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RELATIVISTIC CORRECTIONS vs. GLUON MASS EFFECTS: TOWARD A CONSISTENT DESCRIPTION OF QUARKONIA DECAYS.

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ABSTRACT

We present a detailed discussion of the relativistic corrections to quarkonia decay widths. The results confirm our previous conclusion that relativistic corrections alone are unable to explain the discrepancy existing till now between perturbative QCD and experiments. We re-analyze the experimental data by using simple phase-space corrections parametrized in terms of two phenomenological parameters M_g (the 'effective gluon mass' at each quarkonium scale) determined by fitting the inclusive photon spectra in radiative decays of the Υ and J/ψ. In this case, all $c\bar{c}$ and $b\bar{b}$ decay widths provide values of α_s which are consistent and in very good agreement with the relative perturbative running from the c-quark to the b-quark mass scale and with the extrapolation from deep inelastic scattering. For the Υ, the value $M_g \simeq 1.2$ GeV is in good agreement with the theoretical expectation that, asymptotically, the effective gluon mass should tend to its theoretical value $m_g \simeq 1.5 \pm 0.3$ GeV related to the gluon condensate $\langle \alpha_s G^2 \rangle$ in a recent calculation of dynamical mass generation in QCD. The smaller value $M_g \simeq 0.66$ GeV found for the J/ψ can also be understood since a gluon mass $m_g \simeq 1.5$ GeV forces the primary gluons of J/ψ decays to fuse into light-quark pairs thus leading to an effective reduction of the measured gluon mass. As previously emphasized, this gluon fusion mechanism explains the structure of the hadronic final states observed in J/ψ decays and their close similarity to the continuum $e^+e^- \to$ hadrons annihilation at comparable center of mass energies. Taking into account gluon mass corrections and small relativistic corrections derived from recent estimates of the pole-masses of the c- and b-quarks, we obtain the following values of the strong coupling constant:

$$\alpha_s(m_b) = 0.200 \pm 0.002^{+0.010}_{-0.006} \qquad \alpha_s(m_c) = 0.302 \pm 0.008^{+0.038}_{-0.050}$$

where the first errors are experimental and the second are estimates of the theoretical systematic error.

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1. Introduction

Since the original suggestion by Appelquist and Politzer[1], the annihilation of heavy quark pairs into gluons or gluons plus a photon has been recognized as an apparently excellent laboratory for testing the basic ideas of QCD and measuring with high precision the strong coupling constant α_s. A particularly favourable case is the annihilation of the Υ into three gluons. Indeed, the decay width $\Gamma(\Upsilon \to ggg)$ is $O(\alpha_s^3)$ and the relevant scale, $M_\Upsilon \sim 9.46$ GeV, being considerably larger than the scale of hadronic masses ~ 1 GeV, gives some confidence in the applicability of perturbative QCD to the process. By considering the relative branching ratio to charged lepton pairs then, as suggested in ref.[1], the non-perturbative physics, contained in a non relativistic model in the value of the bound state heavy $Q\bar{Q}$ pair wave function at the origin, cancels, leading to a definite prediction depending only on α and α_s. Furthermore, the above branching ratio is now measured with very high accuracy, namely[2]:

$$\frac{\Gamma(\Upsilon \to ggg)}{\Gamma(\Upsilon \to l^+l^-)} = 32.6 \pm 0.8 \qquad \text{(experiment)}$$

If the theoretical prediction of α_s at a scale $\mu \sim m_b \sim \frac{M_\Upsilon}{2}$, derived from the recent high precision analysis of deep inelastic scattering[3] (DIL)

$$\alpha_s^{Th}(\frac{M_\Upsilon}{2}) = 0.209 \pm 0.006 \tag{1}$$

is, however, used in the formula giving the perturbative QED/QCD prediction[4-6] for heavy quarkonia decays (V=J/ψ,Υ; Q^2=4/9,1/9; $k_V \sim 1.6, 0.4$ and $l = e, \mu$)

$$\frac{\Gamma(V \to ggg)}{\Gamma(V \to l^+l^-)} = \frac{10(\pi^2-9)\alpha_s^3}{81\pi Q^2\alpha^2(M_V)}[1+\frac{\alpha_s}{\pi}k_V] \tag{2}$$

($\alpha^{-1}(M_{J/\psi}) \sim 133.7$, $\alpha^{-1}(M_\Upsilon) \sim 132.1$) we obtain the theoretical prediction

$$\frac{\Gamma(\Upsilon \to ggg)}{\Gamma(\Upsilon \to l^+l^-)} = 50.3 \pm 4.5 \qquad \text{(theory)}$$

with a rather large (4σ) difference between theory and experiment. In the corresponding J/ψ case the discrepancy is even bigger. Indeed, the experimental value[7]

$$\frac{\Gamma(J/\psi \to ggg)}{\Gamma(J/\psi \to l^+l^-)} = 10.1 \pm 0.9 \qquad \text{(experiment)}$$

lies a factor of 5.5 (4.7σ) below the theoretical prediction:

$$\frac{\Gamma(J/\psi \to ggg)}{\Gamma(J/\psi \to l^+l^-)} = 55.5 \pm 9.6 \qquad \text{(theory)}$$

based on Eq.(2) and the DIL estimate[3]

$$\alpha_s^{Th}(\frac{M_{J/\psi}}{2}) = 0.326 \pm 0.017 \tag{3}$$

The very large difference between perturbative QCD predictions and experimental results indicates that some aspect of the theoretical framework leading to Eq.(2) requires modification. The traditional solution to this problem has been to invoke large relativistic corrections. Indeed, potential models indicate (we set everywhere c=1) that v^2 ~0.08, 0.24 for the Υ, J/ψ respectively and a purely "ad hoc" correction factor of the form $(1 + Cv^2)$ with $C \sim -3.4$ is found to bring theory and experiment into fair agreement for both Υ and J/ψ[3,5,6]. Three remarks are in order. First, such a procedure clearly removes any possibility of a precise measurement of α_s from quarkonia decays. Second, the value of v^2 is not, by itself, always indicative of the magnitude of the effect in the various observables. For instance, in the recent calculation by Beyer et al. [12] a detailed comparison between fully relativistic calculations and non relativistic approximation is presented by using a constituent quark-antiquark potential model including spin dependent terms whose parameters are determined from a fit to the heavy meson mass spectrum. By inspection of Table 4 of ref.[12], in the case of the J/Ψ leptonic width, one finds only a 7% difference between the fully relativistic calculation $\Gamma^{rel.}(J/\psi \to l^+l^-) = 5.33$ keV and its non relativistic approximation $\Gamma^{non\ rel.}(J/\psi \to l^+l^-) = 5.72$ keV (to be compared with the experimental result[2]

$\Gamma^{exp}(J/\Psi \to e^+e^-) = 5.36^{+0.21}_{-0.28}$ keV). Note that for the Υ the effect is only 1% smaller, namely $\Gamma^{rel.}(\Upsilon \to l^+l^-) = 1.24$ keV, $\Gamma^{non\ rel.}(\Upsilon \to l^+l^-) = 1.32$ keV (to compare with the experimental result[2] $\Gamma^{exp}(\Upsilon \to \mu^+\mu^-) = 1.34 \pm 0.04$ keV). Therefore, the assumption of strongly asymmetric relativistic corrections for the J/ψ and the Υ system, made in the "ad hoc" ansatz above, is not borne out by actual calculations. One may contrast the good agreement between theory and experiment found here for the leptonic widths with the large discrepancies mentioned above in the ratio of widths in Eq.[2]. This indicates that modifications are required in the theoretical description of the decay process $V \to ggg$ rather than in the that of the quarkonium bound state. The effective gluon mass effects discussed in Ref.[10] and in Section 3 below give just such a modification. Finally, if one really believes in the essential role of the relativistic effects, their presence has to be consistently taken into account in *all* processes. A simultaneous fit to the various observable quantities can determine whether relativistic corrections are responsible for the serious discrepancy between Eq.(2) and the experimental data. This analysis will be presented in Section 2 where we shall make essential use of the results obtained by Bodwin, Braaten and Lepage (BBL)[13]. They have made substantial progress towards resolving the problem of double counting of QCD corrections (a perturbative expansion in α_s) and relativistic corrections in powers of v^2. Our results, will confirm the conclusions of our recent paper[10] that relativistic corrections alone *cannot* lead to a consistent description of quarkonium decays within perturbative QCD (in this sense, our analysis contradicts the claim of the recent review paper by Schuler[14]).

After analyzing the effect of relativistic corrections, we shall review in Section 3 an alternative explanation for the observed discrepancy between experiments and perturbative QCD based on the phase space modification associated with an effective mass for the primary gluons. This affects both the main ggg and gg decay channels of 1^- and 0^- resonant states and the photon spectrum of the radiative decays. This picture, described in detail in ref.[10], leads to a remarkable consistency of the experimental data with the expected perturbative running of α_s once the phase space suppression

factors for the various decay channels are properly taken into account. In Section 3 we also consider the combined effect of relativistic corrections predicted from the parameters of the $Q\bar{Q}$ bound state (the mass of the state and the constituent quark mass) and of the effective gluon mass. The relativistic corrections are found to be small.

Finally, in Section 4, we present a summary of our conclusions. Some technical details of the fitting procedure of the photon spectrum in the radiative decays $J/\psi \to \gamma + X$ and $\Upsilon \to \gamma + X$, where it is essential to take properly into account the resolution in the photon energy of the various experiments, are described in an Appendix.

2. Relativistic corrections in quarkonia decays.

Our analysis of the relativistic corrections in heavy quarkonia decays is based on the approach of ref.[13] that we now briefly recapitulate. BBL introduce an effective theory, called non-relativistic quantum chromodynamics (NRQCD), based on the separation of the problem into "slow" variables, associated with a non-relativistic field theory of heavy quarks with mass m, and "fast" variables, corresponding to the usual relativistic QCD of the gluon and light quark fields. Factorization involves separating the short-distance physics of the annihilation process at scales $\sim 1/m$ from the non-relativistic physics of the quarkonium system over scales $1/(mv)$, v being the heavy-quark velocity in the quarkonium rest frame. The theory is made precisely equivalent to full QCD through the addition of an infinite set of local operators which can be organized into a hierarchy according to their dependence on v^2. For instance, in the case of the η_c and J/ψ decay widths, the BBL results are (LH=light hadrons)

$$\Gamma(\eta_c \to LH) = \frac{2Im\ f_1(^1S_o)}{m^2} < \eta_c|O_1(^1S_o)|\eta_c > + \frac{2Im\ g_1(^1S_o)}{m^4} < \eta_c|P_1(^1S_o)|\eta_c > + O(v^4\Gamma) \quad (4)$$

$$\Gamma(J/\psi \to LH) = \frac{2Im\ f_1(^3S_1)}{m^2} < \psi|O_1(^3S_1)|\psi > + \frac{2Im\ g_1(^3S_1)}{m^4} < \psi|P_1(^3S_1)|\psi > + O(v^4\Gamma) \quad (5)$$

In Eqs.(4,5) the imaginary parts $Im\ f_1$ and $Im\ g_1$ contain the short-distance physics and can be

expressed in powers of $\alpha_s(m)$. On the other hand the matrix elements of the relevant dimension-four operators O_1, which give the leading contribution, and of the next-to-leading dimension-six operators P_1, which represent $O(v^2)$ corrections to the decay widths, contain non perturbative effects and can be computed, in principle, by using lattice simulations or potential models of the "slow" NRQCD regime. To $O(v^2)$ all matrix elements entering the ${}^1S_o$ and ${}^3S_1$ decay widths can be expressed in terms of three independent non-perturbative quantities, namely $|\overline{R_{\eta_c}}|^2$, $|\overline{R_\psi}|^2$ and $Re(\overline{R_s}^* \cdot \overline{\nabla^2 R_s})$ where :

$$R_{\eta_c}(|\vec{x}|) = \sqrt{\frac{2\pi}{3}} < o|\chi^+(-\frac{\vec{x}}{2})\psi(\frac{\vec{x}}{2})|\eta_c > \tag{6}$$

$$R_\psi(|\vec{x}|) = \sqrt{\frac{2\pi}{3}}\ \vec{\epsilon} < o|(\chi^+(-\frac{\vec{x}}{2})\vec{\sigma}\psi(\frac{\vec{x}}{2})|\psi > \tag{7}$$

($\vec{\epsilon}$ denoting the polarization vector of the J/ψ) and R_s denotes the mean value :

$$R_s = \frac{R_{\eta_c} + 3R_\psi}{4} \tag{8}$$

The local operators $(\chi^+\psi)$ and $(\chi^+\vec{\sigma}\psi)$ refer to a renormalization scale Λ and, as suggested by the overline, $\overline{R_{\eta_c}}(\Lambda)$ and $\overline{R_\psi}(\Lambda)$ have the meaning of radial wavefunctions averaged over regions of size $1/\Lambda$ centered at the origin. ϕ (χ) are the Pauli spinor fields of the heavy quark (antiquark).

