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LIGHT-CONE LAX PAIRS FOR THE W-GRAVITY THEORIES

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ABSTRACT

We present the Lax pairs, whose compatibility conditions yield the constraints and the Ward identities of the W_N-gravities in the light-cone gauge. The constraints specify the ghost structure of the theory and take in to account the possible presence of the cosmological constants. The examples of ordinary (W_2) and W_3 gravities are worked out as special cases of this general formulation.

Study of the induced gravity in the light-cone gauge has been the subject of considerable interest after Polyakov's observation of a hidden $SL(2,R)$ Kac-Moody symmetry in this problem[1]. Subsequently, the works of Knizhnik, Polyakov and Zamolodchikov[2] and others[3] have unravelled the full consequences of this symmetry. Although the non-local effective action for the induced gravity,

$$S = \alpha \int \sqrt{-g}\,(R\,\Box^{-1}P - \lambda)\,d^2x, \tag{1}$$

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becomes local both in the conformal and the light-cone gauges, the problem of quantization is much more tractable in the latter case. Here λ is the cosmological constant and $\alpha \propto c$, c being the conformal anomaly of the matter sector. The light-cone gauge is characterised by the line element

$$ds^2 = dx^+ dx^- + h(x^+, x^-) dx^{+2}, \tag{2}$$

where the metric $h(x^+, x^-)$ transforms under the residual coordinate transformations as

$$\delta h = \epsilon h' + \dot{\epsilon} - h\epsilon'. \tag{3}$$

The dot and the prime denote the derivatives with respect to x^+ and x^- variables. The simplicity of the light-cone approach can be easily seen from the equation of constant curvature, which yields a constraint

$$R = \partial_-^2 h = -2\lambda, \tag{4}$$

in contrast to the dynamical Liouville equation in the conformal gauge. The constraint equation yields the equation of motion for the induced gravity $\partial_-^3 h = 0$, allowing the expansion of $h(x^+, x^-)$ as

$$h(x^+, x^-) = J^-(x^+) - 2J^0(x^+)x^- + J^+(x^+)x^{-2}; \tag{5}$$

here $J^-(x^+)$, $J^0(x^+)$ and $J^+(x^+)$ are the currents of the $sl(2,R)$ Kac-Moody algebra.

Geometrical meaning and the origin of the current algebra symmetry have become clearer after imbedding the light-cone gravity in a gauge theory[4]. Both the Virasoro and Kac-Moody symmetries of the effective matter and the gravity sector can be obtained from the residual gauge transformations of a suitably gauge-fixed $SL(2,R)$ connection. Interestingly, the Ward identity and the constraint emerge from the compatibility equation of a pair of Lax operators involving this connection. This compatibility equation is nothing but the zero curvature condition of the $SL(2,R)$ gauge field. This approach is in complete parallel to the Lax pair formulation of the integrable, non-linear equations of KdV type and hence might shed light on the origin of these hierarchies in the lattice model approach to 2d-gravity[5].

In this talk, I will describe the appropriate generalisation of the above observations to give a proper definition of the so called W-gravity[6]. It is worth noting that a consistent covariant formulation and the geometrical interpretation of this theory is still lacking. Briefly, in case of the W-gravity, the effective matter sector is characterised in the light-cone gauge by the W_N algebras[7] which are generalisations of the Virasoro algebra. The corresponding generalised gravity sector has the $SL(N,R)$ Kac-Moody symmetry. It should be mentioned at this point that, although in principle, it is possible to obtain an effective

action in this approach by integrating the anomaly equation(zero-curvature condition)[8], it is not necessary for the purpose of getting the correlation functions. Once the constraints have been obtained and the underlying current-algebra symmetry becomes transparent, the Ward identities can be solved iteratively to yield all the relevant correlators[9].

