— UdeM-LPN-TH-119 —

Fermilab Library
0 1160 0022119 6

UdeM-LPN-TH-119
October 1992

Loop Regularization and Renormalization in QED Theory

Zhong-Hua Wang[1]
Laboratoire de Physique Nucléaire, Universite de Montréal,
C. P. 6128, Succursale A, Montréal(Québec) H3C 3J7, Canada.

Han-Ying Guo
Institute of theoretical physics, Academia Sinica, Beijing 100080, China.

ABSTRACT

We study the QED theory by using the loop regularization method and find that the regularized theory is covariant and can lead to all the results that are derived by using other regularization methods.

In our last paper[1], we developed a new method for regularization and renormalization in the field theory. By using this method, we successfully got the renormalized ϕ^4 field theory to the second order of the coupling constant.

In this paper, we would like to generalize this method to the gauge field theories. As an example, we study the QED theory at the one loop order by using this method.

The action of the theory is

$$S(A,\psi) = \int d^4x\{-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{2}(\partial\cdot A)^2 + \bar{\psi}(i\not{\partial} - m)\psi - e\bar{\psi}\not{A}\psi\} \tag{1}$$

where ψ represents the chiral fermion field, A the $U(1)$ gauge field, $\bar{\psi} = \psi^+\gamma_0$, $F_{\mu\nu} = (\partial_\mu A_\nu - \partial_\nu A_\mu)$ and $\not{a}$ is the abbreviation of $a_\mu\gamma^\mu$. In this paper, we choose the Feynman gauge $\lambda = 1$ for simplicity. In this case the Feynman rules of the theory are:

$$\begin{aligned} &= (\tfrac{i}{\not{p}-m})_{\alpha\beta}, \\ &= \tfrac{-ig_{\mu\nu}}{k^2}, \\ &= -ie(\gamma_\mu)_{\beta\alpha}. \end{aligned} \tag{2}$$

The divergent degree of the 1PI diagram with L loops, I_f internal fermion lines and I_g gauge field lines is

$$\delta = 4L - I_f - 2I_g. \tag{3}$$

[1] E-mail: wang@lps.umontreal.ca

Hence, the divergent 1PI diagrams at the one loop order are those that are shown in fig. 1. In order to get the renormalized QED theory at the one loop order, we have to calculate the divergent diagrams (a), (b) and (c) (fig. 1). Their integral expressions in the momentum space are

$$\begin{aligned}(a) &= -e^2 \int \frac{d^4p}{(2\pi)^4} Tr[\gamma_\mu \frac{1}{\not p-\not k-m}\gamma_\nu \frac{1}{\not p-m}],\\ (b) &= -e^2 \int \frac{d^4k}{(2\pi)^4}(\gamma^\mu \frac{1}{\not p-m}\gamma_\mu)_{\beta\alpha}\frac{1}{(p-k)^2},\\ (c) &= -e^3 \int \frac{d^4l}{(2\pi)^4}(\gamma^\rho \frac{1}{\not l-\not p-m}\gamma_\mu \frac{1}{\not l-m}\gamma_\rho)_{\beta\alpha}\frac{1}{(p-l)^2}.\end{aligned} \tag{4}$$

Obviously, all of them are ultra-violet divergent. (a), (b) and (c) are quadratically, linearly and logarithmically divergent recpectively.

In order to let these integrals at the one loop order to be meaningful, we have to regularize them. The spirit of the regularization is the same as that in the ϕ^4 field theory[1]. In the QED theory, the regularization method can be stated as follows: we first make a shift of the electron mass by μ and regard the term $-\mu\bar\psi\psi$ as the perturbation term together with $\bar\psi \not A\psi$. The new action turns out to be

$$\hat S(A,\psi,\mu) = S(A,\psi) - \int d^4x \mu\bar\psi\psi. \tag{5}$$

So besides (2), we can have another vertex whose Feynman rule is:

$$= -i\mu. \tag{6}$$

The role of this vertex is to cut the fermion propagator into several parts regarding the number of such kind of vertices in the original fermion propagator. In order to regularize the divergent 1PI amplitude at the L-th loop order $\Gamma_L^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g})$ with n_f external fermion lines and n_g external gauge field lines, we first calculate

$$\Gamma_L^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g};q;\mu) \tag{7}$$

which is the summation of the diagrams with all possible $2q$ insertions of vertices (6) in the fermion lines of each loop of the original diagram. Because each insertion of (6) decreases the divergent degree by 1, so the divergent degree of (7) turns to be:

$$\delta' = (4-2q)L - I_f - 2I_g. \tag{8}$$

When q is large enough, $\Gamma_L^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g};q;\mu)$ is convergent and well defined. After calculating $\Gamma_L^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g};q;\mu)$, we make an analtytic continuation of q from the integer to the complex number and when $q\to 0$, we get the original amplitude, i.e.:

$$\Gamma_L^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g}) = \lim_{q\to 0}\Gamma_L^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g};\mu;q). \tag{9}$$