By using the above results and taking into account the known $O(\alpha_s)$ corrections[4-6] in the coefficients of the leading dimension-four operators we obtain the following expressions

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{256\pi\alpha^2}{27M_{\eta_c}^2}\ |\overline{R_{\eta_c}}|^2(1 - 3.4\frac{\alpha_s}{\pi} - \frac{r}{3} + O(v^4, \alpha_s^2, \alpha_s v^2)) \tag{9}$$

$$\Gamma(\eta_c \to gg) = \frac{32\pi\alpha_s^2}{3M_{\eta_c}^2}\ |\overline{R_{\eta_c}}|^2(1 + 4.8\frac{\alpha_s}{\pi} - \frac{r}{3} + O(v^4, \alpha_s^2, \alpha_s v^2)) \tag{10}$$

$$\Gamma(J/\psi \to e^+e^-) = \frac{64\pi\alpha^2(M_{J/\psi})}{9M_{J/\psi}^2}\ |\overline{R_\psi}|^2(1 - \frac{16\alpha_s}{3\pi} - \frac{r}{3} + O(v^4, \alpha_s^2, \alpha_s v^2)) \tag{11}$$

$$\Gamma(J/\psi \to ggg) = \frac{160\alpha_s^3(\pi^2 - 9)}{81M_{J/\psi}^2}\ |\overline{R_\psi}|^2(1 - 3.7\frac{\alpha_s}{\pi} - 4.32r + O(v^4, \alpha_s^2, \alpha_s v^2)) \tag{12}$$

In Eqs.(9-12) we have introduced the quantity r defined in the BBL formalism through

$$r = \frac{4}{m^2|\overline{R_s}|^2} Re(\overline{R_s}^* \cdot \overline{\nabla^2 R_s}) \tag{13}$$

and used the Keung and Muzinich[15] (KM) results to extract the relative coefficient of the explicit $O(v^2)$ correction in the $J/\psi \to ggg$ decay channel, not reported in ref.[13]. To compare with ref.[15] one has to set $r = \frac{\epsilon}{m}$ where the binding energy ϵ is defined from the quarkonium mass through $\epsilon = M - 2m$. Note that overall relativistic corrections to decay widths, due to the difference $M - 2m$, are effectively reabsorbed into an unobservable redefinition of the BBL quantities $|\overline{R_{\eta_c}}|^2$ and $|\overline{R_\psi}|^2$ while additional relativistic corrections to heavy-quark spin symmetry are implicitly included by allowing for $\overline{R_{\eta_c}} \neq \overline{R_\psi}$. In the explicit $O(v^2)$ correction r, on the other hand, one can use the average value $\overline{R_s}$ since the difference is $O(v^4)$.

In the absence of a lattice simulation or some specific phenomenological model of NRQCD one cannot predict the non perturbative quantities in Eqs.(9-12). However, BBL argue as follows. Eqs.(9-12) provide a consistent set of four equations in the four unknown quantities $|\overline{R_{\eta_c}}|^2$, $|\overline{R_\psi}|^2$, $\alpha_s = \alpha_s(m)$ and r, including both $O(\alpha_s)$ and $O(v^2)$ relativistic corrections, which can be directly compared with the experimental data. The experimental determination of α_s from this *four-parameter* fit may produce a sizeable modification of its error, as compared to the simple non-relativistic approximation, and this may reduce the observed $\sim 4\sigma$ discrepancy reported in Section 1. Notice, however, that the rather large renormalization effect expected in the physical quantities $\overline{R_{\eta_c}}(\Lambda \sim mv)$ and $\overline{R_\psi}(\Lambda \sim mv)$, as compared to their "bare", zero-range values for $\Lambda = m$ does not influence the determination of α_s.

Before reporting the results of fits to the experimental data, we discuss in a general way the effect of the relativistic correction parameter r ($\approx$ binding energy) on the α_s determination. For instance, the sign of r is crucial to understand the possible effect of the relativistic corrections when attempting to extract α_s from quarkonia decay rates. If r were positive (i.e. $M > 2m$), then the value of α_s extracted from Eqs.(9-12) would be increased with respect to the simple non-relativistic approximation. On the contrary if r were negative, the value of α_s would be further reduced with respect to the DIL predictions in Section 1. In this context, we note that, quite

independently on the possible effects on α_s, negative values of r are clearly favoured from several theoretical investigations of the quarkonium system. For instance, from the analysis of Tidard and Ynduráin[16], where the quantum-field-theoretical meaning of the heavy quark mass of the potential models (related to the "pole mass" definition) has been clarified, we get $m_c = 1,570 \pm 26$ MeV and $m_b = 4,906 \pm 90$ MeV. Also, in the very recent paper by Narison[17] the "dressed masses entering the non-relativistic Balmer formula including higher order α_s corrections" are found to be $m_c = 1,620 \pm 100$ MeV and $m_b = 4,940 \pm 130$ MeV. In all cases, and especially for the Υ system, negative values of r are favoured.

To gain further insight on the possible values of r we can investigate its effect on J/ψ and Υ radiative decays. Let us neglect, for the moment, the role of the $O(\alpha_s)$ perturbative corrections. Then, from ref.[15] we obtain the relativistic correction to the photon spectrum in the radiative decay $V \to \gamma gg$ ($z = \frac{2E_\gamma}{M_V}$)

$$\frac{1}{\Gamma}\frac{d\Gamma}{dz} = c_N^{-1}[(1+\frac{5r}{3})f_o(z) - \frac{r}{12}f_1(z)] \tag{14}$$

In Eq.(14), c_N is the normalization factor reported in ref.[14], namely

$$c_N = (\pi^2-9)[1+r(\frac{5}{3}-\frac{1}{4}\frac{(9\pi^2-68)}{(\pi^2-9)})] \sim (\pi^2-9)(1-4.32r)$$

and $f_o(z)$ and $f_1(z)$ denote the semi-inclusive distributions obtained by integrating over the 2 massless gluon phase-space the fully differential spectrum for a massless 3-body final state reported in Eq.(3.5) of ref.[15] with the result

$$f_o(z) = 4(1-z)[\frac{1}{z^2} - \frac{(1-z)}{(2-z)^3}]\ln(1-z) + \frac{2z(1-z)}{(2-z)^2} + \frac{4}{z} - 2 \tag{15}$$

$$f_1(z) = \frac{16(1-z)(-56+164z-176z^2+104z^3-32z^4+5z^5)}{z^2(2-z)^4} +$$

$$\frac{16(112-552z+1176z^2-1324z^3+824z^4-272z^5+39z^6)}{z^3(-2+z)^5}\ln(1-z) \tag{16}$$

By analyzing the radiative decays, rather than performing a global fit to the four decay widths in eqs.(9-12), we use Eqs.(14-16) (also allowing for a multiplicative correction factor taking into account leading log QCD corrections from Ref.[9]) and fit r from the experimental data for the photon spectrum. This alternative strategy represents a consistency check of the relativistic correction approach to quarkonia decays. However, for positive r the photon spectrum obtained from Eqs.(14-16) is not positive definite (it becomes negative in the range $0 < z < r$). In this case, for instance, the extrapolation to the full z-range of the MARKII experimental result[8] $B(J/\psi \to \gamma gg) = (4.1 \pm 0.8)10^{-2}$ for $z \geq 0.6$ cannot be meaningfully performed for positive r. Thus, before attempting to extract r from the experimental data, one should keep in mind that positive values of r give unphysical results for some values of z and are strongly disfavoured by the recent quark mass estimates of Refs.[16,17]. In the fits to the photon spectra described below negative values of the photon spectrum are set to zero. Since all experimental data have $z > 0.2$, and the fitted value of r is smaller than this, the fits are not sensitive to the unphysical region of the spectrum.

Let us now compare with the experimental data. We first point out that in the case of the J/ψ radiative decays the fit to the MARKII data[8] with Eqs.(14-16) (or including also the QCD correction factor from ref.[9]), giving a value $r \sim 0.3$, has a totally unacceptable CL since we find $\frac{\chi^2}{\mathrm{DF}} = 17$. In this situation, where the measured photon spectrum in the process $J/\psi \to \gamma + X$ cannot be reproduced, we do not attempt to use the fitted value of r in the inclusive J/ψ branching ratio analysis.

To compare with Eqs.(9-12) we use the following experimental values:

$$\Gamma(\eta_c \to \gamma\gamma) = 8.1 \pm 2.0 \text{ keV } [18]$$

$$\Gamma(\eta_c \to gg) = 10.3 \pm 3.6 \text{ MeV } [2]$$

$$\Gamma(J/\psi \to e^+e^-) = 5.36 \pm 0.28 \text{ keV } [2]$$

$$\Gamma(J/\psi \to ggg) = 54.1 \pm 5.4 \text{ keV } [7]$$

The result of the four parameter fit of Eqs.(9-12) to the above data is

$$|\overline{R_{\eta_c}}|^2 = 0.058 \pm 0.011 \text{ GeV}^3 \tag{17}$$

$$|\overline{R_{\psi}}|^2 = 0.061 \pm 0.009 \text{ GeV}^3 \tag{18}$$

$$\alpha_s(m_c) = 0.189 \pm 0.035 \tag{19}$$

$$r = 0.012^{+0.070}_{-0.153} \tag{20}$$

By comparing with the DIL predictions in Section 1, we see that relativistic corrections are unable to remove the $\sim 4\sigma$ discrepancy with respect to Eq.(3). Indeed, in this fit, their effect is modest as deduced from the small central value of r and the good agreement between the values of the form factors in Eq.(17) and Eq.(18).

Let us now compare with Υ decays. In this case we find, analogously to Eqs.(9-12) for the charmonium system, the BBL-KM predictions

$$\Gamma(\Upsilon \to e^+e^-) = \frac{16\pi\alpha^2(M_\Upsilon)}{9M_\Upsilon^2}\,|\overline{R_\Upsilon}|^2(1 - \frac{16\alpha_s}{3\pi} - \frac{r}{3} + O(v^4, \alpha_s^2, \alpha_s v^2)) \tag{21}$$

$$\Gamma(\Upsilon \to ggg) = \frac{160\alpha_s^3(\pi^2-9)}{81M_\Upsilon^2}\,|\overline{R_\Upsilon}|^2(1 - 4.9\frac{\alpha_s}{\pi} - 4.32r + O(v^4, \alpha_s^2, \alpha_s v^2)) \tag{22}$$

$$\Gamma(\Upsilon \to \gamma gg) = \frac{128\alpha_s^2\alpha(\pi^2-9)}{81M_\Upsilon^2}\,|\overline{R_\Upsilon}|^2(1 - 7.4\frac{\alpha_s}{\pi} - 4.32r + O(v^4, \alpha_s^2, \alpha_s v^2)) \tag{23}$$

We first determine r by fitting Eqs.(14-16) to the data on the inclusive photon spectrum from ARGUS[19] and CRYSTAL BALL[20]. In the case of ref.[20] we have used the results of their "method 2" which, besides suppressing the π^o background by an order of magnitude with respect to their "method 1", allows a measurement of the direct photon spectrum in a larger region of z. The fits, taking into account experimental resolution effects, which dramatically change the spectrum shape (see the Appendix) give the following results:

ARGUS $\quad$ r = 0.164 ± 0.011 $\quad$ CL = 0.03

$$\text{CBALL} \qquad r = 0.155 \pm 0.014 \qquad \text{CL} = 0.08$$

Including the QCD correction factor from ref.[9] we find a very small change in the r-values and a slight improvement in the quality of the fit to the experimental data, namely

$$\text{ARGUS} \ (\text{LL} - \text{QCD}) \qquad r = 0.154 \pm 0.013 \qquad \text{CL} = 0.18 \tag{24}$$

$$\text{CBALL} \ (\text{LL} - \text{QCD}) \qquad r = 0.138 \pm 0.016 \qquad \text{CL} = 0.27 \tag{25}$$

All of these fit results are compared to the experimental data in Fig.1 which also shows the theoretical predictions, taking into account experimental resolution effects, for $r = 0$ i.e. the tree level QCD prediction for massless gluons. This possibility is completely excluded by the experimental data (CL $= 3 \times 10^{-10}$, 7×10^{-8} for ARGUS, CRYSTAL BALL). By averaging the two very consistent results in Eqs.(24,25) we find the final result

$$r \ (\text{LL} - \text{QCD}) = 0.148 \pm 0.010 \tag{26}$$

which should be the relevant value to be used in Eqs.(21-23) for the analysis of the Υ decay widths.

