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IMPACT OF f_B AND $B_{K,B}$ ON THE CKM PARAMETERS[a]

C. Q. GENG and P. TURCOTTE

Lab. de Phys. Nucl., Univ. de Montréal, Montréal, PQ, H3C 3J7

ABSTRACT

The uncertainties of the B-meson decay constant f_B and bag parameters $B_{K,B}$ are reviewed. The effects of these parameters on the CKM matrix elements are examined. The implications on rare kaon decays are presented.

In the standard model, quarks of different flavors are mixed in the charged weak currents by an unitary matrix, V, the Cabibbo-Kobayashi-Maskawa (CKM) matrix[1]. A phenomenological form of this matrix, given by Wolfenstein, is[2]

$$V \simeq \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho-i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2-iA^2\lambda^4\eta & A\lambda^2(1+i\lambda^2\eta) \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \tag{1}$$

where $\lambda \simeq 0.22$ is the Cabibbo angle extracted from the element $|V_{us}| = 0.2205 \pm 0.0018$. It contains three unknown parameters A, ρ and η constrained by the experimental data on V_{cb}, $|V_{ub}/V_{cb}|$, ϵ and the $B_d^0 - \bar{B}_d^0$ mixing parameter defined as $x_d \equiv \Delta M/\Gamma$, which give[3]

$$|V_{cb}| \;:\; A = 0.91 \pm 0.14\,, \tag{2}$$

$$|V_{ub}/V_{cb}| \;:\; \sqrt{\rho^2+\eta^2} = 0.50 \pm 0.18\,, \tag{3}$$

$$|\epsilon| \;:\; A^2\eta[0.8 + 1.43A^2(1-\rho)B(x_t)] = (0.525 \pm 0.006)\frac{1}{B_K}\,, \tag{4}$$

$$x_d \;:\; A^2[(1-\rho)^2+\eta^2]B(x_t) = (3.1 \pm 0.6)\left(\frac{130\ MeV}{f_B\sqrt{B_B}}\right)^2, \tag{5}$$

respectively, where $B(x_t) = \frac{x_t}{4}\left[1 + \frac{3-9x_t}{(x_t-1)^2} + \frac{6x_t^2 \ln x_t}{(x_t-1)^3}\right]$ with $x_t = m_t^2/M_W^2$. The value of $|V_{cb}|$ in Eq. (2) is obtained by combining the results from the inclusive and exclusive decays while in Eq. (3) we have included both experimental and theoretical errors.

[a]Talk presented by C. Q. Geng at the *KEK workshop on rare kaon decays*, KEK, Dec. 10-11, 1991.

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Table 1: Values of B_K in various methods.

Method	B_K	Author (Date)
Vacuum Insertion	1	
Chiral Symmetry[4]	0.33 ± 0.2	Donoghue et al (1982)
Hadronic Sum Rules[5]	0.33 ± 0.09	Pich et al (1985)
	0.39 ± 0.10	Prades et al (1991)
QCD Sum Rules[6]	0.84 ± 0.08	Reinders et al (1987)
	0.50 ± 0.22	Bilic et al (1988)
	0.58 ± 0.16	Decker (1989)
	0.74 ± 0.17	Papadopoulos et al (1991)
Large N Limit[7]	0.66 ± 0.10	Bardeen et al (1988)
Lattice QCD[8]	0.88 ± 0.20 [a]	Gavela et al (1988)
	1.03 ± 0.14 [a]	Bernard et al (1989)
	0.94 ± 0.01 [b]	Kilcup et al (1990)
	0.83 ± 0.01 [b]	Kilcup et al (1990)
	0.75 ± 0.04 [b]	Kilcup et al (1990)
	0.92 ± 0.03 [c]	Kilcup et al (1990)

[a]Calculated with (a) Wilson; (b) Staggered; (c) staggered unquenched 2-flavors fermions, respectively.

