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Quark Deconfinement at Finite Temperature

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Abstract

A proposal is discussed for an order parameter to characterize quark deconfinement at finite temperature. This phase transition is related to that of chiral-symmetry restoration using Finite Energy QCD sum rules at $T \neq 0$. Implications for dilepton production in heavy ion collisions are briefly discussed.

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1 Introduction

In this talk I discuss a proposal for an order parameter to characterize quark deconfinement at finite temperature. The motivation is to find an order parameter such that the phase transition could be studied analytically, as opposed to numerically, e.g. from simulations of QCD on the lattice. Clearly, since we are still unable to perform analytical (first principle) calculations in QCD at hadronic scales, such an order parameter has to be introduced phenomenologically. Having done this I shall discuss how one can study the transition to quark deconfinement in the framework of Finite Energy QCD Sum Rules. In the process, an important relationship emerges between the colour deconfinement phase transition and that of chiral-symmetry restoration. Finally, I shall briefly discuss some implications for dilepton production in heavy ion collisions.

2 Chiral-Symmetry and its Restoration

I begin by discussing some of the symmetries of the QCD Lagrangian

$$\mathcal{L}_{QCD} = \bar{\psi}(i\gamma^\mu D_\mu - M)\,\psi - \frac{1}{4}\,F_{\mu\nu}\,F^{\mu\nu}\,, \tag{1}$$

where

$$F^{\mu\nu} = \partial^\mu G^\nu - \partial^\nu G^\mu + i\,g_0[G^\mu, G^\nu]\,, \tag{2}$$

with

$$\begin{cases} G^\mu &= \frac{1}{2}\,\lambda^a\,G^\mu_a \\ D^\mu &= \partial^\mu + i g_0 G^\mu \end{cases} \quad (a = 1, \cdots 8) \tag{3}$$

and M the quark mass matrix. For the purposes of this talk I consider only two quark flavours (up and down). Among the various symmetries of $\mathcal{L}_{QCD}$ one finds a (global) $SU(2)_V$ and an $SU(2)_R \otimes SU(2)_L$ symmetry. These Lagrangian symmetries are explicitly broken by the quark masses. In fact, the vector (I-spin) Noether current

$V^i_\mu = \bar\psi\gamma_\mu\,\tau^i\psi$, and the axial-vector current $A^i_\mu = \bar\psi\,\gamma_\mu\,\gamma_5\,\tau^i\psi$ have divergences

$$\partial^\mu V_\mu = i(m_d - m_u)\bar d u \,, \tag{4}$$

$$\partial^\mu A_\mu = i(m_d + m_u)\bar d\gamma_5 u \,. \tag{5}$$

In the limit $m_u = m_d = 0, SU(2)_V$, and $SU(2)_R \otimes SU(2)_L$ become exact Lagrangian symmetries. However, this is not what is usually meant by chiral-symmetry restoration, which refers to the symmetry of the vacuum. Given a Lagrangian symmetry one must investigate how it is realized in the states, starting with the vacuum. According to whether the Noether charges $Q^i = \int\, d^3x\; J^i_0(\vec x, t)$ annihilate the vacuum or not, one has a Wigner-Weyl or a Nambu-Goldstone realization of the Lagrangian symmetry. In the former case particles are classified according to the irreducible representations of the symmetry group, as in e.g. $SU(2)_V$. The Nambu-Goldstone realization (spontaneous symmetry breaking) corresponds to a *hidden* symmetry, as the vacuum does not share the symmetry of the Lagrangian. This is the case for $SU(2)_R \otimes SU(2)_L$ since e.g. there are no (quasi) degenerate parity doublets in the particle spectrum. No particle classification is possible in this phase as the vacuum is *contaminated* by an arbitrary number of massless (Nambu-Goldstone) bosons carrying non- trivial quantum numbers. In the case of $SU(2)_R \otimes SU(2)_L$ the three emerging Nambu-Goldstone bosons are readily identified with the pion $(\pi^\pm, \pi^0)$, which decays to the (hadronic) vacuum through the axial-vector current, i.e.

