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REGGE TRAJECTORIES IN QCD*

CHARLES B. THORN†

Department of Physics
University of Florida, Gainesville, FL 32611

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UNIVERSITY OF FLORIDA
Gainesville, Florida 32611

ABSTRACT

Weak coupling QCD predicts the large negative t behavior of Regge trajectories. We review these calculations and discuss how they square with available data. Discrepancies between calculation and experiment might be resolved if higher energy fixed target experiments can be done at the Tevatron or the SSC.

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The strong interactions have posed a challenging problem to theoretical physics for a long time. For 20 years the consensus has been that the theory which describes them is QCD: $SU(3)$ gauge fields coupled to at least 6 "flavors" of spin 1/2 quarks in the fundamental representation of the gauge group. However, testing the QCD predictions for much of the classic experimental data on the strong interactions, *e.g.* masses (or resonance energies), decay rates (or resonance widths), and cross sections at accessible energies, requires the solution of QCD in the strong coupling regime. Despite many heroic attempts at an analyical solution, none have succeeded (even for $N_c \to \infty$), and presently the only available attack on this strong coupling problem is through "brute force" lattice simulations. To date computers can only handle relatively small lattices, and so a definitive confrontation of theory with the vast majority of experimental information must await the development of more powerful computers.

In the meantime, however, physicists have exploited the fact that some experimental information depends only on short distance physics, which in QCD, by virtue of asymptotic freedom, is controlled by weak coupling. Examples of this are well-known: scaling violations in deep inelastic lepto-production, some aspects of jet physics, the high energy limit of hadron production from electron-positron annihilation, etc. It is obviously important to extend the reach of "weak-coupling" QCD as far as possible. Today, I would like to focus on aspects of the particle spectrum of QCD amenable to weak coupling approximations[⋆]. The spectrum of a theory is one of the most clearcut "fingerprints" one can imagine. It would therefore be very desirable to be able to compare the spectrum of QCD with that of nature. For the most part the strong coupling problem has frustrated such a task, but if we can isolate aspects of the spectrum describable by the weak coupling approximation, it would be a step toward the goal.

⋆ I avoid calling this perturbative QCD because bound state spectra are strictly speaking nonperturbative in nature, although there are definitely situations (*e.g.* atoms and molecules) where weak coupling approximations can describe them.

In a strict sense, the only bound states in QCD controlled by weak coupling are those containing only extremely massive quarks. Then we expect the levels to be Coulombic in character for those levels whose associated Bohr radius is both much smaller than $1/\Lambda_{QCD}$, the intrinsic length scale associated with the gluonic force, and much larger than the Compton wavelength of the massive quarks:

$$\frac{\Lambda_{QCD}}{m_q} << \alpha_s(m_q) << 1.$$

Since α_s decreases logarithmically with m_q these inequalities are achieved for large enough quark mass. Whether the quark masses Nature provides us with are large enough is of course nothing we have control over, but we can hope that m_b and m_t are sufficiently large.

But beyond this well-studied example, there is an indirect aspect of bound state spectra for which weak-coupling QCD has something definite to say, namely the so-called Regge trajectories that interpolate between bound states with varying spin. For example, the leading Regge trajectory associated with the bound state spectrum of hydrogen is given by (Recall that $E_{n\ell} = -Ry/(\ell + n)^2$)

$$\alpha_{Hyd}(E) = \ell(E) = -1 + \sqrt{\frac{Ry}{-E}}.$$

A comment is in order here: the excited levels of hydrogen are unstable resonances and the above formula uses the excellent approximation which neglects this. Similarly, if we are going to consider such interpolating trajectories in QCD, it will be clearest if we can define an approximation in which the meson resonances are stable. The stability of the hydrogen levels is exact only for vanishing fine structure constant (with $m\alpha^2$ fixed!). The corresponding stable resonance limit of QCD is t'Hooft's limit $N_c \to \infty$ with $N_c g^2$ fixed. In the real world $\alpha \approx 1/137 \approx .007$ which makes the stable hydrogen approximation very good whereas $1/N_c \approx 0.33$ which is not so promising. Nonetheless the conceptual clarity of such an approximation scheme makes it very desirable. In the large N_c limit the "QCD string" formed by the spatial separation of color cannot break so this limit can be used to *define* what we mean by the QCD string.

