5.4}$$

The couplings A and B have dimension $(mass)^{-1}$; the B term contributes to the ratio 5.1 via the mixing $\pi_0 - \eta'$ and $\eta - \eta'$, in the same way as the β coupling in 3.18. There are no terms with the insertion of two σ; the two P and C conserving candidates

$$\begin{aligned}& \epsilon_{\mu\nu\rho\lambda}\left[< J'\sigma^{\mu\tau}\bar{J}\sigma_{\tau}{}^{\nu} > + < \bar{J}\sigma^{\mu\tau}J'\sigma_{\tau}{}^{\nu} >\right] v^{\rho}\partial^{\lambda} < M(\Sigma - \Sigma^{\dagger}) >; \\ & \epsilon_{\mu\nu\rho\lambda}\left[< J'\sigma^{\mu\nu}\bar{J}\sigma^{\rho\lambda} > + < \bar{J}\sigma^{\mu\nu}J'\sigma^{\rho\lambda} >\right] < M(\Sigma - \Sigma^{\dagger}) >\end{aligned} \tag{5.5}$$

are both vanishing. Using the lagrangian 5.4 and taking into account the mixings 3.15 we can calculate the ratio 5.1, which is quite similar to the ratio 3.22

$$R = \frac{27}{16}\left[\frac{\vec{p_\pi}}{\vec{p_\eta}}\right]^3 \left[\frac{m_d - m_u}{m_s - 1/2(m_u + m_d)}\right]^2 \left[\frac{1 + \dfrac{2B}{3A}\dfrac{\lambda f_\pi}{m^2_{\eta'} - m^2_{\pi 0}}}{1 + \dfrac{B}{A}\dfrac{\lambda f_\pi}{m^2_{\eta'} - m^2_{\eta}}}\right]^2 \tag{5.6}$$

If we neglect the mixings $\pi_0 - \eta'$ and $\eta - \eta'$ ($\lambda = 0$) or the direct coupling of η' (B=0) 5.6 reduces to 5.3.

Eq. 5.6 can receive corrections from electromagnetic contributions to the transition $\psi' \to J/\psi\pi_0$. It has been shown that such corrections are suppressed [11], [12]. A second type of corrections is associated with higher order terms in the light-quark mass expansion (the lagrangian 5.4 is the first order of such expansion); a discussion can be found in ref. [13].

We consider now two hadronic decay modes for the recently discovered [14] $^1P_1$ state of charmonium. These processes are:

$$^1P_1 \to J/\psi\pi\pi \tag{5.7}$$

$$^1P_1 \to J/\psi\pi_0 \tag{5.8}$$

The first one is, at the leading order, spin breaking but chiral conserving, while the second one breaks both symmetries. Therefore one could naively expect an enhancement of 5.7 respect to 5.8. Voloshin [15] suggested that the isospin violating transition 5.8 could be an order of magnitude stronger than the two pion transition 5.7. Kuang Tuan and Yan [16] prediction is quite different, but the E760 Collaboration [14] has set the upper limit

$$\frac{\Gamma(^1P_1 \to J/\psi\pi\pi)}{\Gamma(^1P_1 \to J/\psi\pi_0)} \leq 0.18 \tag{5.9}$$

This result is consistent only with the prediction of Voloshin. We now give an estimate of the partial widths for these decays in our approach.

For the decay ${}^1P_1 \to J/\psi\ \pi\pi$ we can write down in general the following terms:

$$a\left[< J_\mu \sigma^{\mu\nu} \bar{J} > + < \bar{J}\sigma^{\mu\nu} J_\mu >\right] < \mathcal{A}_\nu (v\cdot\mathcal{A}) > \tag{5.10}$$
$$i\ b\left[< J^\mu \sigma_{\mu\rho} \bar{J}\sigma^{\rho\nu} > - < \bar{J}\sigma_{\mu\rho} J^\mu \sigma^{\rho\nu} >\right] < \mathcal{A}_\nu (v\cdot\mathcal{A}) >$$

with a and b arbitrary coefficients with dimension $mass^{-1}$. The contributions of the "heavy" factors of these operators to the matrix element of the process under study are of the same form so that one obtains

$$4(a+b)\epsilon^{\alpha\beta\nu\delta} H_\alpha K_\beta v_\delta \tag{5.11}$$

where H is the field describing the J/ψ resonance and K the ${}^1P_1$.