$$\langle 0|A^i_\mu(0)|\pi^j(p)\rangle = i\, f_\pi\, p_\mu\, \delta^{ij} \,, \tag{6}$$

with $f_\pi = 93.2$ MeV. In the limit $m_u = m_d = 0$ the axial-vector current is strictly conserved and, hence, $f_\pi\, \mu^2_\pi = 0$. In the Nambu-Goldstone phase

$$\mu^2_\pi \propto (m_u + m_d) \to 0 \,, \tag{7}$$

$$f^2_\pi \propto \langle 0|\bar u u + \bar d d|0\rangle \neq 0 \,, \tag{8}$$

with the proportionality constants such that $2f^2_\pi\mu^2_\pi = (m_u + m_d)\langle 0|\bar u u + \bar d d|0\rangle$. A phase transition from a Nambu-Goldstone to a Wigner-Weyl mode (**chiral-symmetry restoration**) is characterized by the vanishing of the order parameter f_π, or alternatively $\langle 0|\bar q q|0\rangle$. Clearly, this can only happen at finite temperature, and if the phase transition does take place this should happen regardless of whether the quark masses are zero or not. The numerical value of the critical temperature, though, is expected to depend on this fact.

The temperature behaviour of $f_\pi(T)$ at low T ($T << \mu_\pi$) has been investigated in chiral perturbation theory with the result [1]

$$f_\pi(T) = f_\pi(0)\left[1 - \frac{T^2}{8f^2_\pi(0)} + \mathcal{O}(T^4)\right] \,, \tag{9}$$

in the chiral limit for three flavours. Chiral-symmetry breaking corrections to (9) have been also calculated [1], together with corrections due to massive states [2]. The low temperature expansion of $\langle\bar q q\rangle_T$ has been carried out up to order $\mathcal{O}(T^6)$ in [2]. As the authors of [1],[2] have pointed out, this low temperature expansion should not be extrapolated to the critical temperature. For instance, if the phase transition is of second order then the critical exponent should be $\frac{1}{2}$, rather than 2 as one would naively obtain from (9). If one is interested in the behaviour of $f_\pi(T)$ near $T = T_c$, other methods should be used, e.g. the composite operator formalism of [3], which reproduces (9) at low T, but gives instead $f_\pi(T) \propto (1 - T/T_c)^{\frac{1}{2}}$ as $T \to T_c$. This feature will become particularly important in Section 4.

3 Quark Deconfinement (Order Parameter)

At zero temperature the shape of a typical hadronic spectral function consists of some delta functions plus resonances with increasing widths, followed by a smooth continuum starting at some threshold energy E_0. It is known from fits to actual data that for $E \gtrsim E_0$ the hadronic spectral functions are well approximated by perturbative QCD; E_0 is thus called the asymptotic freedom (A.F.) threshold. In Fig.1 I show the results of a numerical fit to experimental data in the vector channel (curve (a)), and in the

axial-vector channel (curve (b)), as extracted from τ- lepton decays [4]. For simplicity I do not show the data points. The resonance peaks correspond to the ρ and the a_1 mesons; the delta function from the pion pole in the axial-vector channel is not shown. One can see that for $s \equiv E^2 \gtrsim 2$ GeV^2 the two spectral functions become smooth and roughly equal. The one-loop perturbative QCD prediction is $\frac{1}{\pi}\mathrm{Im}\Pi_V(s) = \frac{1}{\pi}\mathrm{Im}\Pi_A(s) = \frac{1}{8\pi^2}$, in excellent agreement with the data. Under the assumption that

