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RADIATIVELY CORRECTED CROSS SECTIONS FOR QCD HARD SCATTERING PROCESSES

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ABSTRACT

Formulae are derived for radiatively corrected hard scattering cross-sections in QCD. Soft and virtual gluon radiation is taken into account in leading log approximation to all orders in α_s. The language used and the formulae obtained are closely analogous to those of QED. Explicit results are given for QCD Compton scattering and gamma gluon fusion.

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Formulae are derived for radiatively corrected hard scattering cross-sections in QCD. Soft and virtual gluon radiation is taken into account in leading log approximation to all orders in α_s. The language used and the formulae obtained are closely analogous to those of QED. Explicit results are given for QCD Compton scattering and gamma gluon fusion.

In a previous letter [1] an analysis was made of integrated jet cross-sections in leptoproduction on a nucleon target. Non singlet Mellin moments of the F_2 structure function were written as the sum of a series of integrated jet cross-sections where N hard gluons (N = 0, 1, ... ∞) are radiated while soft and virtual gluons are summed in Leading Log (LL) approximation to all orders in α_s.

The aim of the present letter is to give general formulae for soft and virtual radiative corrections (summed to all orders in α_s in LL approximation) to differential cross-sections for QCD hard scattering processes. The corresponding QED formulae have been known for nearly thirty years, and the discussion and notation follows closely that used for the QED case [2, 3].

To make the link between QED and QCD the case of electron potential scattering in QED, which is closely analogous to the "2jet" cross-section in non singlet electroproduction calculated in Ref. [1] is first treated by the resummation methods of Ref. [1]. Comparison with the QED result of Ref. [2] on the one hand and the corresponding QCD result of Ref. [1] on the other then indicates how the QED formulae is modified for QCD in this special case.

Similar modifications to the well known general QED formulae, then give general results for the QCD case. As an example explicit formulae are given for QCD Compton scattering and gamma gluon fusion in real or virtual photon nucleon collisions.

To discuss potential scattering following the method of Ref. [1] a Gribov-Lipatov-Altarelli-Parisi [4] splitting function $P_{ee}(z)$ with "hard" (H) and "virtual + soft" (V,S) components is introduced :

$$P^{H}_{ee}(z) = \left(\frac{1+z^2}{1-z}\right) \theta(z_{max} - z) \tag{1a}$$

$$P^{V,S}_{ee}(z) = \left[2\ln\varepsilon + \frac{1}{2}\right] \delta(1-z) \tag{1b}$$

(The notation used here follows closely that of Ref. [1]).

Defining an electron density q^e and structure function F^e_2 at $O(\alpha^0)$ by :

$$F^e_2(x) = xe^2_e q^e(x) = xe^2_e \delta(1-x) \tag{2}$$

$[e_e$ = charge of the electron$]$

and using the relation between the scattering cross-section and the structure function :

$$\sigma = \frac{4\pi^2\alpha}{Q^2} F_2 \tag{3}$$

the integrated cross-section $\Sigma(k_0)$ for electron potential scattering with energy loss in soft photons $< k_0$, is found to be :

$$\Sigma(k_0) = \exp\left(\frac{\beta_e}{4}\right)\left(\frac{k_0}{\hat{E}}\right)^{\beta_e}\sigma_0 \tag{4}$$

where :

$$\beta_e = \frac{2\alpha}{\pi}\,\ell n\,\frac{Q^2}{m_e{}^2} \tag{5}$$

and σ_0 is the uncorrected γe scattering cross-section :

$$\sigma_0 = \frac{4\pi^2\alpha}{Q^2}\,x e_e^2\,\delta(1-x) \tag{6}$$

$\hat{E}$ is the incoming electron energy in the γe rest frame and k_0 is the maximum value of the summed soft photon energy $\sum_{i=1}^{\infty} k_i$ in the same frame. The factor $k_0/\hat{E}$ is related to the resolution parameter ε introduced in [1] :

$$\varepsilon \equiv \frac{\hat{s}_0}{\hat{s}_0 + Q^2} = \frac{\sqrt{\hat{s}_0}}{2\hat{E}} \leq \frac{k_0}{\hat{E}} \tag{7}$$

