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Superconductivity in a planar field theory through Kosterlitz-Thouless mechanism

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We present a 2+1 dimensional model which exhibits superconductivity through the Kosterlitz-Thouless (KT) mechanism with a non-zero critical temperature T_c. The model has three phases: a confined-vortex phase which is superconducting at low temperatures, an intermediate-temperature phase with deconfined vortices, and an unbroken-symmetry phase at high temperatures. The ground state does not break P or T and the ratio of the energy gap to T_c is bigger than the conventional BCS value.

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The study of various models exhibiting planar superconductivity at non-zero temperatures is obviously of considerable practical and theoretical interest. From the theoretical point of view this is due to the fact that the conventional explanation of this phenomenon based on spontaneous breaking of electromagnetism is not applicable to a planar world. This result follows from the works of Hohenberg,[1] Mermin and Wagner[2] and Coleman:[3] the critical temperature for breaking of a continuous symmetry in $2+1$ dimensions is strictly zero. To circumvent this problem, one usually invokes a small interplanar coupling or takes recourse in unconventional mechanisms, *e.g.*, anyon superconductivity.[4]

In this letter, we analyse a 2+1 dimensional relativistic model with four-fermion interaction (reminiscent of the BCS theory) and show that it posseses a superconducting ground state at nonvanishing temperature, because of the vortex confinement mechanism of Kosterlitz and Thouless.[5] The motivation to study this model lies in the appearance of relativistic fermions as the relevant low-energy degrees of freedom in the strong coupling limit of the Hubbard and related models,[6] widely believed to be relevant for the high-T_c superconductors. Furthermore, four-Fermi couplings also arise naturally in the above-mentioned models in the presence of doping.[7] The goal of the study is to capture the generic properties of this class of models using large-N renormalizability of the four-Fermi couplings in 2+1 dimensions[8] and to find the similarities and dissimilarities with the standard BCS theory. Finally, although not conclusive, there is some experimental indication as to the KT nature of the phase transition in the high-T_c materials,[9] giving urgence to the analysis of various models exhibiting such behaviour.

The model we investigate is described by the Lagrangian

$$\mathcal{L} = \bar{\psi}_\alpha(i\partial\!\!\!/ - eA\!\!\!/)\psi_\alpha - \frac{1}{4\lambda N}\bar{\psi}_\alpha\psi^c_\alpha\bar{\psi}^c_\beta\psi_\beta. \tag{1}$$

Here $\lambda > 0$, $\psi^c = C\bar{\psi}^T = i\gamma^2\psi^*$ is the charge conjugate field, and the flavor indices α, β range from 1 to N. Coupling to the photon field endows the model with a U(1) gauge symmetry. The choice of four-fermi term in (1) was motivated by an attempt to mimic BCS superconductivity, where the interaction $\psi^\dagger_\uparrow\psi^\dagger_\downarrow\psi_\downarrow\psi_\uparrow$ leads to a charge-two condensate, in a relativistic setting. The more obvious four-fermi term $(\bar{\psi}\psi)^2$ found, for instance, in the Gross-Neveu model,[10] leads to a neutral condensate.

In what follows, we will use a Hubbard-Stratonovitch transformation to linearize the four-Fermi term, and find the low-energy effective theory for the auxiliary field ϕ in the large-N approximation. As a first step, the effective potential at zero temperature will be computed to study the question of radiative breaking of the U(1) symmetry *à la* Coleman-Weinberg.[11] If λ is below a certain critical value, we will see that ρ attains a nonzero expectation value. Next, the situation at finite temperature will be explored. We will see that $\rho \equiv |\phi|$ still has a nonzero expectation value up until a certain critical temperature T_1 (where a second-order phase transition occurs). Below T_1, the nature of the phase is rather inevident, since the field ϕ, being charged, cannot have an expectation value. In fact, we will find two separate phases, separated by a second critical temperature $T_{\rm KT} < T_1$. Both these phases contain a high density of vortices. However, below $T_{\rm KT}$ vortices and antivortices come in tightly-bound pairs, whereas in the region $T_{\rm KT} < T < T_1$ the vortices are essentially free. This confinement/deconfinement has a dramatic effect on the properties of the system. We will see that above $T_{\rm KT}$ (but below T_1) the vortices'

contribution to the current-current correlation function serves to cancel the no-vortex contribution, and we find no Meissner effect, while below $T_{\rm KT}$ the vortex-antivortex pairs make essentially no contribution, and we are left with the no-vortex contribution, leading to the Meissner effect.

