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Review of QCD Sum Rule Estimates of Leptonic Decay Constants of Charm and Beauty Mesons

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Abstract

We review various estimates of the leptonic decay constants of heavy flavour pseudoscalar mesons in the framework of relativistic and non-relativistic QCD sum rules.

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In this talk I shall concentrate on estimates of the leptonic decay constants

$$\langle 0|A_\mu|P(k)\rangle = i\,\sqrt{2}\,f_P\,k_\mu \tag{1}$$

($P = D, D_s, B, B_s$) in the framework of QCD sum rules [1]. In order to estimate f_P one considers the two-point function

$$\psi_5(q^2) = i\int d^4x\, e^{iqx}\langle 0|T\left(\partial^\mu\, A_\mu(x)\,\partial^\nu\, A^\dagger_\nu(0)\right)|0\rangle\ ,$$

where $\partial^\mu\ A_\mu(x) = (m_Q + m_q)\ :\ \bar{q}(x)i\gamma_5\ Q(x)$: with $q(Q)$ being the light (heavy) quark field and $m_q(m_Q)$ its corresponding QCD (current) mass. The function $\psi_5(Q^2), Q^2 = -q^2$, satisfies a dispersion relation

$$\psi_5(Q^2) = \frac{1}{\pi}\int ds\,\frac{\mathrm{Im}\psi_5(s)}{s+Q^2} + \text{subtractions} \tag{2}$$

defined up to two subtractions, which can be disposed of by taking at least two derivatives in (2). In this fashion one obtains the Hilbert power moments, which at $Q^2 = 0$ become

$$\varphi^n(0) = \frac{(-)^{n+1}}{(n+1)!}\left(\frac{d}{dQ^2}\right)^{n+1}\psi_5(Q^2)|_{Q^2=0} = \frac{1}{\pi}\int_0^\infty \frac{ds}{s^{n+2}}\,\mathrm{Im}\psi_5(s)\ . \tag{3}$$

The point $Q^2 = 0$ is appropriate for heavy-light quark currents, to the extent that φ^n can be computed in perturbative QCD, adding non-perturbative power corrections which fall off by powers of the heavy quark mass. These corrections are parametrized by vacuum expectation values of the quark and gluon fields in the QCD Lagrangian, and are organized according to their dimension. For instance, in the limit $m_q \to 0$, well justified for D and $B_{u,d}$ mesons, the perturbative contribution to $\varphi^n(0)$ to order $\mathcal{O}(\alpha_s)$ is given by [2]

$$\varphi^{(n)}(0)|_{PT} = \frac{3}{8\pi^2}\left(\frac{1}{m_Q^2}\right)^{n-1} B(n,3)[1 + a_n^0\,\alpha_s], \tag{4}$$

where $B(x,y)$ is the beta function, $\alpha_s \equiv \alpha_s(m_Q^2)$, and a_n^0 are the rational numbers

$$\frac{3\pi}{4}\, a_n^0 - \frac{\pi^2}{6} = 1 - \frac{2}{n+1} - \frac{6}{n+2} + \sum_{r=1}^{n+2} \left[\frac{1}{r^2} + \left(\frac{3}{2} - \frac{1}{n} \right.\right.$$

$$\left.\left. - \frac{1}{(n+1)} - \frac{1}{(n+2)} + \frac{3}{(n+3)} \right) \frac{1}{r} \right] . \tag{5}$$

The non-perturbative part, always in the limit $m_q \to 0$, becomes

$$\begin{aligned} \varphi^{(n)}(0)|_{NP} &= \frac{-m_Q < \bar{q}q >}{m_Q^{2n+2}} \left[1 - \frac{< \alpha_s G^2 >}{12\pi m_Q < \bar{q}q >} - \frac{1}{4}(n+2)(n+1)\frac{M_0^2}{m_Q^2} \right. \\ &- \left. \frac{4}{81}(n+2)(n^2+10n+9)\pi\, \alpha_s\, \rho\, \frac{\langle \bar{q}q \rangle}{m_Q^3} \right] \end{aligned} \tag{6}$$

where $\langle \alpha_s G^2 \rangle \simeq 0.06$ GeV^4, $M_0^2 \simeq 0.5$ GeV^2, $\rho \simeq 3$, and $\langle \bar{q}q \rangle$ (1 GeV) $\simeq$ - 0.016 GeV^3. In the case of the D_s meson, where the approximation $m_q = 0$ should not be made, the full expressions given in [2] must be used for $\varphi^{(n)}(0)$. Finally, the hadronic spectral function appearing on the r.h.s. of (3) is parametrized by the ground state pseudoscalar meson pole plus a continuum starting at some threshold s_0. This continuum is expected to be well approximated by the QCD spectral function, computed in perturbation theory, provided s_0 is high enough, i.e.