The constraints from $|\epsilon|$ in Eq. (4) and x_d in Eq. (5) depend on the mass of t-quark as well as the inputs of B_K and $f_B^2 B_B$, respectively, which have large theoretical uncertainties. For the t-quark mass, the limits are given by

$$89 < m_t < 200\ GeV \tag{6}$$

from the CDF data and the absence of large radiative corrections to electroweak processes, respectively while a recent fit to electroweak data leads to

$$m_t = 140 \pm 40\ GeV. \tag{7}$$

The bag parameter B_K in Eq. (4), which arises from the ignorance of the matrix element $< K^0|[\bar{d}\gamma_\mu(1-\gamma_5 s)]^2|\bar{K}^0 >$, varies in the range $0.3 - 1.2$ for different theoretical calculations as shown in Table 1. However, in our discussions we use

$$B_K = 0.8 \pm 0.2\,, \tag{8}$$

obtained from the large N limit and lattice QCD. The effect of other ranges of B_K is remarked at the end. For the parameter B_B, it is expected to be O(1) in most of models such as the Lattice QCD. We assume

$$B_B \simeq 1 \tag{9}$$

with all uncertainties being hidden in f_B.

Table 2: Values of f_B in various methods.

Method	f_B [MeV]	Author (Date)
QCD Sum Rules[10]	~ 241	Mathur et al (1981,1984)
	115 ± 15	Shifman et al (1986)
	185 ± 19	Narison (1987)
	$150 - 210$	Dominguez et al (1987)
	170 ± 20	Reinders et al (1988)
	130 ± 50	Pich (1988)
	148 ± 26	Rosner (1990)
Potential Models[11]	125	Krasemann (1980)
	75	Suzuki (1985)
	229	Sinha (1986)
	$260 - 300$	Silverman et al (1988)
	155 ± 15	Godfrey et al (1990)
Relativistic Model[12]	195	Mendel et al (1989)
Lattice QCD[13]	~ 120	Gavela et al (1988)
	$105 \pm 17 \pm 30$	Bernard et al (1988)
	183 ± 28	Hamber (1989)
Recent Lattice QCD[14]	$310 \pm 25 \pm 50$ [*a]	Allton et al (1991)
	320 ± 20 [*b]	Bernard et al (1991)
	$188 - 246$	Alexandrou et al (1991)

*Calculated in the static limit, i.e., $m_b \to \infty$ which reduce to (a) 233 ± 42 and (b) 240 ± 15 with 25% $1/m_b$ corrections.

The factor f_B has been estimated based on various QCD models and the results are summarized in Table 2. The results vary considerably. The recent lattice QCD calculations[14] indicate that f_B is most likely larger than 200 MeV. However, before these results, it was widely accepted that 100 $MeV < f_B <$ 200 MeV although larger values of f_B were obtained in the QCD sum rule and potential model. For example, in Ref. 9, we used $f_B B_B^{\frac{1}{2}} = 130 \pm 25\ MeV$ in our study on the range of the CKM mixing parameters and rare K decays.

To illustrate the effects of both lower and higher f_B, we choose

$$f_B = 130 \pm 40\ MeV \tag{10}$$

and

$$f_B = 250 \pm 50\ MeV \tag{11}$$

in our discussions.

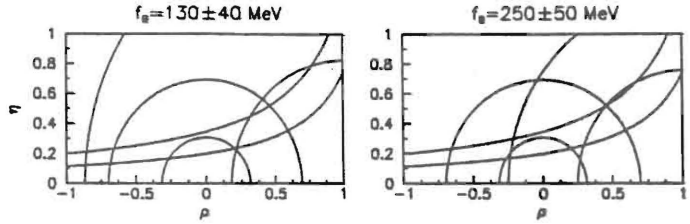

Fig. 1. Constraint in the $\rho - \eta$ parameter space from $|V_{ub}/V_{cb}|$, ϵ, and the $B_d^0 - \bar{B}_d^0$ mixing for $A = 0.91$ and $m_t = 140\ GeV$.

In Fig. 1, the intersection of the three constraints in Eqs. (3)-(5) gives the allowed regions in the $\rho - \eta$ parameter space with the center values of A in (2) and m_t in (7) and Eqs. (8)-(9). From the figure we see that the lower (higher) f_B in Eq. (10) (Eq. (11)) favors negative (positive) ρ and slightly smaller η.

To get proper and meaningful errors for the CKM parameters, we use the χ^2 minimization program MINUIT in the rest of discussions.