The linearized Lagrangian reads

$$\mathcal{L} = \bar{\psi}_a(i\partial\!\!\!/ - eA\!\!\!/)\psi_a + \frac{1}{2}\left(\phi^*\bar{\psi}^c_a\psi_a - \phi\bar{\psi}_a\psi^c_a\right) - \lambda N\phi^*\phi; \tag{2}$$

integrating over the fermions yields the effective action for the condensate ϕ and gauge fields:

$$e^{iS_{\rm eff}[A,\phi,\phi^*]} = \int \mathcal{D}\bar{\psi}\mathcal{D}\psi\, e^{iS}.$$

The effective potential can be computed using the method of Coleman and Weinberg;[11] from this, the gap equation for $\rho \equiv |\phi|$ (which is the fermion mass) is

$$\frac{\delta V_{\rm eff}}{\delta\rho} = 0 = 2\lambda\rho - \frac{\rho}{2\pi}(\sqrt{\Lambda^2+\rho^2} - |\rho|). \tag{3}$$

where Λ is a cutoff. We see that if ($\lambda < \lambda_c \equiv \Lambda/4\pi$), there is a solution for $\rho \neq 0$. (Notice that the four-fermi coupling constant is $\sim 1/\lambda$, so that $\lambda < \lambda_c$ is a strong coupling régime.) There are two interesting limits: (a) $\lambda \lesssim \lambda_c$, where $|\rho| \cong 4\pi(\lambda_c - \lambda)$, in which case the mass gap is small; and (b) $\lambda \ll \lambda_c$, where $|\rho| \cong 2\pi\lambda_c^2/\lambda$, in which case the coupling and the mass gap are large and the fermion mass plays the role of the cutoff.

The next term in a gradient expansion is the two-derivative term for the scalar field; using a diagrammatic method[12] similar to Coleman and Weinberg, the coefficient is found to be $5/96\pi|\rho|$, the important point being that it is positive.

Thus, we obtain the following effective Lagrangian near the critical point ($\lambda \lesssim \lambda_c$):

$$\mathcal{L}_{\rm eff}/N = \frac{5}{96\pi|\rho|}|(\partial_\mu - 2ieA_\mu)\phi|^2 - (\lambda - \lambda_c)\phi^*\phi - \frac{1}{6\pi}(\phi^*\phi)^{3/2}. \tag{4}$$

In addition one has the usual Maxwell term, and possibly a Chern-Simons term. However one easily checks that the latter does not arise; thus, we do not get P or T violation.

To investigate the situation at finite temperature, the effective potential can be calculated using the imaginary-time formalism. While the gap equation itself is not particularly transparent, it can be solved for the temperature T_1 at which ρ vanishes, yielding

$$4\pi\lambda = \Lambda + 2T_1\log(1 + e^{-\Lambda/T_1}) - 2T_1\log 2. \tag{5}$$

In the phase $\lambda \lesssim \lambda_c$, $T_1 = 2\pi(\lambda_c - \lambda)/\log 2$; in the extremely strong coupling phase $\lambda \ll \lambda_c$ and $T_1 \approx \pi\lambda_c^2/\lambda$ can be large. The order parameter approaches zero smoothly, in both cases.

To establish the KT nature of the phase below T_1, let us compute the *vev* of the charged order parameter ϕ. For that purpose we observe that the order parameter can be written as $\phi = \rho e^{i\theta}$, ρ being the massive mode and θ representing the massless excitation. Using the long wavelength propagator for θ and under the approximation that ρ is nearly constant at low temperatures, we find $\langle\phi\rangle \simeq \rho\langle e^{i\theta}\rangle = \rho e^{-\langle\theta^2\rangle/2} \simeq \text{const}(\beta\mu)^\eta$. Here, μ is an infrared cutoff introduced in the θ propagator; this originates from the fact that at finite temperature the $n = 0$ mode (in the imaginary-time formalism $(k^0)_n = 2n\pi/\beta$) is a massless field, which introduces an infrared divergence in the correlation function. In the limit $\mu \to 0$, $\langle\phi\rangle = 0$ since $\eta = 1/4\pi\beta K$ and $K \equiv 5\rho/48\pi$ is positive. We have checked[13] that beyond the leading order in $1/N$, this neutral order parameter is not affected by

infrared divergences. Since the corrections are down by $1/N$, in what follows we use the one-loop value for ρ. Note that the separation of ϕ in to pion-like modes *i.e.*, $\phi = \phi_1 + i\phi_2$ is not stable above $T = 0$.[14]