So $\Gamma_L^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g};\mu;q)$ can be regarded as the regularized form of $\Gamma_L^{(n_f,n_g)}(p_1,\cdots,p_{n_f};k_1,\cdots,k_{n_g})$. The renormalization of the theory is the same as in the usual approach. Namely, we subtract the divergent part of the vertices of the theory at each loop order by adding the relevant counterterms to the original action. The new action is

the renormalized one. All the renormalized 1PI correlation functions calculated from the renormalized action are finite when $q\to 0$.

Regularization and evaluation of divergent integrals:

Now we set about to calculate the one loop divergent diagrams (a), (b) and (c) by using the regularization method stated above.

In order to calculate the divergent diagram (a), we attach to one of the internal fermion lines in the divergent loop with i and to another one with $2q-i$ vertices (6) and the diagram (a) is turned into (a_i')(fig. 2).

The amplitude (a_i') can be expressed as

$$(a_i') = -\mu^{2q}e^2\int \frac{d^4p}{(2\pi)^4} Tr[\gamma_\mu\Big(\frac{1}{\not p-\not k-m}\Big)^{i+}\gamma_\nu\Big(\frac{1}{\not p-m}\Big)^{2q-i+1} \tag{10}$$

and the regularized amplitude of (a) is defined as

$$\Pi_{\mu\nu} = \sum_{i=0}^{2q}(a_i'). \tag{11}$$

We can prove that the regularized amplitude (11) is covariant for q large enough. Because

$$\begin{aligned}k^\mu\Pi_{\mu\nu} &= -\mu^{2q}e^2\int\frac{d^4p}{(2\pi)^4}\sum_{i=0}^{2q}Tr[\not k\Big(\frac{1}{\not p-\not k-m}\Big)^{i+1}\gamma_\nu\Big(\frac{1}{\not p-m}\Big)^{2q-i+1}]\\ &= -\mu^{2q}e^2\int\frac{d^4p}{(2\pi)^4}(\sum_{i=1}^{2q+1}-\sum_{i=0}^{2q})Tr[\Big(\frac{1}{\not p-m}\Big)^{2q-i+1}\Big(\frac{1}{\not p-\not k-m}\Big)^{i}\\ &= -\mu^{2q}e^2\int\frac{d^4p}{(2\pi)^4}Tr[\Big(\frac{1}{\not p-\not k-m}\Big)^{2q+1}\gamma_\nu-\Big(\frac{1}{\not p-m}\Big)^{2q+1}\gamma_\nu].\end{aligned} \tag{12}$$

When q is large enough, this integral is convergent, we can make a shift of the integral variable and the two terms in (12) cancel with each other. Hence the regularized form (11) is covariant for q large enough.

The calculation of $(a_i')s$ are very tedious. Here we only present the calculation of (a_0'). Other $(a_i')s$ can be calculated in a similar way.

By using the Feynman parametrization, (a_0') can be reexpressed as:

$$(a_0') = -\mu^{2q}e^2\int_0^1 d\alpha(1+2q)\alpha^{2q}\int\frac{d^4p}{(2\pi)^4}\frac{Tr[\gamma_\mu(\not p-\alpha\not k+m)\gamma_\nu(\not p+(1-\alpha)\not k+m)^{1+2q}]}{[p^2+\alpha(1-\alpha)k^2-m^2]^{2+2q}} \tag{13}$$

where we have made a shift of $p\to p+(1-\alpha)k$ and q, at this stage, is still regarded as an integer. Hence, we can expand the trace term in (13) into power series with respect to p. After that, we make an extension of q from the integer to the complex number and when