We start with a gauge field A_μ ($\mu = +, -$), valued in the Lie algebra $sl(N,R)$. First, the A_- component of the connection will be fixed in such a way that, the remaining degrees of freedom in A_-, the so-called w_i's, will transform as Virasoro quasi-primary fields under the residual gauge transformations. In the second step, we define the A_+ sector by introducing a new family of fields, the h_i's, which are analogs of the metric field for the w_N generators. In parallel to the case of ordinary gravity, we assume that these fields originate from a covariant action which involves couplings of symmetric fields (generalised metrics) $h_{\mu\nu}$, $h_{\mu\nu\rho}$ *etc* with the tensors (stress-energy and higher spin generators) $w_{\mu\nu}$,$w_{\mu\nu\rho}$ *etc*. This action has enough symmetries to gauge-fix all the components except the h_i's and w_i's mentioned above.

Our objective now is to obtain, the relevant Ward identities and the consistent constraints involving the h_i's from the compatibility of the Lax pair $\partial_- + A_-$ and $\partial_+ + A_+$ or equivalently from the zero-curvature condition. We require the constraints and the dynamical equations of the Ward identity to appear respectively in the diagonal and the upper row of the curvature matrix. All the other entries of this matrix are zero, at least weakly. This procedure determines most of the components of A_+; the remaining freedom is related with the constraint equations themselves and the symmetry properties of the h_i's under diffeomorphism. This leads to the last step, similar to the first one, where we require the fields $h_i, i > 2$ to transform like currents under the fundamental Virasoro symmetry:

$$\delta(\rho) h_{i,i>2} = -(i-1) h_i \rho' + h_i' \rho. \tag{6}$$

For h_2, we want to maintain Eq.(3), at least for the finite subgroup of diffeomorphism. This requirement and Eq.(6) imply that $\partial^i h_i$ are dimensionless quantities and, hence, the constraint equations

$$\Phi_i \equiv \kappa_i \partial^i h_i - \lambda_i \sim 0, \quad \forall i = 2, 3, ..., N \tag{7}$$

are meaningful. This gives us the opportunity to introduce N "cosmological constants" λ_i and N free parameters κ_i.

We now propose an algorithm that realizes for any $sl(N,R)$ algebra the gauge choice

mentioned above. Let us write

$$A_- = \begin{pmatrix} 0 & -W_2 & -W_3 & -W_4 & \dots & & -W_N \\ c_1 & 0 & \dots & \dots & & & 0 \\ 0 & 1 & 0 & \dots & & & 0 \\ 0 & 0 & 1 & 0 & \dots & & 0 \\ \vdots & & & & & & 0 \\ 0 & 0 & & & & 1 & 0 \end{pmatrix}, \tag{8}$$

and

$$A_+ = \begin{pmatrix} \dots & \dots & \dots & \dots & \dots \\ H_2 & \dots & \dots & \dots & \\ H_3 & \dots & \dots & & \\ H_4 & \dots & & & \\ \vdots & & & & \\ H_N & \dots & & & \end{pmatrix}. \tag{9}$$

We will put $c_1 = 1$ for convenience; this parameter scales the central charge. It is well-known that the W_i's are *not* quasi-primary fields and the H_i's do not transform as definite spin currents. Let us consider the following change of variables:

$$\begin{cases} W_i = \sum_{j=2}^{i} (-1)^{i-j} A_{ij}(N) \partial^{i-j} w_j, \text{ with } A_{ij}(N) = \frac{\binom{i-1}{i-j}\binom{N-j}{i-j}}{\binom{i+j-1}{i-j}} \\ H_i = \sum_{j=i}^{N} B_{ji}(N) \partial^{i-j} h_j, \text{ with } B_{ij}(N) = (-1)^{i-j} \frac{\binom{i-1}{i-j}\binom{N-j}{i-j}}{\binom{2i-2}{i-j}}, \end{cases} \tag{10}$$

that we will justify later. The gauge field A_- being thus completely defined, the zero-curvature condition yields the following recurrent relations on the matrix elements a_{ij} of A_+

$$\begin{cases} a_{i,j+1} - a_{i-1,j} = a'_{i,j} - H_i W_j, \\ a_{k,k} = \sum_{i=2}^{k} (a'_{i,i-1} - H_i W_{i-1}) - \sum_{i=2}^{k} \frac{N+1-i}{N} (a'_{i,i-1} - H_i W_{i-1}), \\ a_{N-i,N} = -(a'_{N-i+1,N} - H_{N-i+1} W_N). \end{cases} \tag{11}$$