$$\frac{1}{\pi} \mathrm{Im}\psi_5(s)|_{HAD} = 2 f_P^2 M_P^4\, \delta(s - m_P^2) + \theta(s - s_0) \frac{1}{\pi}\, \mathrm{Im}\psi_5(s)|_{PT} . \tag{7}$$

By taking the ratio of any two consecutive moments one obtains an expression for M_P^2 as a function of s_0, the latter being *a- priori* unknown. The calculation will be meaningful if M_P does not depend strongly on s_0, i.e. there should be a relatively wide range of values of s_0 leading to a value of M_P with a reasonably small spread. This is certainly the case for D and D_s where one obtains, using the first two moments ($n = 1, 2$)

$$s_0 = 2M_D^2 - 3M_D^2 \quad , \quad M_D = 1.85 \pm 0.15 GeV , \tag{8}$$

$$s_0 = 2M_{D_s}^2 - 3M_{D_s}^2 \quad , \quad M_{D_s} = 1.9 \pm 0.1 GeV , \tag{9}$$

to be compared with the experimental values: $M_D|_{EXP} = 1.87$ GeV, and $M_{D_s}|_{EXP} = 1.97$ GeV. The method is not accurate enough to reproduce the small $D_s - D$ mass splitting, although it does lead to different values for f_D and f_{D_s}, as these are not determined by ratios of moments but rather by the moments themselves. Results of this analysis [3] are listed in Table 1.

Table 1: Fully Relativistic QCD Sum Rule Estimates

Author	f_B/f_π	f_D/f_π	f_{D_s}/f_π	Type of Sum Rule
Aliev & Eletsky [4]	1.02 ± 0.11	1.33 ± 0.19	—	Laplace
Dominguez & Paver [3]	1.35 ± 0.25	1.70 ± 0.20	2.1 ± 0.1	Hilbert Moments
Narison [5]	1.38 ± 0.14	1.31 ± 0.12	1.65 ± 0.15	Laplace & Hilbert
Reinders [6]	1.29 ± 0.15	—	—	Laplace
Schilcher & Wu [7]	0.97 ± 0.21	1.33 ± 0.10	1.46 ± 0.10	Green
Experiment [8]	—	< 2.2	2.4 ± 0.4	

Turning to the B and B_s mesons, one finds that the masses now depend more strongly on s_0, i.e. the stability region is substantially reduced. For instance, with $s_0 \simeq (1.1 - 1.3)\, M_B^2$ one finds $M_B = 5.2 \pm 0.2$ GeV ($M_B|_{EXP} = 5.27$ GeV). It must be pointed out that non-perturbative corrections are rather small at this mass scale, so that f_B is essentially determined by the perturbative part of the moments. The estimate of f_{B_s} in this framework gives [3]

$$\frac{f_{B_s}}{f_B} \simeq \frac{M_{B_s}}{M_B} . \tag{10}$$

It is possible to choose a different kernel in the dispersion relation (2) and obtain other types of QCD sum rules, e.g. the Laplace sum rules

$$2f_P^2\frac{M_P^4}{m_Q^2}exp(-M_P^2/M^2) = \int_{m_Q^2}^{s_0} ds\, exp(-s/M^2)\frac{1}{\pi}\mathrm{Im}\psi_5(s)|_{QCD}$$

$$+ exp(-m_Q^2/M^2)\left[C_4\langle 0_4\rangle - \frac{1}{4}\frac{m_Q}{M^2}\left(1-\frac{m_Q^2}{2M^2}\right)C_5\langle 0_5\rangle + \frac{1}{6M^2}\left(2-\frac{m_Q^2}{2M^2}-\frac{m_Q^4}{6M^4}\right)C_6\langle 0_6\rangle\right], \tag{11}$$