$q \to 0$, we find that the terms contributing to (13) are

$$
\begin{aligned}
& Tr[\gamma_\mu(\not p - \alpha \not k + m)\gamma_\nu(\not p + (1-\alpha)\not k + m)^{1+2q}] \\
& = 4\{(p^2)^q[2p_\mu p_\nu - g_{\mu\nu}p^2 + \alpha(1-\alpha)(k^2 g_{\mu\nu} - 2k_\mu k_\nu) + (1+2q)m^2 g_{\mu\nu}] \\
& +2q(1-\alpha)(1-2\alpha)(p^2)^{q-1}p\cdot k[p_\mu k_\nu + k_\mu p_\nu - p\cdot k g_{\mu\nu}] \\
& +q(1-\alpha)^2(p^2)^{q-1}k^2[2p_\mu p_\nu - g_{\mu\nu}(p^2)] - 2q(1-\alpha)^2(p^2)^{q-2}(p\cdot k)^2[2p_\mu p_\nu - g_{\mu\nu}(p^2)] \\
& +qm^2(p^2)^{q-1}[2p_\mu p_\nu - g_{\mu\nu}(p^2)]\} \\
& +\textit{other terms that do not contribute to the intrgral } (13).
\end{aligned}
\tag{14}
$$

At this stage, (13) can be integrated by using the following formulas:

$$
\begin{aligned}
& \int \frac{d^4p}{(2\pi)^4}\frac{(p^2)^\beta}{(p^2+M^2)^A} = \frac{i}{(4\pi)^2}\frac{\Gamma(2+\beta)\Gamma(A-2-\beta)}{\Gamma(A)}(M^2)^{2+\beta-A} \\
& \int \frac{d^4p}{(2\pi)^4}\frac{(p^2)^\beta p_\mu p_\nu}{(p^2+M^2)^A} = \frac{i}{(4\pi)^2}\frac{1}{4}g_{\mu\nu}\frac{\Gamma(3+\beta)\Gamma(A-3-\beta)}{\Gamma(A)}(M^2)^{3+\beta-A} \\
& \int \frac{d^4p}{(2\pi)^4}\frac{(p^2)^\beta p_\mu p_\nu p_\rho p_\sigma}{(p^2+M^2)^A} = \frac{i}{(4\pi)^2}\frac{1}{24}(g_{\mu\nu}g_{\rho\sigma} + g_{\mu\rho}g_{\nu\sigma} + g_{\mu\sigma}g_{\nu\rho})\frac{\Gamma(4+\beta)\Gamma(A-4-\beta)}{\Gamma(A)}(M^2)^{4+\beta-A}.
\end{aligned}
\tag{15}
$$

From (14) and (15), we get:

$$
\begin{aligned}
& \int \frac{d^4p}{(2\pi)^4}\frac{Tr[\gamma_\mu(\not p-\alpha\not k+m)\gamma_\nu(\not p+(1-\alpha)\not k+m)^{1+2q}]}{[p^2+\alpha(1-\alpha)k^2-m^2]^{2+2q}} \\
& = \frac{4i}{(4\pi)^2}\frac{1}{\Gamma(2+2q)}\{-\tfrac{1}{2}g_{\mu\nu}\Gamma(3+q)\Gamma(-1+q)[\alpha(1-\alpha)k^2-m^2]^{1-q} \\
& +\Gamma(2+q)\Gamma(q)[\alpha(1-\alpha)(k^2g_{\mu\nu}-2k_\mu k_\nu) + (1+\tfrac{3}{2}q)m^2 g_{\mu\nu} \\
& +\tfrac{q}{3}(1-\alpha)(2-5\alpha)k_\mu k_\nu - q(1-\alpha)(-\tfrac{7}{6}\alpha+\tfrac{2}{3})k^2 g_{\mu\nu}][\alpha(1-\alpha)k^2-m^2]^{-q}\} + o(q).
\end{aligned}
\tag{16}
$$

By using the expansion

$$
\frac{\Gamma(3+q)\Gamma(-1+q)}{\Gamma(2+2q)} = -\frac{2}{q} - 1 + o(q), \qquad \frac{\Gamma(2+q)\Gamma(q)}{\Gamma(2+2q)} = \frac{1}{q} - 1 + o(q).
$$

when $q \to 0$, we get:

$$
\begin{aligned}
& \lim_{q\to 0}\int \frac{d^4p}{(2\pi)^4}\frac{Tr[\gamma_\mu(\not p-\alpha\not k+m)\gamma_\nu(\not p+(1-\alpha)\not k+m)^{1+2q}]}{[p^2+\alpha(1-\alpha)k^2-m^2]^{2+2q}} \\
& = \frac{4i}{(4\pi)^2}\{(k^2g_{\mu\nu}-k_\mu k_\nu)[\tfrac{2\alpha(1-\alpha)}{q} - 2\alpha(1-\alpha)\ln[\alpha(1-\alpha)k^2-m^2)] \\
& +\tfrac{1}{3}(1-\alpha)(2+\alpha)k_\mu k_\nu - \tfrac{1}{3}(1-\alpha)^2k^2g_{\mu\nu}\} + o(q).
\end{aligned}
\tag{17}
$$

