UdeM-LPN-TH-101A

Fermilab Library

0 1160 0022120 4

UdeM-LPN-TH-101A

June 1992

CP VIOLATION AND QUARK MASS MATRICES

G. Bélanger

Laboratoire de Physique Nucléaire, Université de Montréal

C.P. 6128 Montréal, PQ, Canada H3C 3J7

and

E. Boridy, C. Hamzaoui and G. Jakimow

Département de Physique, Université du Québec à Montréal

C.P. 8888, Succ. A, Montréal, P.Q. Canada H3C 3P8

Abstract

Mass matrices are discussed in the vanishing limit of CP violation in the standard model in order to disclose possible patterns. A particular mass pattern emerges in the limit $J = 0$ and $\omega = 0$. In this case, we obtain $|V_{cb}| \simeq (\sqrt{2}m_s/m_b)$ which is in a very good agreement with the data. This new value of $|V_{cb}|$ coincides with the findings of Rosner and Worah in the context of a composite model and with Giudice in the context of grand unified supersymmetric theories.

The issue of quark masses and mixings through the weak interactions is interesting and puzzling at the same time. The standard model, in spite of its extraordinarily success, does not give any explanation of such fundamental issues as the number of generations and how they mix. To discover realistic quark mass matrices where one has a relation between masses and mixing elements is in general very difficult.[1] This is obviously due to the many free parameters contained in the theory. However, taking into account the observed hierarchy enjoyed by the Kobayashi-Maskawa (KM) matrix[2], we can actually derive some genuine patterns. We can obtain interesting relations between masses and mixings in the limit $J = 0$ and $\omega = 0$ (the symmetric KM matrix). For example in the case where $H_u = M_u M_u^\dagger$ is diagonal and one of the element of $H_d = M_d M_d^\dagger$ vanishes as a consequence of the absence of CP violation, we obtain that $|V_{cb}| = \sqrt{2} m_s / m_b$. The purpose of doing that is to unveil some realistic pattern in the mass matrices, however note that we are not switching off CP violation alltogether, it will be reinstored later on.

In general we consider non-hermitean mass matrices, we use H_u and H_d. There are 10 observable parameters among the masses and mixings, six quark masses and four KM elements. Unnecessary parameters can be eliminated since the physics is invariant under unitary transformations of H_u and H_d :

$$H_u \to U H_u U^\dagger, H_d \to U H_d U^\dagger \tag{1}$$

We can choose U to diagonalise H_u and reduce the number of parameters in H_d from nine to seven. This is a straightforward exercise and the results are

$$H_d = \begin{pmatrix} a & d & e \\ d & b & f+ig \\ e & f-ig & c \end{pmatrix} \tag{2}$$

and the other possible form for H_d is

$$H_d = \begin{pmatrix} a & d & ie \\ d & b & f+ig \\ -ie & f-ig & c \end{pmatrix} \tag{3}$$

where a, b, c, d, e, f and g are real numbers.[2] A similar result is obtained in the up quark sector when U diagonalizes H_d. We will study these two cases separately,

$$1) H_u^D, H_d$$

$$2) H_u, H_d^D$$

and see what realistic mass matrices can be derived in the limit $J = 0$ and $\omega = 0$.

Before we deal with the mass matrices, we show that certain elements in the Kobayashi-Maskawa matrix are not sensitive to the values of the measure of CP violation, J, and the asymmetry parameter ω. The Kobayashi-Maskawa matrix is written in terms of the four independent parameters

$$\alpha \equiv |V_{us}| \ , \quad \beta \equiv |V_{cb}| \ , \quad \gamma \equiv |V_{ub}| \ , \quad \omega \equiv |V_{cd}|^2 - |V_{us}|^2 \ , \tag{4}$$

The parameter ω is given by

$$\omega = |V_{cd}|^2 - |V_{us}|^2 = |V_{ts}|^2 - |V_{cb}|^2 = |V_{ub}|^2 - |V_{td}|^2 \tag{5}$$

and represents the deviation from the symmetric KM matrix, in which $|V_{ij}| = |V_{ji}|$.

The smallest elements in the KM matrix are $|V_{ub}| \approx .004$ and $|V_{td}| \approx .01$ (note that there are large error bars on these two parameters). The quantity J and the asymmetry parameter ω are even smaller $O(10^{-5})$. Since $|V_{us}| \approx .22$ and $|V_{cb}| \approx .045$,[3] it is clear that

$$|J|, |\omega| \ll |V_{cb}| \ll |V_{us}|$$

We can then write

$$|V_{us}| = |V_{us}|_{(J=\omega=0)} + |V_{us}|_{(J,\omega)} \tag{6}$$

and similarly for $|V_{cb}|$. The correction term can be safely neglected and these two matrix elements can be evaluated at $J = \omega = 0$.