We now perform a fit to Eqs.(21-23) of the experimental data:

$$\Gamma(\Upsilon \to e^+e^-) = 1.34 \pm 0.04 \text{ keV}^{[2]}$$

$$\Gamma(\Upsilon \to ggg) = 43.7 \pm 1.7 \text{ keV}^{[2]}$$

$$\Gamma(\Upsilon \to \gamma gg) = 1.28 \pm 0.10 \text{ keV}$$

In the case of the $\Upsilon \to \gamma gg$ decay width we have used, for consistency, the same sample of data analyzed to extract the r-values in Eqs.(24-25), namely the ARGUS result[19]

$$\frac{\Gamma(\Upsilon \to \gamma gg)}{\Gamma(\Upsilon \to ggg)} = (3.00 \pm 0.22)10^{-2}$$

and the CRYSTAL BALL result[20]

$$\frac{\Gamma(\Upsilon \to \gamma gg)}{\Gamma(\Upsilon \to ggg)} = (2.7 \pm 0.5)10^{-2}$$

We have obtained the results

$$|\overline{R_\Upsilon}|^2 = 0.516 \pm 0.023 \text{ GeV}^3 \tag{27}$$

$$\alpha_s(m_b) = 0.169 \pm 0.009 \tag{28}$$

$$r = -0.039^{+0.029}_{-0.036} \tag{29}$$

with, again, a $\sim 4\sigma$ discrepancy in the value of α_s with respect to the DIL value in Eq.(1). Notice that the value of the parameter r obtained from the inclusive decay widths in Eq.(29) is inconsistent at the 6σ level with Eq.(26) where, in order to reproduce the observed softening of the photon spectrum at large z, one needs a substantial positive r.

No improvement is obtained if $O(\alpha_s)$ effects and relativistic corrections to quarkonia decay widths are combined into a factorized form to simulate the effect of the unknown $O(\alpha_s)$ corrections in the imaginary parts of the coefficients g_1 in Eqs.(4,5). For instance, by replacing Eqs.(9-12) with the corresponding expressions :

$$\Gamma(\eta_c \to \gamma\gamma) = \frac{256\pi\alpha^2}{27M^2_{\eta_c}} |\overline{R_{\eta_c}}|^2(1 - 3.4\frac{\alpha_s}{\pi})(1 - \frac{r}{3}) \tag{30}$$

$$\Gamma(\eta_c \to gg) = \frac{32\pi\alpha_s^2}{3M^2_{\eta_c}} |\overline{R_{\eta_c}}|^2(1 + 4.8\frac{\alpha_s}{\pi})(1 - \frac{r}{3}) \tag{31}$$

$$\Gamma(J/\psi \to e^+e^-) = \frac{64\pi\alpha^2(M_{J/\psi})}{9M^2_{J/\psi}} |\overline{R_\psi}|^2(1 - \frac{16\alpha_s}{3\pi})(1 - \frac{r}{3}) \tag{32}$$

$$\Gamma(J/\psi \to ggg) = \frac{160\alpha_s^3(\pi^2 - 9)}{81M^2_{J/\psi}} |\overline{R_\psi}|^2(1 - 3.7\frac{\alpha_s}{\pi})(1 - 4.32r) \tag{33}$$

the result of the fit to the experimental data is

$$|\overline{R_{\eta_c}}|^2 = 0.058 \pm 0.011 \text{ GeV}^3 \tag{34}$$

$$|\overline{R_\psi}|^2 = 0.061 \pm 0.009 \text{ GeV}^3 \tag{35}$$

$$\alpha_s(m_c) = 0.190 \pm 0.035 \tag{36}$$

$$r = 0.016^{+0.096}_{-0.200} \tag{37}$$

Analogously, if Eqs.(21-23) for the Υ are replaced by

$$\Gamma(\Upsilon \to e^+e^-) = \frac{16\pi\alpha^2(M_\Upsilon)}{9M_\Upsilon^2}\,|\overline{R_\Upsilon}|^2(1-\frac{16\alpha_s}{3\pi})(1-\frac{r}{3}) \tag{38}$$

$$\Gamma(\Upsilon \to ggg) = \frac{160\alpha_s^3(\pi^2-9)}{81M_\Upsilon^2}\,|\overline{R_\Upsilon}|^2(1-4.9\frac{\alpha_s}{\pi})(1-4.32r) \tag{39}$$

$$\Gamma(\Upsilon \to \gamma gg) = \frac{128\alpha_s^2\alpha(\pi^2-9)}{81M_\Upsilon^2}\,|\overline{R_\Upsilon}|^2(1-7.4\frac{\alpha_s}{\pi})(1-4.32r) \tag{40}$$

we find the values

$$|\overline{R_\Upsilon}|^2 = 0.505 \pm 0.026\ \mathrm{GeV}^3 \tag{41}$$

$$\alpha_s(m_b) = 0.164 \pm 0.011 \tag{42}$$

$$r = -0.086^{+0.061}_{-0.074} \tag{43}$$

The r value here is still inconsistent (a difference of 3.8σ) with the value in Eq.(26) derived from the inclusive photon spectrum. In conclusion, on the basis of the BBL-KM approach to relativistic corrections, and on the basis of *all* the available experimental data, no consistent description of quarkonia decay widths in perturbative QCD is possible. Indeed,

i) the values of $\alpha_s(m_c)$ and $\alpha_s(m_b)$ in Eqs.(19,28,36,42) are not consistent within $3-4\sigma$ with the predictions from DIL in Eqs.(1,3).

ii) for the bottomonium the value of the parameter r obtained from the fits to the inclusive decay widths is not consistent at the 4-6 σ level with the value in Eq.(26) extracted from the photon spectrum in the radiative decay $\Upsilon \to \gamma + X$. Moreover, in the case of the J/ψ the attempt to describe the photon spectrum on the basis of the relativistic-correction approach fails completely.

iii) the fitted relativistic corrections to the inclusive widths, which are not effectively reabsorbed into the values of the average quantities $|\overline{R_{\eta_c}}|^2$, $|\overline{R_\psi}|^2$ and $|\overline{R_\Upsilon}|^2$, turn out to be modest for *both* J/ψ and Υ decays. In the latter case, the fit shows a preference for negative values of r (see Eqs.(29,43)). While this might be expected on the basis of the interpretation of r as a

binding energy and the value of the pole mass of the b-quark[16,17] $m_b \sim 4.92 \pm 0.10$ GeV (leading to $r \sim -0.077 \pm 0.040$) the fit then gives values of $\alpha_s(m_b)$ which are even smaller than in the simple non-relativistic approximation ($\alpha_s(m_b) = 0.181 \pm 0.003$).

The results of this section confirm our point of view, as discussed in ref.[10] and in the Introduction, that relativistic corrections alone cannot lead to a consistent picture of quarkonia decays within perturbative QCD. Here, our conclusions differ substantially from ref.[21]. There, the Bethe-Salpeter equation is used to compute the relativistic corrections to the $Q\bar{Q} \to ggg$ and $Q\bar{Q} \to \gamma gg$ annhilation. In these specific channels, these authors find large corrections ($\delta = -69\%$ for the J/ψ and $\delta = -31\%$ for the Υ) with respect to the radial wave function at the origin of the zero-range approximation. By *assuming* that the modifications in the other decay channels are negligible, they deduce the values $\alpha_s(m_c) = 0.28 \pm 0.01$ and $\alpha_s(m_b) = 0.20 \pm 0.01$, thus concluding that the relativistic effects can bring perturbative QCD in good agreement with the experiments. However, as previously remarked, in the BBL formalism a large fraction of the relativistic effects is effectively reabsorbed in the matrix elements of the heavy-quark operators averaged over distances $1/\Lambda \sim 1/(mv)$. This effect, contained in the definition of the form factors $|\overline{R_{\eta_c}}|^2$, $|\overline{R_\psi}|^2$, and $|\overline{R_\Upsilon}|^2$, affects *all* decay channels and only the explicit r-dependent terms influence the determination of α_s. Thus, in our opinion, to provide a precise estimate of the effects that *cannot* be reabsorbed into a simple redefinition of the form factors, the authors of ref.[21] should check the validity of their claim with an explicit calculation of the corrections to the zero-range approximation in the other J/ψ and Υ decay channels. The following additional remarks may also be made on the work presented in Ref.[21]:

a) The results for α_s presented depend crucially on the factorisation ansatz used, i.e. on the effect of actually uncalculated but assumed higher order relativistic and QCD corrections. Assuming, as they do, that their correction δ represents an *observable* relativistic effect (affecting

only the γgg and ggg decay channels) amounts to the identification

$$\delta = -4.32r + O(v^4)$$

If instead of the factorisation ansatz actually used in ref.[21] the BBL formalism is used, as in Eqs.(9-12) and (21-23), by inserting the $O(\alpha_s)$ correction[6] one obtains the expression for the $J/\psi \to \gamma gg$ decay width

$$\Gamma(J/\psi \to \gamma gg) = \frac{512\alpha_s^2\alpha(\pi^2 - 9)}{81M_{J/\psi}^2} |\overline{R_\psi}|^2(1 - 6.7\frac{\alpha_s}{\pi} + \delta + O(\alpha_s^2, \alpha_s\delta))$$

Now to $O(\alpha_s, \delta)$, by simply imposing the obvious physical condition that $\Gamma(J/\psi \to \gamma gg)$ should be positive, the value[21] $\delta = -0.69$ implies that:

$$\alpha_s(\frac{M_{J/\psi}}{2}) \leq 0.145$$

This is roughly a factor of 2 smaller than the value $\alpha_s(m_c) = 0.28 \pm 0.01$ quoted above and is clearly inconsistent with perturbative QCD. However, the only difference between the factorisation ansatz for the relativistic and QCD corrections used in Ref.[21] and the BBL approach is a different assumption about uncalculated higher order $O(\alpha_s\delta)$ corrections.

b) The value of $\Gamma(J/\psi \to \gamma gg)/\Gamma(J/\psi \to ggg)$ used in Ref.[21], taken from Ref.[6], is a factor ≈ 2 smaller than an estimate taking properly into account the measured form of the photon spectrum[7]. Thus, independently on any assumption about the structure of the higher order terms, the derived value of α_s is about a factor of 2 too high.

c) Since the photon radiated in the process $V \to \gamma gg$ is an external on-shell photon the appropriate value of α is $\alpha(0)$ not $\alpha(M_V)$ as assumed in Ref.[21]. This implies a negative correction of several percent to the values of $\alpha_s(\frac{M_V}{2})$ quoted in Ref.[21].

d) The authors of Ref.[21] can easily check the consistency of their approach (which is essentially a calculation to all orders in v^2 of the relativistic correction to the born term) by explicitly

calculating the inclusive photon spectra in J/ψ and Υ decays and comparing with the experimental data. To facilitate such theoretical comparisons we give below in Table IV the inclusive photon spectra, derived from the fits to the experimental data to be described in the following Section, and corrected for the effect of experimental resolution.

3.The gluon mass and quarkonia decays.

The discrepancy between perturbative QCD and experimental quarkonia decay widths may stimulate basically different lines of research. On one hand, one may investigate whether the neglected $O(\alpha_s^2)$, $O(\alpha_s v^2)$, $O(v^4)$ corrections can bring theory and experiment into agreement by providing an explicit calculation of these effects. On the other hand, one may explore the alternative possibility that the discrepancy has to be searched for elsewhere and try to take into account, although in a semi-phenomenological way, the non-perturbative properties of QCD such as gluon condensation.