For the decay ${}^1P_1 \to J/\psi\pi_0$ we can list three independent terms in the lagrangian

$$i\ c\ \epsilon_{\mu\nu\rho\sigma}\left[< J^\mu \sigma^{\nu\rho} \bar{J} > + < \bar{J}\sigma^{\nu\rho} J^\mu >\right] v^\sigma < M\Sigma - M\Sigma^\dagger >; \tag{5.12}$$
$$d\ \epsilon_{\mu\nu\rho\sigma}\left[< J^\mu \sigma_\tau{}^\rho \bar{J}\sigma^{\nu\tau} > - < J^\mu \sigma^{\nu\tau} \bar{J}\sigma_\tau{}^\rho >\right] v^\sigma < M\Sigma - M\Sigma^\dagger >;$$
$$e\ \epsilon_{\mu\nu\rho\sigma}\left[< \sigma^{\mu\tau} J_\tau \sigma^{\nu\rho} \bar{J} > - < \sigma^{\nu\rho} J_\tau \sigma^{\mu\tau} \bar{J} >\right] v^\sigma < M\Sigma - M\Sigma^\dagger >$$

with c, d, e arbitrary dimensionless coefficients. The contribution of the heavy part to the matrix element of the process ${}^1P_1 \to J/\psi\ \pi_0$ sums up to

$$8(c+d+2e) H\cdot K \tag{5.13}$$

For the ratio of the partial widths for ${}^1P_1 \to J/\psi\ \pi_0$ and for ${}^1P_1 \to J/\psi\ \pi\pi$ we find:

$$\frac{\Gamma({}^1P_1 \to J/\psi\pi\pi)}{\Gamma({}^1P_1 \to J/\psi\pi_0)} = 1.7\times 10^{-2}\left(\frac{a+b}{c+d+2e}\right)^2 GeV^2 \tag{5.14}$$

Due to our ignorance of the coefficient ratio in 5.14 we cannot give an exact prediction. However one can try an estimate. If we use our previous argument about $\sigma^{(\pm)}_{\mu\nu}$ insertions, we expect a to be of order $1/M_c$ and b of order $1/M_c^2$. On the other hand, the coefficients in 5.12 are expected to be proportional to Λ_χ ($\Lambda_\chi \approx 1\ GeV$), and furthermore we expect c of order $1/M_c$ and d, e of order $1/M_c^2$. Therefore, except for possible cancellations, it seems reasonable to assume:

$$\frac{a+b}{c+d+2e} \approx \frac{1}{\Lambda_\chi} \tag{5.15}$$

leading to a rough estimate

$$\frac{\Gamma({}^1P_1 \to J/\psi\pi\pi)}{\Gamma({}^1P_1 \to J/\psi\pi_0)} \approx 2\times 10^{-2} \tag{5.16}$$

This would provide for a possible explanation of the relative suppression of the two pion channel. Finally we notice that in our approach the decays ${}^1P_1 \to J/\psi\ \pi\pi$ and ${}^1P_1 \to J/\psi\ \pi_0$ can be related to other processes: ${}^3P_0 \to {}^1S_0\pi_0$ and ${}^3P_1 \to {}^1S_0\pi\pi$. An analogous estimate in the case of bottomonium leads to:

$$\frac{\Gamma({}^1P_1 \to \Upsilon\pi\pi)}{\Gamma({}^1P_1 \to \Upsilon\pi_0)} \approx 2.6\times 10^{-2} \tag{5.17}$$

6 Conclusions

There is no theoretical basis for heavy-quark symmetry as asymptotic symmetry of bound quarkonium in the limit of infinite quark mass. Nevertheless, in a preasymptotic region, expectedly including charmonium and bottomonium, numerical discussion, plus a number of successful applications, show the practical usefulness of adopting the heavy-quark formalism to describe a certain class of processes, which do not violate the Zweig rule, and furthermore only by limiting to (broken) heavy-quark spin symmetry, that is excluding heavy-quark flavor symmetry. The processes are characterized by predominantly soft gluon-exchanges (SEA regime), both for the essential of the bound state description and for the occurring dynamical gluon exchanges. The usefulness of the description appears in particular in conjunction with use of the chiral expansion for the light pseudoscalars, allowing for the construction of an effective chiral lagrangian for charmonium states and light pseudoscalars, which has been successfully applied to study of hadronic transitions [2], [4].