where

$$\int_{m_Q^2}^{s_0} ds\, exp(-s/M^2)\frac{1}{\pi}\mathrm{Im}\psi_5(s)|_{QCD} = \frac{3}{8\pi^2}\{m_Q^4[E_1(m_Q^2/M^2) - E_1(s_0/M^2)] + M^2(M^2+m_Q^2)$$

$$exp(-m_Q^2/M^2) - M^2(M^2+s_0)exp(-s_0/M^2) - 2m_Q^2M^2[exp(-m_Q^2/M^2) - exp(-s_0/M^2)] + O(\alpha_s)\}\ , \tag{12}$$

and

$$E_1(z) = \int_z^\infty \frac{exp(-t)}{t}\, dt\ . \tag{13}$$

Estimates based on these sum rules [4]-[6] have claimed somewhat smaller values of f_D and f_B than the ones obtained through Hilbert transforms. However, the point we raised in [3] was that although eq. (11) does exhibit a better (softer) behaviour on s_0, it has conceptual as well as numerical disadvantages. First, the power corrections in eq. (11) appear dominated by $C_4\langle 0_4\rangle$ on account of the smallness of $C_5\langle 0_5\rangle$ and $C_6\langle 0_6\rangle$ rather than because of the usual Laplace suppressions; notice that the terms multiplying $C_5\langle 0_5\rangle$ and $C_6\langle 0_6\rangle$ are roughly comparable. This means that inside the "sum rule window" ($M^2 \simeq 0.8 - 2.1$ GeV^2 for P = D and $M^2 \simeq 3.3 - 4.2$ GeV^2 for P = B) the Laplace variable M^2 has lost the unambiguous character of the short-distance expansion parameter it usually has in the familiar applications to light quark systems. A second cause of discomfort is the pronounced sensitivity of the perturbative contribution eq. (12) to the values of m_Q and M^2. In fact, the dependence on m_Q is in this case exponential. For instance, in the case of f_B a 4% increase in m_b from $m_b =$ 4.6 GeV to $m_b = 4.8$ GeV produces a change in the perturbative contribution of a factor of 2-3 for $M^2 \simeq$ 3-5 GeV^2. On the other hand, for fixed s_0 and m_Q, eq. (12) changes by a factor of 8-16 for P = D and by a factor of 40 for P = B, inside the "sum rule window" in M^2.

The corresponding changes in the non-perturbative piece are roughly a factor of 3 and 20 (in the same direction), respectively. In the case of f_D these extreme variations are offset by corresponding large variations in the exponential on the LHS of eq. (11), so that in the end the result for f_D appears somewhat stable. For f_B the huge variations of the perturbative and non-perturbative contributions are not entirely offset by the variation of the exponential on the LHS of eq. (11), so that uncertainties are beyond the 100% level. We wish to point out that such a dangerous situation is not encountered in the usual applications of Laplace sum rules to light quark systems. Also, these problems do not affect the Hilbert transform power moments at $Q^2 = 0$, which in our view, are more reliable to treat charm and beauty mesons as they lead to more stable predictions.In any case, all estimates of f_B are in reasonable agreement. This is not quite the case for f_D and f_{D_s}. Since the latter are more sensitive to non-perturbative contributions, different choices of values for the vacuum condensates (particularly the dimension $d = 5$ quark gluon condensate, through the parameter M_0^2 in eq. (6)), explain some of the discrepancies. The recent extraction of f_{D_s} from experimental data on inclusive and exclusive B meson decays [8] appears to favour the Hilbert moment estimate [3]. Since f_{D_s} is correlated with f_D, the Hilbert moment estimate of the latter would also appear to be favoured.