Now (4) will be analysed in the London limit, to compute the contribution of the phase θ to the photon effective action. With ρ fixed, (4) appears to be a trivial quadratic action for θ. However, it must be remembered that θ is a multivalued variable, and naively performing a Gaussian integration over it fails to capture the contribution of the vortices. This is remedied[15] by the replacement $\partial_\mu\theta \to \partial_\mu\theta - i\varphi^*\partial_\mu\varphi$ with $\varphi^*\varphi = 1$; the new θ is now treated as a single-valued field and φ describes the vortex dynamics. The action is rewritten in terms of an auxiliary current J_μ:

$$\mathcal{L}_{\rm eff}/N = -\frac{1}{2K}J_\mu J^\mu - J_\mu\left(\partial^\mu\theta - i\varphi^*\partial^\mu\varphi + 2eA^\mu\right). \tag{6}$$

Since θ is single-valued, it can be integrated out, giving the constraint $\partial^\mu J_\mu = 0$, which is readily solved by $J_\mu(x) = \epsilon_{\mu\nu\lambda}\partial^\nu a^\lambda$, where the field $a^\mu(x)$ is defined globally. The Lagrangian now becomes

$$\mathcal{L}_{\rm eff}/N = -\frac{1}{4K}f_{\mu\nu}^2 - 2e\epsilon_{\mu\nu\lambda}a^\mu\partial^\nu A^\lambda - 2\pi a^\mu J_\mu^{(v)}, \tag{7}$$

where $J_\mu^{(v)} = (2\pi i)^{-1}\epsilon_{\mu\nu\lambda}\partial^\nu(\varphi^*\partial^\lambda\varphi)$ is the conserved vortex current. The globally defined gauge field can now be integrated out in the Landau gauge, yielding

$$-\frac{1}{2\pi^2 KN}\mathcal{L}_{\rm eff} = J_\mu^{(v)}(x)\frac{g^{\mu\nu}}{\partial^2}J_\nu^{(v)}(x) + \frac{2e}{\pi}J_\mu^{(v)}\frac{g^{\mu\nu}}{\partial^2}\epsilon_{\nu\lambda\rho}\partial^\lambda A^\rho + \frac{e^2}{2\pi^2}F^{\mu\nu}\frac{1}{\partial^2}F_{\mu\nu}. \tag{8}$$

The last term in $\mathcal{L}_{\rm eff}$ is the contribution of the topologically trivial piece and is relevant for superconductivity. The contribution of this term to the electromagnetic current-current

correlator is:

$$\langle j^\mu(x)j^\nu(y)\rangle = -4e^2KN\left(\frac{\partial^\mu\partial^\nu}{\partial^2} - g^{\mu\nu}\right)\delta^3(x-y), \tag{9}$$

indicating a pole at zero momentum and hence superconductivity. This term in combination with the Maxwell action yields a mass $M_v^2 = 4e^2KN$ for the photon and hence leads to the Meissner effect. Since the scalar mass near the critical point is $M_s^2 = \rho^3 N/\pi K$, the nature of the superconductivity (type I or type II) can in principle be inferred from $M_s/M_v = 48\pi\sqrt{\lambda_c - \lambda}/5e$ which is independent of N. As mentioned before, near the critical point $\lambda_c - \lambda$ is small; however, in 2+1 dimensions the electromagnetic coupling is dimensionful (as is λ), and without knowledge of its value nothing meaningful can be said about the nature of the superconductivity.

Below we will show that at low temperatures, when the vortices are confined, their contribution to the $F^{\mu\nu}\partial^{-2}F_{\mu\nu}$ term is zero and hence they do not affect the Meissner effect. In contrast, above the KT phase transition temperature the vortex contribution to this term is crucial: it exactly cancels the contribution from the single-valued part (that appearing in (8)), thereby destroying superconductivity.