Technically Regge trajectories are defined as the locations of poles (or more generally any singularities) in the partial wave amplitudes $f_j(E)$ which have been continued to complex j. For example, if

$$f_j(E) \sim \frac{\beta_j(E)}{j - \alpha(E)},$$

$\alpha(E)$ is a Regge trajectory. If j_0 is a physical value of angular momentum, and $\alpha(M) = j_0$, then there is a pole in the physical partial wave amplitude $f_{j_0}(E)$ at $E = M$, *i.e.* there is a particle of mass M and spin j_0 in the theory. This explains why the Regge trajectories interpolate the mass spectrum of the theory. Of particular importance are the leading Regge trajectories (those with the largest $Re\ \alpha(E)$) in a given quantum number channel. They interpolate the particle states with maximum angular momentum for each mass. In other words they interpolate the ground states for each angular momentum and set of internal quantum numbers.

In the relativistic context of quantum field theory, it is appropriate to use center of mass energy squared in some channel rather than the energy as the variable. When considering the resonances contributing to a scattering amplitude we might pick one of the Mandelstam variables s, t, or u as this new variable. Let us consider a leading t channel trajectory $\alpha(t)$ in QCD. We certainly cannot expect to be able to use weak coupling methods to calculate $\alpha(t)$ for all t. At best we could only hope to get away with this for $|t| >> \Lambda^2_{QCD}$. However, if confinement holds and we consider the stable resonance limit, the high mass hadrons must lie on asymptotically linear trajectories, $\alpha(t) \sim \alpha' t$ as $t \to +\infty$, where $\alpha' = 1/2\pi T_0$ and T_0 is the infinite distance limit of the confining force. We surely cannot get $\alpha(t)$ for large positive t from weak coupling physics!

The opposite limit $t \to -\infty$ is an altogether different story. This is because for $t < 0$ $\alpha(t)$ can be directly measured in high energy scattering experiments. In two to two scattering processes, the Regge high energy limit is defined as $s \to +\infty$ with t fixed. This limit is controlled by the leading Regge trajectory in the t channel, and the scattering amplitude behaves in that limit as

$$A(s,t) \underset{s \to \infty}{\sim} \beta(t) s^{\alpha(t)}.$$

The double limit $s, t \to \infty$ should probe the short distance structure of the theory, and hopefully might be describable in part by weak coupling. To succeed such an approach must make use of a factorization of the hard and soft aspects of the process. If we think of the hadron as a bag of quarks, high momentum transfer processes should occur via hard quark-quark scattering. If so, soft (strong coupling) physics would be contained in wave function factors representing the probability of finding a quark in each participating hadron. These are then multiplied by the scattering amplitude for the core constituent scattering process. Perturbative QCD makes statements about the powers of s, t, and u that occur in this core process. If this reasoning is correct, the large negative t behavior of the Regge trajectories of QCD should be describable with weak coupling techniques.

This program has been pursued by Lipatov and his collaborators. I will simply quote their results for the Pomeron (glueball) trajectories supplemented by some recent results on quark-antiquark (meson) trajectories by McGuigan and me. Then I want to call attention to some puzzles arising from trying to square these results with currently available experimental information. Some of these puzzles might be resolved with new experiments at the higher energies available at the Tevatron and SSC.

<u>Lipatov's QCD Pomeron</u> This is obtained by summing leading log ($\alpha \log s = O(1)$) contributions to the gluon-gluon scattering amplitude at fixed coupling and then incorporating running coupling effects [1]. Define $\lambda(Q^2) = N_c \alpha_s(Q^2)/\pi$, then*

$$\alpha_P^a(t) \sim 1 + (4 \ln 2)\lambda(-t) + \cdots.$$

Since this calculation gives $\alpha_P > 1$, it leads to a violation of the Froissart

* When running coupling effects are neglected the singularity in the angular momentum plane is actually a fixed cut. After consistently incorporating asymptotic freedom this cut is changed to a distribution α_P^a of poles whose a dependence appears only at order $\alpha_s^{5/3}$ [2, 3].

bound and hence is not consistent with s channel unitarity. One tentative interpretation is that the calculation is good for the eikonal χ which is then inserted in the impact parameter representation

$$A(s,t) = 8\pi s \int db J_0(b\sqrt{-t}) \frac{e^{2i\chi(s,t)} - 1}{2i}.$$

Consider for example that the Pomeron contribution to the scattering amplitude (and hence also the eikonal) for color singlets is of order $1/N_c^2$. Thus the Lipatov pomeron might be a good picture of the large N_c limit of QCD, which satisfies only tree unitarity and is not subject to the Froissart bound.