These equations determine all the elements of A_+ in terms of the $N-1$ fields H_i's. Let us ignore the constraints for the moment and concentrate on the symmetries. It is rather obvious that the issue of the gauge preserving symmetries $\delta A_- \equiv \partial_- \epsilon + [A_-, \epsilon]$ is similar to the zero-curvature condition since, on the constant elements of the gauge field in Eq.(8), the gauge preserving condition amounts to solving $\partial_- \epsilon - \partial_+ A_- + [A_-, \epsilon] = 0$. Therefore,

the matrix ϵ is determined by its first column

$$\epsilon = \begin{pmatrix} \dots & \dots & \dots & \dots & \dots \\ \epsilon_2 & \dots & \dots & \dots & \\ \epsilon_3 & \dots & \dots & & \\ \epsilon_4 & \dots & & & \\ \vdots & & & & \\ \epsilon_N & \dots & & & \end{pmatrix}, \tag{12}$$

and the recurrent equations (11) can be used to compute all the other components. The knowledge of the ϵ leads to the proper variations of the w_i's and of the h_i's. In order to fulfill the requirement that w_i's transform as Virasoro quasi-primary fields the following redefinition of the gauge parameter is necessary:

$$\epsilon_i = \sum_{j=i}^{N} B_{ji}(N) \partial^{i-j} \rho_j, \tag{13}$$

with the matrix $B(N)$ defined in Eq.(10). More precisely, looking only at the variations induced by ρ_2, the gauge transformations lead to the following expressions

$$\begin{cases} \delta w_2 = 2 w_2 \rho'_2 + w'_2 \rho_2 + C(N) \rho'''_2, \\ \delta w_{2k+1} = (2k+1) w_{2k+1} \rho'_2 + w'_{2k+1} \rho_2 + \sum_{l=1}^{k-1} C^l_{2k+1}(N) \Omega^{2k+1}_l, \\ \delta w_{2k} = 2k\, w_{2k} \rho'_2 + w'_{2k} \rho_2 + \sum_{l=1}^{k-1} C^l_{2k}(N) \Omega^{2k}_l. \end{cases} \tag{14}$$

We have defined the central charge $C(N) = \frac{(N-1)N(N+1)}{12}$ and

$$\Omega^q_l = \sum_{i=0}^{2(l-1)} \frac{(-1)^i \sigma_i(l)}{\binom{2q-4l+i-1}{i}} w^{(i)}_{q-2l} \rho_2^{(2l-i+1)}, \tag{15}$$

where $\sigma_i(l)$ are given by:

$$\sigma_0(l) = 1, \ \sigma_1(l) = 2(l-1)(2l+1), \ \sigma_2(l) = l(l-1)(2l+1)(2l-3),$$

and,

$$\sigma_{i\geq 3}(l) = \frac{\binom{2l+1}{i}\binom{2(l-1)}{i-3}\binom{2l+i-3}{3}}{\binom{i}{3}}. \tag{16}$$

Let us note that the derivatives appearing on the parameter ρ_2 are always greater or equal to three which implies that we have defined the gauge field A_- in terms of quasi-primary

fields. Futhermore, the dependence on N, i.e. on the central charge, is completely contained in the coefficients $C_k^l(N)$. A lengthy calculation leads to the following conjecture[10]

$$C_k^l(N) = \frac{(l-1)!}{2^l(2l+1)}\binom{N-k+2l}{2l}\binom{k-l-2}{l-1}\frac{(2(k-l)-1)l\,N+(k-l)(k-l-1)+l^2}{\prod_{i=1}^{l}(2(k-i)-1)} \tag{53}$$

Finally, let us mention that we obtain the correct symmetry varitions of the h_i's for $i > 2$:

$$\delta h_{i>2} = -(i-1)h_i\,\rho_2' + h_i'\rho_2.$$

However, the variation of h_2 involves higher derivative terms which vanish only in the finite subgroup of the diffeomorphisms.

