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ON THE INFRA-RED STRUCTURE OF QCD, THE Λ PARAMETER AND THE QUARK AND GLUON MASSES

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ABSTRACT

In perturbative QCD the non-abelian gluon self coupling generates logarithmic mass singularities in diagrams with internal gluon lines, rendering the massless gluon theory divergent. The divergences are localised in a universal running coupling constant and are removed, in conventional QCD phenomenology, by an effective gluon mass implicit in the Λ parameter. Recent work suggesting an effective gluon mass of ~ 1.2 GeV is summarised and a conjecture is made on the nature of the confinement mechanism.

In conventional QCD the SU(3) colour symmetry is assumed to be exact and the related gauge bosons, the gluons, are in consequence strictly massless at the perturbative level [1, 2]. It was conjectured at the same time that QCD was proposed that the Infra Red (IR) divergences generated by massless gluon exchange might be responsible for quark confinement (Infra-Red-Slavery) [1-3].

The infra-red structure of perturbative QCD was extensively investigated in the literature during the middle seventies. It was found that certain diagrams containing the non-abelian gluon self-coupling lead to an IR divergent result, even for fully inclusive quantities. After reviewing this work (which seems largely forgotten today) it is pointed out that in conventional QCD phenomenology using the Λ parameter the gluon is tacitly assumed to be massive, so avoiding the IR divergences of the massless theory.

Finally, recent work in which phenomenological evidence for an effective gluon mass of ~ 1 GeV has been obtained, is briefly mentioned, and a conjecture on an alternative explanation of quark confinement to "Infra-Red-Slavery" is made.

Studies in perturbation theory of the IR behaviour of QCD fall into two broad categories :

1) Processes with external gluon lines and associated vertex diagrams, responsible for the generation of leading-logarithmic terms of the form $[\alpha_s \ln (Q/m_g)]^n$ where Q is the physical scale of a process and m_g is the gluon mass, which should be considered, in the massless theory, as an infra-red regulator. α_s is the on-shell fine structure constant of QCD [4-7].

2) Processes with internal gluon lines, where gluon loops or vertex corrections generate non-leading logarithms of the form : $\alpha_s^n[\ln(Q/m_g)]^m$, $1 \leq m \leq n - 1$ [8-11].

   For (1) (analogous to the case of soft photon radiation in QED) all authors find exact cancellation of IR singular terms between real and virtual diagrams of a gauge invariant set, at the lowest non-trivial order. General arguments were given that this cancellation should occur order-by-order to all orders of perturbation theory [5].

   For (2) three studies [9-11] find an IR divergent result, a conclusion recently independently confirmed by the writer [12], whereas one [8] finds an IR finite result. This discrepancy is discussed below.

Feynman diagrams corresponding to processes of type (1) are shown in Fig 1a, b) and of type (2) in Fig. 1c, d). Figs 1a, b, c) correspond to different radiative

corrections to the same basic process, quark-quark scattering, that was considered in Ref. [6, 11]. Fig 1d), considered in Ref. [9, 10], gives a next-to-leading contribution to the anomalous magnetic moment of a coloured quark. The IR divergences for $m_g \rightarrow 0$ of the QCD diagrams Fig. 1c, d) have a similar structure to the mass singularities for $m_e \rightarrow 0$ of the QED diagrams shown in Fig. e, f) respectively. These latter give radiative corrections to Mφller scattering and the anomalous magnetic moment of the muon.

For clarity in the subsequent discussion it is remarked that the IR structure of QCD in the limit $m_g \rightarrow 0$ has two different aspects according to whether the process considered is of type (1) or (2) :

(i) "True" IR singularities associated with diagrams such as Fig. 1a, b), are connected with the possible vanishing of the 4-momentum vector of a real, massless gluon. These will be referred to as "λ_g singularities", and are strictly analogous to the IR singularities of QED as $m_\gamma \rightarrow 0$.

(ii) IR singularities related to the non-abelian gluon self coupling as in Fig. 1c, d). Since these are analogous to the $m_e \rightarrow 0$ mass singularities of QED as occurring in the diagrams shown in Fig. 1e, f), these are called "m_g "singularities".

Finally one may remark that QCD contains mass singularities related to the vanishing of the quark masses. "m_q singularities". Typical processes having such singularities are obtained from Figs 1e, f) with the replacements lepton $\rightarrow$ quark, photon $\rightarrow$ gluon.

