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MASS AND MIXING ANGLE PATTERNS IN THE STANDARD MODEL AND ITS MINIMAL SUPERSYMMETRIC EXTENSION*

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Dedicated to John Klauder on his sixtieth birthday

ABSTRACT

Using renormalization group techniques, we examine several interesting relations among masses and mixing angles of quarks and leptons in the Standard Model of Elementary Particle Interactions as a function of scale. We extend the analysis to the minimal Supersymmetric Extension to determine its effect on these mass relations. For a heavy top quark, and minimal supersymmetry, most of these relations can be made to agree at one unification scale.

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INTRODUCTION

Most of the parameters in the Standard Model of the Strong, Weak, and Electromagnetic Interactions are to be found in the Yukawa sector of the theory where they serve to parametrize quark and lepton masses, as well as the interfamily mixings of the quarks, and CP violation. Historically, of these thirteen parameters, only one was ever predicted[1], the charmed quark mass, but only after an inspired guess on the value of a strong (*i.e.* presently uncalculable) matrix element.

Theoretical guesses on the nature of physics beyond the Standard Model have been formulated, using as inspiration the idea of Grand Unification[2] which emerged from the observed pattern of the quantum numbers of the elementary particles. When applied in conjunction with the renormalization group[3], this idea has proven extremely fruitful. Recent work indicates that the experimental values of the gauge couplings are such that all three couplings evolve to the same value[4] at shorter distances when supersymmetry is included at SSC scales. Without supersymmetry, the gauge couplings meet two at a time, forming a small GUT triangle in the plot of their evolution as a function of scale.

This encouraging situation, hinting at a Super Grand Unified Theory, should be matched by concomitant simplicity in the other parameters of the theory. To that purpose we present a comparative analysis of possible relations among Yukawa couplings at shorter distances both in the Standard Model itself and in its minimal supersymmetric extension.

In the context of the $SU(5)$ Grand Unified Theory[5], several mass relations were proposed based on simple assumptions on the possible Higgs structure.The first of these (assuming only a **5** Higgs representation) leads to the equality between the τ-lepton and b quark Yukawas or masses at the GUT scale:

$$m_b = m_\tau \; . \qquad \text{RELATION I}$$

This relation, when folded into the running of the masses with distance is not inconsistent with experiment, due to the fact that QCD provides through the anomalous dimension of the quark mass, the required factor to bring it in rough agreement at experimental scales[6]. This relation, if applied to the lighter two families, is off by a factor of ten. A new scheme was proposed[7] with a slightly more complicated Higgs structure (using a **45** representation in conjunction with the **5**). It replaces the above with the more complicated relations

$$m_d = 3m_e \ , \qquad \text{RELATION IIa}$$

$$3m_s = m_\mu \ . \qquad \text{RELATION IIb}$$

These are typical of the SU(5) types of model in which the charge -1/3 quarks and the charged leptons Yukawa couplings appear with the same quantum numbers.

The situation concerning the mixing angles is equally intriguing. It was noticed long ago that there existed a near numerical equality between the square of the tangent of the Cabibbo angle and the ratio of the down to the strange quark masses (determined from current algebra). This Oakes relation reads

$$\tan\theta_c \approx \sqrt{\frac{m_d}{m_s}} \ . \qquad \text{RELATION III}$$

It has provided the central inspiration in the search for Yukawa matrices. It was found[8] that very general classes of matrices with judiciously chosen textures (*i.e.* zeroes in the right places) reproduced this relation, at least approximately.

In a model based on $SO(10)$[9] it was found[10] that these three different relations could all be obtained at the same time, with the required texture being enforced naturally by discrete symmetries at the GUT scale. As a consequence of the model, the mixing of the third family with the two lighter ones was dictated exclusively through the Yukawa matrices of the charge 2/3 quarks. Accordingly an Oakes-like relation for the mixing of the second and third families ensued[10]

$$V_{cb} = \sqrt{\frac{m_c}{m_t}}\ ; \qquad\qquad \text{RELATION IV}$$

it relates the "23" matrix element of the CKM matrix to the ratio of the charm quark mass to the top quark mass. This relation, if true, presents us with the very exciting possibility of predicting the mass of the top quark. It is known that the top quark mass is somewhere in between 100 and 200 GeV, the lower limit being set by direct experimental searches, the upper by the radiative effect of the top quark mass on the ratio of neutral to charged current processes.