The constraints and the"cosmological constants" can now be added to the A_+ sector. We consider the following shift of the previous gauge field A_+:

$$A_+ \to A_+ + \begin{pmatrix} 0 & \lambda_2-\kappa_2\partial^2 h_2 & 0 & 0 & \dots & 0 \\ 0 & 0 & \lambda_3-\kappa_3\partial^3 h_3 & 0 & \dots & 0 \\ 0 & 0 & 0 & \lambda_4-\kappa_4\partial^4 h_4 & \dots & \\ \vdots & & & & & \vdots \\ 0 & \dots & & & 0 & \lambda_N-\kappa_N\partial^N h_N \\ 0 & \dots & & & & 0 \end{pmatrix}. \tag{17}$$

The resulting zero curvature equations exhibit the expected dynamical equations on the fields w_k with, a shift in the coefficients of the higher derivative of the h_i's. The diagonal entries of F_{+-} now exhibit the expected $N-1$ equations $\Phi_i \sim 0$.

It is easy to show that the presence of the constraints restricts the gauge preserving symmetry to that of the finite subgroup of the diffeomorphisms. Of course, such restriction is also seen in the quantum regime where the relevance of the constraints is well understood in terms of the ghost system. Let us only mention that, similar to Eq.(5), the operators h_i's are expanded in terms of $sl(N,R)$ current operators

$$h_i = \sum_{n=2}^{2i-2} x^{-n}\,J_i^{(n)}(x^+). \tag{18}$$

Such expansions are legitimate by the equations $\partial^{2i-1}h_i = 0$ that generalize the constant curvature equation and are easily obtainable from the dynamical equations with the w_i's

equal to zero. Then, the equations $\Phi_i \sim 0$ lead to the following system of constraints

$$J_i^{(i)} \sim \text{const}, \qquad \text{and} \qquad \begin{cases} J_i^{(i+1)} \sim 0, \\ J_i^{(i+2)} \sim 0, \\ \vdots \\ J_{i,i>2}^{(j)} \sim 0, \\ \vdots \\ J_i^{(2i-2)} \sim 0. \end{cases} ,i = 2,3,\dots,N \tag{19}$$

Dimensional analysis of this set of constraints completely describes the weight spectrum of the necessary ghost systems. The resulting central charge contribution is[11] $c_{ghost} = -n^2(n+1)^2 + n(n+1) + 2$.

In the following we will work out the examples of the much studied ordinary(w_2) and the w_3 gravities in light of the above general formulation for the sake of consistency and clarity. In the former case the gauge choice

$$A_- = \begin{pmatrix} 0 & -t \\ \lambda_- & 0 \end{pmatrix}, \quad A_+ = \begin{pmatrix} -\frac{1}{2}h' & \lambda_+ - (\frac{1}{2\lambda_-}+\kappa)h'' - th \\ \lambda_- h & \frac{1}{2}h' \end{pmatrix} \tag{21}$$

yields $\delta t = 2t\epsilon' + \epsilon t' + \frac{1}{2\lambda_-}\epsilon'''$ under the residual gauge transformations that maintain A_-. This variation is easily recognized as the variation of the stress-energy tensor in a conformal field theory, with a central charge c equal to $\frac{1}{2\lambda_-}$. Furthermore, all the parameters of the gauge variation are not independent:

$$\begin{cases} \epsilon^- = +\dfrac{1}{\lambda_-}\epsilon, \\ \epsilon^0 = -\dfrac{1}{2}\partial\epsilon, \\ \epsilon^+ = -(\dfrac{1}{2\lambda_-}\partial^2 + t)\epsilon; \end{cases} \tag{22}$$

and it can be easily checked that $h(x^+,x^-)$ does transform as the light-cone metric field:

$$\delta h = -h\epsilon' + \epsilon h' + \dot{\epsilon}. \tag{23}$$

The zero curvature condition $F = 0$ provides the constraint equation and the gravitational Ward identity given respectively by

$$\partial_-^2\,h = \frac{\lambda_+}{\kappa}, \tag{24a}$$

and

$$\partial_+ t = \Big((c+\kappa)\partial_-^3 + \partial t + t\partial\Big)h. \tag{24b}$$