A general analysis of mass singularities in QED was made by Kinoshita [13]. It was found that the unrenormalised amplitude for a total transition probability (unlike those for partial transition probabilities) was free of mass singularities. However, after charge renormalisation mass singularities reappear (even in total transition probabilities) in a one-to-one relationship with terms that, before renormalisation, were Ultra Violet (UV) divergent. Similar behaviour was demonstrated for the m_g singularities of QCD by Cvitanovic [10].

It is then straight forward, using the renormalisation group, to obtain the general structure of mass singularities in QED or the m_g, m_q singularities of QCD.

For example in QED the mass singular next-to-leading radiative correction to the muon decay rate Γ_μ is given [14] by the replacement :

$$\alpha \rightarrow \alpha(m_\mu) \tag{1}$$

in the first order radiative correction :

$$\delta^{(1)}_{\Gamma_\mu} = \frac{\alpha}{2\pi}\left[\frac{25}{4} - \pi^2\right] \tag{2}$$

where $\alpha(\mu)$ is the one-loop solution of the renormalisation group equation for the QED β-function [15], m_μ is the mass of the muon and α is the QED fine structure constant ($\alpha^{-1} = 137.036$).

The β-function renormalisation group equation reads :

$$\mu\frac{\partial a}{\partial \mu} = \beta(a) = -\ ba^2 + \ldots \tag{3}$$

$$b = -\frac{2}{3}, \qquad a = \alpha/\pi$$

Explicitly :

$$\alpha(m_\mu) = \frac{\alpha}{1 + b\frac{\alpha}{\pi}\ln\left(\frac{m_\mu}{m_e}\right)} = \alpha\left[1 - b\frac{\alpha}{\pi}\ln\left(\frac{m_\mu}{m_e}\right) + \ldots\right] \tag{4}$$

Note in (4) the non-leading nature $\approx \alpha^2 \ln\left(\frac{m_\mu}{m_e}\right)$ of the mass-singular logarithm.

Another QED example is the final state radiative correction to the total cross section for the process $e^+e^- \rightarrow e^+e^-$ by s-channel photon exchange [12]. Here the mass singular next-to-leading correction is given by the replacement, in the lowest order correction :

$$\alpha \rightarrow \alpha\left(\sqrt{s}\right) \tag{5}$$

where $\sqrt{s}$ is the total center-of-mass energy.
For the quark-quark scattering process in QCD the mass singular contribution

of the complete gauge invariant set of diagrams, including Fig 1c), and the (separately) gauge invariant quark loop contribution is given [11] by the replacement :

$$\alpha_s \rightarrow \alpha_s\left(\sqrt{t}\right) \tag{6}$$

in the lowest order gluon exchange diagram. Here -t is the squared 4 - momentum transfer in the scattering and :

$$\alpha_s\left(\sqrt{t}\right) = \frac{\alpha_s}{1 + \frac{\alpha_s}{\pi}\left[\frac{11}{2}\ln\left(\frac{\sqrt{t}}{m_g}\right) - \frac{n_f}{3}\ln\frac{\sqrt{t}}{m_q}\right]} \tag{7}$$

n_f is the number of active quark flavours with mass m_q. For $m_g = m_q = m$, (7) may be written as :

$$\alpha_s(\sqrt{t}) = \frac{\alpha_s}{1 + b_s \frac{\alpha_s}{\pi} \ln\frac{\sqrt{t}}{m}} \tag{8}$$

$$b_s = 11/2 - n_f/3$$

where b_s is the leading coefficient in the perturbation expansion of the QCD β-function whose renormalisation group equation is given by (3) with the replacements $\beta \rightarrow \beta_s$, $a \rightarrow a_s$, $b \rightarrow b_s$.

It was demonstrated in Ref. [9, 10] that m_g singular contributions to the quark anomalous magnetic moment are similarly controlled by the QCD β-function. The replacement in this case is :

$$\alpha_s \rightarrow \alpha_s(M_q) \tag{9}$$

where M_q is the mass of the external quark line in Fig. 1d).