In order to introduce our analysis, we argue as follows. The relativistic correction approach is based on the assumption that the failure of the perturbative predictions depends on the physics *before* the short-distance annihilation process. On the other hand, if gluon condensation, leading to dynamical gluon mass generation is a true property of the QCD ground state, one expects sizeable modifications in the properties of the gluon *final* states. The point of view that the discrepancy has to be searched for in the description of final states where gluons are produced is supported by the simple observation (already mentioned above) that when considering decays into purely photonic or leptonic final states such as $\eta_c \to \gamma\gamma$ and $J/\psi \to e^+e^-$ one finds a good agreement between the theoretical prediction

$$\frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(J/\psi \to l^+l^-)} \sim \frac{4M_{J/\psi}^2}{3M_{\eta_c}^2} \sim 1.44$$

and the experimental result

$$\frac{\Gamma(\eta_c \to \gamma\gamma)}{\Gamma(J/\psi \to l^+l^-)} = 1.51 \pm 0.38$$

Motivated by the original paper by Parisi and Petronzio[22], we have presented in ref.[10] a phenomenological analysis of quarkonia decay widths based on the introduction of an *effective* gluon mass. Let us briefly review the theoretical motivation for a non-vanishing gluon mass. After, we shall further clarify the meaning of the word "effective".

As mentioned in ref.[10], the introduction of a gluon mass naturally arises by considering gauge-invariant truncations of the Schwinger-Dyson equations of QCD[23]. Here we report further evidence obtained by Kogan and Kovner (KK) in ref.[24]. These authors perform a variational calculation of the vacuum wave functional in an SU(N) pure Yang Mills theory in 3+1 dimensions (see also ref.[25]). In the (exactly gauge invariant) KK wave functional, correlations among the gluon field at different points in 3-dimensional space are governed by the inverse equal-time scalar propagator $G^{-1}(\vec{x}-\vec{y})$ whose Fourier transform $\tilde{G}^{-1}(|\vec{k}|)$ in the limit $|\vec{k}| \to 0$ is a constant M taken as a free parameter to be determined by minimization of the energy. The variational procedure includes the perturbative ground state (corresponding to $\tilde{G}^{-1}(|\vec{k}|) = |\vec{k}|$) and gluon condensation corresponds to the situation where the absolute minimum occurs for $M \neq 0$ which has the meaning of a gluon mass. The basic KK results are the following:

a) the energy of the vacuum is minimised when:

$$\alpha_s(M) = \alpha_c = \frac{\pi}{4N} = 0.262 \tag{44}$$

($\alpha_s(M)$ being the perturbatively calculated running coupling constant)

b) the relation with the gluon condensate

$$\langle \alpha_s G^2 \rangle = \frac{N}{120\pi} M^4 \tag{45}$$

The meaning of M as a gluon mass is further confirmed by the following argument. By expressing

α_s in terms of the conventional Λ parameter of QCD and using the KK one loop relation

$$M = \Lambda_{QCD} \exp(\frac{24}{11}) \tag{46}$$

one finds

$$\alpha_s(Q) = \frac{\alpha_c}{1 + \frac{\alpha_c}{\pi}\frac{11}{2}\ln\frac{Q}{M}}$$

If instead $\alpha_s(Q)$ is re-written in an on-shell scheme[26] (see also ref.[23]) where the Λ parameter is replaced by an on-shell coupling constant α_s^0 and a gluon mass m_g one obtains

$$\alpha_s(Q) = \frac{\alpha_s^0}{1 + \frac{\alpha_s^0}{\pi}[\frac{11}{2}\ln\frac{Q}{m_g} - \frac{n_f}{3}\ln\frac{Q}{m_q}]} \tag{47}$$

It can be seen from Eq.(47) that in a pure Yang Mills theory where $n_f = 0$, the identification of α_s^0 with α_c then gives simply:

$$M = m_g \tag{48}$$

By using the experimental value $\langle\alpha_s G^2\rangle = 0.04 \pm 0.02$ GeV^4 of ref.[16] one deduces from Eq.(45) $m_g = 1.50^{+0.15}_{-0.25}$ GeV and by using the different estimate of ref.[17], namely $\langle\alpha_s G^2\rangle = 0.06 \pm 0.03$ GeV^4 , we find $m_g = 1.66^{+0.18}_{-0.26}$ GeV. These numbers should be treated with caution since, as pointed out by KK, higher order corrections may change the lowest order result for $\alpha_s(M)$, Eq.(44), by $O(\alpha_s^2)$ (that is as much as 25 %). Also the KK analysis neglects the contribution of quark pairs to the QCD vacuum. It turns out, however, that the predictions of KK:

$$\alpha_s^0 = \alpha_c = 0.26 \pm 0.06 \tag{49}$$

$$m_g \sim 1.5 \pm 0.3 \text{ GeV} \tag{50}$$

are in remarkably good agreement with the values obtained for these parameters in a next-to-leading order perturbative QCD analysis of a number of different processes with low physical scales ≈ 2 GeV[27] :

$$\alpha_s^0 = 0.30 \pm 0.05 \tag{51}$$

$$m_g = 1.5^{+1.2}_{-0.6} GeV \tag{52}$$

It should be noted however that the error on m_g quoted in Eq.(50) takes into account only the uncertainty in the value of the gluon condensate not any theoretical uncertainties in the KK analysis.

Let us consider the implications of these results for our quarkonium analysis:

i) the above estimate of the "critical" α_s in Eq.(49) lies precisely in the range expected for quarkonia decays (see Eqs.(1,3))

ii) a value $m_g \sim 1.5$ GeV implies sizeable phase-space modifications of the $\Upsilon \to ggg$ decay width, where the typical energy of the gluon jets is $E_g \sim \frac{M_\Upsilon}{3} \sim 3$ GeV.

iii) The corresponding effect at the J/ψ has to be dramatic since in this case $E_g < m_g$!

In the J/ψ case, in particular, the primary gluons must be off-shell. If they split into secondary gluons the latter must be even more highly virtual, resulting in a strong supression of this process. Also, their individual splitting into light-quarks should be modest, their independent fragmentation being suppressed by additional powers of E_g/m_g. Rather, as emphasized in ref.[10], the primary gluons in the $J/\psi \to ggg$ and $J/\psi \to \gamma gg$ decays will mainly produce hadrons through their *fusion* into light-quark pairs as in the diagram shown in Fig.2. As first suggested in ref.[8], such a dominance of the gluon fusion mechanism can explain why the multiplicity of charged particles measured in the hadronic state X of the decay $J/\psi \to \gamma + X$ is the same as in the continuum e^+e^- annihilation at the same center of mass energy since in both cases the primary partonic state is the same ($q\bar{q}$). A similar annihilation of the 3-gluon system into light $q\bar{q}$ pairs would also allow to understand the similarity of the inclusive hadronic final state in e^+e^- annihilation on and just below the J/ψ resonance[8]. We find remarkable that, without any explicit calculation, the gluon mass hypothesis allows to predict the qualitative features of the *observed* structure of the final states in J/ψ decays.

Let us now address the question concerning quantitative estimates of m_g from quarkonia

decays. In ref.[10] we employed the simple approximation where only the phase space restrictions due to a non-vanishing m_g were imposed on the tree-level perturbative QCD predictions. For instance, in the case of the $V \to \gamma gg$ and $V \to ggg$ decays we considered the effect on the tree-level spectrum

$$\frac{1}{\Gamma}\frac{d^2\Gamma}{dx_1 dx_3} = \frac{1}{(\pi^2-9)}[(\frac{1-x_1}{x_2 x_3})^2 + (\frac{1-x_2}{x_3 x_1})^2 + (\frac{1-x_3}{x_1 x_2})^2] \tag{53}$$

of imposing the appropriate kinematical restrictions on the 3-body Dalitz plot. In the γgg case ($\eta = \frac{2M_g}{M_V}$, $x_i = \frac{2E_i}{M_V}$; i=1,2; $x_3 = z = \frac{2E_\gamma}{M_V}$; E_i and E_γ being respectively the gluon and photon energies) we obtain the kinematical limits :

$$x_1 + x_2 + z = 2 \tag{54}$$

$$0 \leq z \leq 1 - \eta^2 \tag{55}$$

$$x_1^{min} \leq x_1 \leq x_1^{max} \tag{56}$$

and

$$x_1^{max} = 1 - \frac{z}{2}[1 - \sqrt{1 - \frac{\eta^2}{(1-z)}}\,] \tag{57}$$

$$x_1^{min} = 1 - \frac{z}{2}[1 + \sqrt{1 - \frac{\eta^2}{(1-z)}}\,] \tag{58}$$

while for the 3-gluon decay we obtain :

$$x_1 + x_2 + x_3 = 2 \tag{59}$$

$$\eta \leq x_3 \leq 1 - \frac{3\eta^2}{4} \tag{60}$$

$$x_1^{max} = 1 - \frac{x_3}{2}[1 - D(x_3, \eta)\,] \tag{61}$$

$$x_1^{min} = 1 - \frac{x_3}{2}[1 + D(x_3, \eta)\,] \tag{62}$$

where

$$D(x_3, \eta) = \sqrt{(1 - \frac{\eta^2}{x_3^2})(1 - \frac{\eta^2}{1 - x_3 + \frac{\eta^2}{4}})} \tag{63}$$

Note that in Eqs.(54-63) the gluon mass is denoted by 'M_g'. This is the 'effective gluon mass' to be discussed below, which is, in general, $< m_g$. In more inclusive quantities, the effect of M_g can be evaluated by integrating the differential distributions over the allowed phase-space. For instance, in the process $V \to \gamma gg$ the photon spectrum is obtained by integrating Eq.(53) (or its leading-log improvement of ref.[9]) with the constraints (54-58). In fully inclusive widths one finds the suppression factors (we use the same notation as in ref.[10], PS=phase-space)

$$f_3^{PS}(\eta) = \frac{\Gamma(V \to ggg)|_{M_g}}{\Gamma(V \to ggg)|_{M_g=0}} \tag{64}$$

$$f_2(\eta) = \frac{\Gamma(\eta_c \to gg)|_{M_g}}{\Gamma(\eta_c \to gg)|_{M_g=0}} \tag{65}$$

$$f_3^{gg\gamma}(\eta) = \frac{\Gamma(V \to \gamma gg)|_{M_g}}{\Gamma(V \to \gamma gg)|_{M_g=0}} \tag{66}$$

where also the photon (or the remaining gluon) energy is integrated according to the kinematical constraints of Eqs.(54-58) (or (59-63)) to give respectively $f_3^{gg\gamma}$ (or f_3^{PS}). For f_2 the calculation of Ref.[22] is used. These correction factors when inserted in the tree-level QCD predictions, give, as shown in ref.[10], sizeable changes in the derived values of α_s. This procedure is based on the assumption that one can decouple the perturbative regime and treat the contribution of the non perturbative structure of the QCD ground state as an external constraint via a single parameter, the effective gluon mass. In this sense, it is similar to the usual (tree-level) factorization which is included as a special case corresponding to $M_g = 0$.

In Ref.[10] the value of M_g was determined from a fit to the photon spectrum of the radiative decays $J/\psi \to \gamma + X$ and $\Upsilon \to \gamma + X$. Both the tree-level spectrum in Eq.(53) (with $x_3 \to z$) and its leading-logarithmically QCD improved version of ref.[9] were used within the phase space limitations imposed by Eqs.(54-58). Before reporting the results of the fits, a remark is in order. It should be clear from our previous discussions of the structure of the final states, that there is an important difference in the role of M_g for Υ and J/ψ decays. In the latter case, in fact, a value

$m_g \simeq 1.5$ GeV does not simply reduce the available phase-space but forces the primary gluons to be off-shell and so to preferentially either fuse or to split into light-quark pairs. Therefore, the experimental photon spectrum in $J/\psi \to \gamma + X$ results from the recoil of a complicated hadronic final state and, in our simple approach, where gluons are treated as external on-shell particles, there is no reason why the value of M_g from the fit should be close to the prediction $m_g \sim 1.5$ GeV. Such effects, however, should be much smaller in the Υ radiative decay where gluons of mass 1.5 GeV are allowed up to $z = 0.9$ and, therefore, the effective gluon mass M_g in Υ radiative decays should be nearer to its predicted value m_g. For this reason, different values of the gluon mass are expected in the two cases and the word "effective" should be used in connection with the values of M_g obtained from the fit.

