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HIGHER ORDER CORRECTIONS TO THE WESS-ZUMINO TERM IN CHIRAL PERTURBATION THEORY [1]

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Abstract

We calculate all the terms needed to cancel the divergences appearing in one-loop calculations involving the Wess-Zumino term. We apply our result to the $P\gamma\gamma$ and $PPP\gamma$ vertices, fixing the finite part of the coefficients of the $O(p^6)$ terms in the lagrangian with the assumption of their saturation by the vector meson contribution. In all the cases the corrections improve the agreement between the theoretical predictions and experimental data.

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The low energy strong and electroweak interactions of the lightest pseudoscalar mesons are conveniently described by Chiral Perturbation Theory [1]. In the limit where the quark masses are set to zero the QCD lagrangian has an $SU(3)_L \times SU(3)_R$ chiral symmetry which is believed to be spontaneously broken into its vector subgroup, $SU(3)_V$. In this spontaneous symmetry breaking process 8 goldstone bosons appear and become the relevant degrees of freedom. The chiral lagrangian is written in terms of the matrix

$$\Sigma = exp\frac{2i}{f}M \quad , \quad M = \begin{pmatrix} \frac{\pi^0}{2}+\frac{\eta}{6} & \pi^+ & K^+ \\ \pi & -\frac{\pi^0}{2}+\frac{\eta}{6} & K^0 \\ K & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} , \tag{1}$$

containing the octet of pseudoscalar mesons, and can be expanded in terms of external momenta and quark masses, which are assumed to be small compared to the chiral symmetry breaking scale. The lowest order lagrangian is $O(p^2)$:

$$L_2 = \frac{f^2}{8}tr(D_\mu\Sigma D^\mu\Sigma^y + \chi\Sigma^y + \chi\Sigma), \tag{2}$$

where $f = f_\pi = 132\ MeV$ and χ contains external scalar and pseudoscalar fields $\chi = B(s - ip)$. The covariant derivative, $D_\mu\Sigma = \partial_\mu\Sigma + iL_\mu\Sigma - i\Sigma R_\mu$, contains external vector fields. Electromagnetic interactions can be introduced with the identification $L_\mu = R_\mu = eQA_\mu$, where A_μ is the photon field and $Q = diag(2/3, -1/3, -1/3)$ is the quark charge matrix. The quark mass matrix, $\mathcal{M} = diag(m_u, m_d, m_s)$ is included in the external scalar field, $s = \mathcal{M} + \cdots$. This allows to fix the constant B in terms of the meson and quark masses.

Loop diagrams contribute at higher order in the expansion according to

$$D = 2 + \sum_d N_d(d-2) + 2N_L, \tag{3}$$

where p^D is the order of the loop diagram, N_d is the number of vertices of $O(p^d)$ contained in the diagram and N_L is the number of loops [2]. The free coefficients of the $O(p^D)$ terms in the lagrangian can be used to absorb the divergences appearing in the $O(p^D)$ loop diagrams. Gasser and Leutwyler found all the $O(p^4)$ counterterms needed to cancel the divergences appearing in one-loop diagrams involving vertices from (2) and constructed the most general lagrangian at $O(p^4)$ [1].

The Wess-Zumino term, which is $O(p^4)$, has to be introduced in order to account for anomalous processes [3,4]. The full action at lowest order in non-anomalous and anomalous processes is given by:

$$S = \int d^4x L_2 - N_c S_{WZ}, \tag{4}$$

where $N_c = 3$ is the number of colors and

$$S_{WZ} = \frac{i}{240\pi^2}\int d\Sigma^{ijklm} tr(\Sigma_i^L\Sigma_j^L\Sigma_k^L\Sigma_l^L\Sigma_m^L) + \frac{i}{48\pi^2}\int d^4x\epsilon^{\mu\nu\alpha\beta}Z_{\mu\nu\alpha\beta}, \qquad (5)$$

where $\Sigma_i^L = \Sigma\partial_i\Sigma^y$. The full expression for $Z_{\mu\nu\alpha\beta}$ can be found in [5]. Witten has shown that the coefficient multiplying the Wess-Zumino action is quantized, due to the impossibility of writing the first term in Eq. (5) in a compact form in four dimensions, together with the topological properties of the $SU(3)$ manifold [4].