First, using $B_K = 0.8 \pm 0.2$ and $B_B = 1$, we fit the parameters A, ρ, η, m_t and f_B with the data in Eqs. (2)-(5) and Eq. (7). The best fit, χ^2_{min}, as function of f_B is shown in Fig. 2. We note that the perfect fits, i.e., $\chi^2_{min} = 0$, occur at $f_B = 135$ and 270 MeV which coincide with the values in Eqs. (10) and (11) respectively. Similarly, we fit the data with f_B fixed as in (10) and (11), respectively and the χ^2_{min} as function of B_K is shown in Fig. 3. For the lower f_B, $\chi^2_{min} \to 0$ for $B_K > 0.3$ whereas for the higher f_B the good fits are in the region around $B_K = 0.8$. It is interesting to pointed out that our results on B_K and f_B agree with that given by the various QCD methods in Tables 1 and 2 respectively. With the program, in Fig. 4, we show the contours in $\rho - \eta$ plane with the parameters in (7)-(9) where the solid and dashed curves correspond to χ^2 equal to one and two standard deviations above $\chi^2_{min}(\rho, \eta)$ respectively.

We now fit the three parameters A, ρ and η with the constraints in Eqs. (2)$-$(5) for a given m_t and fixed ranges of f_B and B_K. In Fig. 5 we show the best fits $\chi^2 = \chi^2_{min}|_{m_t}$ with $B_K = 0.8 \pm 0.2$. From Fig. 5 we see that $\chi^2_{min} = 0$ occur at $m_t \simeq 135$ (138) GeV for the lower (higher) f_B. The CKM parameters A, ρ and η corresponding to the best fits for different values of m_t are displayed in Fig. 6 (solid curves) where the dotted and dashed curves are the upper and lower bounds, respectively, for values of $\chi^2 = \chi^2_{min} + 1$. We emphasize that for the best fits the ρ parameter is always negative (positive) with the lower (higher) f_B which is consistent with Fig. 1.

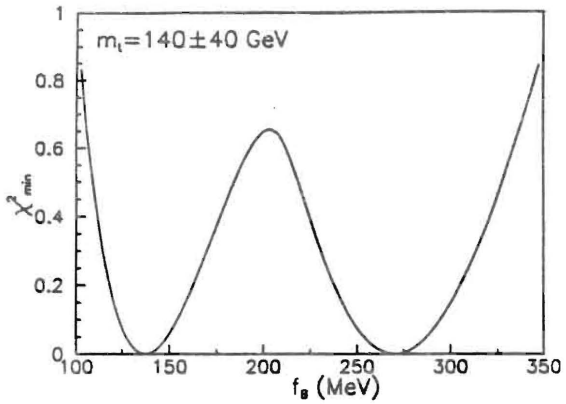

Fig. 2. The best fits χ^2_{min} as function of f_B for $B_K = 0.8 \pm 0.2$ and $m_t = 140 \pm 40\ GeV$.

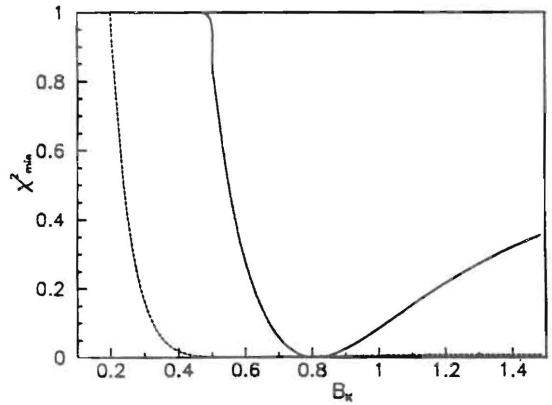

Fig. 3. The best fits χ^2_{min} as function of B_K for $f_B = 250 \pm 50\ MeV$ (solid curve) and $f_B = 130 \pm 40\ MeV$ (dashed curve) and $m_t = 140 \pm 40\ GeV$.

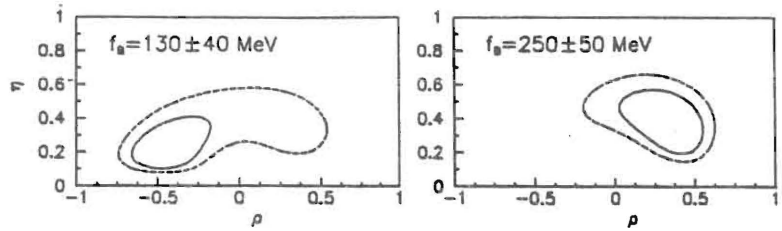

Fig. 4. The contour curves in $\rho - \eta$ plane with $B_K = 0.8 \pm 0.2$ and $m_t = 140 \pm 40\ GeV$. The dashed and dotted curves are boundaries taking into account a change in χ^2 by one and two units with respect to $\chi^2_{min}(\rho, \eta)$ respectively.