In two recent papers Narison [9], [10] has claimed that in order to circumvent the problem of stability in s_0 for B-mesons one should search for stability in n, i.e. the order of the Hilbert moment. In [9] the value $f_B/f_\pi = 1.72 \pm 0.16$ is found, for $s_0 = 50 - 60$ GeV^2 and $n = 7 - 8$, while in [10] $f_B/f_\pi = 1.64 \pm 0.35$ for $s_0 = 35 - 55$ GeV^2 and $n = 3 - 8$. If correct, these results would render QCD sum rule estimates somewhat meaningless. Stability criteria should also, in themselves, be more or less stable. On account of this, we have repeated the calculations outlined in [9], [10], using exactly the same values of the various parameters. We have been unable to reproduce the results claimed in [9],[10]. In fact, we obtain instead that $f_B/f_\pi \simeq 1.36$, for $n = 1 - 7$ and $s_0 = 33$ GeV^2, the latter value being the one that leads to the correct mass of the B-meson. In Fig.1 I show the results for M_B^2 as a function of n for $s_0 = 30$ GeV^2 (curve (a)), and $s_0 = 60$ GeV^2 (curve (b)). Figure 2 shows f_B

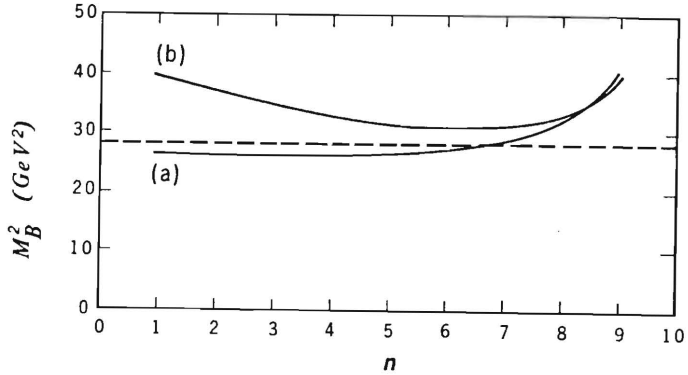


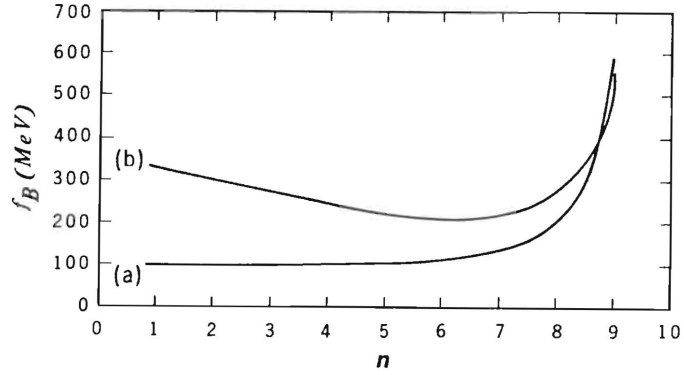
Figure 1:

as a function of n for the same choices of s_0. Both these curves are different from those in [9],[10], particularly the one for f_B. In addition, it is clear from Fig.1 that in order to reproduce the experimental value of M_B (broken line) s_0 should be close to $s_0 \simeq 30\text{GeV}^2$, instead of $s_0 \simeq 55$ GeV^2 as advocated in [9]. In fact, we find that $s_0 = 33$ GeV^2 leads to the correct value of M_B, as shown in Fig.3, and gives the stable result $f_B/f_\pi \simeq 1.36$, as shown in Fig.4. Changes in the values of the various parameters, within the errors adopted in [9],[10], produce an uncertainty in f_B of about 10%. However, the true error is much bigger due to the sensitivity of f_B to changes in s_0. In summary, nothing is gained by analyzing M_B and f_B as a function of n, as the results are extremely stable and reproduce the old values [3],[6].

A related issue, of current interest in weak hadronic physics involving heavy flavours is that of the connection between f_P and the $(\bar{Q}q)$ bound- state wave function at the origin, leading to the asymptotic scaling law (modulo logarithms)

$$f_P = \frac{const.}{\sqrt{M_P}} \,. \tag{14}$$

Some time ago we studied the M_P dependence of f_P in the framework of Hilbert moment QCD sum rules in [11]. Our results indicate that the charm quark does not seem heavy