The claim in Ref. [8] of the IR finiteness of type (2) processes in QCD is now commented upon. In fact Ref. [8] does not explicitly discuss QCD, but rather argues by analogy with the supposed IR finiteness of a QED quantity, the photon proper self energy function. It is claimed that this quantity $d(k^2/M^2, m^2/M^2, g(M))$, renormalised at space like transfer $\mu^2 = -M^2$, is IR finite as $m \rightarrow 0$. In fact

d is divergent as a consequence of the IR divergent nature of g(M) as $m \to 0$ (see Eqn (4) above, $g^2/4\pi = \alpha$, $m = m_e$). Otherwise, it may be remarked that d is renormalisation scale invariant. Since it is manifestly divergent for $m \to 0$ using on-shell renormalisation, it is divergent for any other choice of renormalisation scale. The procedure of renormalisation removes only the UV divergences, not the IR ones [13].

It is often stated in the literature that suitably chosen physical quantities may be defined in QCD for which finite perturbative predictions can be obtained even for massless quarks and gluons "because of the KLN (Kinoshita-Lee-Nauenberg) theorem". In fact, such "Infra-Red-Safe" quantities are only so with respect to the λ_g singularities, yielding leading logarithmic terms as discussed in Ref. [4-7]. Kinoshita (for QED) [13] and Korthales Altes, De Rafael and Cvitanovic [9, 10] (for QCD) pointed out the IR divergent nature of the contributions of diagrams such as Fig. 1c-f). Such diagrams will contribute to the leading radiative correction of any process whose Born term contains an internal photon or gluon line. Lee and Nauenberg's [16] demonstration of the vanishing of mass singularities in QED applies only to diagrams such as Fig. 1a, containing external massless lines i.e. to λ_γ, λ_g singularities.

The "KLN Theorem" then has two distinct aspects :

(i) The IR finiteness, to all orders in perturbation theory, of suitably defined physical quantities with respect to λ_γ, λ_g divergences.

(ii) The IR divergent nature of the "m_e singular" terms in QED and the "m_g, m_q singular" terms in QCD.

The IR divergent terms are in a one-to-one relationship with terms that are UV divergent (before renormalisation) and, using renormalisation group arguments may be grouped into a process independent "running coupling constant" at a well defined process dependent scale. Examples of this are given in Eqns (1), (5), (6), (9) above.

Nothing stated above is new. All the results have existed in the published literature for almost twenty years now. Even so it is commonly stated that gluons are massless at the perturbative level in QCD, and furthermore it is clear that the comparison of "Infra-Red-Safe" perturbatively calculated quantities with experimental data has lead to an enormously successful and consistent description of many different physical observables [17]. In addition the vast majority of these calculations have used the elegant gauge-invariant Dimensional Regularisation Procedure (DRP) of t'Hooff and Veltman [18] in which both gluons and quarks are, apparently, assumed to be massless. How can this be reconciled with the IR

divergent behaviour pointed out in Ref. [9-12] ? To the writer's knowledge there was no subsequent discussion in the literature of the IR divergent nature of perturbative QCD with massless gluons after the publication of references [9-11]. In Ref. [10] it was suggested that the solution to the IR problem might be found in the breakdown of perturbation theory, a route that has since been followed by many authors seeking non perturbative generation of an "effective" or "constituent" gluon mass [19]. On the other hand a possible solution within perturbation theory was suggested in the second Ref. [9]: "the actual value of λ (the gluon mass) can be considered as a physical parameter (much as the Rydberg Constant replaces the infra-red cut-off in the Lamb Shift calculation in QED)".

The second of these two approaches is the one that has actually (though tacitly) been adopted in subsequent phenomenology, by retaining perturbation theory and introducing the constant "Λ" with the dimension of mass to parametrise the running coupling constant of QCD, where all the IR singularities are, by the KLN theorem, localised. Comparing the expression (7) for $\alpha_s(\mu)$ with the conventional one-loop formula containing the parameter $\Lambda^{(1)}_{n_f}$ (the superscript denotes the order, the subscript the number of active quark flavours):

$$\alpha_s(\mu) = \frac{\pi}{b_s} / \ln\left(\frac{\mu}{\Lambda^{(1)}_{n_f}}\right) \tag{10}$$

the following expression for the gluon mass is obtained :

$$m_g = \Lambda^{(1)}_5 \exp\left\{\frac{2}{11}\left[\frac{\pi}{\alpha_s} + \frac{5}{3}\ln\left(\frac{\overline{m}}{\Lambda^{(1)}_5}\right)\right]\right\} \tag{11}$$

