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A QUANTUM GRAVITATIONAL MECHANISM FOR EXITING INFLATION*

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ABSTRACT

I discuss the proposal by Nick Tsamis and myself that quantum gravity may mediate a naturally slow exit from inflation. The emphasis is on our recently completed two loop computation of the effective Hubble constant. When this result is properly checked and independently confirmed it will provide a proof that the mechanism functions in perturbation theory.

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1. Inflation Is Great — But How Do You Stop It?

This is a report on my work with Professor Nicholas Tsamis of the University of Crete on quantum gravity with a non-zero cosmological constant.[1–5] Because this theory differs so radically from conventional quantum gravity — in which the cosmological constant is fine tuned to zero — we distinguish it with the name, "quantum cosmological gravity," or QCG for short. The Lagrangian of QCG is:

$$\mathcal{L} = \frac{1}{16\pi G}\Big(R - 2\Lambda\Big)\sqrt{-g} + \text{counterterms} \tag{1}$$

Although the correct perturbative formulation of QCG was itself a worthy problem, our solution of which[1,2,4,5] will stand even if it lacks applications, the primary interest that QCG holds for Nick and me is the possibility that it can explain a curious hierarchy known as, "the problem of the cosmological constant."[6] To understand this let me review the parameters of QCG, their values and how they affect the universe around us.

The parameter G is Newton's constant, and it controls the strength with which gravity couples to matter. The loop counting parameter of quantum gravity (conventional or cosmological) is $\kappa^2 \equiv 16\pi G$ in the usual unit system for which $\hbar$ and c are one. Depending upon how one deploys factors of $\hbar$ and c the parameter κ can be variously regarded as a length, a time, or an inverse mass:

$$\kappa \sim 10^{-33}\text{cm} \sim 10^{-43}\text{s} \sim \Big(10^{19}\text{GeV}\Big)^{-1} \tag{2}$$

These extreme values are responsible for the ubiquitous belief that quantum gravity is negligible at any scale we can access experimentally.

The parameter Λ is called the cosmological constant, and it controls the global expansion or contraction of the universe. To be more precise, consider the invariant element, $ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu$ — the quantity which gives the physical distance between infinitesimally close coordinate points. For a homogeneous, isotropic and spatially flat universe the invariant element can be expressed in a co-moving coordinate system for which the "time" coordinate really measures physical time durations at fixed spatial position:

$$ds^2 = -dt^2 + a^2(t)d\vec{x}\cdot d\vec{x} \tag{3}$$

Classical gravity predicts that if the universe is empty and Λ is positive then the scale factor is $a(t) = e^{Ht}$, where $H^2 \equiv \frac{1}{3}\Lambda$. Even if the universe isn't initially homogeneous and isotropic, and even if it contains a finite density of normal matter, the late time behavior still tends locally to this form. This sort of cosmological evolution is known as *inflation* because of its similarity to the history of wages and prices during the last Democratic Administration.[7]

Inflation means that the physical distance between observers at rest on fixed spatial coordinates is actually increasing exponentially. Consider observers A, B,

C, and so on, all initially at rest with respect to each other and strung out at equal coordinate separations along the x axis. If A sees his physical distance from B grow, and B sees *his* physical distance from A and C grow, then A must obviously see his physical distance from C grow even faster. The effect builds up so rapidly that it isn't possible even for light waves to cross the gap between observers after the physical distance separating them reaches the critical value of H^{-1}. This separation is known as the *Hubble radius*, and causality obviously prevents any physical effect from propagating beyond it.

Inflation is very unlike what we observe today. In fact one can derive an impressive bound on H by simply noting that humans are capable of seeing their feet. Observations of the wider universe improve this by many orders of magnitude:[8]

$$H_{\rm today} \lesssim 10^{-60}\kappa^{-1} \tag{4}$$

The problem of the cosmological constant is why the dimensionless product of the two gravitational parameters should be so small. There is no larger hierarchy in fundamental physics. By comparison, the discrepancy between the electroweak scale and the conjectured scale of grand unification is less than one fourth as large.