and hence

$$
\begin{aligned}
& \lim_{q\to 0}(a_0') = \\
& e^2\frac{4i}{(4\pi)^2}\{(k_\mu k_\nu - k^2 g_{\mu\nu})[\tfrac{1}{3q} + \tfrac{1}{9} - 2\int_0^1 \alpha(1-\alpha)\ln\tfrac{\alpha(1-\alpha)k^2-m^2}{\mu^2}] + \tfrac{7}{18}k_\mu k_\nu - \tfrac{1}{9}k^2 g_{\mu\nu}\} + o(q).
\end{aligned}
\tag{18}
$$

We find that (a_0') is not covariant because of the last two terms in (18). But as we have proved that $\sum_{i=0}^{2q}(a_i')$ is covariant for q large enough. So it should be covariant for all the coefficients of $\sum_{i=0}^{2q}(a_i')$ expanded with repect to q. It can be shown that the divergent part of (a_i') when $q \to 0$ do not depend on i, furthermore, the dimension of (a_i') for all i equal to 2, so we can conclude that for all i, (a_i') can be expressed as

$$
\begin{aligned}
(a_i') = & e^2\frac{4i}{(4\pi)^2}(k_\mu k_\nu - k^2 g_{\mu\nu})[\tfrac{1}{3q} - 2\int_0^1 \alpha(1-\alpha)\ln\tfrac{\alpha(1-\alpha)k^2-m^2}{\mu^2} + o(q)] \\
& +e^2\frac{4i}{(4\pi)^2}(c_i k_\mu k_\nu - d_i k^2 g_{\mu\nu} + f_i m^2 g_{\mu\nu})
\end{aligned}
\tag{19}
$$

where c_i, d_i and f_i are some constants and in general $c_i \neq d_i$. From (12), we get

$$
\sum_{i=0}^{2q} c_i = \sum_{i=0}^{2q} d_i = C, \quad \sum_{i=0}^{2q} f_i = 0
\tag{20}
$$

where C is a constant which is not important for the phisical result which will be shown later. Hence we get

$$
\begin{aligned}
& \Pi_{\mu\nu} = \textstyle\sum_{i=0}^{2q}(a_i') \\
& = (1+2q)e^2\frac{4i}{(4\pi)^2}(k_\mu k_\nu - k^2 g_{\mu\nu})[\tfrac{1}{3q} - 2\int_0^1 \alpha(1-\alpha)\ln\tfrac{\alpha(1-\alpha)k^2-m^2}{\mu^2} + o(q)] \\
& +e^2\frac{4i}{(4\pi)^2}C(k_\mu k_\nu - k^2 g_{\mu\nu}) \\
& = e^2\frac{4i}{(4\pi)^2}(k_\mu k_\nu - k^2 g_{\mu\nu})[\tfrac{1}{3q} + \tfrac{2}{3} + C - 2\int_0^1 \alpha(1-\alpha)\ln\tfrac{\alpha(1-\alpha)k^2-m^2}{\mu^2} + o(q)].
\end{aligned}
\tag{21}
$$

Let us denote $\Pi_{\mu\nu} \equiv ie^2(k_\mu k_\nu - k^2 g_{\mu\nu})\Pi(k^2)$, we find $\Pi(k^2) = \Pi(0) + \Pi_c(k^2)$ where

$$
\Pi_c(k^2) = -\frac{1}{2\pi^2}\int_0^1 \alpha(1-\alpha)\ln[1 - \frac{\alpha(1-\alpha)k^2}{m^2}].
\tag{22}
$$

This result is the same as that derived by using other regularization methods[2].