According to Eq. (3) loop diagrams involving one vertex from the Wess-Zumino lagrangian and one from L_2 are $O(p^6)$. The divergent terms appearing in these one-loop calculation can be found expanding the matrix Σ around its classical value and using the heat kernel expansion and dimensional regularization. After a tedious calculation one obtains [5]:

$$\begin{aligned}
Z^{WZ}_{oneloop} = & -\frac{1}{16\pi^2(d4)}\Big(\frac{N_cN_f}{72\pi^2f^2}\epsilon^{\mu\nu\alpha\beta}tr\Big(\Big[D^\lambda L_{\lambda\mu} + \Sigma D^\lambda R_{\lambda\mu}\Sigma^y \\
& + \tfrac{1}{2}[\mathcal{L}^\lambda, L_{\lambda\mu} - \tilde{R}_{\lambda\mu}] + \tfrac{i}{4}[\mathcal{L}^\lambda, \Sigma D_\lambda D_\mu\Sigma^y - D_\lambda D_\mu\Sigma\Sigma^y] \\
& - \tfrac{i}{4}[\mathcal{L}_\mu, \Sigma D^2\Sigma^y - D^2\Sigma\Sigma^y]\Big] \\
& \times[\mathcal{L}_\nu\mathcal{L}_\alpha\mathcal{L}_\beta - \tfrac{i}{2}\mathcal{L}_\nu(L_{\alpha\beta} + \tilde{R}_{\alpha\beta}) - \tfrac{i}{2}(L_{\nu\alpha} + \tilde{R}_{\nu\alpha})\mathcal{L}_\beta]\Big) \\
& - \frac{iN_c}{48\pi^2f^2}\epsilon^{\mu\nu\alpha\beta}\Big(tr\Big(\Big[\frac{N_f^2}{4N_f}(\chi\Sigma^y + \Sigma\chi) - \frac{N_f}{4}\mathcal{L}_\lambda\mathcal{L}^\lambda\Big] \qquad (6) \\
& \times\Big[\tilde{R}_{\mu\nu}L_{\alpha\beta} - L_{\mu\nu}\tilde{R}_{\alpha\beta} + i(\tilde{R}_{\mu\nu} - L_{\mu\nu})\mathcal{L}_\alpha\mathcal{L}_\beta - i\mathcal{L}_\mu\mathcal{L}_\nu(\tilde{R}_{\alpha\beta} - L_{\alpha\beta})\Big]\Big) \\
& - \frac{i}{2N_f}tr((\chi\Sigma^y + \Sigma\chi)[\mathcal{L}_\mu\mathcal{L}_\nu(\tilde{R}_{\alpha\beta} - L_{\alpha\beta}) - (\tilde{R}_{\mu\nu} - L_{\mu\nu})\mathcal{L}_\alpha\mathcal{L}_\beta]) \\
& + \tfrac{i}{2}tr(\mathcal{L}_\nu(\tilde{R}_{\alpha\beta} - L_{\alpha\beta}))tr(\mathcal{L}_\mu(\Sigma\chi + \chi\Sigma^y - \mathcal{L}_\lambda\mathcal{L}^\lambda)) \\
& + \tfrac{i}{2}tr(\mathcal{L}_\lambda\mathcal{L}_\mu)tr(\mathcal{L}^\lambda\mathcal{L}_\nu(\tilde{R}_{\alpha\beta} - L_{\alpha\beta}) + \mathcal{L}^\lambda(\tilde{R}_{\nu\alpha} - L_{\nu\alpha})\mathcal{L}_\beta))\Big),
\end{aligned}$$

where $\mathcal{L}_\mu = \Sigma D_\mu\Sigma^y$, $V_{\mu\nu} = \partial_\mu V_\nu + \partial_\nu V_\mu + i[V_\mu, V_\nu]$ for $V_\mu = L_\mu$, R_μ and $\tilde{R}_{\mu\nu} = \Sigma R_{\mu\nu}\Sigma^y$. Notice that all the terms in Eq. (6) are $O(p^6)$, so the coefficient of the Wess-Zumino term is not renormalized. This is the case because the tree level S_{WZ} already saturates the inhomogeneous part of the anomalous Ward identities, so that all the remaining contributions have to satisfy the homogeneous Ward identities.