The current smallness of H is even harder to understand if one accepts the view that the very early universe actually underwent a period of inflation somewhere between the electroweak and the GUT scales:*

$$10^{-34}\kappa^{-1} \lesssim H_{\rm inflation} \lesssim 10^{-6}\kappa^{-1} \tag{5}$$

The best evidence for this is the observed homogeneity and isotropy of today's universe. The cosmic microwave background radiation from different parts of the sky is seen to have the same temperature to within about one part in a hundred thousand.[9] Without inflation one must accept the coincidental establishment of regions of nearly identical thermal equilibria which are only now coming into causal contact with one another. Inflation evades this problem by blowing the observed universe up from a single region so tiny that causal processes could have established an initial thermal equilibrium across it. Inflation would also disperse unwanted relics such as primordial black holes and magnetic monopoles, and it would explain why the currently observed energy density is so near to the critical density needed for closing the universe.**

The main disadvantage of inflation is that one must somehow get the effective Hubble constant to fall from the relatively large value (5) it must have had during

* Most inflationary cosmologists prefer the larger value. The electroweak limit arises because the known physics below this scale does not violate CP strongly enough to explain the observed asymmetry between baryons and anti-baryons.

** This particular explanation may actually be too good because inflation predicts $\rho = \rho_{\rm crit}$ whereas a variety of different observations seem to give $\rho < \rho_{\rm crit}$ by one to two orders of magnitude. The matter is still hotly debated.

inflation to the extremely small value (4) it has now. Whatever mechanism is invoked to perform this task must be powerful because it has to slow the expansion rate by at least 26 — and probably by more than 54 — orders of magnitude. On the other hand, this powerful mechanism must operate on a very slow time scale because a minimum of something under 60 e-foldings of inflation is needed to explain the observed degree of isotropy in the cosmic microwave background. Finally, this powerful but very slow mechanism must not embarrass low energy phenomenology with observable but hitherto unseen light quanta.

2. Our Proposal

It is almost irresistible to suspect that the mechanism we seek is an infrared effect mediated by very weakly coupled massless quanta. The strength of this effect would derive from the coherent and causal superposition of weak interactions occurring throughout the past light cone as it extends back to the onset of inflation. The long time lag between the beginning of inflation and its effective extinction would derive from the need for the invariant volume of the past light cone to become enormously large in order to overcome the natural weakness of the interaction. Only massless quanta could mediate interactions coherently over such a large region. Since we have presumably observed the long range forces associated with all massless quanta, it is highly desirable to seek the conjectured mechanism in one of the conventional models of low energy phenomenology. Finally, the model we seek must be capable of distinguishing between flat space and a gravitational background of the form (3) with a time varying scale factor. The precise formulation of this requirement is that the desired model must violate conformal invariance.

The conditions stated above are necessary for a plausible mechanism but they are obviously not sufficient. Happily they do narrow the field to a unique contender: QCG. Proving — even at the level of perturbation theory — that QCG actually manifests the desired mechanism has consumed the better part of my research effort and Nick's over the past three years. Since I have explained our older work at some length in another conference report[10] I will confine myself here to the barest summary of previous conclusions:

(1) *QCG can be used reliably as a quantum field theory provided the phenomenological focus is on the far infrared.*[3,5] Although QCG is not perturbatively renormalizable, one can of course absorb ultraviolet divergences by adding ever higher local counterterms. This gives reliable predictions for the infrared because the physics of large spatial and temporal separations is dominated by non-local and ultraviolet finite terms which are unaffected by local counterterms. This is a standard idea behind the effective field theory of massless particles. It is why Bloch and Nordsieck were able to solve the infrared problem of QED[11] long before anyone understood renormalization. It is why Weinberg was able to do the same for conventional quantum gravity,[12] and why his result will stand no matter what the correct fundamental theory of gravity turns out to be. And it

is why the late Gerald Feinberg and our own Joe Sucher were able to use Fermi theory to compute the long range force due to neutrino exchange.[13]

(2) *QCG corrections become strong at late times in an inflating universe.*[3–5] There are two reasons for this:

a) QCG allows massless gravitons to self-interact through a coupling of dimension three; and

b) The inflationary expansion of physical distances induces a corresponding redshift in physical momenta, and so populates the infrared ever more heavily.