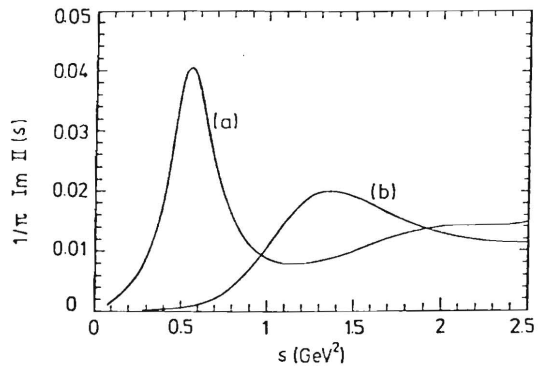


Figure 1:

quark deconfinement does take place at some critical temperature T_d, one would expect that by increasing T from $T = 0$ the resonance peaks in the spectral function should become broader. At $T = T_d$ the resonance widths would then become infinite, signalling quark deconfinement. This resonance melting with increasing temperature would be accompanied by a shift of the asymptotic freedom threshold s_0 towards threshold. In this picture the resonance width, and/or the asymptotic freedom threshold provide a suitable order parameter. In the complex s-plane, bound states (e.g. the pion) correspond to poles of the S-matrix lying on the real axis, and resonances correspond to poles located in the second Riemann sheet, their distance to the real axis being measured by the width Γ_R. At finite temperature the form of the Green's function will

be

$$G(E,T) \propto \frac{1}{E - M_R(T) + \frac{i}{2}\Gamma_R(T)} \,. \tag{10}$$

In the picture described above, an increase in the temperature will shift **all** poles farther away from the real axis as $\Gamma_R(T)$ increases. While the mass M_R may depend on T, it is not a relevant order parameter. With the second Riemann sheet poles infinitely far from the real axis, and the A.F. threshold correspondingly close to the origin, the spectral function is then described entirely by the QCD continuum extrapolated down to the kinematic threshold. One may interpret such a phase as a deconfined phase where all hadrons originally contributing to the spectral function have melted into quarks or quark-antiquark pairs.

I wish to stress that the relevant order parameter for deconfinement should be the width and not the mass. Claims have been made occasionally in the literature that the mass should vanish at the critical temperature. I believe this to be incorrect for the following reason. Let us consider a stable hadron, i.e. one with zero width, at zero temperature such as the pion or the nucleon. If with increasing temperature the mass goes to zero, and nothing happens to the width, then at the critical temperature one would still see a peak in the hadronic spectral function at zero energy. The hadron has not disappeared from the spectrum, and hence has not melted! The only way a particle can melt is by having a temperature dependent width such that $\Gamma_R \to \infty$ as $T \to T_c$. What happens to the mass (defined as the position of the pole on the real axis) is irrelevant for this argument. Once the resonance becomes infinitely broad, the spectral function becomes smooth and should be well approximated by the quark degrees of freedom, i.e. by perturbative QCD.

Since this proposal was first made [5] independent theoretical evidence has become available. In fact, Leutwyler and Smilga [8] have shown in the framework of the virial expansion that: (a) at low temperatures ($T < 50$ MeV) and in the chiral limit ($\mu_\pi = 0$) the mass of the nucleon $M_N(T) \simeq M_N(0)$, and its width $\Gamma_N(T) \simeq \Gamma_N(0) \equiv 0$; (b)

At higher temperatures both $M_N(T)$ and $\mu_\pi(T)$ **increase** slightly (by 4% and 1%, respectively, up to $T \lesssim 160$ MeV), while $\Gamma_N(T)$ and $\Gamma_\pi(T)$ increase substantially, e.g. at $T = 160$ MeV the ratio of width to mass (imaginary to real part of the propagator) is about 43% for the pion and 20% for the nucleon. Since the pion and nucleon widths are strictly zero at $T = 0$, this is quite a dramatic effect. Additional evidence for the approximate constancy of M_N and μ_π follows from the sigma model [9], which also gives increases over $M_N(0)$ and $\mu_\pi(0)$ at the level of a few percent. Further support comes from a recent calculation of $\mu_\pi(T)$ in the composite operator formalism [10], showing approximate constancy of the pion mass over a wide range of temperatures, with a tendency to **increase** near the critical temperature.