<u>$q\bar{q}$ Reggeons</u> In the same spirit McGuigan and I [4] investigated the quark-antiquark trajectories in large N_c QCD. In this case the leading log sum is an expansion in powers of $N_c\alpha_s \ln^2 s$ and the departure of the trajectory from the constant value 0 is of order $\sqrt{\lambda}$:

$$\alpha^r_{q\bar{q}} \sim \lambda^{1/2}(-t) - 2\lambda^{5/6}(-t)\left[\frac{11}{32}\left(r\pi + \frac{3\pi}{4}\right)\right] + \cdots,$$

with $r = 0, 1, 2, \ldots$.

The t dependence in these formulas is probably too subtle to be resolved by experiment. The fact that the leading quark-antiquark trajectories, which include the ρ, A_2, ω, f' trajectories, are predicted to approach 0 *from above* is a striking prediction of these calculations which one would hope could be seen in the data. It is well-known from the early days of Regge phenomenology that all of these trajectories are roughly linear down to values of t where they have crossed the axis $\alpha = 0$. More recent data on the ρ trajectory measured in the reaction $\pi^- + p \to \pi^0 + X$ [5], which extend to much larger values of $-t$, are consistent with the early data but also show the trajectories flattening off at around $t = -2$ to a value between $-1/2$ and -1. Since these values are inconsistent with the QCD calculations, some discussion is needed. Of course the first possibility is that something is wrong with the idea of calculating the large $-t$ behavior of the trajectories within perturbative QCD. This seems to me, however, unlikely because this assumption seems quite analogous to the one

used to justify applying weak coupling methods to the bound state problem of heavy quarks: one can think of a Regge trajectory at large $-t$ as giving the mass of a system with very large spacelike momentum. Also in a scattering process to which these trajectories contribute, the limit $t \to -\infty$ approaches fixed angle scattering, for which perturbative QCD is generally believed to correctly give the power of the energy dependence but not, of course, the overall normalization. The lowest order diagrams for the core constituent scattering process include some that behave as s^0.

Assuming the validity of the calculations, we must attribute the discrepancy to the failure of the experiments to reach high enough values of s and/or t. The logical possibility that the leading trajectory actually dips below zero at moderate t but then increases to zero at very high $-t$ is probably nonsense physically: that would mean that at very high energies, increasing the momentum transfer $-t$ would lead to an enormous increase in the scattering cross section. I therefore think the most likely possibility is that the values of s are simply not high enough to pick out the true leading trajectory and nonleading trajectories are still very important.

In fact the authors of Ref.[5] noticed some energy dependence in their extracted ρ trajectory. It is of course necessary to use a range of energies to extract a trajectory from the data. Using data in the range $140\ GeV < E < 200\ GeV$ led to a *lower* ρ trajectory than when the data was restricted to therange $160\ GeV < E < 200\ GeV$. One can try to parametrize this situation by fitting a sum of two powers to the data. I have been attempting this with the help of Mikaelian [6]. The quality of the data don't justify trying to detect the t dependence of the QCD prediction so we assume a behavior

$$A(s,t) \sim \beta_1(t)s^0 + \beta_2(t)s^{\alpha(t)}$$

where the first term is a crude representation of the hard parton QCD prediction and the second term is meant to parametrize the soft hadronic part of the process. In effect the authors of Ref.[5] assume that $\beta_1 = 0$. Since they get a reasonable fit we can conclude that β_1 should be relatively small. A value

of $\beta_2/\beta_1 \sim 20$ gives a rough account of the energy dependence the authors of Ref.[5] found in their extracted ρ trajectory. We conclude that the data are roughly consistent with a relatively small contribution from the QCD trajectory, but this is far from definitive. Clearly better data at even higher energy could clarify the situation. For extremely high s say 4000 GeV^2 corresponding to a pion beam with energy 2 TeV the s^0 term should stand out clearly. For a fixed target experiment these energies for a secondary pion beam would require the LHC or SSC. However at Fermilab a 500 GeV pion beam is feasible. We think serious consideration should be given to performing this type of experiment along with the more fashionable colliding beam experiments.

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