Physicists are very familiar with the notion of the infrared being dominated by massless particles with self-interactions of the lowest possible dimension. One might expect the massless gravitons of QCG to be even more potent in the infrared than the massless gluons of QCD with their dimension four self-interactions. In fact we see a secular effect at late times rather than a static long range force because infrared phenomena in QCG are so powerful that they engender a breakdown of the background. Note as well that even though the momentum redshift is diluted by the decoupling of wavelengths as they pass the Hubble radius, there is still a powerful secular effect due to the ever increasing number of modes whose physical wavelengths are *just reaching* the Hubble radius.

(3) *QCG dominates the physics of late time inflation with respect to any matter theory whose phenomenological viability does not require fine tuning.*[3,5] This is because all the other known massless particles are conformally invariant on the classical level. They therefore cannot distinguish locally between de Sitter space and flat space, and their self-interactions have dimension four.

(4) *If one assumes that the natural vacuum of QCG suffers only perturbatively small corrections then asymptotic graviton scattering amplitudes are infrared divergent even at tree order.*[4] This is a trivial consequence of the fact that QCG vertices grow at late times in a way that is not compensated by either external wavefunctions or propagators. This result is sometimes put down as an obvious error owing to the curious fact that the rate of infrared *divergence* in the cutoff time happens to be much stronger than the rate at which the expectation value of the background can possibly *grow* with causal time evolution. Note first that there is nothing inconsistent about this provided both effects become non-perturbatively large at late times. Note as well that although expectation values must be used to follow time evolution, in-out amplitudes provide a perfectly good check on the consistency of any guess one may care to make for this time evolution. For example, our result debunks the oft-made suggestion that the background suffers only perturbatively small corrections at late times.

(5) *Modulo some tensor algebra, QCG corrections act to slow the inflationary expansion rate by an amount which becomes non-perturbatively large at late times.*[5]

3. Our Two Loop Computation

This summer's work has been devoted to removing the qualification, "modulo some tensor algebra," from the fifth conclusion stated above. Starting from a homogeneous and isotropic free vacuum on $T^3 \times R$, we follow the evolution of the expectation value of the invariant element in co-moving coordinates:

$$\Big\langle 0 \Big| \, g_{\mu\nu}(t,\vec{x}) \, dx^\mu dx^\nu \, \Big| 0 \Big\rangle = -dt^2 + a^2(t) \, d\vec{x} \cdot d\vec{x} \tag{6}$$

Our choice of state and our organization of perturbation theory enforces the initial condition $a(0) = 1$; we impose $\dot{a}(0) = H$ as a renormalization condition. The subsequent time evolution of $a(t)$ is determined by the dynamics of QCG. From this time evolution we define the (generally coordinate invariant) effective Hubble constant as:

$$H_{\rm eff}(t) \equiv \frac{d \ln(a)}{dt} \tag{7}$$

I am happy to report the following result — so new that it has not yet appeared elsewhere[14] — from a two loop calculation that consumed most of this summer:

$$H_{\rm eff}(t) = H \Bigg\{ 1 - 2 \Big(\frac{\kappa H}{\pi} \Big)^4 \Big[(Ht)^3 + \text{subdominant} \Big] + O\Big(\kappa^6 H^6 \Big) \Bigg\} \tag{8}$$

What we actually computed was the amputated 1-point function for a conformally rescaled version of the graviton field in conformal coordinates. We then attached the external propagator, undid the conformal rescaling, and transformed the result back to co-moving coordinates. The calculation was done using a modified version of the diagrammatic formalism developed by Schwinger for computing expectation values[15]. This technique is manifestly causal in that the only net contributions come from interaction vertices which reside on or within the past light cone of the observer who measures the amputated field. Our modification of Schwinger's formalism is infrared finite because the observer's past light cone has a finite invariant volume, although this invariant volume grows as we observe at later and later times. This growth, and the fact that graviton propagators do not suppress large loops, is the origin of the secular effect so prominent in our result (8).