We can now apply our result to some physical processes for which experimental data are available. In particular, we are going to discuss the vertices $P\gamma\gamma$ and $PPP\gamma$. A more detailed discussion can be found in [5,6,7,8]. We first concentrate on the decays π^0, η, $\eta \to \gamma\gamma^{()}$.

The physical η is known to be a mixture of the octet and singlet states, η_8 and η_1, with mixing angle $\theta \sim -20^0$ [9] (we will take $\theta = -19.5^0$, corresponding to $\sin\theta = -1/3$). Consequently, one cannot make any prediction for the physical η without considering the singlet part. The simplest way of introducing the η_1 is in a nonet symmetry context. This can be easily achieved enlarging the Σ matrix defined in Eq. (1) with the singlet part.

The one-loop diagrams contributing to π^0, η, $\eta \to \gamma\gamma^{()}$ are the ones shown in Fig. 1, together with the ones for wave-function and f renormalization. The result of the one-loop calculation for real photons turns out to be finite [8] in such a way that all the effects of the loops and non-anomalous L_4 terms amount to change f into f_{π,η_8,η_1}. The values of f_{η_8} and f_{η_1} can be fixed from the experimental values $f_K/f_\pi = 1.22$ and $\rho_\eta = 3m_\pi^3\Gamma(\eta \to \gamma\gamma)/8m_\eta^3\Gamma(\pi^0 \to \gamma\gamma) = 0.62 \pm 0.05$ to be $f_{\eta_8}/f_\pi = 1.3$ and $f_{\eta_1}/f_\pi = 1.1$. In this way one obtains the prediction:

$$\rho_\eta = \frac{3m_\pi^3\Gamma(\eta \to \gamma\gamma)}{m_\eta^3\Gamma(\pi^0 \to \gamma\gamma)} = 2.6, \qquad (7)$$

in very good agreement with the experimental value: $\rho_\eta^{exp} = 2.4 \pm 0.4$.

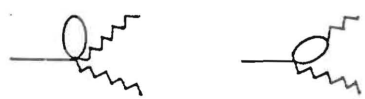

Fig. 1.- One-loop Feynman diagrams contributing to the decay $M \to \gamma\gamma$.

The situation is very different when one of the photons is allowed to be off mass shell, as it is the case in the decay $\eta \to \mu^+\mu\gamma$. In this case, the second diagram in Fig. 1 gives a contribution proportional to k^2. This contribution has some divergent terms, as well as finite ones. The divergences can be removed introducing the $O(p^6)$ terms from the lagrangian with the divergent coefficients found in Eq. (6). The finite part of the coefficients, however. are free constants in Ch.P.T.. In order to fix these constants and to be able to predict the k^2-dependence of the $\eta \to \mu^+\mu\gamma$ amplitude we have to make some extra assumption.

In a recent paper, Ecker and collaborators have found that the finite part of the coefficients of the next to leading order terms, $O(p^4)$, in the non-anomalous lagrangian are saturated by the low-lying resonance contribution at $\mu = m_\rho$ [10]. In addition, they have

found that vector mesons provide the dominant contribution. We will assume here that this is still the case for the anomalous sector, i.e. that the finite part of the coefficients of the $O(p^6)$ terms is saturated by the vector meson contribution.

We have introduced vector mesons into the chiral lagrangian using the hidden symmetry approach of Bando et al. [11] assuming ideally mixed nonet symmetry. The coefficients of the $O(p^6)$ terms in the chiral lagrangian can be related to the coefficients appearing in the lagrangian for vector mesons, once these have been integrated out. In particular, the relevant coefficients for $\eta \to \mu^+\mu\gamma$ are fixed from the ones contributing to the radiative decays of vector mesons, such as $\omega \to \pi^0\gamma$. In this way we can predict the slope parameter to be [5]:

$$\frac{1}{A(M \to \gamma\gamma)} \frac{d}{dk^2} A(M \to \gamma\gamma)|_{k^2=0} = 1.96\ GeV^2. \tag{8}$$

The experimental value for the slope parameter measured in the decay $\eta \to \mu^+\mu\gamma$ is $(1.9 \pm 0.4)\ GeV^2$ [12], in very good agreement with the just obtained theoretical prediction. Notice that, although the loop result and the $O(p^6)$ coefficients are μ-dependent, the physical result is independent of μ.