4 Relationship between Deconfinement and Chiral-Symmetry Restoration

I consider the retarded two-point function involving the axial-vector current

$$\Pi_{\mu\nu}(\omega, \vec{p}) = i \int d^4x \, e^{ip\cdot x}\theta(x_0) \ll [A_\mu(x), A^\dagger_\nu(0)] \gg , \tag{11}$$

where

$$\ll A \cdot B \gg \equiv \sum_n exp(-E_n/T)\langle n|A \cdot B|n\rangle / Tr(exp(-H/T)) , \tag{12}$$

with $|n\rangle$ a complete set of eigenstates of the Hamiltonian H. The philosophy of QCD sum rules [11] is to calculate the two-point function in the deep euclidean region in perturbative QCD, add non-perturbative effects parametrized in terms of vacuum expectation values of the quark and gluon fields appearing in the QCD Lagrangian, and relate the result to the hadronic spectral function by means of a dispersion relation.

In order to find the QCD behaviour of (11) in the deep euclidian region we assume a dilute quark gas at temperature T with zero chemical potential. The virtual "quanta" associated to the local current $A_\mu(x)$ will convert into $q\bar{q}$ pairs for $p^2 = \omega^2 - \mathbf{p}^2 > 4m_q^2$ while for space-like momenta $(\omega^2 - \mathbf{p}^2) < 0$ there will be "scattering" of these quanta by

the quarks in the gas. These two distinct processes contribute to the spectral function as follows

$$\frac{1}{\pi}\mathrm{Im}\Pi^+_{\mu\nu}(\omega, \mathbf{p}) = \sum_q \int LIPS(\omega, \mathbf{p}, E_1, \mathbf{p}_1 E_2, \mathbf{p}_2)$$

$$\times \langle 0|A_\mu|q\bar{q}\rangle\langle q\bar{q}|A_\nu|0\rangle \times \{[1 - n_F(E_1)][1 - n_F(E_2)] - n_F(E_1)n_F(E_2)\} , \tag{13}$$

for annihilation, and

$$\frac{1}{\pi}\mathrm{Im}\Pi^{(-)}_{\mu\nu}(\omega, \mathbf{p}) = \sum_q \sum_{\bar{q}} \int LIPS(\omega, \mathbf{p}, E_1, \mathbf{p}_1 - E_2 - \mathbf{p}_2)$$

$$\times \langle q|A_\mu|\bar{q}\rangle\langle \bar{q}|A_\nu|q\rangle \times \{n_F(E_1)[1 - n_F(E_2)] - n_F(E_2)[1 - n_F(E_1)]\} \tag{14}$$

for scattering. In the above equations $n_F(E)$ is the Fermi-Dirac distribution function, and the phase space is

$$LIPS(\omega, \mathbf{p}, E_1, \mathbf{p}_1 E_2 \mathbf{p}_2) = \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3}$$

$$\times \delta(\omega - E_1 - E_2)\delta^{(3)}(\mathbf{p} - \mathbf{p}_1 - \mathbf{p}_2) . \tag{15}$$

It must be stressed that the above two contributions to the spectral function, arising from different physical mechanisms, are unrelated. The first contribution ("annihilation") leads to the usual right- and left-hand cuts in the complex energy plane, and does not vanish at $T = 0$. The second piece ("scattering") leads to a new cut centered around $\omega = 0$, vanishes at $T = 0$, and is unrelated to duality.