These four relations can all be satisfied if one takes the Yukawa Mixing matrices to be of the form[10], shown here in a specific gauge

$$\mathbf{Y}_u = \begin{pmatrix} 0 & A & 0 \\ A & 0 & B \\ 0 & B & C \end{pmatrix} ,$$

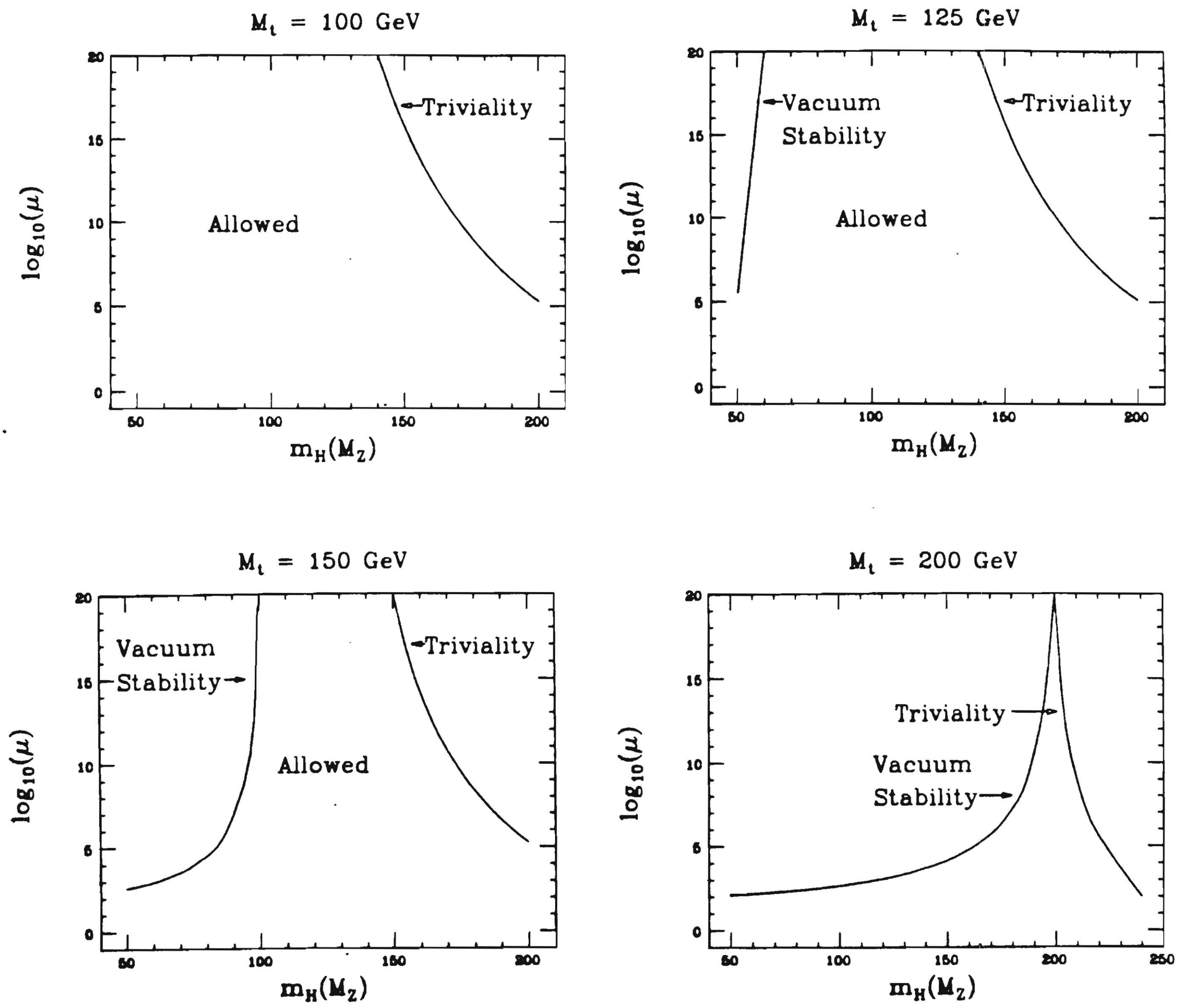
$$\mathbf{Y}_d = \begin{pmatrix} 0 & R & 0 \\ R & S & 0 \\ 0 & 0 & T \end{pmatrix} \ ; \ \mathbf{Y}_e = \begin{pmatrix} 0 & R & 0 \\ R & -3S & 0 \\ 0 & 0 & T \end{pmatrix} .$$