A similar situation is found when studying the $PPP\gamma$ vertices. In particular, we will discuss the processes $\gamma\pi \to \pi^0\pi$ and $\eta \to \pi^+\pi\gamma$. For the first process we have studied the quantity $F^{3\pi}$ defined through the amplitude:

$$A(\gamma\pi \to \pi^0\pi) = iF^{3\pi}\epsilon^{\mu\nu\alpha\beta}A_\mu p_1 p_2 p_0, \tag{9}$$

where $p_1(P_2)$ is the momentum of the incoming (outgoing) π and p_0 is the π^0-momentum. At lowest order, one has $F^{3\pi} = 9.4 GeV^3$, in fairly good agreement with the experimental value $F^{3\pi} = 12.9 \pm 0.9 \pm 0.5\ GeV^3$ [13]. The agreement between the lowest order prediction and the experimental value is worst for the $\eta \to \pi^+\pi\gamma$ decay width, where we have $\Gamma(\eta \to \pi^+\pi\gamma) = 35\ eV$ to be compared to $\Gamma_{exp}(\eta \to \pi^+\pi\gamma) = 53 \pm 10\ eV$ [14]. Moreover, the observed photon energy spectrum is softer than the predicted one [15].

The loop diagrams contributing to $\gamma\pi \to \pi^0\pi$ are shown in Fig. 2, while only the first three diagrams contribute to $\eta \to \pi^+\pi\gamma$. The result of the loop calculation is divergent in both cases, but the divergences can, again, be removed introducing the appropriate terms from Eq. (6). The finite part of the coefficients of the $O(p^6)$ terms in the lagrangian is again fixed from the radiative decays of vector mesons.In this way we obtain $\Gamma(\eta \to \pi^+\pi\gamma) = 47\ eV$ at next to leading order, improving the ageement with the experimenatl data. In addition, the loop diagrams and the $O(p^6)$ tree level contribution depend on the photon energy in such a way that the predicted energy spectrum is softer

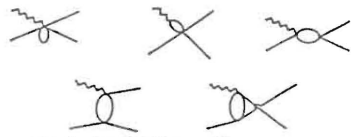

Fig. 2.- One-loop Feynman diagrams contributing to the process $\gamma\pi^0 \to \pi^-\pi^0$. Only the first three diagrams contribute to the decay $\eta \to \pi^+\pi^-\gamma$.

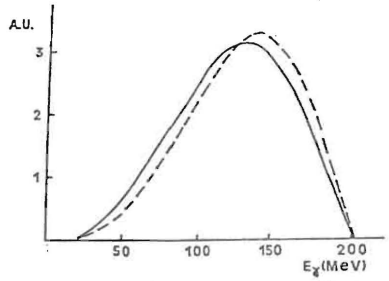

Fig. 3.- Photon energy spectrum in the decay $\eta \to \pi^+\pi^-\gamma$. The dashed line is the lowest order prediction. Both curves are normalized to the same number of events.

than the obtained at lowest order (see Fig. 3) [7]. Unfortunately, a direct comparison with the published experimental data cannot be made because they are affected by the detection efficiency [15]. The correction to $F^{3\pi}$ is complex and depends on the phase space variables. The absolute value of $F^{3\pi}$ increases with respect to the tree level prediction and the correction ranges from a 7% at $p_{12}^2 = (p_1 + p_2)^2 = 0.7m_\pi^2$ and $p_{02}^2 = (p_0 + p_2)^2 = 4m_\pi^2$ to a 12% at $p_{12}^2 = 3.5m_\pi^2$ and $p_{02}^2 = 13m_\pi^2$, corresponding to the largest values of the kinematical variables for which experimental data are available [7].

In conclusion, we have discussed the next to leading order corrections to anomalous processes. They include one-loop contributions and tree level contributions from the $O(p^6)$ lagrangian. We have found all the divergent coefficients of the $O(p^6)$ terms needed to cancel all the divergences appearing in one-loop calculations involving the Wess-Zumino term and one vertex from L_2. We have studied the vertices $P\gamma\gamma$ and $PPP\gamma$, fixing the finite part of the coefficients of the $O(p^6)$ terms through the assumption of their saturation by the vector meson contribution. In all the cases the corrections improve the agreement between the theoretical predictions and experimental data.

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