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Superconductivity in a relativistic field theory with a charge-2 condensate

R. MacKenzie, P.K. Panigrahi and S. Sakhi

Laboratoire de Physique Nucléaire
Université de Montréal
Montréal, Qué H3C 3J1
Canada

We consider a 2+1 dimensional model with relativistic fermions interacting via a four-Fermi term. Our goal is to mimic the procedure done in studying BCS superconductivity, namely, linearizing the interaction with an auxiliary field, and integrating out the fermions to obtain the one-loop effective action for the auxiliary field, which is then viewed as a condensate. We examine the effective potential to see if symmetry breaking occurs, which it does at low temperatures, and also look at the two-derivative term to ensure that its coefficient is positive, yielding an acceptable truncation of the derivative expansion. We also study the finite temperature corrections to the effective potential and find that the symmetry is restored at a critical temperature through a second order phase transition. The critical exponent for the order parameter is one-half, which agrees with the mean field result.

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Introduction

In discussions of BCS superconductivity by field theorists,[1,2] a common approach is the following. The starting point is the second-quantized BCS Lagrangian, including the effective electron-electron interaction:

$$\mathcal{L}_{\rm BCS} = \psi_\uparrow^\dagger(i\partial_t - \epsilon(p))\psi_\uparrow + \psi_\downarrow^\dagger(i\partial_t - \epsilon(p))\psi_\downarrow - \lambda\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow. \tag{1}$$

Here $\epsilon(p)$ is the kinetic energy, including a chemical potential and coupling to electromagnetism, if desired. This has the obvious U(1) phase symmetry $\psi \to e^{i\theta}\psi$. One then argues that the low-energy behaviour of the theory is best described in terms of a condensate field rather than the fermions. Physically this is connected to the fact that there is a gap in the fermion spectrum below the critical temperature, as a result of which long-wavelength excitations of the condensate are much cheaper to create than fermionic excitations.

In order to achieve an effective theory for the condensate, one replaces the four-Fermion interaction in (1) by a Yukawa-type interaction with a scalar field, a step known as a Hubbard-Stratonovich transformation. Consider then the following Lagrangian:

$$\mathcal{L}_1 = \psi_\uparrow^\dagger(i\partial_t - \epsilon(p))\psi_\uparrow + \psi_\downarrow^\dagger(i\partial_t - \epsilon(p))\psi_\downarrow + \sqrt{\lambda g}(\phi^*\psi_\downarrow\psi_\uparrow + \phi\psi_\uparrow^\dagger\psi_\downarrow^\dagger) + g\phi^*\phi. \tag{2}$$

The equation of motion of ϕ^* is

$$g\phi + \sqrt{\lambda g}\psi_\downarrow\psi_\uparrow = 0;$$

inserting this in (2) we recover (1). We also note from this that ϕ has charge two, if ψ has charge one.

The advantage of (2) is that its bilinear nature in the fermion fields permits one to eliminate the fermion, at least formally. In the language of path integrals, the generating functional of (2) is

$$Z = \int \mathcal{D}\psi^\dagger \mathcal{D}\psi \mathcal{D}\phi^* \mathcal{D}\phi \, e^{iS_1}, \tag{3}$$

where $S_1 = \int d^4x \, \mathcal{L}_1$. Notice that the functional integral over the scalar field can be done, yielding an "effective action" for the fermion which is none other than the integral of (1). From (3), one can perform the fermion functional integral:

$$Z = \int \mathcal{D}\phi^* \mathcal{D}\phi \, e^{iS_{\rm eff}(\phi^*,\phi)},$$

where

$$e^{iS_{\rm eff}[\phi^*,\phi]} = \int \mathcal{D}\psi^\dagger \mathcal{D}\psi e^{iS_1} \tag{4a}$$

and

$$S_{\rm eff} = -i{\rm Tr}\log\begin{pmatrix} p_0 - \epsilon(p) & -\sqrt{\lambda g}\phi \\ \sqrt{\lambda g}\phi^* & p_0 + \epsilon(p) \end{pmatrix} + \int d^4x \, g\phi^*\phi. \tag{4b}$$

Here the matrix, if sandwiched between $\psi^\dagger = (\psi_\uparrow^\dagger, \psi_\downarrow)$ and $\psi = (\psi_\uparrow, \psi_\downarrow^\dagger)^T$, gives the fermion-dependent part of (2).