This form has been recently rediscovered by several groups[11,12], and some of our analysis overlaps with their work. Although derived with specific and sometimes complicated Higgs structures in mind, these relations may well prove sturdier than the theories which generated them. In the following, we first examine these relations in the context of the Standard Model at varying distances all the way down to Planck length. We then extend the analysis to the minimal supersymmetric extension of the Standard Model, and compare the effect of this extension on their relative validity. A more thorough treatment is in preparation[13].

RUNNING THE STANDARD MODEL TO PLANCK

In this section we make use of the numerical techniques and routines developed in a previous work[14]. We first use experiment to fix the parameters of the Standard Model at lower energies. We then use these values as initial conditions in the renormalization group running to lower length scales, using the $\overline{MS}$ scheme. Our incomplete knowledge of the Standard Model parameters forces us to repeat the analysis for a range of allowed values of the top quark and Higgs masses. In these runs, we take $g_3(M_Z) = 1.191$ and the physical bottom quark mass $M_b = 4.89$ GeV.

Let us summarize the salient features of the renormalization group running in the Standard Model. At the one loop level, the gauge couplings are unaffected by the other couplings in the theory. On the other hand, the Yukawa couplings are affected at one loop by both the gauge and Yukawa coupling constants. Since the top Yukawa coupling is at least as big as the gauge couplings at low energy, it means that the Yukawa runnings are sensitive to mostly the top Yukawa and the QCD gauge couplings. Thus we can expect that the mass and mixing relations we have just described to be sensitive to the value of the top quark mass. On the other hand, the Higgs quartic self coupling enters the running of the other couplings only at the two loop level, and its effect on the quark and lepton parameters is expected to be small. However its own running is very sensitive to the top quark mass; it can become negative as easily as it can blow up. The former leads to vacuum instability, the latter, called the triviality bound leads to strong self-interaction of the Higgs. The following graphs summarize these bounds for representative values of the top quark mass. It is amusing to note that it is for comparable value of the top and Higgs masses that these bounds are least effective, but it is important to emphasize that a high value of the top with a relatively low value of the Higgs necessarily indicates the presence of new physics within reach of the SSC.



SCALES OF EXPECTED NEW PHYSICS IN THE STANDARD MODEL

RELATION (I)

This relation is the most natural one in the $SU(5)$ theory, and it could be expected to be valid at scales where the Standard Model gauge couplings are the closest to one another. We examine its validity for different physical values of the top and Higgs masses in the Standard Model. The results are summarized in the following tables:

Standard Model: $m_t = 100$ GeV; $m_H = 100$ GeV						
Scale (GeV)	6×10^7	10^{11}	10^{13}	10^{15}	10^{17}	10^{19}
$\frac{m_b}{m_\tau}$	1.001	.858	.798	.752	.716	.688

Standard Model: $m_t = 150$ GeV; $m_H = 150$ GeV						
Scale (GeV)	1.5×10^7	10^{11}	10^{13}	10^{15}	10^{17}	10^{19}
$\frac{m_b}{m_\tau}$	1.003	.818	.758	.711	.675	.651

Standard Model: $m_t = 190$ GeV; $m_H = 180$ GeV						
Scale (GeV)	4×10^6	10^{11}	10^{13}	10^{15}	10^{17}	10^{19}
$\frac{m_b}{m_\tau}$	1.001	.763	.699	.649	.611	.581

Their noteworthy feature is that this simplest of the SU(5) relation tends to be valid at an energy scale many orders of magnitude removed from that at which the gauge couplings tend to converge, which is around 10^{15} GeV.