On the hadronic side, at low temperatures $T < \mu_K$ only pions from the gas will contribute to the spectral function. Aside from the pion pole contribution to the "annihilation", or right- and left-hand discontinuities in the complex ω-plane, there is a "scattering" piece in the hadronic spectral function from the center cut arising from the scattering of the current off the pions in the plasma. The latter involves an integral of $\langle\pi|A_\mu|2\pi\rangle\langle 2\pi|A_\nu|\pi\rangle$ weighted by Bose factors. However, since $A_\mu \propto \varphi(\varphi^+ \overleftrightarrow{\partial}_\mu \varphi)$ this term appears at order T^4/f_π^2 in $\mathrm{Im}\Pi(S)$. Hence, a parametrization in terms of the pion pole plus a continuum modelled by perturbative QCD should be a good approximation

to the hadronic spectral function. In this way one obtains the following Finite Energy QCD sum rule (FESR) [7]

$$8\pi^2 f_\pi^2(T) = \frac{1}{2}\int_{4m_q^2}^{S_0(T)} dz^2 v(z)[3 - v^2(z)]\, th\left(\frac{z}{4T}\right)$$
$$+ \int_{4m_q^2}^{\infty} dz^2 v(z)[3 - v^2(z)] n_F\left(\frac{z}{2T}\right) , \qquad (16)$$

where $v(z) = (1 - 4m_q^2/z^2)^{\frac{1}{2}}$. Equation (16) is an eigenvalue equation fixing $S_0(T)$ once $f_\pi(T)$ is known independently. Notice that even if $f_\pi(T)$ were to be assumed constant at low temperatures, $S_0(T)$ would still be a decreasing function of T on account of the quark "scattering" piece of $\mathrm{Im}\Pi(S)$, i.e. the second term on the right-hand side of (16). The advantage of our choice of Green's function (11) should be evident: at low temperatures ($T < \mu_K$) apart from $f_\pi(T)$ there are no other unknown T-dependent hadronic parameters such as masses and widths. For instance, QCD sum rules for the correlator of two vector currents at $T \neq 0$ [6],[12], involve $m_\rho(T)$ and $\Gamma_\rho(T)$ which, unlike $f_\pi(T)$, are a priori unknown and model- dependent.

In the chiral limit the FESR (16) takes the simple form

$$S_0(T=0) - \frac{10}{3}\pi^2 T^2 = \int_0^{S_0(T)} dS\, th\left(\frac{\sqrt{S}}{4T}\right) , \qquad (17)$$

where $S_0(T=0) = 8\pi^2 f_\pi^2(0)$. In [7] we solved the FESR using the low temperature expansion for $f_\pi(T)$ from [1], i.e. eq.(9). The solution to (17) is illustrated in Fig.2. The solution to (16) is qualitatively the same, except that $S_0(T)$ vanishes at a slightly higher temperature, viz. $S_0(T_d) = 0$ for $T_d \simeq 135$ MeV ($\mu_\pi = 0$), and $T_d \simeq 170$ MeV ($\mu_\pi \neq 0$). However, one should keep in mind that (9) is not supposed to be extrapolated all the way up to the critical temperature for chiral-symmetry restoration (T_c). In fact, as shown in [3], if one uses the expression for $f_\pi(T)$ from the composite operator formalism, $S_0(T)$ vanishes at practically the same temperature as $f_\pi(T)$, i.e. $T_d \simeq T_c$, as seen from Fig.3 (reproduced from [3]). Independent confirmation of this result may be obtained by using the first Weinberg sum rule at $T \neq 0$, which in the

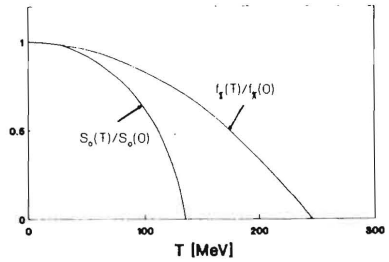


Figure 2:

chiral limit is given by

$$\frac{1}{\pi}\int_0^{S_0(T)} dS[\mathrm{Im}\,\Pi_V(S,T) - \mathrm{Im}\,\Pi_A(S,T)] = f_\pi^2(T) \qquad (18)$$

This sum rule was studied in [7] using eq. (9), the result being essentially the same as with the FESR (17). However, the authors of [3] obtained $T_d/T_c \simeq 0.99$ using the more accurate expression for $f_\pi(T)$ from the composite operator formalism.