In this note we have explored, within such an approach, transitions which proceed either by breaking the chiral symmetry, or by breaking the heavy-quark spin symmetry, or both symmetries. For processes such as ${}^3P'_{J'} \to {}^3P_J\pi^0$ or ${}^3P'_{J'} \to {}^3P_J\eta^0$ one can relate among them π^0 and η^0 emission and estimate the suppression factors in terms of the current quark masses. A manifestation of the heavy quark spin breaking is in the observed level splittings, reproduced in this approach through spurion-type spin-breaking insertions, and corresponding to the standard spin-spin, spin-orbit, and tensor splittings. We have also calculated the partial widths of the heavy-quark spin-breaking transitions $\psi' \to J/\psi\pi^0$ and $\psi' \to J/\psi\eta^0$, whose ratio is directly related to quark masses, and the p-state to s-state transitions ${}^1P_1 \to J/\psi\pi\pi$ and ${}^1P_1 \to J/\psi\pi^0$, both spin-breaking, the first one chiral-conserving, the second one chiral-violating. Recent and forthcoming improvements in the experimental limits and expected future availability of heavy meson factories will make comparison with data more precise and theoretically informative.

Table Captions

Table I: Matrix elements for spin-spin (a), spin-orbit (b), and tensor term (c) in S, P, D states of quarkonium.

Table II: Masses (in MeV) of S and P states of charmonium and bottomonium. All values are experimental, except $^1P_1$ and $^1S_0$ states for bottomonium [17]. Data are from [18], except for $^1P_1$ state of charmonium [14].

References

[1] N.Isgur and M.B.Wise, Phys. Lett. **B232** (1989) 113; ibidem **B237** (1990) 527; M.B.Voloshin and M.A.Shifman, Sov.J.Nucl.Phys. **45** (1987) 292; ibidem **47** (1988) 511; H.D.Politzer and M.B. Wise, Phys. Lett. **206B** (1988) 681; ibidem **208B** (1988) 504; E.Eichten and B.Hill, Phys. Lett. **234B** (1990) 511; H.Georgi, Phys.Lett. **240B** (1990) 447; B.Grinstein, Nucl. Phys. **B339** (1990) 253; A.F.Falk, H.Georgi, B.Grinstein and M.B.Wise, Nucl. Phys. **B343** (1990) 1.

[2] R.Casalbuoni, A.Deandrea, N.Di Bartolomeo, F.Feruglio, R.Gatto and B.Nardulli, Phys.Lett. **B 302** (1993) 95

[3] T.M.Yan, Phys.Rev. **D22** (1980) 1653

[4] R.Casalbuoni, A.Deandrea, N.Di Bartolomeo, F.Feruglio, R.Gatto and B.Nardulli, UGVA-DPT in preparation.

[5] for recent reviews see for instance: J.Kirkby, CERN-PPE/92-30 (1992) for a tau-charm factory; and SLAC-400, Conf/9204126, UC-414, ed. D.Hitlin, for B-factories.

[6] W.Buchmueller and S.-H.H.Tye, Phys.Rev. **D24** (1981) 132

[7] C.Quigg and J.L.Rosner, Phys. Lett. **B71** (1977) 153

[8] A.K.Grant and J.L.Rosner, preprint E.Fermi Institute, November 1992

[9] A.F.Falk, B.Grinstein, M.E.Luke, Nucl.Phys. **B357** (1991) 185

[10] B.L.Ioffe and M.A.Shifman, Phys.Lett. **B 95** (1980) 99

[11] J.F.Donoghue and S.F.Tuan, Phys.Lett. **B 164** (1985) 401

[12] K.Maltman, Phys.Rev. **D 44** (1991) 751

[13] J.F.Donoghue and D.Wyler, Phys.Rev. **D 45** (1992) 892

[14] E760 Collaboration, Phys. Rev. Lett. **69** (1992) 2337

[15] M.B.Voloshin, Sov. J. Nucl. Phys. **43** (1986) 1011

[16] Y.P.Kuang, S.F.Tuan, T.M.Yan, Phys. Rev. **D37**, (1988) 1210

[17] L.P.Fulcher, Phys.Rev. **D 39** (1989) 295

[18] Particle Data Group, Phys.Rev. **D45**, Part 2 (1992)

Table I

State	a	b	c
$^1S_0$	-3/4	0	0
$^3S_1$	1/4	0	0
$^1P_1$	-3/4	0	0
$^3P_0$	1/4	-2	-4
$^3P_1$	1/4	-1	2
$^3P_2$	1/4	1	-2/5
$^1D_2$	-3/4	0	0
$^3D_1$	1/4	-3	-2
$^3D_2$	1/4	-1	2
$^3D_3$	1/4	2	-4/7

Table II

State	$c\bar{c}$	$b\bar{b}$
$^1S_0$	2980	9420
$^3S_1$	3097	9460
$^1P_1$	3526	9901
$^3P_0$	3415	9860
$^3P_1$	3510	9892
$^3P_2$	3556	9913