In our parametrization, the KM matrix is given by

$$V_{KM}(J=\omega=0) = \begin{pmatrix} \sqrt{1-\alpha^2-\gamma^2} & \alpha & \gamma \\ -\alpha & \sqrt{1-\alpha^2-\gamma^2} - \frac{\gamma\beta}{\alpha} & \beta \\ \gamma & -\beta & \frac{\alpha\beta}{\gamma} - \sqrt{1-\alpha^2-\gamma^2} \end{pmatrix}, \tag{7}$$

with the following constraint imposed by the vanishing CP violation

$$2\alpha\beta\gamma\sqrt{1-\alpha^2-\gamma^2} = \alpha^2\beta^2 + \beta^2\gamma^2 - \alpha^2\gamma^2 \tag{8}$$

which leads to

$$|V_{ub}|/|V_{cb}| \simeq |V_{us}|/2. \tag{9}$$

This is in a very good agreement with the experimental central value. Probabably one of the main reason why CP violation in the standard model is small is due to the preferred central value of $|V_{ub}|/|V_{cb}| \simeq 0.11$.

In the following we will work only in the case $J = \omega = 0$, the details of how to recover the general case and the consequences of reintroducing the CP violation will be discussed in a future publication.[4] The measure of CP violation is given by

$$J = \frac{deg}{(m_s^2 - m_d^2)(m_b^2 - m_d^2)(m_b^2 - m_s^2)} \tag{10}$$

We start with the case where H_u is diagonal, then $H_d = V_{KM} H_d^D V_{KM}^\dagger$. Although H_d is unknown, it is interesting to see which form leads to reasonable values for the KM angles. From the equation above, J vanishes when either d or e or g vanish.

One can easily show that among these possibilities, the only realistic one is when $d = 0$, in Eq. 2. In this case we obtain the following relation between mixing angles and masses

$$(m_s^2 - m_d^2)^2 |V_{us}|^2 |V_{cs}|^2 = (m_b^2 - m_d^2)^2 |V_{ub}|^2 |V_{cb}|^2. \tag{11}$$

Replacing the value of $|V_{ub}|$ in Eq. 9, we obtain

$$|V_{cb}| = \sqrt{2} m_s / m_b \tag{12}$$

and

$$|V_{us}|^2 = \frac{2}{3} \frac{(a - m_d^2)}{(m_s^2 - m_d^2)} \tag{13}$$

Note that $|V_{cb}|$ agrees very well with the measured value. This new value coincides with the findings of Rosner and Worah in the context of composite models and with Giudice in the context of grand unified supersymmetric theories.[5] This result is general as it is valid for both forms of H_d. Some comments are in order. Since we have five parameters in the mass matrix and three can be eliminated using the invariants, we are left with two parameters, say a and b. In the mixing matrix we also have two parameters, $|V_{us}|$ and $|V_{cb}|$ which in general are functions of a and b. It turns out that the dependence of $|V_{cb}|$ on a and b is contained in $|V_{cs}|$. Since this element is almost one, $|V_{cb}|$ depends very mildly on a and b. On the other hand $|V_{us}|$ has a strong dependence on a but not b. This behaviour is useful when reconstructing the mass matrix in the general case where J and ω are different than zero. No realistic value for the KM parameters are obtained if any other of the elements of H_d vanish.

In the case where H_d is diagonal we can show that none of the elements of H_u can vanish without contradicting experiment. We are left with six parameters in H_u and $|V_{us}|$ and $|V_{cb}|$ are given as a function of the elements of H_u, say a,b and d. The two mixing matrix elements depend strongly on these three parameters, in contrast with the previous case.

We have studied mass matrices in the limit of vanishing CP violation and we have shown that some of the KM elements can be calculated safely in this limit. We obtained that $|V_{cb}| \simeq (\sqrt{2}m_s/m_b)$ which is in good agreement with the measured value. We have shown also that the dependence of the KM elements on the unknowns in the mass matrix is very mild. Such a feature is very useful in building mass matrices consistent with the data.

Acknowledgments

The authors would like to thank W. Marciano, J.-M. Frère and D. Atwood for useful discussions. One of us (C.H.) would like to thank W. Marciano and the Physics Department at the Brookhaven National Laboratory, where part of this work was done, for their warm hospitality. This work was supported in part by the Natural Science and Engineering Research Council of Canada.

References

1. H. Fritzsch, Phys. Lett. **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979); L. F. Li, Phys. Lett. **84B**, 461 (1979); B. Stech, Phys. Lett. **130B**, 189 (1983); M. Shin, Phys. Lett. **145B**, 285 (1984); M. Gronau, R. Johnson, and J. Schechter, Phys. Rev. Lett. **54**, 2176 (1985).

2. M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

3. Particle Data Group, Phys. Rev. **D45**, III-65 (1992).

4. G. Bélanger, E. Boridy, C. Hamzaoui and G. Jakimow, work in progress.

5. J. Rosner and M. Worah, U. of Chicago preprint EFI 92-12, March 1992; G. F. Giudice, U. of Texas at Austin preprint UTTG-5-92, March 1992.