Next, the effective action can be evaluated in a gradient expansion. The lowest-order term, with no derivatives, is the effective potential for the condensate; at finite temperature, the result is

$$V_{\rm eff} \sim a \log \frac{T}{T_c}\phi^*\phi + b(\phi^*\phi)^2, \tag{5}$$

where a, b are positive parameters, the details of which are not important to us. One sees from the effective potential that ϕ attains a nonzero vacuum expectation value below a certain critical temperature. When that happens, electromagnetism is broken and superconductivity results. The phase transition is of second order.

In this paper, we essentially want to repeat this calculation in a relativistic setting. Although we do not have a specific condensed-matter system as a starting point, we were motivated in this investigation in part by other recent works where relativistic fermionic field theories (usually in 2+1 dimensions) have arisen as effective models of strongly coupled electronic systems on a plane.[3] These continuum Dirac fermions have two[3] or more[4] species and describe the long wavelength excitations of an underlying lattice Hamiltonian, with linear dispersion relation. Couplings with scalar[5] and gauge fields (both abelian[6,7] and non-abelian[8]) can also arise because of the residual interactions and constraints at the lattice level. Four-fermion couplings have been considered in Ref.9. Recently, gauge theories, and in particular those with statistics-changing Chern-Simons (C-S) couplings, have been closely examined after Laughlin's suggestion regarding their possible relevance to high-temperature superconductivity.[10] Models with couplings to neutral scalars and CP^1 (or O(3)) bosons have also been studied to settle the questions of spontaneous breaking of parity[11,12,13] and bound state formation, and to find the occurrence of superconductivity.[14]

Probably the first situation in which a calculation similar to the above was done in a relativistic setting was the work of Nambu and Jona-Lasinio.[15] This work, which is actually approximately concurrent with BCS, introduced the idea of dynamical symmetry breaking in the context of the pion-nucleon system. A four-Fermi interaction was shown to break chiral symmetry, leading to what are now known as Nambu-Goldstone bosons, which Nambu and Jona-Lasinio identified with the pion.

Another, more recent field theory which mimics the BCS theory as discussed above is

the Gross-Neveu model,[16] a 1+1-dimensional model described by the following Lagrangian:

$$\mathcal{L}_{\rm GN} = \bar{\psi}_i i\partial\!\!\!/ \psi_i + \frac{g^2}{2}\left(\bar{\psi}_i\psi_i\right)^2 . \tag{6}$$

Again one often introduces an auxiliary field to remove the four-Fermi interaction, replacing the last term in (6) with

$$g\phi\bar{\psi}_i\psi_i - \frac{1}{2}\phi^2. \tag{7}$$

It is well known that the discrete chiral transformation $\psi \to \gamma_5\psi$ is broken spontaneously. This symmetry breaking, together with the fact that the model is asymptotically free, makes it a good testing ground for QCD (although given that it is in 1+1 dimensions makes the connection rather tenuous).

One obvious difference between the Gross-Neveu model and the BCS theory is that the "condensate" ϕ in (7) is a real (uncharged) scalar, while that in BCS is doubly charged. One can of course couple a charged scalar to fermions via an interaction of the form

$$\bar{\psi}\psi|\phi|^2,$$

but this is not terribly reminiscent of BCS.

In order to find a closer analog we must find a doubly-charged bilinear in the fermion. This can be done using the charge-conjugate fermion field, $\psi^c = C\bar{\psi}^T = i\gamma^2\psi^*$. The appropriate relativistic generalization of (1), which we will analyze in detail, is thus

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/)\psi + \frac{g^2}{4}\bar{\psi}\psi^c\bar{\psi}^c\psi. \tag{8}$$

Coupling to the photon field endows the model with a U(1) gauge symmetry.

In what follows, we will remove the four-fermi interaction in favour of a Yukawa-type coupling to an auxiliary field ϕ. Our goal is then to perform a derivative expansion of the effective theory for ϕ. As a first step, the effective potential will be computed at the one-loop level to study the question of radiative breaking of the U(1) symmetry *à la* Coleman-Weinberg.[17] After answering this question in the affirmative, finite temperature effects will then be incorporated to show the restoration of the symmetry at a critical temperature, the phase transition being of second order. In contrast, it is interesting to point out that most of the C-S theories give rise to so-called semi-superconductivity, where there is no phase transition.