In deriving (4), (5) care must be taken to include all relevant electron mass terms [5] in the angular integrations. Equation (4) may be compared with the result of Ref. [2] :

$$\Sigma(k_0) = \frac{\exp\left(\frac{\beta_e}{4}\right)}{\Gamma(1+\beta'_e)}\left(\frac{k_0}{\gamma_E\hat{E}}\right)^{\beta'_e}\sigma_0 \tag{8}$$

where :

$$\beta'_e = \beta_e - 2\alpha/\pi$$

and γ_E is the Euler constant 1.781. In LL approximation $\beta'_e = \beta_e$. The additional normalisation factor in (8) as compared to (4) :

$$F(\beta'_e) = \frac{(\gamma_E)^{-\beta'_e}}{\Gamma(1+\beta'_e)} \tag{9}$$

takes into account correlated emission of soft photons. If one photon of energy k_1 is emitted the condition :

$$\sum_{j>1} k_j < k_0 - k_1$$

reduces the probability of emission of the photons with energy k_2, k_3, ... [2, 3]. This effect is not included in (4) and therefore not in the LL QCD evolution equations, which correspond to independent soft gluon emission. It may be shown by taking the large Q^2 limit of equation (16) of Ref. [1] that the number of jets becomes Poisson distributed with mean value :

$$< N_{jet} > = 2C_F b \, \ell n \frac{Q^2}{\hat{s}_0} L(Q^2, Q_0^2) \tag{10}$$

where : $b = \dfrac{6}{11N_C - 2N_f}$ $\qquad C_F = \dfrac{4}{3}$, $N_C = 3$

N_f = number of active flavours

$$L(Q^2, Q_0^2) = \ell n[\ell n(Q^2/\Lambda^2)/\ell n(Q_0^2/\Lambda^2)]$$

so that in this limit the gluons are independently emitted. The effect of the analogous normalisation factor $F(\beta_q)$ (see below for the definition of β_q) in QCD on the Q^2 evolution of moments of the F_3 structure function has been investigated in Ref. [6].

The QCD formula directly comparable to (4), (8) is (including also the normalisation factor $F(\beta_q)$) [1] :

$$\Sigma(k_0) = F(\beta_q) \exp\left(\frac{3\beta_q}{4}\right)\left(\frac{k_0}{\hat{E}}\right)^{\beta_q} \sigma_0 \tag{11}$$

where :

$$\sigma_0 = \frac{4\pi^2\alpha}{Q^2} e_q^2 x \, [q(x, Q_0^2) + \bar{q}(x, Q_0^2)] \tag{12}$$

and

$$\beta_q = 2C_F b L(Q^2, Q_0^2) \tag{13}$$

(b, L are defined after (10) above).

The double log factor in β_q has its origin in the running coupling constant at scale t_i in the angular integrals occurring in the perturbation series in α_s for $q(x, Q^2)$ [1].

The above expression for β_q agrees with that of Ref. [6]. However the authors of that paper did not consider explicitly jet production cross-sections or radiative corrections to them.

Equation (11) gives the well known [7] analytical formula for the quark density near $x = 1$ on making the replacement :

$$\left.\begin{aligned} &\frac{k_0}{\hat{E}} = 1 - \hat{x} \Rightarrow 1 - x \\ &\hat{x} = Q^2/(Q^2 + \hat{s}) \quad , \quad x = Q^2/(Q^2 + s) \end{aligned}\right\} \tag{14}$$

Differentiating (11) w.r.t. k_0 leads to the differential cross-section for soft gluon energy k in association with hard "current quark" and "spectator diquark" jets :

$$\frac{d\sigma}{dk} = \frac{\beta_q F(\beta_q)}{k} \exp\left(\frac{3\beta_q}{4}\right)\left(\frac{k}{\hat{E}}\right)^{\beta_q} \sigma_0 \tag{15}$$

Comparing (5), (13) it can be seen that the QCD β function differs (apart from a colour factor) from the QED one by the replacement of a log factor by a loglog one. In the QED case the lower limit of the angular integral that gives the log factor is determined by the electron mass. The same will be true for the loglog factor of QCD for the case of heavy quarks. For light quarks (u, d, s) as considered in Ref. [1], the lower limit is determined by the physical cut-off Q_0^2 on the quark virtuality, chosen so that only the phase space region where perturbation theory is applicable is included in the angular integral.