RELATION (II)

We now turn to the more complicated relation between the masses from the two lighter families. Again we run the predictions for different values of the top and Higgs masses. The results are summarized in the following tables

Standard Model: $m_t = 100$ GeV; $m_H = 100$ GeV				
Scale (GeV)	10^{13}	10^{15}	10^{17}	10^{19}
$\frac{m_d}{3m_e}$	1.445	1.376	1.30	1.25
$\frac{3m_s}{m_\mu}$	1.30	1.22	1.17	1.12

Standard Model: $m_t = 150$ GeV; $m_H = 150$ GeV				
Scale (GeV)	10^{13}	10^{15}	10^{17}	10^{19}
$\frac{m_d}{3m_e}$	1.43	1.35	1.29	1.25
$\frac{3m_s}{m_\mu}$	1.28	1.21	1.16	1.12

Standard Model: $m_t = 190$ GeV; $m_H = 180$ GeV				
Scale (GeV)	10^{13}	10^{15}	10^{17}	10^{19}
$\frac{m_d}{3m_e}$	1.42	1.34	1.28	1.23
$\frac{3m_s}{m_\mu}$	1.28	1.21	1.15	1.11

We see that these relations are never satisfied at any scale in the Standard Model, and they are also quite insensitive to the value of the top. The formula for the second family provides the better agreement, but it never gets below 11%. From these tables, one can easily read off the ratio of the determinants of the charge -1/3 to charge -1 mass matrices. We note that for the lowest top mass, these two determinants are equal at 10^{18} GeV, while for the highest they converge at 10^{15} GeV.

RELATION (III)

We find that the Oakes relation to be quite independent of scale. The reason is that the Cabibbo angle does not run, and the ratio of light quarks is essentially unaffected by QCD, since both are far away from the Pendleton-Ross infrared fixed point. Further one finds that their numerical values are pretty much independent of the value of the top quark mass and of the Higgs mass. However the agreement is spectacular, hovering around the 4% level. We only present a representative example for one case:

Standard Model: $m_t = 100$ GeV; $m_H = 100$ GeV					
Scale (GeV)	10^7	10^{13}	10^{15}	10^{17}	10^{19}
$\tan\theta_C\sqrt{\frac{m_s}{m_d}}$	1.038	1.038	1.038	1.038	1.038

RELATION (IV)

Since this relation involves the top quark mass directly, it could be used to fix its value. On the other side of the equation, the experimental value of the "23" element of the CKM matrix, V_{cb} is known to within $\sim 10\%$ only[15]

$$V_{cb} = 0.043 \pm 0.006 \ .$$

In the following we use the central value. It is interesting to see under what conditions this relation can be made to hold[16], in particular if it is satisfied for a top quark anywhere in its allowed range between 100 and 200 GeV. In this case, because of the Pendleton-Ross fixed point the ratio of the two quark masses runs appreciably in the infrared region. For a top quark in its lower range, 100 – 150 GeV, this relation fails over all scales, so we start with a 190 GeV top in the following tables. The value of V_{cb} at all scales is obtained by running the angles; the particular value used for the CP-violating phase produces no appreciable difference in our results.

Standard Model: $m_t = 190$ GeV; $m_H = 180$ GeV					
Scale (GeV)	10^{11}	10^{13}	10^{15}	10^{17}	10^{19}
$\sqrt{\frac{m_c}{m_t}}$	.0565	.0559	.0555	.0552	.0548
V_{cb}	.0502	.0513	.0519	.0529	.0534

Standard Model: $m_t = 200$ GeV; $m_H = 195$ GeV					
Scale (GeV)	10^{13}	10^{14}	10^{15}	10^{17}	10^{19}
$\sqrt{\frac{m_c}{m_t}}$	.0536	.0533	.0530	.0525	.0519
V_{cb}	.0527	.0534	.0539	.055	.0561

We see that it is only for a very heavy top quark that this relation can be fulfilled. Of course things get better if we use the largest experimentally allowed value of V_{cb}. In particular, for a yet heavier top quark mass, the region of agreement spans more scales.