Were ϕ a true dynamical field, rather than an auxiliary field, we would argue that the two-derivative term induced by the fermion loop serves only to renormalize the already-present, kinetic term for ϕ. However, being an auxiliary field, it has no kinetic term and the induced two-derivative term is physically significant as it stands. In order for the expansion up to two derivatives to be sensible, therefore, the coefficient of this term must be positive. We check this, and find that it is.

We restrict our attention to 2+1 dimensions, for two reasons. First, many interesting relativistic analogs of lattice spin systems are in 1+1 and 2+1 dimensions. Second, as has been emphasized recently by Rosenstein, *et al.*,[12] the strong ultraviolet divergences which arise in 3+1 dimensions are prohibitive, leading one to consider lower dimensions. The point is that, although naively non-renormalizable, four-fermi theories in 2+1 dimensions can be made renormalizable in an appropriate large-N limit. However, the physics of 1+1 dimensions is in many ways unique, and in particular can be *very* different from

3+1 dimensions, perhaps too much so in order for what is learned in 1+1 to be useful in 3+1 dimensions. So we are left with 2+1 dimensions. It must be admitted, furthermore, that it has not escaped our attention that high-temperature superconductivity is a planar phenomenon.

We use a diagrammatic approach to compute the derivative expansion,[18] making use of the interpretation of the effective action in terms of one-particle-irreducible Green's functions. The approach is essentially a generalization of the method devised by Coleman and Weinberg to compute the effective potential. An alternative approach[19] using an expansion of the trace-log representation of the effective action is also employed as a check.

It is worth pointing out at this point that the effective Lagrangian we arrive at, with additional scalar self-couplings, has been analysed before for the purpose of finding zero-modes of the Dirac equation in the presence of a vortex, either without[20,21] or with[22,23] a C-S term. Hence our investigation supplements these previous works.

II. One-loop effective action at zero temperature

As mentioned above, our analysis of the Lagrangian (8) begins by first decoupling the four-fermion term by introducing auxiliary fields ϕ and ϕ^*. The new Lagrangian reads

$$\mathcal{L} = \bar{\psi}(i\not{\partial} - e\not{A})\psi - \frac{ig}{2}\left(\phi^*\bar{\psi}^c\psi - \phi\bar{\psi}\psi^c\right) - \phi^*\phi; \tag{9}$$

substituting the equations of motion for ϕ and ϕ^* back in (9) reproduces (8). It is worth pointing out that to maintain gauge invariance of the Yukawa term in (9), the charge of the complex field has to be $q = 2e$.

The effective action for the scalar and gauge fields is attained by integrating over the fermions:

$$e^{iS_{\rm eff}[A,\phi,\phi^*]} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{iS}.$$

In fact, the dependence of $S_{\rm eff}$ on A can be deduced from gauge invariance, so often in what follows we set A to zero.

A nonlocal object, $S_{\rm eff}$ cannot be computed exactly. However, if our goal is to describe the low-energy, long-wavelength excitations of the condensate, it is reasonable to attempt a gradient expansion of $S_{\rm eff}$. Indeed, the equivalent of the trace-log in (4) can be computed as a derivative expansion directly. However, a perhaps more physical approach is to appeal to the interpretation of the effective action as an object which incorporates the effects of scalar field interactions mediated by internal fermion lines. Thus, $S_{\rm eff}$ is the generating functional of one-particle irreducible vertex functions:

$$\begin{aligned} S_{\rm eff}[\phi,\phi^*] = -i\sum_n \frac{1}{n!^2}\int \frac{d^3p_1}{(2\pi)^3}\frac{d^3q_1}{(2\pi)^3}\cdots\frac{d^3p_n}{(2\pi)^3}\frac{d^3q_n}{(2\pi)^3}(2\pi)^3\delta^3(\sum p_i + q_i) \\ \times\, \Gamma^{(n,n)}(p_1,\cdots,p_n;q_1,\cdots,q_n)\bar{\phi}(p_1)\bar{\phi}^*(q_1)\cdots\bar{\phi}(p_n)\bar{\phi}^*(q_n) \end{aligned} \tag{10}$$