A general formula is now given for the QCD β function which specialises correctly to (13) for the case of the 2-jet cross-section in non singlet lepto-

production. For completeness the corresponding QED formula [2, 3, 8] is first given :

QED

$$\beta^{QED} = -\sum_{i>j} \beta_{ij}^{QED} \tag{16a}$$

$$\beta_{ij}^{QED} = \eta_i \eta_j Q_i Q_j \frac{2\alpha}{\pi} \left[\frac{\omega_{ij}}{\omega'_{ij}} \ln \frac{(\omega_{ij} + \omega'_{ij})}{m_i m_j} - 1 \right] \tag{16b}$$

$$\omega_{ij} = p_i \cdot p_j \quad , \quad \omega'_{ij} = \sqrt{\omega_{ij}^2 - m_i^2 m_j^2}$$

QCD

$$\beta^{QCD} = -\sum_{i>j} \beta_{ij}^{QCD}$$

$$\beta_{ij}^{QCD} = \eta_i \eta_j < Q_i^a Q_j^a > 2b \frac{\omega_{ij}}{\omega'_{ij}} L\ (\omega_{ij} + \omega'_{ij} \quad , m_i m_j) \tag{17a}$$

[for m_i , $m_j \gg \Lambda$]

$$\beta_{ij}^{QCD} = \eta_i \eta_j < Q_i^a Q_j^a > 2b\ L(2\omega_{ij}\ ,\ Q_0^2) \tag{17b}$$

[for i, j = light quarks or gluons]

Q_i (Q_i^a) are electric (colour) charges and $\eta_i = +1$ (-1) for an outgoing (incoming) particle with 4-vector p_i in the general hard scattering process :

$$l + m \rightarrow p + q + \ldots$$

Each pair (ij) chosen among all the incoming and outgoing particles l, m, p, q, ... contributes a term to the β function. The general inclusive radiatively corrected cross-section has exactly the same form as (8) or (11) :

$$\Sigma(k_0) = F(\beta)\ \exp\ (v) \left[\frac{k_0}{\hat{E}}\right]^{\beta} \sigma_0 \tag{18}$$

$$\beta = \beta^{QED}, \quad \beta^{QCD}; \qquad v = v^{QED}, \quad v^{QCD}$$

Here σ_0 is the uncorrected hard scattering cross-section, $\hat{E}$ is the energy of an incoming parton in the hard scattering C.M. system and k_0 is the maximum soft gluon energy in this system. The parameter v is determined by the LL part of the $O(\alpha)$ or $O(\alpha_s)$ virtual correction to the hard scattering process.

The colour factors in (16) $< Q_i^a Q_j^a >$ (summed over final state and averaged over initial state colour indices) may be read off directly from the $|\text{matrix element}|^2$ of the process :

$$l + m \rightarrow p + q + \ldots + g_s$$

where g_s is an additional soft gluon. For the case of 3-jet production in collisions of a real or virtual photon with a nucleon the relevant formulae are given in Ref. [9]. Representative Feynman diagrams for the hard scattering processes contributing to the non singlet (singlet) F_2 structure function in virtual photon nucleon collisions are shown in Fig. 1a (1b).