Putting all these results together, it is hard to arrive at a unified picture in the context of the Standard Model. The scale at which one relation tends to be satisfied does not coincide with that at which the other is valid. Still, the disagreement is never too large, which makes us hope that small course corrections in the running of the parameters may make most if not all of these relations hold simultaneously at a unified or similar scales. It is remarkable that for a top quark at the upper reaches of its allowed range, the long life of the bottom quark lends plausibility to the $SO(10)$-inspired relation.

RUNNING THE SUPERSYMMETRIC STANDARD MODEL TO PLANCK

As is well known, the Standard Model shows no apparent inconsistencies until perhaps the Planck scale, where quantum gravity enters the picture. Thus the nature of the physics that is to be encountered in between our scale and the Planck scale is a matter of theoretical taste. At one extreme, the value of the gauge couplings may be interpreted to infer new phenomena every two orders of magnitude. At the other, there is the possible desert suggested by GUTs; however, the absence of new phenomena over many orders of magnitude cannot be understood (perturbatively) unless one generalizes the Standard Model to be Supersymmetric at an experimentally accessible scale. This particular scenario is bolstered by the fact that with such "low energy" supersymmetry, the three gauge couplings of the Standard Model meet at a scale of $\approx 10^{16}$ Gev at the perturbative value of $1/26$[4]. In the following we restrict ourselves to this particular scheme in investigating the fate of these four relations among masses and mixing angles.

The collapse of the "GUT Triangle" in the supersymmetric extension fixes two scales, one is that at which the gauge couplings unify, the other denotes the threshold of supersymmetry. Minimal supersymmetry[17] implies two Higgs doublets, and eliminates the feisty quartic self-coupling of the Standard Model. Accordingly, even in the limit where only one Higgs is light, there appears an extra parameter, the ratio of the vacuum values of these two doublets, parametrized by an angle β

$$\tan\beta = \frac{v_u}{v_d} \ .$$

In a previous publication[18], it was shown that with supersymmetry it becomes possible to assume that relation (I) is valid at gauge unification. This fixes the angle β in terms of the top quark mass and the mass of the lightest neutral Higgs. These results are displayed for two scales of supersymmetry. For $M_{SUSY} = 1$ TeV, the gauge unification occurs between a low of 6.92×10^{15} GeV and a high of 1.26×10^{16} GeV, corresponding to $g_3 = 1.171$ and $g_3 = 1.197$, respectively. The error bars in the strong coupling allow for a SUSY scale as high as 10 TeV, with unification at 6.46×10^{15} GeV. The values in the table are obtained for the lower g_3.

Super-Standard Model: $M_{SUSY} = 1$ TeV		
$g_3 = 1.171$; $M_{GUT} = 6.92 \times 10^{15}$ GeV		
β (degrees)	M_H(GeV)	M_t(GeV)
50	57	154
60	81	171
70	102.5	183
80	116.5	190.5
85	120	192

Super-Standard Model: $M_{SUSY} = 1$ TeV		
$g_3 = 1.197$; $M_{GUT} = 1.26 \times 10^{16}$ GeV		
β (degrees)	M_H(GeV)	M_t(GeV)
50	60	159
60	84	176
70	105	188
80	119	196
85	123	198

Super-Standard Model: $M_{SUSY} = 10$ TeV		
β (degrees)	M_H (GeV)	M_t(GeV)
50	86	163
60	112.5	180
70	134	191.5
80	148	198
85	151	199.6