Alternatively, simply by recognizing $S_{\rm eff}$ as a functional of ϕ, it can be expressed in terms of the gradients of ϕ and ϕ^*:

$$S_{\rm eff}[\phi,\phi^*] = \int d^3x\,\left(-V_{\rm eff}(\phi) + Z(\phi)\partial_\mu\phi\partial^\mu\phi^* + \cdots\right). \tag{11}$$

The key feature of the first expansion is that each term is calculable: $\Gamma^{(n,n)}$ are the one-fermion-loop diagrams with n external ϕ's and ϕ^*'s. On the other hand, the second is a convenient way of capturing the long-wavelength behaviour of the scalar field as induced

by the fermion. In what follows we will compute the first two terms in the derivative expansion through judicious combinations of the 1PI Green's functions.

We concentrate first on the effective potential. By rewriting (11) in momentum space, the other terms vanish at zero momentum, so $V_{\rm eff}$ can be calculated from Green's functions at zero momentum. It is not difficult to see that

$$V_{\rm eff}(\phi) = V_0 + i\sum_1^\infty \frac{(\phi^*\phi)^n}{n!^2}\Gamma^{(n,n)}(0,\cdots 0;0,\cdots 0). \tag{12}$$

The sum over all zero-momentum diagrams appears formidable, but as shown by Coleman and Weinberg, it is actually quite doable. The only unusual twist in the present case is actually in the Feynman rules: the $\phi\bar{\psi}\psi^c$ coupling, for example, gives a vertex with an incoming ϕ and two outgoing fermions.

After some work, the effective potential is found to be (up to a constant)

$$V_{\rm eff} = V^{(0)} + i\int \frac{d^3k}{(2\pi)^3}\log\left(1 - \frac{g^2\phi^*\phi}{k^2}\right),$$

where $V^{(0)}$ is the classical potential, *i.e.*, the last term in (9). $V_{\rm eff}$ is linearly divergent; imposing a cutoff Λ, we get

$$V_{\rm eff} = \left(1 - \frac{\alpha}{\alpha_c}\right)\phi^*\phi + \frac{4g^3}{3\pi}(\phi^*\phi)^{3/2}, \tag{13}$$

where $\alpha = g^2/4$ and $\alpha_c = \pi^2/2\Lambda$.

We see that if $\alpha < \alpha_c$, the symmetry is not broken, while for $\alpha > \alpha_c$ it is broken, the auxiliary field developing vacuum expectation value

$$v \equiv |\phi| = \frac{\pi}{8g}\frac{\alpha - \alpha_c}{\alpha\alpha_c}.$$

Note that if we suppose that $\Lambda/g^2 \gg 1$, $v \to \Lambda/4\pi g$.

Below we will determine the temperature dependence of the effective potential; for now we continue with the zero-temperature derivative expansion. If we are to have a sensible Ginzburg-Landau theory for the condensate, the two-derivative term must have a positive coefficient. The computation is very similar to the effective potential. The Z term from (11) is, using a Taylor expansion for $Z(\phi)$,

$$-\sum_0^\infty \frac{Z_n}{n!}\int\left(\int \frac{d^3p_i}{(2\pi)^3}\frac{d^3q_i}{(2\pi)^3}\right)^n (2\pi)^3\delta^3(\sum p_i + q_i)$$
$$\times\, p_0\cdot q_0\bar{\phi}(p_1)\cdots\bar{\phi}(p_n)\bar{\phi}^*(q_1)\cdots\bar{\phi}^*(q_n)$$

This time, it is clearly Green's functions with two nonvanishing external momenta which contribute. After some work, one finds

$$Z(\phi) = \frac{5g^2}{96\pi\mu}, \tag{14}$$

where $\mu = gv$ is the induced fermion mass. (14) is blissfully positive.