Because it is usually not possible to distinguish q, $\bar{q}$, g jets only the C.M. scattering angle (modulo π) and the total effective mass $\left(\sqrt{\hat{s}}\right)$ of the high p_T jets can be measured. The observable differential jet cross-section is then symmetrical for the exchange $\cos\hat{\theta} \leftrightarrow \cos(\pi - \hat{\theta})$ or equivalently $\hat{t} \leftrightarrow \hat{u}$ where (Fig. 1) :

$$Q^2 = -q^2 \qquad \hat{s} = (p_0 + q)^2$$

$$\hat{t} = -(q - p')^2 \qquad \hat{u} = -(p_0 - p')^2$$

$$s = (P + q)^2$$

The radiatively corrected 3 jet cross-section for the QCD Compton scattering process (Fig. 1a) is :

$$\frac{d\Sigma_{NS}}{d\hat{s}\,d|\cos\hat{\theta}|} = \frac{x}{2\hat{x}} \sum_q q_0 \left(\frac{x}{\hat{x}}\right) \left[S_{NS}\,(\hat{s}, \hat{t}) \frac{d\sigma}{d\hat{t}} (\hat{s}, \hat{t})_{NS} + \hat{t} \rightarrow \hat{u} \right]$$

$$q_0(z) = q(z, Q_0^2) \tag{19}$$

where [10] :
$$\frac{d\sigma}{d\hat{t}}(\hat{s},\ \hat{t})_{NS} = \frac{d\sigma}{d\hat{t}}(\gamma^* q \to gq)$$

$$= \frac{8\pi\alpha\alpha_s}{3\hat{s}} e_q^2 \left\{ \frac{1+\hat{x}^2}{\hat{t}} + \frac{\hat{x}}{Q^2}\left[8\hat{x}(1-\hat{x}) - 2\hat{x}^2\right] + \hat{t}\,\frac{\hat{x}^2}{Q^4}\left[\hat{x}^2 + (1-\hat{x})^2 - 8\hat{x}(1-\hat{x})\right]\right\} \tag{20}$$

$$S_{NS} = F(\beta_{NS})\ \exp\ (v_{NS})\ \left(\frac{k_0}{\hat{E}}\right)^{\beta_{NS}} \tag{21}$$

$$\beta_{NS} = 2b\left\{\left(C_F - \frac{1}{2}N_c\right) L(\hat{u},\ Q_0^2) + \frac{N_c}{2}\left[L(\hat{s},\ Q_0^2) + L(\hat{t},\ Q_0^2)\right]\right\} \tag{22}$$

$$v_{NS} = 2b\left[\frac{C_F}{2} + N_c + C_F\ \ell n\,\frac{\hat{t}}{\hat{s}} - N_c\ \ell n\,\frac{\hat{t}}{\hat{u}}\right] L(\hat{t},\ Q_0^2) \tag{23}$$

Similarly, for the photon gluon fusion process of Fig. 1b :

$$\frac{d\Sigma_S}{d\hat{s}\,d|\cos\hat{\theta}|} = \frac{x}{\hat{x}}\sum_q G_0\left(\frac{x}{\hat{x}}\right) S_S(\hat{s},\ \hat{t})\ \frac{d\sigma}{d\hat{t}}(\hat{s},\ \hat{t})_S \tag{24}$$

$$G_0(z) = G(z,\ Q_0^2) = \text{gluon density}$$

and [10] :
$$\frac{d\sigma}{d\hat{t}}(\hat{s},\ \hat{t})_S = \frac{d\sigma}{d\hat{t}}(\gamma^* g \to q\bar{q})$$

$$= \frac{\pi\alpha\alpha_s}{(\hat{s}+Q^2)^2} e_q^2 \left\{\left[\hat{x}^2 + (1-\hat{x})^2\right]\left[\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right] + 8\hat{x}(1-\hat{x})\right\} \tag{25}$$

$$S_S = F(\beta_S)\ \exp\ (v_S) \left(\frac{k_0}{\hat{E}}\right)^{\beta_S} \tag{26}$$

$$\beta_S = 2b \left\{ \left(C_F - \frac{1}{2} N_c\right) L\left(\hat{s},\ Q_0^2\right) + \frac{N_c}{2} \left[L\left(\hat{t},\ Q_0^2\right) + L\left(\hat{u},\ Q_0^2\right)\right]\right\} \tag{27}$$

$$v_S = b \left[\frac{C_F}{2} + N_c \left(1 + \frac{1}{2}\ \ell n\ \frac{\hat{t}}{\hat{s}} + \frac{1}{2}\ \ell n\ \frac{\hat{u}}{\hat{s}}\right)\right] \left[L\left(\hat{t},\ Q_0^2\right) + L\left(\hat{u},\ Q_0^2\right)\right] \tag{28}$$