The vortex current is conserved and the winding number of φ is always an integer, allowing us to write $J_0^{(v)} \equiv \rho_v(x) = \sum_a m_a\delta(x-x_a)$ and $J_v^i(x) = \sum_a m_a\dot{x}_a^i\delta(x-x_a)$, where m_a is the integer vorticity. Considering only static configurations ($J_v^i = 0$) and looking at the $n = 0$ mode (the rest are massive and hence damped), at finite temperature we get

$$S_{\rm eff}/N = \pi K\beta\sum_{a\dot{a}} m_a m_{\dot{a}}\ln|\vec{r}_a - \vec{r}_{\dot{a}}| - 2eK\beta\sum_a m_a\int d^2r B(\vec{r})\ln|\vec{r} - \vec{r}_a|; \tag{10}$$

without the interaction term this is the action of the familiar XY model. The confining-deconfining phase transition in this model occurs at $\pi KN\beta \doteq 2$, which after taking into

account the finite temperature corrections gives $x\tanh(x)$=8/N with $x = \rho/2T_{\rm KT}$. Solving the above equation gives $\rho N/16 < T_{\rm KT} < \rho N/4\sqrt{2}$. Since this analysis is valid for $T_{\rm KT} < T_1$, this implies $N < 4$. Below we will show, first with a physical argument and then with a direct calculation, that this KT transition is actually a transition from a superconducting state to a non-superconducting one, and so the superconducting critical temperature T_c is actually equal to $T_{\rm KT}$. Thus, the physically relevant quantity, $2\rho/T_c$, is restricted to the range $\frac{8\sqrt{2}}{N} < 2\rho/T_c < \frac{32}{N}$. In the physically interesting theories $N = 2$;[6] notice that in this case the above ratio is higher than the BCS value of 3.52. This happens because the gap ρ vanishes at a temperature higher than $T_{\rm KT}$.

Assuming that the interaction with the electromagnetic field is small, the effect of the vortices on superconductivity can be easily inferred. Physically, in the confined phase, a slowly varying magnetic field sees pairs of tightly bound vortices with opposite vortex charges and hence the net contribution from each pair vanishes, leaving untouched the contribution of the single-valued part. Concretely, in the static limit

$$\langle j^i(q)j^j(-q)\rangle = \frac{\delta^2 S_{eff}}{\delta A^i(q)\delta A^j(-q)} = \left(\delta^{ij} - \frac{q^i q^j}{q^2}\right)q^2\langle\rho(q)\rho(-q)\rangle. \tag{11}$$

In the confined phase, a straightforward calculation yields $\langle\rho(q)\rho(-q)\rangle \sim q^{p-2}$, where $p > 4$,[16] indicating that the zero-momentum pole in $\langle j^i(q)j^j(-q)\rangle$ does not get a contribution from the confined vortices. This also implies that in $\mathcal{L}_{\rm eff}$ the $F^{\mu\nu}\partial^{-2}F_{\mu\nu}$ term, and hence the Meissner effect, is not affected by the vortex sector. However, above the KT temperature, when the vortices are free and the interaction between them is screened like in a plasma we expect rather substantial contribution from that sector. In this phase,

directly integrating out the J^μ variable yields:

$$\mathcal{L}_{\rm eff}^{\rm vortex} = NKe^2F^{\mu\nu}\frac{1}{\partial^2}F_{\mu\nu}, \tag{12}$$

with precisely the opposite sign to that of the previous contribution. Hence, this confirms the physical argument given above that in the deconfined phase the superconductivity is destroyed and $T_c = T_{KT}$. We note that this analysis is also applicable to other models[17], but unlike these ones we do not have gauge fields except the photon.

To conclude, the model presented here exhibits superconductivity through the KT mechanism without parity breaking and is based on a relativistic field theory which arises naturally in many strongly correlated electronic systems. The gap to T_c ratio is high as compared to the BCS value because of the fact that KT phase transition temperature is lower than that of the neutral order paramater responsible for the gap in the fermion spectrum. This indicates that the metallic phase might have interesting properties. Finally, we remark that the relevance of KT transitions to quasi-two-dimensional systems has been discussed previously;[18] such an extension to the model described here is currently in progress and will be described elsewhere.

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