The results of the fits to the inclusive photon spectra in the decays of the Υ and the J/ψ (which are shown in Figs.3,4 respectively) are reported in Table I. Here LO (Lowest Order) refers to fits to the tree level formula (53), whereas the HO (Higher Order) fits include also the correction factor from Ref.[9]. Taking for the Υ the weighted average of the ARGUS and CRYSTAL BALL HO fits, (which give somewhat better quality fits) the following results are obtained for the effective gluon mass[10].

$$J/\psi \to \gamma + X \qquad M_g = 0.66 \pm 0.08 \text{ GeV} \tag{67}$$

$$\Upsilon \to \gamma + X \qquad M_g = 1.17 \pm 0.08 \text{ GeV} \tag{68}$$

As expected on the basis of the previous discussion, the value of M_g from Υ decay is close to, but less than the predicted value $m_g \approx 1.5$ GeV.

As an independent check of the consistency of the approach the value of M_g deduced from the process $J/\psi \to \gamma + X$ was used to describe *all* $c\bar{c}$ decay widths and the value of M_g deduced from the process $\Upsilon \to \gamma + X$ was used in the analysis of *all* $b\bar{b}$ decay widths. This analysis has been fully reported in ref.[10] and we shall simply quote the main conclusions. By taking into account the

suppression factors in Eqs.(64-66) and using Eqs.(67,68) one finds the results presented in Table II. For instance, in the case of the very precise branching ratios $\frac{\Gamma(V\to ggg)}{\Gamma(V\to l^+l^-)}$ one finds the function f_3^{PS} shown in fig.1 of ref.[10]. Numerically we find from Eqs.(67,68)

$$f_3^{PS}(\eta) = 0.619 \pm 0.037 \qquad (\Upsilon)$$

$$f_3^{PS}(\eta) = 0.250^{+0.10}_{-0.08} \qquad (J/\psi)$$

Note that in the range of η relevant for J/ψ and Υ decays (say $0.15 \leq \eta \leq 0.45$) the function $f_3^{PS}(\eta)$ is, to good accuracy, a straight line and can be approximated by

$$f_3^{PS}(\eta) = 1.13 - 2.07\eta$$

leading to a *linear* pattern of scaling violations parametrized by the effective gluon mass (we shall return to this point in the next section). The weighted average value of α_s from Table II for charmonium is $\alpha_s(m_c) = 0.297 \pm 0.022$, and for bottomonium $\alpha_s(m_b) = 0.210 \pm 0.003$. The quoted errors are due only to experiment and the uncertainty on M_g. These values are already in excellent agreement with the DIL predictions in Eqs.(1) and (3).

We have seen above that good internal consistency is obtained among all observed charmonium and bottomonium branching ratios when the phase space constraints imposed by a non-vanishing gluon mass are taken into account. Two questions remain to be answered :

1) What is the size of the relativistic corrections, which have been neglected in the above discussion of gluon mass effects ?

2) What is the effect of the $O(\alpha_s)$ radiative corrections in the presence of a non-vanishing gluon mass ?

We first discuss 1) and after will make some brief comments on 2). In our discussion of the relativistic corrections we shall assume the validity of the $O(v^2)$ calculation of Ref.[15] and of the recent estimates of the c- and b-quark pole-masses from Refs.[16,17]. Refs.[16,17] give very

consistent values of m_b (see Section 2 above) and the weighted average value of $r = r_b$ was used:

$$r_b = -0.076 \pm 0.029 \qquad (\Upsilon)$$

For the J/ψ on the other hand somewhat different values of $r = r_c$ are found:

$$r_c = -0.027 \pm 0.032 \qquad (J/\psi \quad Ref.[16])$$

$$r_c = -0.088 \pm 0.118 \qquad (J/\psi \quad Ref.[17])$$

These values are actually consistent within the quoted errors, but in the following we use the more precise value from Ref.[16]. Since the relativistic corrections change the shape of the inclusive photon spectrum (Eq.(14)) fits for M_g to the experimental data were performed for the Υ including also relativistic correction according to Eq.(3.5) of ref.[15]. In these fits the kinematical limits of Eqs.(54-58) were taken into account. For $r_b = -0.076$ the changes in the fitted values of M_g were $\simeq$ -10 MeV and so negligible in comparison to the fitting errors quoted in Eq.(68). Hence, in the following, the values of M_g from Eqs.(67-68) are used.

When including the relativistic corrections to the inclusive $\Upsilon \to ggg$ decay width, we find

$$\frac{\Gamma(\Upsilon \to ggg)}{\Gamma(\Upsilon \to e^+e^-)} = \frac{10(\pi^2-9)\alpha_s^3}{9\pi\alpha^2(M_\Upsilon)} \frac{1-4.32r_b}{1-\frac{r_b}{3}} f_3^{PS}(\eta, r_b) = 32.6 \pm 0.8 \tag{69}$$

where $f_3^{PS}(\eta, r)$ is obtained from Eq.(64) by fully integrating Eq.(3.5) of ref.[15] over the 3-gluon phase-space with the constraints (59-63). We find for $M_g = 1.17 \pm 0.08$ GeV and $r_b = -0.076 \pm 0.029$

$$f_3^{PS}(\eta, r_b) = 0.581^{+0.012}_{-0.009} \pm 0.036 \qquad (\Upsilon)$$

where the second error is due to the variation in M_g. Therefore, we obtain the final determination

$$\alpha_s(m_b) = 0.200 \pm 0.002^{+0.005}_{-0.004} \pm 0.004 \qquad (\Upsilon) \tag{70}$$

Here and in the following the errors in α_s are, in order, experimental, and due to the uncertainty in r and M_g. The upper value $\alpha_s(m_b) = 0.205$ corresponds to $r_b = -0.047$ (or $m_b = 4.84$ GeV) and

the lower value $\alpha_s(m_b) = 0.196$ corresponds to $r_b = -0.105$ (or $m_b = 5.0$ GeV). We can check the value in Eq.(70) against the perturbative running down to the c-quark pole-mass scale precisely determined by Tidard and Ynduráin[15] to be $m_c = 1570 \pm 26$ MeV (implying $r_c = -0.027 \pm 0.032$). Using the exact two loop, four flavour, Renormalisation Group Equation[10] (RGE) we find the theoretical prediction

$$\alpha_s(1.57 \text{ GeV}) = 0.303 \pm 0.004 \pm 0.009 \qquad (\Upsilon \text{ evolved to } m_c) \tag{71}$$

By using the analogue of Eq.(69), namely

$$\frac{\Gamma(J/\psi \to ggg)}{\Gamma(J/\psi \to e^+e^-)} = \frac{5(\pi^2 - 9)\alpha_s^3}{18\pi\alpha^2(M_{J/\psi})} \frac{1 - 4.32 r_c}{1 - \frac{r_c}{3}} f_3^{PS}(\eta, r_c) = 10.1 \pm 0.9 \tag{72}$$

and the values $r_c = -0.027 \pm 0.032$, and $M_g = 0.66 \pm 0.08$ GeV we find

$$f_3^{PS}(\eta, r_c) = 0.240^{+0.010}_{-0.007} \pm 0.10 \qquad (J/\psi)$$

where the second error is due to M_g. From Eq.(72) we obtain (errors are as in Eq.(70))

$$\alpha_s(m_c) = 0.302 \pm 0.008 \pm 0.008 \pm 0.037 \qquad (\mathrm{J}/\psi) \tag{73}$$

in excellent agreement with Eq.(71) even without invoking the rather large additional error (± 0.037) induced by the uncertainty in the value of M_g at the J/ψ mass scale.

It remains now to discuss the rôle of the $O(\alpha_s)$ correction neglected until now in Eqs.(69,72). First a general remark. Since large suppression factors, due to the effect of the non-zero gluon mass occur in the tree level amplitudes[10,22] even stronger suppression is to be expected in contributions from higher order diagrams involving gluon radiation. Indeed, by integrating the higher order differential distributions (analogous to Eq.(53)) over the corresponding phase-space limitations (analogous to Eqs.(54-58) or to Eqs.(59-63)) one gets similar powers of $\frac{E_g}{M_g}$. This implies that the $O(\alpha_s)$ corrections are most likely smaller than those actually calculated (see Eq.(2)) for the

massless gluon case. Assuming the values of $\alpha_s(m_b)$, $\alpha_s(m_c)$ derived above from Eqs.(70,73), the relative changes in the value of α_s are (+4.2 % , -1.3 %) for the Υ and (-11.4 % , -11.0 %) for the J/ψ. The first figure quoted in each case corresponds to the BBL formalism (Eqs.(11,12,21,22)) the second one to näive factorisation (Eqs.(32,33,38,39)). These conservative estimates are of the same order as the total error on the values of α_s quoted above.

The overall situation is now summarised in Fig.5 and Table III where the consistency of the high precision experimental data is compared to the predictions of perturbative QCD, including relativistic corrections, both with and without gluon mass corrections. For this comparison the results for α_s at the scale $\frac{M_{J/\psi}}{2}$ are evolved, using the RGE mentioned above, to the scale $\frac{M_\Upsilon}{2}$. Including the gluon mass corrections excellent agreement is seen both with the perturbative QCD evolution and with the DIL measurement of α_s.

The results of the global fits of Section 2 above using the BBL/KM formalism to take into account both $O(v^2)$ and $O(\alpha_s)$ QCD corrections are also shown in Fig. 5. Within errors the data are consistent with perturbative QCD evolution, but the weighted average value for the J/ψ and the Υ:

$$\alpha_s(\frac{M_\Upsilon}{2}) = 0.165 \pm 0.008$$

lies 4.4σ below the DIL result (1). Finally Fig. 5 shows the result for α_s deduced from the fit to the inclusive photon spectrum in Υ decays, including relativistic corrections, but without gluon mass corrections (Eqn.(14)). As described above in Section 2 a good fit to the photon spectrum is obtained with $r_b = 0.148 \pm 0.010$ (corresponding to $m_b = 4.40 \pm 0.02$ GeV). Using this value of r_b in Eq.(69) with $\eta = 0$ and $f_3^{PS} = 1$ gives:

$$\alpha_s(\frac{M_\Upsilon}{2}) = 0.252 \pm 0.002 \pm 0.010$$

where the first error is experimental and the second one due to the fitting error on r_b. This value is inconsistent at the level of 6.8σ with the result obtained from the global fit to the Υ branching

ratios. Thus only when the effects of a non-vanishing gluon mass are taken into account can a consistent description of all experimental data on quarkonium decays be obtained within the framework of perturbative QCD. Explicit calculation using the BBL/KM formalism shows, as we conjectured in Ref.[10], that relativistic corrections have only a marginal effect on α_s determinations from quarkonium branching ratios.

Our final values of α_s are:

$$\alpha_s(m_b) = 0.200 \pm 0.002^{+0.010}_{-0.006} \tag{74}$$

$$\alpha_s(m_c) = 0.302 \pm 0.008^{+0.038}_{-0.050} \tag{75}$$

where the first error is purely experimental and the second takes into account the uncertainties due to r, M_g and the uncalculated $O(\alpha_s)$ corrections.

4. Conclusions.

In this paper we have presented a detailed discussion of the relativistic corrections to quarkonia decay widths. To this end, we have made essential use of the papers by Bodwin, Braaten and Lepage[13] and Keung and Muzinich[15]. Our results, reported in Section 2, confirm our original point of view[10] that relativistic corrections alone are unable to explain the discrepancy between perturbative QCD and experimental data. In fact we find that the large 'ad hoc' relativistic correction used until now to explain the discrepancy between QCD predictions and the measured quarkonium branching ratios, is without theoretical foundation, and unable to give a consistent description of all experimental data. We find instead a consistent description of *all* available experimental data with relativistic corrections that are small and even of opposite sign to the 'ad hoc' correction.