It is worth emphasising that for taking into account the constraint with the possible presence of a cosmological constant, the parameter κ should be nonvanishing. It should be pointed out that it is precisely the case in the quantum Ward identity of the 2d-gravity. In this case however, the nonvanishing κ is exactly calculable. In our gauge choice with $\kappa \neq 0$, the gauge preserving symmetries are restricted to those described by ϵ such that $\partial\epsilon = 0$ if $\lambda_+ \neq 0$ or $\partial^2\epsilon = 0$ if $\lambda_+ = 0$. The parameter of the residual coordinate invariance is restricted as can be odserved in the work of KPZ[2].

In the W_3 case we define the gauge field A_-:

$$A_- = \begin{pmatrix} 0 & -W_2 & -W_3 \\ c_1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \tag{26}$$

where the c_1 is a constant and the W_i's are dynamical fields with dimension i (the dimension of $\frac{\partial}{\partial x_-}$ is $+1$).

As they stand, the dynamical fields in A_- do not satisfy the Virasoro symmetry properties mentioned before. The gauge transformations that preserve Eq.(26) are found to be described by the functions ϵ^- and ϵ^{--} or, more conveniently, by the arbitrary functions ρ_2 and ρ_3:

$$\begin{cases} \epsilon^- = c_1\rho_2 - \frac{c_1}{2}\rho_3', \\ \epsilon^{--} = c_1\rho_3, \end{cases} \tag{27}$$

and, defining the w_i's by the following change of variables

$$W_2 = w_2, \quad W_3 = w_3 - \frac{1}{2}w_2'. \tag{28}$$

It is easy to check that,

$$\begin{aligned} \delta(\rho_2)\,w_2 &= 2w_2\,\rho_2' + \rho_2\,w_2' + \frac{2}{c_1}\rho_2''', \\ \delta(\rho_2)\,w_3 &= 3w_3\,\rho_2' + \rho_2\,w_3', \\ \delta(\rho_3)\,w_2 &= 3w_3\,\rho_3' + 2\rho_3\,w_3', \\ \delta(\rho_3)\,w_3 &= -\frac{1}{12}\big(2w_2'''\,\rho_3 + 9w_2''\,\rho_3' + 15w_2'\,\rho_3'' + 10w_2\,\rho_3'''\big) \\ &\quad - \frac{2c_1}{3}\big({w_2}^2\rho_3' + w_2w_2'\,\rho_3\big) - \frac{1}{6c_1}\rho_3''''. \end{aligned} \tag{29}$$

We can consistently construct a Poisson bracket structure between the w_i's, starting with the Virasoro algebra satisfied by the generator of diffeomorphisms w_2:

$$\{w_2(x), w_2(y)\} = \big(c\partial^3 + w_2\partial + \partial w_2\big)\delta(x-y) \tag{30}$$

with $c = \frac{2}{c_1}$. The primary field w_3 can be interpreted as the generator of the spin-3 transformations:

$$\begin{aligned} &\{w_3(x), w_3(y)\} = -\frac{1}{12}\big(c\partial^5 + 2w_2\partial^3 + 3\partial w_2\partial^2 + 3\partial^2 w_2\partial + 2\partial^3 w_2 + \frac{16}{c}w_2\partial w_2\big)\delta(x-y), \\ &\{w_2(x), w_3(y)\} = (w_3\partial + 2\partial w_3)\delta(x-y) \\ &\{w_3(x), w_2(y)\} = (2w_3\partial + \partial w_3)\delta(x-y). \end{aligned} \tag{31}$$

As can be seen from the Poisson bracket $\{w_3(x), w_3(y)\}$, there are non-linear terms which go to zero in the so-called "classical limit" $c \to \infty$.