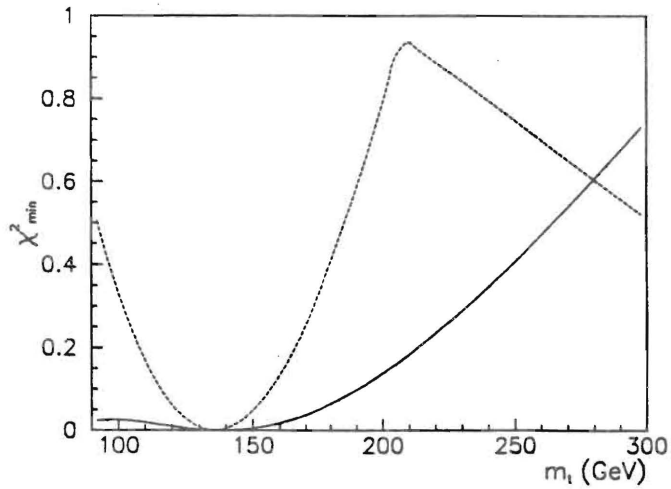

Fig. 5. The best fits χ^2_{min} as function of m_t for $f_B = 250 \pm 50\ MeV$ (solid curve) and $f_B = 130 \pm 40\ MeV$ (dashed curve) and $B_K = 0.8 \pm 0.2$.

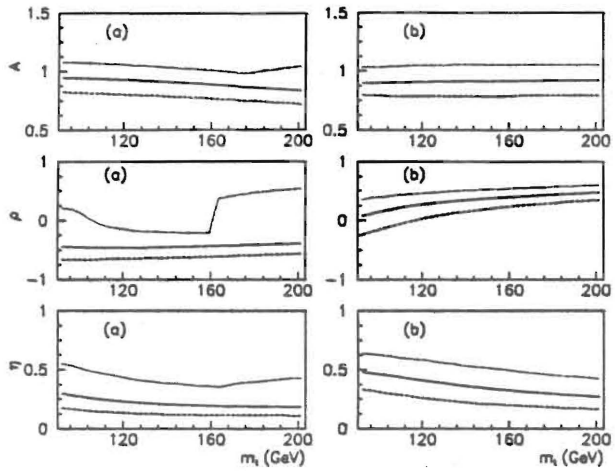

Fig. 6. Values of parameters A, ρ and η allowed by fits as function of m_t with (a) $f_B = 130 \pm 40$ and (b) $f_B = 250 \pm 50$ MeV and $B_K = 0.8 \pm 0.2$. The solid curves correspond to the best fits, χ^2_{min}. The dashed and dotted curves are boundaries taking into account a change in χ^2 by one unit with respect to χ^2_{min}.

We now study the consequences of the fits for rare kaon decays. We concentrate on the following crucial modes: $K_L \to \mu\bar{\mu}$, $K^+ \to \pi^+\nu\bar{\nu}$, $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0\nu\bar{\nu}$. The formulas for the standard model contributions to each process are given by[9]

$$
\begin{aligned}
Br(K_L \to \mu\bar{\mu})_{SD} &= 4.06 \times 10^{-10} A^4 (1-\rho)^2 C_\mu^2(x_t)\,, \\
Br(K^+ \to \pi^+\nu\bar{\nu}) &= 10^{-6}[C_\nu(x_c) + 3.3 \times 10^{-3} A^2 (1-\rho) C_\nu(x_t)]^2 \\
&\quad + 1.08 \times 10^{-11} A^4 \eta^2 C_\nu^2(x_t)\,, \\
Br(K_L \to \pi^0 e^+ e^-)_{dir} &= 2.6 \times 10^{-14} A^4 \eta^2 [C_V^2(x_t) + C_A^2(x_t)]\,, \\
Br(K_L \to \pi^0\nu\bar{\nu})_{dir} &= 4.61 \times 10^{-11} A^4 \eta^2 C_\nu^2(x_t)
\end{aligned}
\tag{12}
$$

where the functions $C_i(x_t)$ are defined in Ref. 9. The predictions for the best fits and the ranges with all possible CKM parameters by allowing a change in χ^2 by one unit with respect to χ^2_{min} are shown in Fig. 7.