The ultraviolet regularization we used corresponds to an exponential cutoff upon coordinate momentum modes. Our interaction vertices[1] and propagators[4,5] have been reported elsewhere. The tensor algebra was done using the program FeynCalc[16] within Mathematica on a SUN SPARC station with 8MB of random access memory. The various scalar loop integrals were performed analytically up to a single conformal time integration and then subjected to an asymptotic expansion. The leading contributions were summed using a computer.

Two of the three powers of Ht in the result (8) derive from the fact that at two loops one can have two interaction vertices integrated over the observer's past light cone, the invariant volume of which grows linearly with his co-moving time.

The final power of Ht has its physical origin in the increasing vacuum correlation imposed by inflation and evident in the so-called, "logarithm piece" of the bare propagator.[4,5] This enhancement enters (8) through whichever of the propagators happens to connect the earlier of the two interaction vertices to the observation point.

I should conclude this section on a note of caution. This is only the second two loop calculation ever done in quantum gravity, and it is the first one for which the ultraviolet finite part of anything was computed. Such a difficult and complicated piece of work will not be completely accepted — and should not be — until it has been subjected to the most rigorous possible checks, and has also been independently confirmed. Although we checked many things we have not yet had the time — nor will we for many months — to verify that the one loop position space self energy obeys the appropriate BRS Ward identity and that its flat space limit agrees with the momentum space result obtained by Capper.[17]

4. The Physics Behind Our Result

Counting the coupling constants at ℓ loops is simple enough, but it is not so easy to infer the powers of Ht — as witness the fact that we have gotten it wrong twice already![3,5] At $\ell = 3$ there will be up to four free interaction vertices, which should each contribute a factor of Ht as they explore the order Ht invariant volume of the observer's past light cone almost without suppression. It is possible to show as well that the seven propagators can contribute up to two undifferentiated logarithms, each of which should give another factor of Ht. This makes the leading three loop term of order $(H\kappa)^6 \times (Ht)^6$. We therefore conclude that perturbation theory must break down long before the secular dependence at two loops can overwhelm the naturally small prefactor of $(H\kappa)^4 \lesssim 10^{-24}$. In fact the two loop term would go non-perturbative at $Ht \gtrsim 10^8$, whereas the three loop term becomes large when $Ht \gtrsim 10^6$. Our best guess for the leading term at ℓ loops is:

$$\Big(\kappa H\Big)^{2\ell}\Big(Ht\Big)^{3\ell-3} \tag{9}$$

This implies that perturbation theory breaks down after:

$$N = Ht \gtrsim 10^4 \tag{10}$$

inflationary e-foldings. Of course this would also be a rough estimate for the lifetime of inflation.

A potentially troublesome point in this analysis is that perturbation theory breaks down when all of the leading terms are still quite small:

$$\Big(\kappa H\Big)^{2\ell}\Big(Ht\Big)^{3\ell-3} \longrightarrow (\kappa H)^2 \lesssim 10^{-6} \tag{11}$$

We therefore require some evidence that the contribution at each order is really such as to decrease the expansion rate. The desired argument is supplied by Feynman's tree theorem,[18] a procedure which serves to decompose loops of locally interacting quanta into sums of tree diagrams. When applied to any of the loops which go to make up the amputated 1-point function we obtain a sum of terms whose physical interpretation is the effect on the background of various amounts of gravitational radiation. In fact, the dominant diagrams at ℓ loops decompose to give ℓ-graviton trees.

To understand the significance of this representation first consider the effect of having an initial graviton of wave length λ. At first there is little because the spatial fluctuation of the graviton couples weakly to the zero mode background. However, it is the physical wavelength that matters and this gets expanded by a factor of $a(t)$ as the universe evolves. Eventually the graviton is so spread out that it has a strong coupling to the background. Except, of course that causality prevents it from doing much for times after $a(t)\lambda > H^{-1}$. The strongest effect is therefore at that time when the physical wave length is comparable to the Hubble radius. Note that this effect is to *resist* the inflationary expansion by virtue of the gravitational self-attraction of the wave. The sign is unambiguous because gravitation is universally attractive.