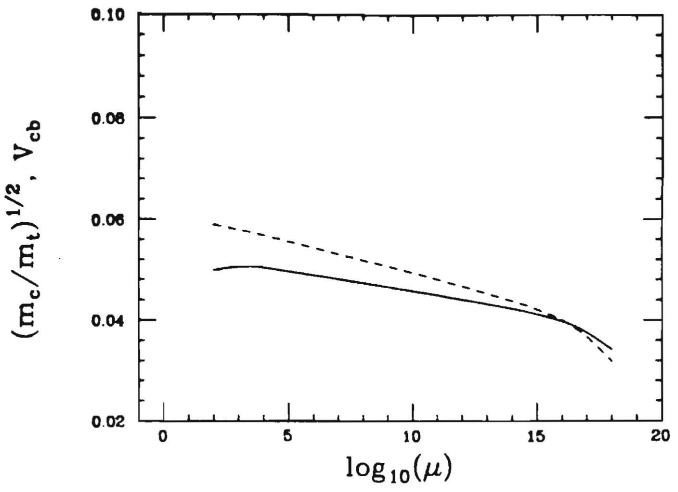
We still have one unknown degree of freedom, the angle β. The strategy of this paper is to fix its value by demanding optimum agreement on the remaining relations (II-IV). The results are again given in terms of tables for the central value of $V_{cb} = .044$. We treat three cases, the first where unification takes place at its lowest value. The second table uses the same value of the SUSY scale but the highest unification scale. Finally the third table uses the maximum SUSY value of 10 TeV, consistent with the error on g_3.

Super-Standard Model: $M_{SUSY} = 1$ TeV					
$g_3 = 1.171$; $M_{GUT} = 6.92 \times 10^{15}$ GeV					
β (degrees)	50	60	70	80	85
$\frac{m_d}{3m_e}$	1.543	1.539	1.534	1.53	1.53
$\frac{3m_s}{m_\mu}$	1.38	1.381	1.379	1.378	1.379
$\sqrt{\frac{m_c}{m_t}}$	.0518	.0485	.0463	.0451	.0450
V_{cb}	.0371	.0362	.0369	.0370	.0370

Super-Standard Model: $M_{SUSY} = 1$ TeV					
$g_3 = 1.197$; $M_{GUT} = 1.26 \times 10^{16}$ GeV					
β (degrees)	50	60	70	80	85
$\frac{m_d}{3m_e}$	1.38	1.38	1.38	1.38	1.37
$\frac{3m_s}{m_\mu}$	1.24	1.24	1.23	1.23	1.23
$\sqrt{\frac{m_c}{m_t}}$	.0457	.0458	.0409	.0398	.0397
V_{cb}	.0351	.0351	.0351	.0350	.0350

Super-Standard Model $M_{SUSY} = 10$ TeV					
$g_3 = 1.171$; $M_{GUT} = 6.46 \times 10^{15}$ GeV					
β (degrees)	50	60	70	80	85
$\frac{m_d}{3m_e}$	1.54	1.54	1.54	1.55	1.55
$\frac{3m_s}{m_\mu}$	1.39	1.39	1.38	1.38	1.38
$\sqrt{\frac{m_c}{m_t}}$	.0489	.0456	.0435	.0424	.0422
V_{cb}	.0373	.0372	.0371	.0372	.0372

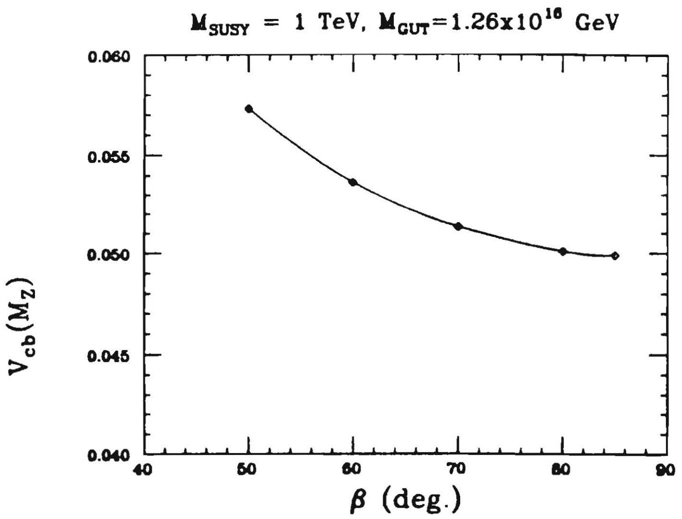
It is clear from the above that the mass relation between the two lightest families are consistently off over all scales. The same is not true for the mixing angle relation. Using the central value for V_{cb}, and supersymmetry, there is no agreement at the unification scale, although it gets close especially in the second case. Thus it lends credence to the fact that with a higher value of the mixing angle, one could satisfy that relation. We have made several runs with a higher value $V_{cb} = .050$. For instance, we find for the higher value of the strong coupling, $g_3 = 1.197$, and $\beta = 85$ degrees, the following graph