The regularization of (b) is similar to (a), i.e., we attach to the internal fermion propagator with $2q$ vertices (6) and the diagram (b) is turned to (b')(fig. 3). When q is large enough, (b') is convergent and it can be regarded as the regularized form of (b). The amplitude of (b') is

$$
(b') = -\mu^{2q}e^2\int \frac{d^4k}{(2\pi)^4}\Big(\gamma^\mu\frac{(\not k+m)^{1+2q}}{(k^2-m^2)^{1+2q}}\gamma_\mu\Big)_{\beta\alpha}\frac{1}{(p-k)^2}.
\tag{23}
$$

By using the same discussions as in the calculation of (a_0'), we find that when $q \to 0$, the terms contributing to the integral (23) are

$$
\lim_{q\to 0}(b') = 2\mu^{2q}\int_0^1(1+2q)\alpha^{2q}\int \frac{d^4k}{(2\pi)^4}\frac{(k^2)^q[\{(1-\alpha)\not p - 2(1+2q)m] + 2(1-\alpha)q(k^2)^{q-1}p\cdot k\not k\}_{\alpha\beta}}{[k^2+\alpha(1-\alpha)p^2-\alpha m^2]^{2+2q}}.
\tag{24}
$$

Take the integral over k, we find that

$$
\begin{aligned}
\lim_{q\to 0}(b') = & \frac{ie^2}{(4\pi)^2}\{\tfrac{1}{q}(\not p - 4m) - \not p[\ln\tfrac{p^2-m^2}{\mu^2} - \tfrac{1}{2} - \tfrac{m^2}{p^2} + (\tfrac{m^2}{p^2})^2\ln(1-\tfrac{p^2}{m^2})] \\
& -m[12 + \tfrac{4m^2}{p^2}\ln(1-\tfrac{p^2}{m^2}) - 4\ln\tfrac{p^2-m^2}{\mu^2}] + o(q)\}.
\end{aligned}
\tag{25}
$$

Similarly, after attaching to the internal fermion propagator with $2q$ vertices (6) in the divergent diagram (c), we get (c'_i) (fig.3) which is the diagram with i vertices (6) in one of the internal fermion line and $2q-i$ vertices in another one. The regularized form of (c) can be represented as

$$(c') = \sum_{i=0}^{2q}(c'_i). \tag{26}$$

When q is large enough, (c') is convergent and when $q=0$, $(c')=(c)$. So (c') can be regarded as the regularized form of (c). The amplitude of (c'_i) can be represented as

$$(c'_i) = -\mu^{2q}e^3\int\frac{d^4l}{(2\pi)^4}\sum_{i=0}^{2q}\Big(\gamma^\rho\frac{(\not l-\not k+m)^{1+i}}{[(l-k)^2-m^2]^{i+1}}\gamma_\mu\frac{(\not l+m)^{1+2q-i}}{(l^2-m^2)^{1+2q-i}}\gamma_\rho\Big)_{\beta\alpha}\frac{1}{(p-l)^2}. \tag{27}$$

By the same discussions as in the calculation of (a'_0), we find that the terms contribute to the integral (27) when $q\to 0$ are

$$(c'_i) = -\mu^{2q}e^3\int\frac{d^4l}{(2\pi)^4}\frac{(l^2)^q\Big(\gamma^\rho(\not l-\not k+m)\gamma_\mu(\not l+m)\gamma_\rho\Big)_{\beta\alpha}}{[(l-k)^2-m^2](l^2-m^2)^{1+2q}(p-l)^2}+o(q). \tag{28}$$

for all i. By using the Feynman parametrization, we find

$$\begin{aligned}
&\lim_{q\to 0}(c') = \textstyle\sum_{i=0}^{2q}(c'_i) = (1+2q)(c'_0)+o(q)\\
&= \tfrac{-ie^3\mu^{2q}}{(4\pi)^2}\gamma_\mu[\tfrac{1}{q}+\tfrac{1}{2}-\textstyle\int_0^1 d\alpha\int_0^{1-\alpha}d\beta\ln\{\beta(1-\beta)k^2+\alpha(1-\alpha)p^2-2\alpha\beta k\cdot p-(1-\alpha)m^2\}]\\
&-\tfrac{ie^3}{2(4\pi)^2}\textstyle\int_0^1 d\alpha\int_0^{1-\alpha}d\beta\frac{\gamma^\rho[(\beta-1)\not k+\alpha\not p+m]\gamma_\mu[\beta\not k+\alpha\not p+m]\gamma_\rho}{\beta(1-\beta)k^2+\alpha(1-\alpha)p^2-2\alpha\beta\cdot p-(1-\alpha)m^2}+o(q).
\end{aligned} \tag{29}$$

Let us denote $(c')=\Lambda_\mu$ which is the vertex correction at the one loop order. The observable part Λ^c_μ of the vertex is defined by the equation:

$$\Lambda_\mu = K\gamma_\mu+\Lambda^c_\mu \tag{30}$$

where K contains the divergent part when $q\to 0$ and Λ^c_μ is convergent. From (28), we are able to derive the anomalous magnetic momentum of the electron. Here we will not give its detailed procedures which can be found in [2]. We can find from our approach that the second-order correction to the magnetic momentum is $\frac{e^2}{8\pi^2}$.