In Section 3, we have provided additional theoretical arguments, based on the recent paper by Kogan and Kovner[24], in support of the introduction of a gluon mass $m_g \approx 1.5$ GeV associated

with a non-vanishing gluon condensate $\langle \alpha_s G^2 \rangle$. This value of m_g allows to explain qualitatively the observed structure of the hadronic final states in J/ψ decays providing the underlying rationale behind the fusion of the primary gluons into light-quark pairs (Fig. 2). While m_g is expected to be a fundamental parameter of QCD, the phenomenological analysis of quarkonia decays requires the introduction of two "effective" gluon masses M_g to describe the phase-space modification of the perturbative final states. By fitting their values to the inclusive photon spectra of the J/ψ and Υ and correcting the theoretical predictions with the appropriate suppression factors, the analysis of all $c\bar{c}$ and $b\bar{b}$ decay widths is consistent and in good agreement with the DIL predictions for $\alpha_s(m_c)$ and $\alpha_s(m_b)$. The change in the effective gluon mass from $M_g = 0.66 \pm 0.08$ GeV for the J/ψ to the Υ value $M_g = 1.17 \pm 0.08$ GeV effectively describes the energy transfer from the primary gluons to the physical hadronic states available at the two center of mass energies. The observed increase of the effective gluon mass from $Q = 3.1$ GeV to $Q = 9.46$ GeV is consistent with the theoretical expectation that, asymptotically, the phenomenological value of M_g should approach its theoretical value $m_g \approx 1.5$ GeV. As very good fits to the experimental data were obtained with only the effective gluon mass as a free parameter, the theoretical curves obtained, before applying resolution corrections, give a good representation of the measured inclusive photon spectra, and so are useful for future theoretical comparisons. These spectra are reported in Table IV and Fig. 6. Also shown in Fig 6. are the tree-level prediction of QCD with massless gluons, the result of Ref.[9] where the leading logarithms in $1 - z$ are summed to all orders in α_s and the prediction of the parton shower model of Ref.[11]. It can be seen that the latter, while describing well the photon spectrum in Υ decays, predicts a spectrum that is too hard for the J/ψ. In this model the non-perturbative cut-off parameter is only 450 MeV, much smaller than the masses of either the J/ψ or the Υ, so that the spectrum shape is predicted to be very similar in the two cases[28]. Actually the cut-off in the $g \to gg$ splitting is $\simeq O(M_{J/\psi})$, which, as discussed above, explains the much softer photon spectrum observed for the J/ψ. The QCD corrections calculated in Ref.[9] near

$z = 1$ are modest, in accordance with the Chudakov[29] effect in QCD. Near $z = 1$ the two primary gluons form a colour singlet current, from which the radiation of wide angle secondary gluons is inhibited by destructive interference. One may note that the prediction of Ref.[11] is not consistent with the Chudakov effect since the correction near $z = 1$ is $\simeq 100\%$.

Quite independently of the concept of a "genuine" gluon mass m_g, our procedure of introducing an "effective" gluon mass M_g as an external constraint on the phase-space of the perturbative final states can be regarded as a simple tree-level extension of the usual factorization procedure which is contained as a special case corresponding to $M_g = 0$. As such, it can be taken as a direct way to estimate the size of the power-law corrections at various center of mass energies Q and, in this sense, our approach is complementary to the modification of the perturbative Feynman rules for quarks and gluons in the presence of gluon and fermion condensates (for a review and a complete list of references see ref.[30]). In contrast to this approach, however, the corrections are directly proportional to $\eta \sim \frac{M_g}{Q}$ and not to $\eta^4 \sim \frac{\langle \alpha_s G^2 \rangle}{Q^4}$, at least in the relevant range of the parameters. In this respect, our analysis is closer to the approach of ref.[31] where the introduction of a gluon mass in the tree level QCD predictions is used to to estimate the size of the power-law corrections to several hadronic shape variables and allows to reproduce[31] the observed $\sim 1/Q$ behaviour of the hadronization corrections in such infrared-safe quantities as $\langle 1 - T \rangle$ and the C-parameter. Another similar application is to the prediction of jet multiplicities at low values of the resolution parameter $y_{\rm cut}$. As shown in Refs.[26,27] a phenomenological gluon mass parameter can replace the conventional Λ parameter in all perturbative QCD applications. However, it is only by considering, as has been done in the present paper, processes whose physical scale is not very large as compared to m_g that the gluon mass parameter itself becomes essential to obtain a consistent description of experimental data. The mechanism of dynamical gluon mass generation proposed in Ref.[24] is a very attractive one and though claimed by the authors to be only qualitative, is in fact in remarkable agreement with the actual phenomenology. However it should not be forgotten that

this is not the only possible mechanism for the generation of a non-zero gluon mass within the context of gauge theories of the strong interaction. A brief review of such broken-colour theories and their experimental status may be found in Ref.[27].

Finally we remark that when all relevant physical effects are properly taken into account quarkonium decays do indeed provide a precise measurement of the strong coupling constant α_s . The parameters necessary to calculate the corrections due to these effects can be determined directly from experiment. By far the most important correction comes from the effect of the non-vanishing gluon mass. The determination of α_s reported in Eq.(74) has a relative error, conservatively estimated, of only 5 % which is comparable to that on the precise DIL result in Eq.(1) that we have used as a benchmark for our analysis.

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7) The value of the ratio $\frac{\Gamma(J/\psi\to ggg)}{\Gamma(J/\psi\to l^+l^-)}$ depends on the magnitude of the $J/\psi \to \gamma gg$ branching ratio which cannot be determined in a model-independent way from the MARK II result[8] $B(J/\psi \to \gamma gg)|_{z\geq 0.6} = (4.1 \pm 0.8)10^{-2}$ with $z = \frac{2E_\gamma}{M_{J/\psi}}$. Indeed, the extrapolation in perturbative QCD (based on the simple relation $B(J/\psi \to \gamma gg)|_{z>z_c} \sim (1 - z_c^2)B(J/\psi \to \gamma gg)$) is ill defined since both the tree level spectrum and the perturbative leading-log improvement based on the Photiadis calculation[9] do not describe the experimental data at all (see fig. 4 and fig. 9 of ref.[8]). As discussed in ref.[10] the R. D. Field[11] Monte Carlo, based on a the parton shower approach, is unable to reproduce the MARK II data. As far as we know, the only consistent description of the experimental photon spectrum is based on the introduction of an effective gluon mass (see ref.[10]). In this case, however, the extrapolation of the MARK II result produces the substantially larger value $B(J/\psi \to \gamma gg) = (11.3^{+3.7}_{-2.8})10^{-2}$ implying $\frac{\Gamma(J/\psi\to ggg)}{\Gamma(J/\psi\to l^+l^-)} = 9.8 \pm 0.6$ rather than $\frac{\Gamma(J/\psi\to ggg)}{\Gamma(J/\psi\to l^+l^-)} = 10.5 \pm 0.5$. Rather than using two different "experimental" values to compare with the various predictions, we prefer to enlarge the error by including the uncertainty due to the choice of the extrapolation procedure and use the average determination $\frac{\Gamma(J/\psi\to ggg)}{\Gamma(J/\psi\to l^+l^-)} = 10.1 \pm 0.9$. In this case, by using the experimental result $\Gamma(J/\psi \to e^+e^-) = 5.36 \pm 0.29$ KeV we find $\Gamma(J/\psi \to ggg) = 54.1 \pm 5.4$ KeV that will be used

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APPENDIX

The method used to take into account the experimental resolution function in the fits is briefly described here.

In a first step a histogram with bin index i and bin width Δz is generated according to the theoretical $d\Gamma/dz$ distribution. A new histogram is then generated by smearing the value of z for each entry of the bin with mean value z_i according to a gaussian resolution function with RMS σ_z corresponding to the experimental resolution. The quantity in the new histogram is then the smeared photon energy z_s where:

$$z_s = z_i + \sigma_z \mathrm{RG} \qquad (A1)$$

and RG is a random number with a gaussian distribution and unit standard deviation. The total number of entries in the original and the smeared histogram are then the same. The bins of the smeared histogram are chosen to be identical to those of the experimental distribution to be fitted. It is used as the theoretical curve in the fit to the data, and is normalised so as to have the same number of entries so that for the fits to both r and M_g there is only a single free parameter . In order to take into account accurately the effect of the resolution smearing the bin width of the input theoretical histogram is chosen to be much narrower ($\Delta z = 0.01$ for ARGUS and CRYSTAL BALL, $\Delta z = 0.001$ for MARK II) than that of the experimental histogram ($\Delta z = 0.05$ for ARGUS and CRYSTAL BALL ,$\Delta z = 0.025$ for MARK II). To reduce statistical fluctuations in the theoretical histogram to a negligible level as compared to the expeimental errors, a large number (10^6) entries is used in the input histogram.

The following resolution functions were used for the different experiments[8,19,20]:

$$\text{MARK II} \qquad \sigma_E/E = \frac{0.12}{\sqrt{E}} \qquad (A2)$$

$$\text{ARGUS} \qquad \sigma_E/E = \sqrt{0.0518 + \frac{0.00423}{E}} \qquad (A3)$$

$$\text{CRYSTAL BALL} \qquad \sigma_E/E = \frac{0.027}{E^{\frac{1}{4}}} \tag{A4}$$

In Eqs.(A2-A4) E is the photon energy in GeV.

FIGURE CAPTIONS

Fig.1 Fits to the inclusive photon spectrum in Υ radiative decays as measured by a) ARGUS, b) CRYSTAL BALL. Dotted lines : lowest order uncorrected QCD prediction. Broken lines : relativistic corrections included according to Eq.(14). Solid lines : Eq.(14) modified according to the QCD calculation of Ref.[9]. In all cases massless gluons are assumed and experimental resolution effects are taken into account as described in the Appendix.

Fig.2 Hadronisation by gluon fusion in the radiative decay of heavy quarks.

Fig.3 Fits to the inclusive photon spectrum in Υ radiative decays as measured by a) ARGUS, b) CRYSTAL BALL. Dashed lines : one parameter fits for M_g using Eqs.(53-58). Solid lines : fits for M_g including also the QCD correction factor from Ref.[9]. Experimental resolution effects are taken into account as described in the Appendix.

Fig.4 Fit to the inclusive photon spectrum in J/ψ radiative decays as measured by the MARK II Collaboration. The solid line is a one parameter fit for M_g using Eqs.(53-58). Experimental resolution effects are taken into account as described in the Appendix.

Fig.5 Values of $\alpha_s(\frac{M_\Upsilon}{2})$ derived from the branching ratio $\frac{\Gamma(V\to ggg)}{\Gamma(V\to l^+l^-)}$. Open squares : results for the massless gluon case, including relativistic corrections, as obtained from the global fits described in Section 2. Solid square : result for the massless gluon case using the relativistic correction parameter r as derived from the Υ inclusive photon spectrum. Open circles : results including both relativistic and gluon mass corrections as described in Section 3. Conservative estimates of the effect of $O(\alpha_s)$ QCD corrections in this case are indicated by the thick solid horizontal bars. The cross-hatched vertical band marked 'DIL' is the deep inelastic scattering result from Eq.(1). In this figure and in Table III no distinction is made between $\frac{M_\Upsilon}{2}$ and $m_b$for different values of m_b related to r_b via $r_b = \frac{M_\Upsilon}{m_b} - 2$.

Fig.6 Normalised inclusive photon spectra. Solid line A : Lowest order QCD prediction with massless gluons. Broken line : QCD prediction of Ref.[9]. Dotted line : prediction of the parton shower

model of Ref.[11]. Solid line B : best fit result for the Υ (Eqs.(53-58), including also the QCD correction of Ref.[9], with $M_g = 1.17$ GeV). Solid line C : Best fit result for the J/ψ (Eqs.(53-58) with $M_g = 0.66$ GeV).

TABLE CAPTIONS

Table I Gluon mass (M_g) values obtained from fits to the photon spectra in the decays : $\Upsilon \to \gamma X$, $J/\psi \to \gamma X$. LO denotes fits to Eq.(53). For the HO fits the QCD correction from Ref.[9] is also included.

Table II α_s values obtained from different branching ratios after including gluon mass corrections. The error on the correction factor f is due to the uncertainty on M_g. The errors on α_s are, respectively, experimental and due to M_g. Relativistic and $O(\alpha_s)$ corrections are neglected.

Table III Values of $\alpha_s(\frac{M_\Upsilon}{2})$ derived from $\frac{\Gamma(V \to ggg)}{\Gamma(V \to l^+l^-)}$ with different treatments of relativistic and gluon mass corrections. See Fig.5 and the text. The DIL value from Eqn.(1) is given for comparison.

Table IV Inclusive photon spectra for the Υ and J/ψ corresponding to the curves B and C respectively of Fig.6.