where $\overline{m} = (m_u m_d m_s m_c m_b)^{\frac{1}{5}}$

In (11) the quark masses for different flavours are explicitly introduced and it is assumed that $m_b \ll \mu \ll m_t$, so that quark threshold effects may be neglected. Inverting (11) gives $\Lambda^{(1)}_5$ as a function of m_g, $\overline{m}$ and α_s :

$$\Lambda_5^{(1)} = m_g \exp\left\{-\frac{6}{23}\left[\frac{\pi}{\alpha_s} + \frac{5}{3}\ln\frac{\overline{m}}{m_g}\right]\right\} \qquad \textbf{(12)}$$

Generalisations of (12) to two-loop order and including threshold factors for the c and b quarks may be found in Ref. [20]. It is clear from (11) that the "Λ" parametrisation of the running coupling constant has effectively introduced a non zero gluon mass into the theory, thus avoiding the IR divergence problem. It might be argued that Λ is indeed "non perturbative" giving a simple parametrisation of the IR behaviour of QCD. According to (10) $\alpha_s(\mu)$ is IR divergent for $\mu = \Lambda$. However, as pointed out in Ref. [10, 12] the IR singularity generated by massless gluons in perturbation theory is of a very different nature. Inspection of (7) shows that $\alpha_s(\mu)$ vanishes in this case for all non zero values of α_s at all non-zero values of μ. As it is said in Ref. [10] the theory is not "asymptotically free" but rather "everywhere free". Thus the problem is not that the coupling constant diverges at small scales as suggested by (10) when $\mu \rightarrow \Lambda$, but rather that the running coupling constant vanishes for all finite values of μ ! This behaviour is in contradiction to "Infra-Red- Slavery" [1-3] so that another explanation must be found for quark confinement. This problem is briefly addressed again at the end of this letter.

It can be seen from (11) that the vanishing of m_g in fact implies the vanishing of Λ, and hence, from [10] the vanishing of $\alpha_s(\mu)$ for all non-zero μ.If quark mass effects are taken into account in the phenomenology as done in Ref. [11, 12, 20] terms such as $\alpha_s \ln(Q/m_q)$ will appear in asymptotic formulae. Simply for consistency (see Eqn (7)) the corresponding gluonic terms $\alpha_s \ln(Q/m_g)$, with $m_g \neq 0$ must also exist. In other words there is no consistent way to introduce effective quark masses into the theory without, at the same time, introducing a (non-zero) effective gluon mass.

If all perturbative QCD analyses so for performed have tacitly introduced massive gluons via the Λ parameter, a problem of internal consistency arises. What other massive gluon effects, apart from those already included in Λ, should be taken into account in the phenomenology ? Three such effects are :

a) Modification of phase space due to the gluon mass.

b) Contributions of longitudinal helicity states to transition amplitudes of processes with external gluon lines.

c) Propagator suppression effects for virtual gluons.

All of these are expected to produce observable consequences only in the low energy regime where m_g/Q is not too small and so to have a negligible effect on QCD phenomenology at high energies. The first calculation (to the best of the writer's knowledge) of a), b) was made by Parisi and Petronzio [22] for the case of J/Ψ and η_c decays into gluons. These authors observed that the introduction of a gluon mass of ~ 0.8 GeV with a value of α_S of 0.35 resolved a number of otherwise unexplained problems in low energy QCD phenomenology. More recent determinations of α_s and m_g have been described elsewhere [12, 20 ,23]. Of particular interest is the analysis using the photon spectra in inclusive radiative decays of quarkonia: $J/\Psi \rightarrow \gamma X$, $\Upsilon \rightarrow \gamma X$. Fits to the recent high precision data on the photon spectrum in $\Upsilon \rightarrow \gamma X$ decays from the ARGUS [24] and Crystal Ball [25] Collaborations suggest a gluon mass of ~ 1.2 GeV [23]. Such a gluon mass leads, by phase space and propagator suppression to a considerable softening of the photon spectrum in $J/\Psi \rightarrow \gamma X$ decays, as first suggested in Ref. [22]. Combining suppression factors calculated in Ref. [22] with the fitted effective gluon mass values from Ref. [23] leads [26] to values of $\alpha_s(M_c)$, $\alpha_s(M_b)$ in good agreement with $\alpha_s(\mu)$ found in deep inelastic scattering. Previously an ad hoc "relativistic correction" of a factor of $\cong 3$ was applied to explain the low value of α_s obtained from the ratio : $\Gamma(J/\Psi \rightarrow ggg)/\Gamma(J/\Psi \rightarrow \ell^+\ell^-)$ [27,28]. In the analysis presented in Ref. [26] no adjustable parameters are used.