Equations (24–27) are symmetric under the exchange $\hat{t} \leftrightarrow \hat{u}$ (corresponding to the exchange $q \leftrightarrow \bar{q}$ in the final state), so that (23) does not require to be symmetrised in $\hat{t}$ and $\hat{u}$. Equations (19), (24) are valid for both real and virtual photon nucleon collisions, while (20), [(25)] apply respectively to the interaction, [production] of light quarks with $m_q^2 \ll \hat{s},\ \hat{t},\ \hat{u}$. The fluxes of longitudinal and transverse photons are assumed to be equal, a good approximation except when s is near to the upper kinematical limit in the electroproduction process.

Differentiating (19) or (24) w.r.t. k_0 gives the differential cross-section for two high p_T jets and the target spectator jet in association with a total energy between k and k + dk in soft gluons.

Information on the angular distribution of the soft gluon energy can be obtained by further differentiation, since β is derived by angular integration over the direction of the radiated energy [2, 3]. The β functions (22), (27) correspond to the case where soft gluons of total energy $> k_0$ are observable over the whole angular region, i.e. they correspond to a "4π" detector. In any practical experimental situation certain angular regions (typically near the beam directions in a collider experiment) are unobserved, so that in these regions no restriction on the radiated gluon energy is possible. This is taken into account by suitably modifying the angular integrations when calculating the β function. This problem is discussed in considerable detail for QED radiative corrections in Ref. [2].

Fig. 2 shows the different contributions to the overall radiative Sudakov factor $S(\beta, v)$ as a function of $\sqrt{\hat{s}}$ for jet production at $\hat{\theta} = \frac{\pi}{2}$ in real photon nucleon collisions where $Q^2 = 0$, $2\hat{s} = \hat{u} = \hat{t}$. A fixed value of 0.1 is taken for $k_0/\hat{E}$. Other parameters are : $Q_0^2 = 2(\mathrm{GeV}/c)^2$, $\Lambda = 200$ MeV/c. Real gluon radiation (the factors $F(\beta)$, $(0.1)^\beta$) reduce the cross-section as $\hat{s}$ increases, whereas the virtual gluon effect (exp(v)) increases it.

The cross-section for fixed $k_0/\hat{E}$ (corresponding roughly to events where the 2 high p_T jets are defined by a fixed parton thrust cut) drops quite rapidly with increasing $\hat{s}$, – by a factor $\simeq 8$ between $\sqrt{\hat{s}} = 10$ and 200 GeV/c^2.

As discussed in Ref. [1] the quark and gluon densities in the cross-section formulae are defined at the scale $Q_0^2 \simeq 1(\mathrm{GeV}/c)^2$ where perturbative QCD corrections are expected to be small. One should not however expect q_0, G_0 to show naive Bjorken scaling near Q_0^2, as in the parton model, because "higher twist" terms of diverse physical origins may be important.

The argument of α_s is assumed throughout to be given by the virtuality (i.e. $|\mathrm{mass}|^2$) of the off shell partons in the dominant Feynman diagrams. This choice [11] may be justified by the consistency found in the diagrammatic calculations with the leading order Operator Product Expansion Renormalisation Group Equation results both for the nucleon [1] and the photon [12] structure functions. The factorisation scale [13, 14] is Q_0^2, a physical cut-off parameter, which has a close correspondence to similar cut-offs in parton cascade models [15].