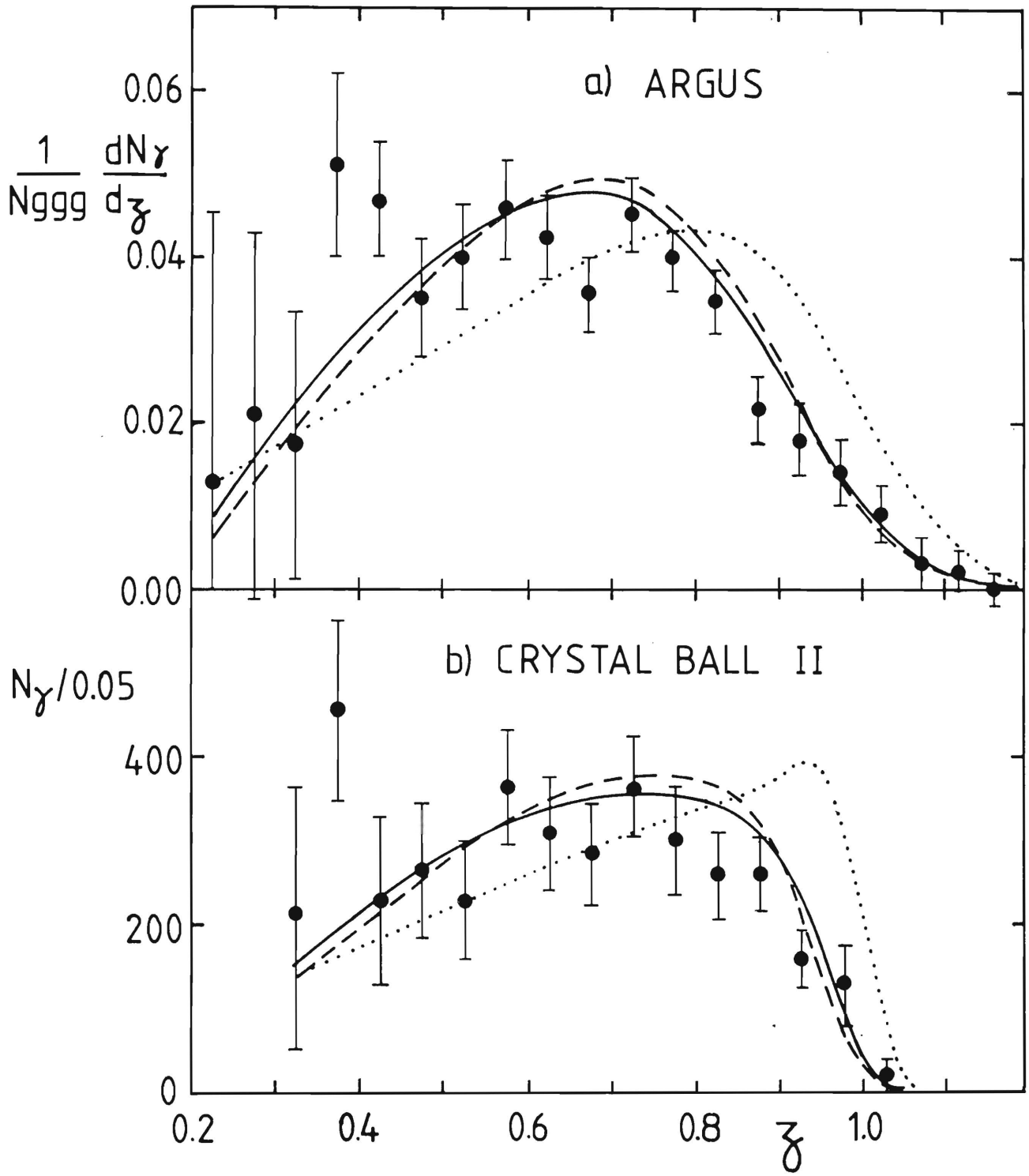
a) ARGUS
b) CRYSTAL BALL II
$\frac{1}{N_{ggg}} \frac{dN_\gamma}{dz}$
$N_\gamma/0.05$
0.06
0.04
0.02
0.00
400
200
0
0.2
0.4
0.6
0.8
1.0
z

Fig. 1

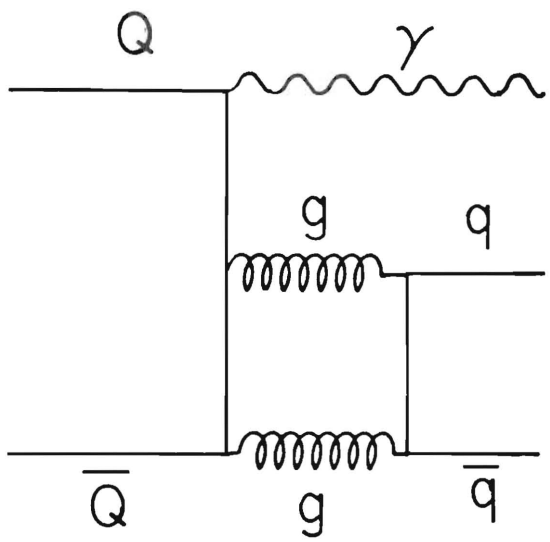

Fig. 2

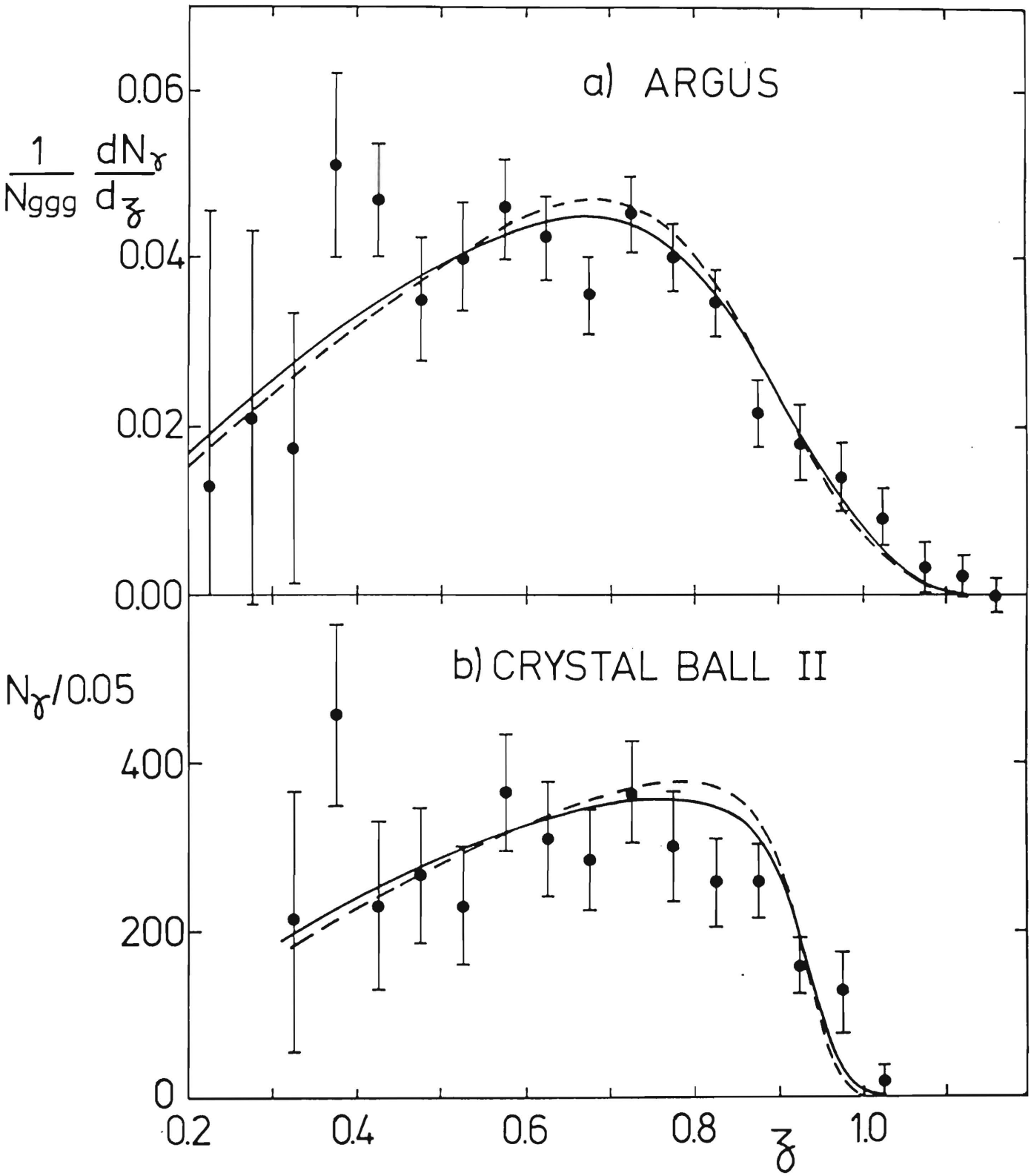
a) ARGUS
b) CRYSTAL BALL II
$\frac{1}{N_{ggg}} \frac{dN_\gamma}{dz}$
$N_\gamma/0.05$
0.06
0.04
0.02
0.00
400
200
0
0.2
0.4
0.6
0.8
1.0
z