Now consider the classical effect of having some initial radiation in an inflating universe. If the initial geometry is non-singular then there must be a rapid fall-off in the short wavelength modes. Suppose the characteristic wave length of the cutoff is λ. What we see is some effect on the background until t such that $a(t)\lambda \gtrsim H^{-1}$, after which there will be nothing much. At late times the universe inflates without disturbance because the maximum wave length has been expanded past the causal horizon.

The crucial difference between the classical process we have just considered and the representation given by the tree theorem is that *the latter involves sums over gravitons whose initial spatial momenta are arbitrarily large.* The quantum effect never cuts off because we never run out of wave lengths which are just redshifting to the Hubble radius. In fact the number of these *increases* with time as $a^2(t)$ by a simple phase space effect.

Note that this argument does more than we bargained for. Not only do we see that each order acts to slow inflation — because gravitation is universally attractive — we can also understand why relaxation is an inherently quantum mechanism. One finds no large classical effect because all well defined initial configurations must be cut off at small wave lengths, and when the smallest wave length has expanded beyond the Hubble radius there can be no more effect by causality. Quantum gravity is just the opposite because the uncertainty principle *forbids* us from constraining both the initial value and the initial time derivative of each mode to vanish. This is what endows quantum cosmological gravity with the unending supply of ultraviolet modes which the locally de Sitter background eventually redshifts to the infrared.

5. The Implications Of Our Mechanism

If Nick and I are right then the bare cosmological constant is not unreasonably small. Further, there is no need for fine tuning of scalar potentials. Our view is that inflation happened in the early universe for no other reason than that the bare cosmological constant is large and positive. The current observed universe derived from a patch of about one Hubble volume which began inflation when its local temperature dropped below the scale $M = \sqrt{H/\kappa}$.

The reason our universe is not observed to be inflating today — at least not rapidly — is that infrared processes in QCG tend to screen the cosmological constant. We saw in the previous section that any vacuum loop contribution to the background can be represented as a sum of tree corrections due to the presence of some initial gravitational radiation. It is the self-gravitation of this radiation which resists inflation. This is why the contributions at all orders have the same sign. The irresistible force of the effect derives from the fact that it scales as powers of the invariant volume of the past light cone, which grows without bound as evolution continues. The slowness of the effect derives from the fact that only the causal and coherent superposition of interactions over an enormous invariant volume can overcome the natural weakness of the gravitational interaction.

All experimentally confirmed matter quanta are either massive or else conformally invariant at the classical level. This means that they give only negligibly small corrections to the classical geometry of an inflating universe. Although certain conjectured light matter quanta may be competitive with QCG for a time, in the end only the graviton is left. This means that QCG makes *unique* predictions for relaxation. Unfortunately, the most interesting predictions of QCG are not easy to obtain because perturbation theory breaks down before a significant amount of relaxation has occurred. In particular, the end of rapid inflation should tell us what reheating temperature was reached; and the last sixty e-foldings govern the magnitude and spectrum of observable density perturbations.

Finally, we would like to suggest two more points of potential experimental contact. The first arises from the fact that the universe should still be experiencing a residual expansion due to its not completely screened bare cosmological constant. It is conceivable that a modified form of perturbation theory could be used to predict this residual expansion rate. There is some evidence that a suitably time varying effective cosmological constant can explain the apparent discrepancy between the age of the oldest galaxies, as inferred by models of stellar evolution, and the age of the universe as inferred by the currently measured Hubble constant.[19]

The second point of potential experimental contact derives from electroweak and QCD chiral symmetry breaking phase transitions. If these occurred after inflation and relaxation at some high scale then they should have induced an effectively negative cosmological constant. We do not understand the process by which deflation is relaxed — or even if it is relaxed — but one would think that the time scale of this gravitational process should be longer than that of either phase transition. If

so then there would have been substantial periods of deflation, which would almost inevitably have affected low scale baryogenesis and nucleosynthesis.

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