The cross-section formulae presented here therefore certain no arbitrary "unphysical" parameters. This may be contrasted with a recent analysis [9] of inclusive high p_T hadron production in real photon hadron collisions, where the complete $O(\alpha_s)$ corrections to the lowest order hard scattering cross-sections ((20), (25) above) are calculated. In Ref. [9] the factorisation scale and the scale of α_s are considered to be "unphysical" (arbitrary) and are determined by optimisation procedures [16, 17, 18] which are effectively ansätze to estimate uncalculated higher order terms in the perturbation series. In the work presented

here the jet cross-sections (radiatively corrected to all orders in α_s) are observable quantities and therefore renormalisation group invariants. Optimisation procedures related to a truncated perturbation series as used in Ref. [9], are therefore not relevant. On the other hand non leading log $O(\alpha_s)$ correction terms to the hard scattering cross-sections (20), (25), as calculated in Ref. [9], are not taken into account in the present paper.

I should like to thank J. Haissinski for introducing me to the work described in Ref. [2, 3] and G. Altarelli and R. Gatto for helpful remarks on a draft version of this paper.

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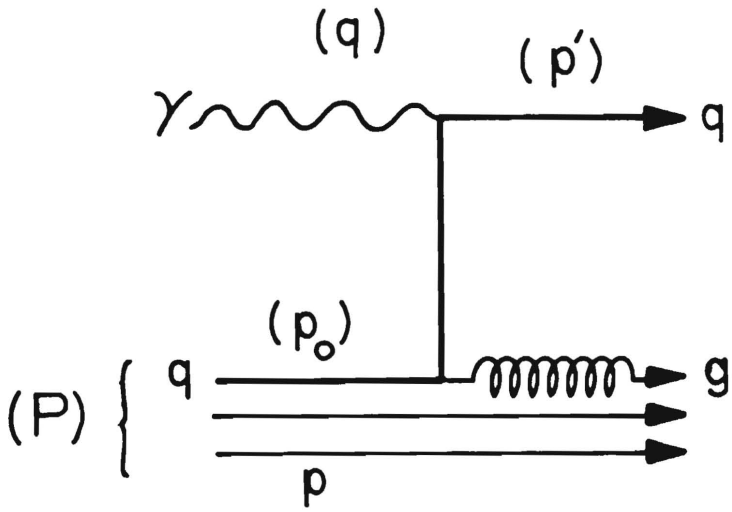
Figure Captions

Fig. 1. Typical Feynman diagrams of the lowest order hard scattering processes contributing to jet production in photon–proton collisions.

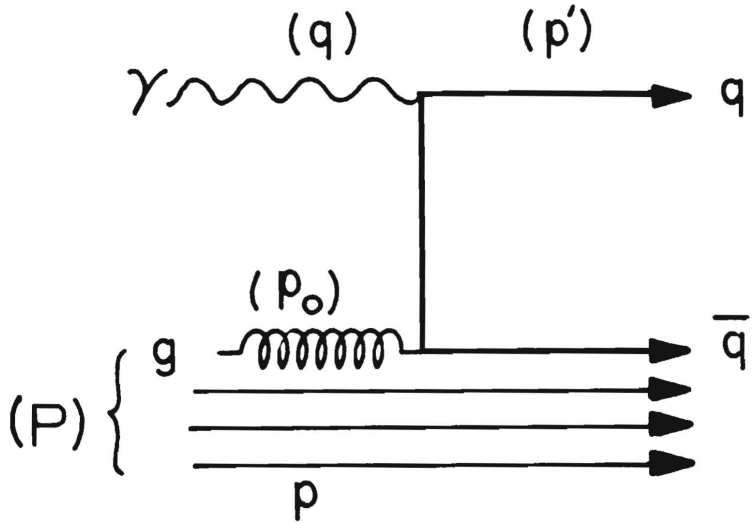
a) QCD Compton scattering, b) photon–gluon fusion. 4–vector definitions are bracketed.