Fig. 3

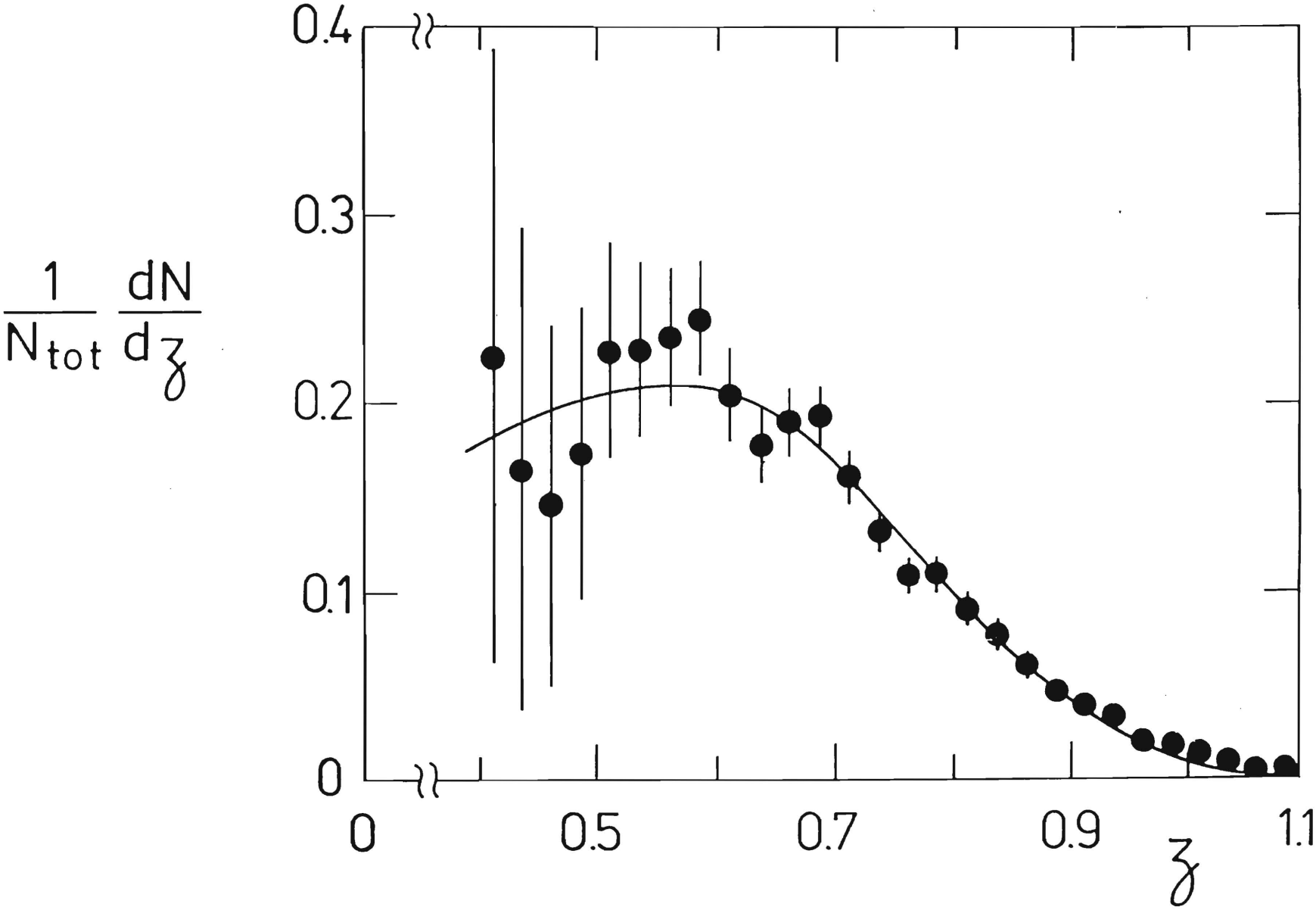

Fig. 4

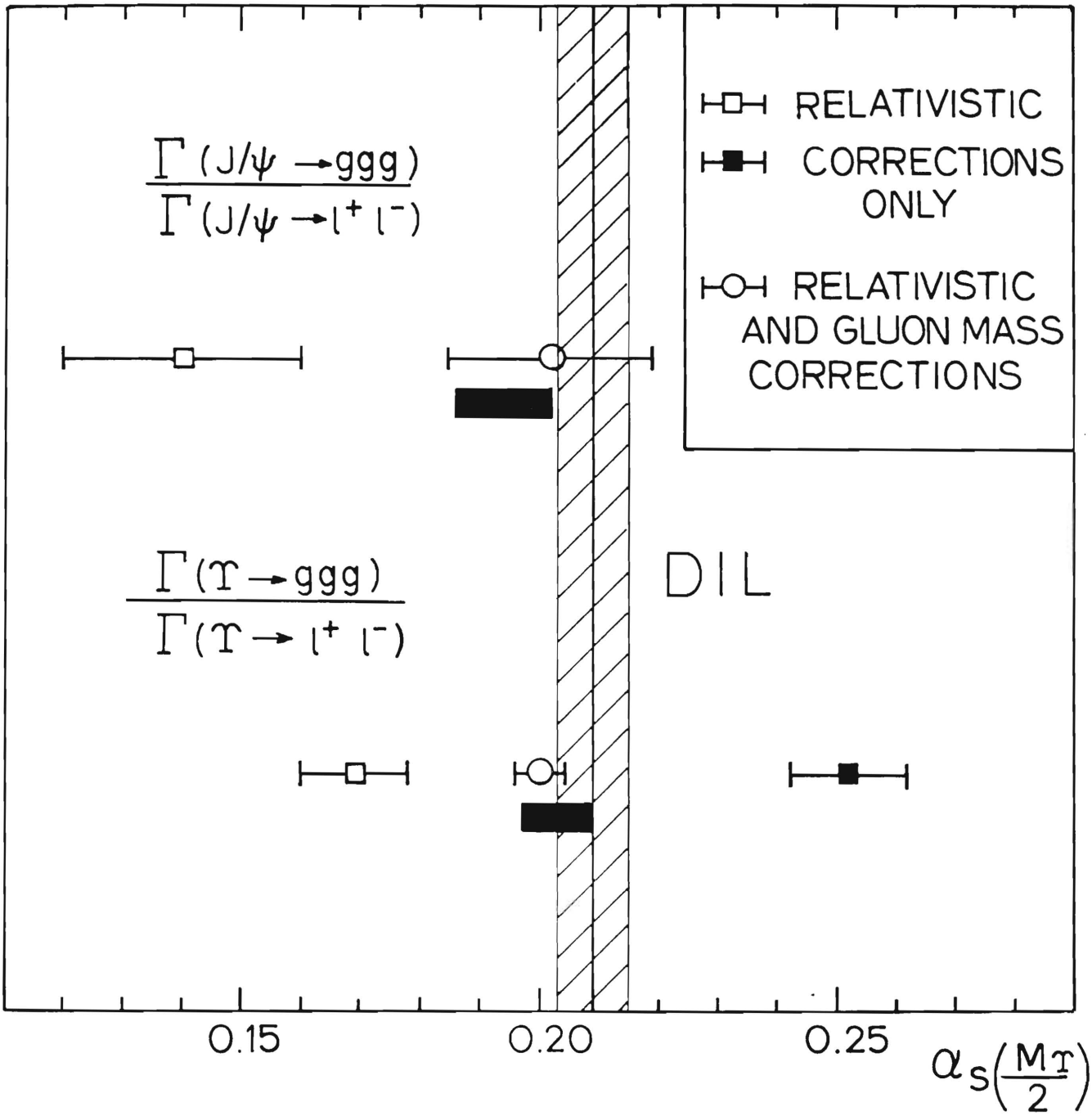
RELATIVISTIC
CORRECTIONS ONLY
RELATIVISTIC AND GLUON MASS CORRECTIONS
Γ(J/ψ → ggg) / Γ(J/ψ → l⁺ l⁻)
Γ(ϒ → ggg) / Γ(ϒ → l⁺ l⁻)
DIL
0.15
0.20
0.25
αs(MϒI/2)

Fig. 5

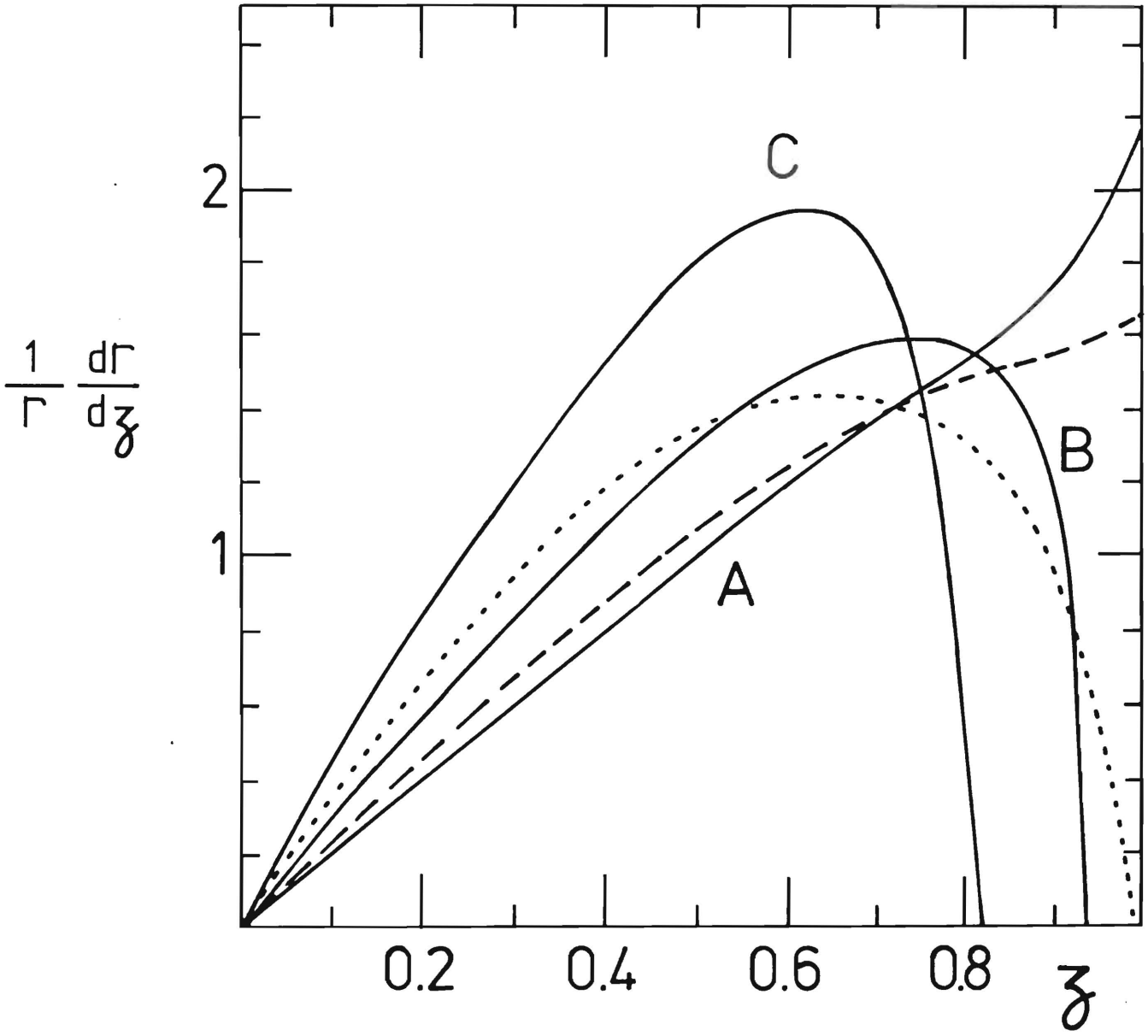

Fig. 6

TABLE I

EXPERIMENT	M_g (GeV)	χ^2_{MIN}	D.F.	C.L. %	FIT TYPE
ARGUS Υ	1.43 $\pm$0.094	19	18	39	LO
	1.21 $\pm$0.10	14.7	18	68	HO
CRYSTAL BALL Υ	$1.20^{+0.12}_{-0.10}$	24	13	3	LO
	$1.10^{+0.12}_{-0.13}$	18	13	16	HO
MARK II J/ψ	$0.66^{+0.07}_{-0.08}$	18.3	26	86	LO

TABLE II

	Exp.	f	$\alpha_s(m_Q)$
$\frac{\Gamma(J/\psi\to ggg)}{\Gamma(J/\psi\to l^+l^-)}$	10.1 ± 0.9	0.25 ± 0.1	$0.31 \pm 0.003 \pm 0.04$
$\frac{\Gamma(J/\psi\to ggg)}{\Gamma(J/\psi\to \gamma gg)}$	$5.3^{+2.1}_{-1.5}$	0.54 ± 0.1	$0.23 \pm 0.09 \pm 0.05$
$\frac{\Gamma(\psi(2S)\to ggg)}{\Gamma(\psi(2S)\to l^+l^-)}$	14.7 ± 5.1	0.38 ± 0.09	$0.30 \pm 0.04 \pm 0.02$
$\frac{\Gamma(\eta_c\to gg)}{\Gamma(\eta_c\to \gamma\gamma)}$	$(1.27 \pm 0.54) \cdot 10^3$	0.69 ± 0.06	$0.30 \pm 0.07 \pm 0.01$
$\frac{\Gamma(\chi_{c2}\to gg)}{\Gamma(\chi_{c2}\to \gamma\gamma)}$	$(1.35 \pm 0.39) \cdot 10^3$	0.76 ± 0.04	$0.29 \pm 0.04 \pm 0.01$
$\frac{\Gamma(\Upsilon\to ggg)}{\Gamma(\Upsilon\to l^+l^-)}$	32.6 ± 0.8	0.620 ± 0.037	$0.214 \pm 0.002 \pm 0.004$
$\frac{\Gamma(\Upsilon\to ggg)}{\Gamma(\Upsilon\to \gamma gg)}$	34.1 ± 3.0	0.84 ± 0.02	$0.236 \pm 0.022 \pm 0.007$
$\frac{\Gamma(\Upsilon(2S)\to ggg)}{\Gamma(\Upsilon(2S)\to l^+l^-)}$	34.3 ± 6.2	0.649 ± 0.034	$0.214 \pm 0.014 \pm 0.004$
$\frac{\Gamma(\Upsilon(3S)\to ggg)}{\Gamma(\Upsilon(3S)\to l^+l^-)}$	25.6 ± 2.8	0.665 ± 0.033	$0.193 \pm 0.007 \pm 0.003$

TABLE III

SOURCE	INPUT	$\alpha_s(\frac{M_\Upsilon}{2})$	REL. CORRN.	M_g CORRN.
J/ψ	Global Fit	0.145 ± 0.020	Yes	No
J/ψ	M_g,m_c	0.202 ± 0.017	Yes	Yes
Υ	Global Fit	0.169 ± 0.009	Yes	No
Υ	Photon Spectrum	0.252 ± 0.010	Yes	No
Υ	M_g,m_b	0.2004 ± 0.0043	Yes	Yes
DIL		0.209 ± 0.006		

TABLE IV

$z = 2\frac{E_\gamma}{M_V}$	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
$\frac{1}{\Gamma}\frac{\Delta\Gamma}{\Delta z}$ J/ψ	0.458	0.831	1.186	1.508	1.797	1.949	1.780	0.576	
$\frac{1}{\Gamma}\frac{\Delta\Gamma}{\Delta z}$ Υ	0.271	0.559	0.831	1.085	1.322	1.492	1.593	1.593	1.237