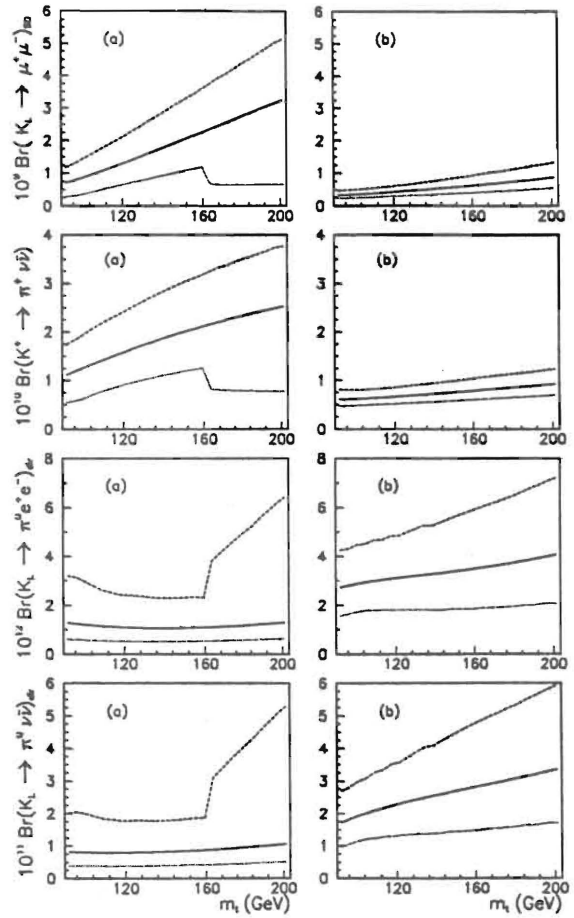

Fig. 7. The short-distance contributions to the branching ratios of $K_L \to \mu\bar{\mu}$; $K^+ \to \pi^+\nu\bar{\nu}$; $K_L \to \pi^0 e^+ e^-$; and $K_L \to \pi^0\nu\bar{\nu}$ as function of m_t. Legend is the same as in Fig. 6.

As can be seen from Fig. 7 for the branching ratios of CP conserving decays, $K_L \to \mu\bar{\mu}$ and $K^+ \to \pi^+\nu\bar{\nu}$, with the higher f_B the upper bounds and the predictions with respect to χ^2_{min} are much smaller than the values with the lower f_B and moreover the allowed ranges become tighter and less sensitive to the change of the t-quark mass. However there is no useful bound on m_t that can be extracted from the new measurements[15] on $K_L \to \mu\bar{\mu}$ at KEK and BNL-AGS for both lower and higher f_B. Conversely, for the larger value of f_B, the direct CP violating contributions to the branching ratios in $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0\nu\bar{\nu}$ decays are enhanced especially for the lower bounds.

Finally, we remark that (a) with a even larger f_B than the value in Eq. (11) the ranges of the first two branching ratios in Eq. (12) become even tighter whereas the last two ones are more enhanced and (b) if we change the center value of B_K in Eq. (8), for example, to 0.7 or 0.9, the CP conserving decays are not much affected but the direct CP violating contributions to the decay branching ratios of $K_L \to \pi^0 e^+ e^-$ and $K_L \to \pi^0\nu\bar{\nu}$ increase by 25% or decrease by 15%.

The above significant effects on both CP conserving and violating parts of rare kaon decays arise from the use of the constraints on the CKM parameters in Eq. (4) and (5), given by the CP violating parameter ϵ and $B_d^0 - \bar{B}_d^0$ mixing where the parameter B_K and f_B enter, respectively. Clearly, a precision measurement on rare kaon decays could provide another or even substitute these constraints. For example, the precision measurement on $K^+ \to \pi^+\nu\bar{\nu}$ mode at a future phase of E787 at BNL-AGS or kaon factory[16] would give a better constraint than Eq. (5) since the standard model prediction on the decay branching ratio in (12) is very clean. This, in turn, would help to justify the case of having f_B in Eq. (10) or (11).

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