In Ref. [12, 23] next-to-leading order QCD analyses of several different low energy processes are used to derive the estimate for the strong fine structure constant :

$$\alpha_s = 0.291 \pm 0.023$$

Taking the gluon mass to be 1.2 GeV [23] and the quark masses to be the "current" values at 1 GeV scale from the Particle Data Group [29], Eqn (6) may be used to predict $\alpha_s(M_Z)$:

$$\alpha_s(M_Z) = 0.132 \text{ (1-loop estimate)}$$

This may be compared with the next-to-leading order estimate using the same quark and gluon masses and α_S value [23] :

$$\alpha_s(M_Z) = 0.128 \pm 0.005 \text{ (2-loop estimate)}$$

and the recent LEP average value [30] :

$$\alpha_s(M_Z) = 0.125 \pm 0.005 \text{ (LEP 1992 average)}$$

Further examples of consequences of an effective gluon mass of $\cong 1$ GeV in low energy phenomenology may be found in Ref. [12, 22, 23], while some discussion of, and references to, gauge theories of the strong interaction with spontaneously broken colour symmetry and massive gluons are given in Ref. [12]. In these theories the colour symmetry is typically broken by the Higgs-Kibble mechanism so that massive scalar fields are introduced. It has however been demonstrated that a massive gluon theory that is gauge invariant and renormalisable may be constructed in the absence of scalar fields [31].

The infra-red behaviour, in the massless limit, of the DRP of 't Hooft and Veltman [18] is now discussed. It is often stated, in this case, that the gluon is massless and that the massless quark limit may be also be taken. As pointed out above, such a statement is inconsistent (see Eqn (12)) if a non-zero value of Λ is introduced to parametrise the running coupling constant. Otherwise an "unphysical" scale μ may appear in the calculation after renormalisation, leading to terms containing $\ln(Q/\mu)$ where Q is the physical scale of the process. It has been shown by Coqueraux [32], for the case of QED at the one-loop level, that the conventional asymptotic on-shell result is recovered from the DRP calculation by the replacement

$$\mu \rightarrow \overline{m}_\ell, \qquad \overline{m}_\ell = (m_1 m_2 \ldots m_{n_\ell})^{\frac{1}{n_\ell}} \tag{13}$$

where n_ℓ is the number of charged leptons of mass m_i with $m_i << Q$.

The corresponding replacement for QCD is [12] :

$$\mu \rightarrow m_g \left[\frac{n_f}{\frac{33}{2} - n_f} \ln \frac{m_g}{\overline{m}} \right] \tag{14}$$

where $\overline{m}$ is defined after Eqn (11) above. It can be seen from (13) and (14) that the massless limit of either QED or QCD corresponds, in the DRP, to $\mu \rightarrow 0$ so that both theories are infra-red divergent in this limit.

Finally a remark on the conjectured "Infra-Red-Slavery" explanation of quark confinement [1-3]. If the IR divergences of QCD are avoided by a non-zero gluon mass, and if, as suggested in Ref. [22] and the recent analysis of Ref. [12, 20, 23] the running strong coupling constant is "frozen" at a value of $\cong 0.3$ for scales below $\cong 2$ GeV, it would seem that the origin of quark confinement must be sought elsewhere than in "Infra-Red-Slavery". It is conjectured here that the confinement mechanism may be analogous to that responsible for the Bloch-Nordsieck Theorem [33] in QED. Just as it is impossible to accelerate a charged particle without the generation of soft photons, so in any attempt to separate the colour charge carried by a quark, the colour field created will have an energetically favoured decay into light colour singlet states (mostly pions) with the generation of new colour dipoles, which subsequently decay in the same manner, as in colour string fragmentation models [34]. Separation of the colour charge would require switching off this damping mechanism for the energy density of the colour field. This would seem to be no more possible than to switch off the damping of the electromagnetic field by photon radiation when an electric charge is accelerated.