Figure 2. Different factors in the Sudakov functions $S(\beta, v)$ for :

(i) QCD Compton scattering, Eqn. (21), solid lines; (ii) photon–gluon fusion, Eqn. (26), dashed lines. See text for the kinematical configuration and parameter values.



a)



b)

Figure 1

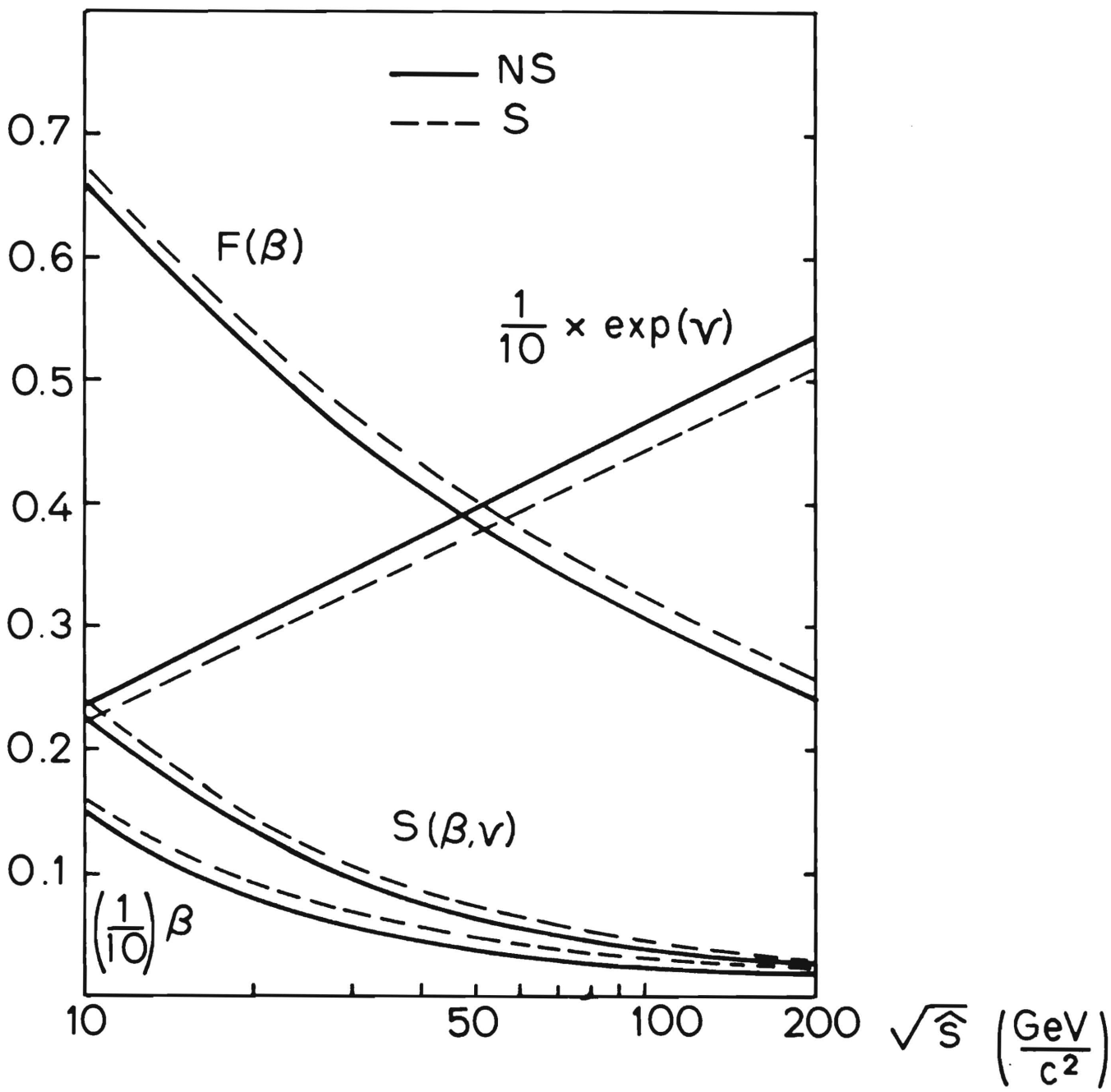


Figure 2