In summary, Finite Energy QCD sum rules at $T \neq 0$ lead to the prediction that $S_0(T)$ vanishes at some critical temperature, which is essentially the same as that for chiral-symmetry restoration : $T_d \simeq T_c \simeq 130$ MeV. According to the interpretation of $S_0(T)$ as a relevant order parameter for quark deconfinement (see Section 3), one can conclude that the QCD-FESR provide evidence for the existence of this phase transition.

At this point it is important to point out that $T = 0$ QCD sum rules cannot be naively extrapolated to finite temperature. In particular, it has been shown in [12] that the lowest dimension FESR, i.e. the one not containing the gluon condensate, is the only one free from contradictions. Higher dimensional FESR, and hence Laplace transform

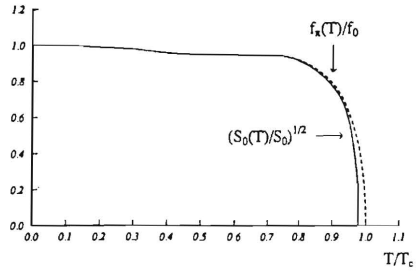


Figure 3:

QCD sum rules, give the wrong temperature dependence of the gluon condensate. The QCD sum rule program breaks down at dimension $d = 4$, and hence claims on the vanishing of hadron masses from Laplace transform QCD sum rules are invalidated. This happens even after taking into account the temperature dependent "scattering" terms discussed at the beginning of this section. Whether additional mechanisms could be brought into play in order to rescue the QCD sum rule program remains an open problem.

5 Dilepton Production in Heavy Ion Collisions

As discussed in Section 3 one expects resonance widths to diverge as the critical temperature for deconfinement is approached. This has important implications for the rates of dilepton production in heavy ion collisions at high energies, as first pointed out in [13]. The point is that when calculating rates in the hadronic phase of the

plasma one should use a temperature dependent pion form factor, i.e.

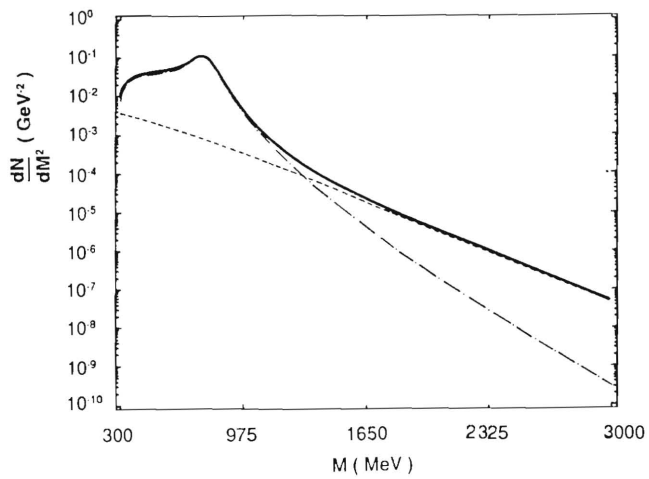
$$|F_\pi(M^2,T)|^2 = \frac{M_\rho^4 + M_\rho^2\Gamma_\rho^2(T)}{(M^2 - M_\rho^2)^2 + M_\rho^2\Gamma_\rho^2(T)}\,, \tag{19}$$

where M is the (dilepton) invariant mass, and M_ρ is taken as a constant. Although the approximate constancy of particle masses with increasing temperature has been shown only for the nucleon and the pion [8]-[10], it seems quite reasonable to assume the same for the ρ-meson. In the following discussion I shall further assume the following simple ansatz for $\Gamma_\rho(t)$