Renormalization at the one loop order:

At this stage, the renormalization of QED theory is straightforward as has been shown in a lot of textbooks[3]. In general, we may add to the original Lagrangian with the counterterms which is of the form:

$$\begin{aligned}
&-\tfrac{1}{4}(Z_3-1)(\partial_\mu A_\nu-\partial_\nu A_\mu)(\partial^\mu A^\nu-\partial^\nu A^\mu)\\
&+i(Z_2-1)\bar\psi\not\partial\psi-\delta m(Z_2-1)\bar\psi\psi\\
&-e(Z_1-1)\bar\psi\not A\psi\}-\tfrac{\lambda}{2}(Z_3-1)(\partial\cdot A)^2.
\end{aligned} \tag{31}$$

The new action is the renormalized one. If we choose

$$\begin{aligned}
Z_3 &= 1-\tfrac{e^2}{(4\pi)^2}\tfrac{4}{3q}, \qquad Z_2 = 1-\tfrac{e^2}{(4\pi)^2}\tfrac{1}{q},\\
Z_1 &= 1-\tfrac{e^2}{(4\pi)^2}\tfrac{1}{q}, \qquad \delta m = (1-\tfrac{e^2}{(4\pi)^2}\tfrac{3}{q})m,
\end{aligned} \tag{32}$$

we find that all the correlation functions got from the renormalized action at the one loop order are finite when $q\to 0$.

If we let $A^\mu_0 = Z_3^{\frac{1}{2}}A^\mu$, $\psi_0 = Z_2^{\frac{1}{2}}\psi$, $e_0 = Z_2^{-1}Z_3^{-\frac{1}{2}}Z_1 e \equiv Z_e^{\frac{1}{2}}e$, $m_0 = m+\delta m$ which are called the bare quantities, then the renormalized action can be expressed in the form (5) with the physical quantities A, ψ, m and e replaced by the corresponding bare quantities respectively. It is remarkable that μ together with λ in (5) are not renormalized.

Similar to the discussions in [1], we can also get the renormalization group equation in the QED theory. It is straightforward and we will not give its detailed expression here.

The method developed here is typical in the gauge field theories. It may shed some light on the application of this method to more complicate field theories such as the QCD theory, the standard model, etc. Further works on these aspects are under progress[4].

Z. H. Wang is obliged to Prof. Daniel Z. Friedman, L. Vinet for their encouragement in the course of this work.

References

1 Zhong-Hua Wang and Han-Ying Guo, Loop regularization and renormalization in ϕ^4 theory.

2 See, for example, C. Itzykson and J.-B. Zuber, "Quantum Field Theory", McGraw-Hill Inc. and references therein.

3 P. Ramond, "Field Theory: A Modern Primer", 2nd edition, Addison-Wesley Publishing Company, Inc.

4 Zhong-Hua Wang and H. Y. Guo, in preparation.

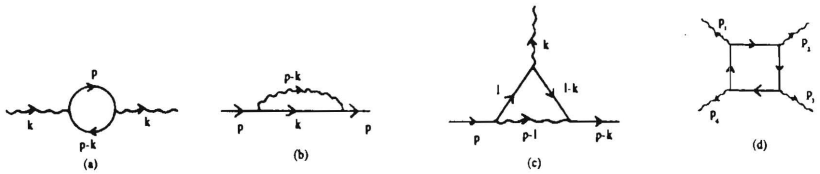

Fig. 1: Divergent diagrams of QED at the 1-loop oder.

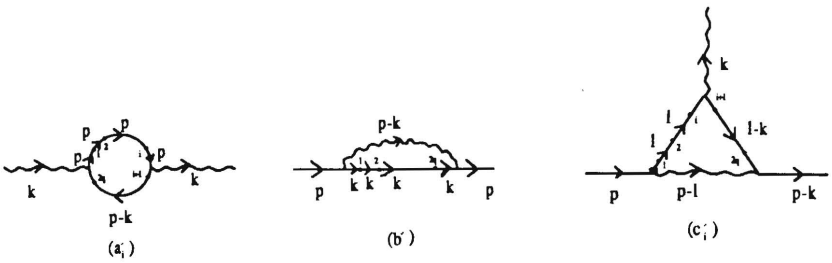

Fig. 2: The regularized diagrams of (a), (b) and (c).