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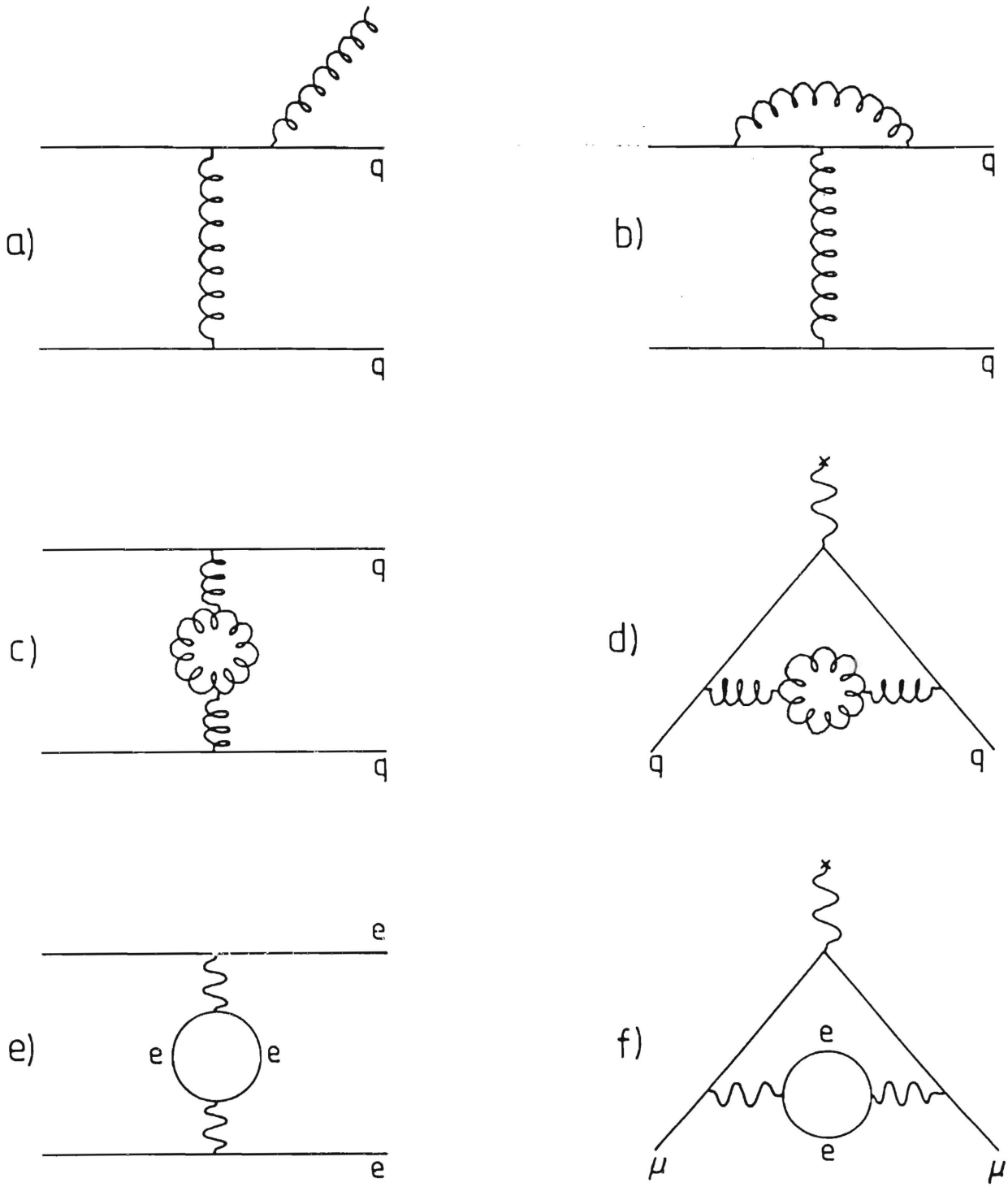
FIGURE CAPTIONS

Figure 1. Some examples of infra-red finite and mass singular processes. Curly lines are gluons, wavy lines photons.The sum of the diagrams in Fig. 1a), 1b), together with others in the gauge invariant set, gives a finite result even for $m_g = 0$. The diagrams of Fig. 1c, d; e, f are logarithmically divergent for $m_g = 0$; $m_e = 0$ respectively.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