The A_+ field is written as,

$$A_+ = \begin{pmatrix} \dots & \lambda_2 + \dots & \dots \\ H_2 & \dots & \lambda_3 + \dots \\ H_3 & \dots & \dots \end{pmatrix}, \tag{32}$$

with $H_i \sim h_i + \dots$.

The components of A_+ are determined in such a way that the curvature is of the following form

$$\begin{pmatrix} \Phi_2 & E_1 & E_2 \\ 0 & \Phi_3 - \Phi_2 & 0 \\ 0 & 0 & -\Phi_3 \end{pmatrix}, \tag{33}$$

with

$$\begin{cases} \Phi_2 = \kappa_2\partial^2 h_2 - \lambda_2, \\ \Phi_3 = \kappa_3\partial^3 h_3 - \lambda_3 \end{cases}, \tag{34}$$

and $E_1 = -\partial_+ w_2 + \cdots$, $E_2 = -\partial_+ w_3 + \cdots$. The constants κ_i are free parameters. Together with the "cosmological constants" λ_i, they represent the presence or the absence of the constraints. Let us notice that the Eqs.(34) are legitimate only if the metric fields h_i have the proper dimensions such that $\partial^i h_i$ is dimensionless. In fact, the zero-curvature condition *alone* does not define a unique A_+ since we obtain

$$\begin{cases} H_2 = c_1 h_2 + a\,h_3', \\ H_3 = c_1 h_3, \end{cases} \tag{35}$$

where a is free. However, choosing this parameter equal to zero spoils the symmetry property of h_2 under the diffeomorphisms. Keeping the interpretation of h_2 as a metric

field, i.e. Eq.(3), gives $a = -\frac{c_1}{2}$. Then, the remaining varations are found to be

$$\begin{aligned}
\delta(\rho_2)\, h_3 &= -\,2h_3\,\rho_2' + \rho_2\, h_3', \\
\delta(\rho_3)\, h_2 &= \frac{1}{6}h_3\,\rho_3''' - \frac{1}{4}h_3'\,\rho_3'' + \frac{1}{4}h_3''\,\rho_3' - \frac{1}{6}h_3'''\,\rho_3 + \frac{4}{3c}w_2\,h_3\,\rho_3' - \frac{4}{3c}w_2\,h_3'\,\rho_3 \\
\delta(\rho_3)\, h_3 &= -\,h_2\,\rho_3' + 2h_2'\rho_3 + \dot{\rho}_3.
\end{aligned} \tag{36}$$

Finally, the dynamical equations $E_{1,2} = 0$, obtained from the zero-curvature condition are the following:

$$\partial_+ w_2 = \big((c+\alpha_2)\partial^3 + w_2\partial + \partial w_2\big)h_2 + \big(\beta_2\partial^4 + (w_3\partial + 2\partial w_3\big)h_3, \tag{37a}$$

$$\begin{aligned}
\partial_+ w_3 =& \big(\alpha_3\partial^3 + 2w_3\partial + \partial w_3\big)h_2 \\
&-\frac{1}{12}\big((c+\beta_3)\partial^5 + (2\partial^3 w_2 + 3\partial^2 w_2\partial + 3\partial w_2\partial^2 + 2w_2\partial^3) + \frac{16}{c}w_2\partial w_2\big)h_3.
\end{aligned} \tag{37b}$$

The α_i's and β_i's are linear combinations[2] of the κ_i's. It is remarkable that they only occur in the shift of the central charges, as in the $sl(2,R)$ case. It is worth mentioning that this system leads to the Boussinesq equation by setting $h_2 = 0$ and $h_3 = \frac{1}{12}$.

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[10] This analysis has been made possible by the use of symbolic programming with Mathematica. Formula (53) has been checked up to N=40, using a program available by e-mail from the following address: lina@lps.umontreal.ca.

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[2] Notice that the explicit expressions for the α_i's and the β_i's are irrelevant in this classical analysis.