Incorporation of electromagnetism is trivial; since we know ϕ has charge $2e$, the kinetic term must be modified to an appropriate covariant derivative; we get effective Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{5g^2}{96\pi\mu}|(\partial_\mu - 2eA_\mu)\phi|^2 - (1 - \frac{\alpha}{\alpha_c})\phi^*\phi + \frac{4g^3}{3\pi}(\phi^*\phi)^{3/2}. \tag{15}$$

III. Finite-temperature effects

We now analyse the temperature corrections to the effective potential in order to determine the nature of the phase transition. In the imaginary-time formalism, finite temperature is achieved by making field configurations periodic or antiperiodic in time,

with period $\beta = 1/T$; in what follows we put κ, the Boltzman constant to one. In Euclidean momentum space this amounts to replacing $\int d^3k/(2\pi)^3$ by $\beta^{-1}\sum_l \int d^2k/(2\pi)^2$, where $k^0 = 2\pi\beta^{-1}(l+1/2)$. The effective potential at finite temperature is then

$$V_{\rm eff}(\phi,\phi^*;T) = \phi^*\phi - \frac{1}{\beta}\sum_{-\infty}^{\infty}\int \frac{d^2k}{(2\pi)^2}\log\left(1+\frac{g^2\phi^*\phi}{\mathbf{k}^2+\frac{4\pi^2}{\beta^2}(l+\frac{1}{2})^2}\right).$$

The calculation is more or less standard;[24] after some work, the result is

$$V_{\rm eff}(\phi,\phi^*;T) - V_{\rm eff}(0;T) = V_{\rm eff}(\phi,\phi^*;0) + \frac{g^2\phi^*\phi}{2\pi\beta}\log\left(1+e^{-\beta g\sqrt{\phi^*\phi}}\right) - \frac{1}{2\pi\beta^3}\int_{\beta g\sqrt{\phi^*\phi}}^{\infty} dt\,\frac{t^2}{e^t+1} + \frac{3\zeta[3]}{4\pi\beta^3}. \tag{17}$$

The critical temperature can be computed from the coefficient of the $\phi^*\phi$ term(this is done by differentiating the $V_{\rm eff}$ twice with respect to ϕ and ϕ^* and subsequently putting ϕ and ϕ^* to zero). After simplification we see that there is indeed a critical temperature (assuming $\alpha > \alpha_c$ so that there is symmetry breaking at zero temperature), given by

$$T_c = \frac{gv}{2\log 2}. \tag{18}$$

The order parameter approaches zero smoothly, with critical exponent $\beta = 1/2$, so this is a second-order phase transition. This behaviour of the order parameter ($\beta = 1/2$ is precisely the mean-field value) also arises in the context of the Gross-Neveu model.[25]

The massgap μ being gv, the physically relavant quantity $2\mu/T_c$ in this theory equals $4\log 2$, this is lower than the BCS value 3.52 .

We have also calculated the two-derivative term at finite temperature to make sure that it is well behaved below and above the critical temperature.

Finally, let us mention that in 3+1 dimensions the effective potential is

$$V_{\rm eff}^{(3+1)} = \frac{1}{32\pi^2}\left(2\Lambda^2m^2 + m^4 log(m^2/\Lambda^2) - \frac{m^4}{2}\right).$$

It is, as mentioned above, much more divergent than in 2+1 dimensions: quadratically rather than linear. This is a serious problem if one hopes to attain large-N renormalizability, as discussed above.[12]

Note that our condensate, $\bar{\psi}\psi^c$ does not transform in a well defined manner under parity and time reversal symmetries. This is due to the phase arbitrariness of the spinors under the above discrete transformations. This leads to the question of the possible presence of a C-S term in the effective action. This term could arise from one-loop diagrams with two external photons carrying momentum (in addition to any number of scalar fields at zero momentum). However it is easy to see that such diagrams can never generate a C-S term. This is essentially due to the fact that there must be an even number of external scalar legs (to preserve gauge invariance), and therefore an even number of fermion propagators. Since the fermion is massless there results a trace of an even number of propagators; however with two-component spinors a C-S term would only arise from the trace of three gamma matrices (proportional to ϵ_{ijk}). In hindsight, this result is perfectly sensible because the photon field and consequently the C-S term is well defined under P and T transformations unlike the spinors.

To conclude, the model presented here exhibits superconductivity without a parity breaking C-S term and is based on a relativistic field theory which arises naturally in many strongly correlated electronic systems. Further analysis of this model using the

large-N method and its connection with the microscopic lattice Hamiltonians is currently under progress and will be reported elsewhere.

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