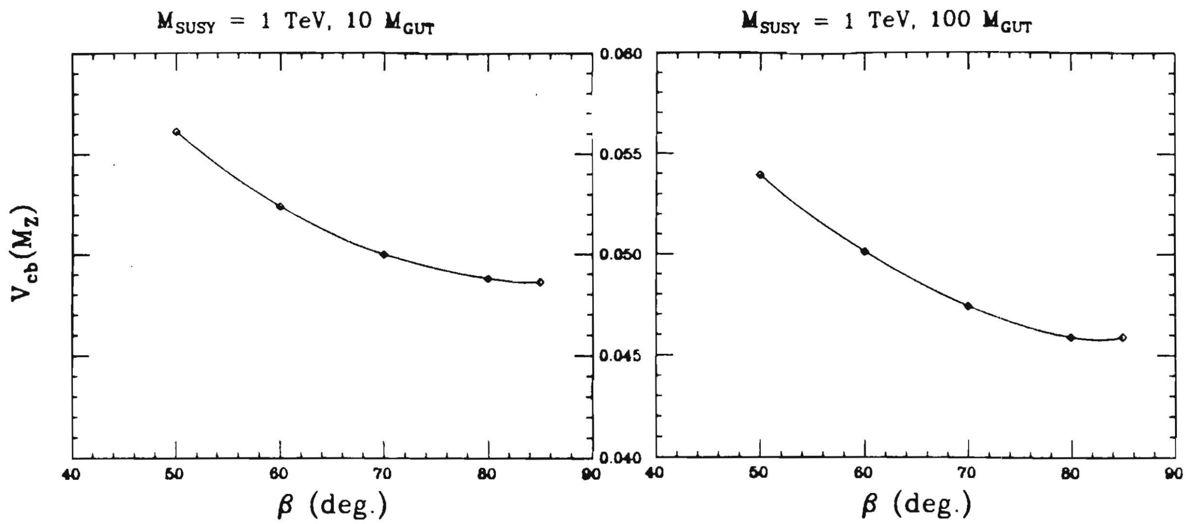


Evolution of V_{cb} (solid line) and $\sqrt{m_c/m_t}$ (dashed line) with $V_{cb} = .050$

If we consider the lower value of the strong coupling constant, the two curves meet closer to the Planck scale. Also, by raising the value of V_{cb} we can change the crossing scale. We just show this particular curve as an illustrative example.

From the theoretical point of view, we do not know exactly where the scale at which the $SO(10)$-inspired relation is valid. It could be much higher than M_{GUT}, the scale of unification of the Standard Model's couplings. Accordingly, we now plot V_{cb} as a function of β, assuming that relation (IV) is valid at M_{GUT}, $10M_{GUT}$, and $100M_{GUT}$, and the higher value of g_3.





V_{cb} as a function of β for relation (IV) valid at different scales

We conclude that it is not impossible to achieve agreement for three out of the four relations. But, for this to be true, several things must occur: one V_{cb} must be larger than its presently measured value; second the top quark mass must be around 190 GeV (if it is a bit lighter, then agreement dictates that the mixing angle should be larger still); third the Higgs mass should hover around 120 GeV. These conclusions are qualitatively correct if one demands maximum agreement. A similar analysis which recently appeared in the literature has reached similar conclusions[12]. However, it is difficult to arrive at a definite number without an exhaustive analysis of the parameter space. We leave this to a future publication[13].

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