$$\Gamma_\rho(T) = \frac{\Gamma_\rho(0)}{1 - T^2/T_d^2}\,, \tag{20}$$

where T_d is the critical temperature for deconfinement. Other critical exponents are, of course, possible but this does not affect the dilepton production rates in any significant way. In Fig. 4 I show the dimuon spectrum for the case of an ^{16}O beam with incident energy of 225 GeV per nucleon on a target of ^{195}Pt. The pion form factor is assumed temperature independent and a common critical temperature for deconfinement (T_d) and chiral-symmetry restoration (T_c) is specified as $T_c = T_d = 200$ MeV. The dashed and dash-dotted lines are the quark-gluon phase (QGP) and the hadronic contributions, respectively; the solid line is the total. The nice feature about this result is that the QGP contribution is some two orders of magnitude larger than the hadronic. If F_π is given a temperature dependence, through the ρ-meson width, the spectrum changes dramatically as shown in Fig.5. The unfortunate result is that the QGP contribution is now comparable to the hadronic. Hence, experimental data would not be offering much information about the QGP, which is what one wishes to probe in these experiments (for a recent related discussion see [14]). Nevertheless, we have shown [13] that the derivative of the average momentum distribution can offer information on the nature of the phase transition, i.e. whether the system goes through a single ($T_c = T_d$) or a double ($T_d < T_c$) phase transition.

In Fig. 6 I show the derivative of the average momentum distribution, for a T-independent pion form factor. The solid curve corresponds to $T_c = T_d = 246$ MeV, and the dashed curve to $T_c = 246$ MeV and $T_d = 200$ MeV. Figure 7 shows the result of



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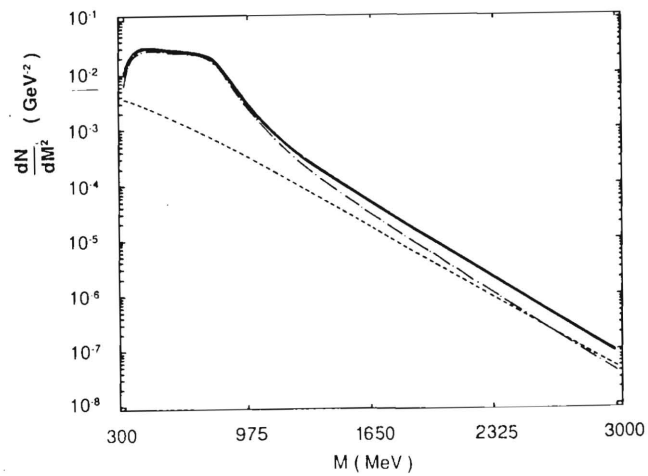
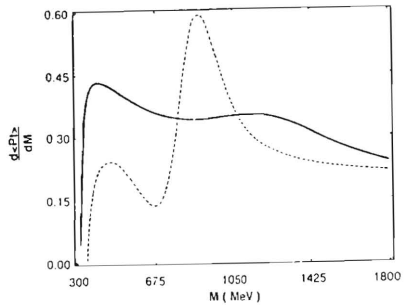


Figure 5:

making F_π temperature dependent. One notices that if $T_c = T_d$, there is quite a dramatic difference between the results for $F_\pi = F_\pi(M^2)$ and those for $F_\pi = F_\pi(M^2, T)$. A measurement of $d < P_t > /dM$ would then be useful to pin down the temperature dependence of the pion form factor.



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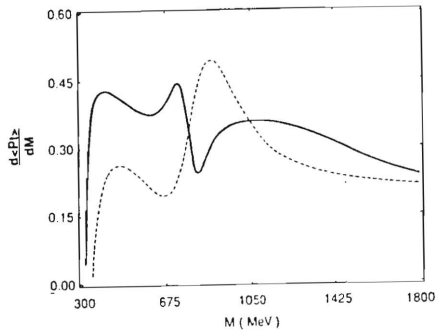

Figure 7:

Acknowledgements

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