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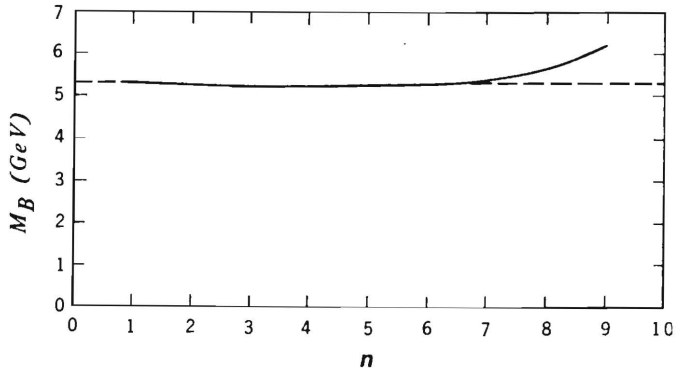

Figure 3:

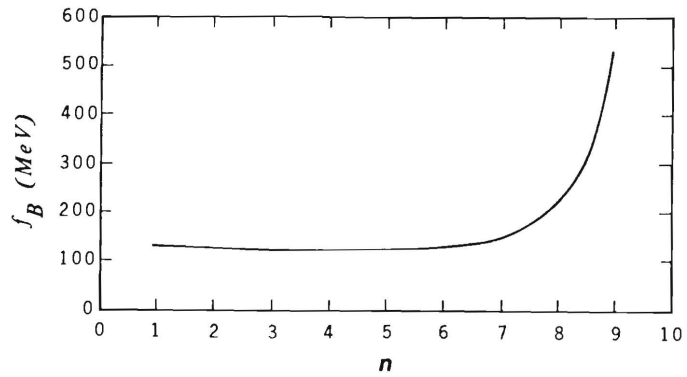

Figure 4:

enough for a straightforward application of (14) from M_D up to M_B. In fact, it is only for $M_P \geq 3$ GeV that one finds a behaviour numerically consistent with (14). Recently, Narison [12] used the results of [9], [10] to reanalyze this problem. In [9] he finds $f_B/f_\pi = 1.72 \pm 0.16$, in [10] $f_B/f_\pi = 1.64 \pm 0.35$, and now in [12] one reads $f_B/f_\pi = 1.50 \pm 0.13$, with all three results claimed to have been obtained in the **same** framework of Hilbert moment QCD sum rules! To the extent that we have been unable to reproduce the results claimed in [9],[10], I find the new result of [12] also incorrect.

Numerical values aside, the scaling law which follows from the Hilbert moments is not the result of an explicit $1/\sqrt{m_Q}$ dependence of f_P but rather a result of various cancelling effects leading to an **effective** $1/\sqrt{m_Q}$ behaviour [11]. In an attempt to remedy this situation we have recently studied Laplace transform QCD sum rules in the infinite mass limit [13],[14], following an earlier suggestion by Shuryak [15]. In our analysis we computed quark mass corrections, which allow us to answer the question: *how heavy is heavy*? In this fashion one finds e.g. that for $Q = c$ mass corrections are at the 100% level, thus confirming Hilbert moment results [11] indicating that asymptotia does not start at the charm quark mass scale. The results for f_B depend on whether one constrains, or not, the calculation so as to

reproduce the correct $B^* - B$ mass splitting, as indicated in Table 2.

Table 2: Non-Relativistic ($m_Q \to \infty$) QCD Sum Rule Estimates

Author	f_B/f_π	Type of Sum Rule
Nasrallah *et al* [19]	0.99 ± 0.15	Green
Dominguez & Paver [13]	1.25 ± 0.15	Laplace - Constrained to reproduce M_B
Dominguez & Paver [14]	0.95 ± 0.05	Laplace - Constrained to reproduce M_B and $M_{B^*} - M_B$
Eletsky & Shuryak [16]	1.18 ± 0.19	Laplace
Bagan *et al* [17]	1.5 - 2.0	Laplace (see text)
Neubert [18]	1.48 ± 0.39	Laplace (see text)

An independent analysis by Eletsky and Shuryak [16] ($B^* - B$ mass splitting unconstrained) is in agreement with [13]. Subsequent analyses by Bagan *et al* [17] and by Neubert [18], using the renormalization group to carry out the summation of logarithms lead to the values shown in Table 2, where we also include an earlier estimate by Nasrallah *et al* [19] based on a slightly different technique.

In order to render the comparison between [13]-[16] and [17]-[18] meaningful I have quoted the results of [17]-[18] for the quantity $\tilde{f}_{stat}/\sqrt{M_B}$, where

$$f_P(M_P) = \frac{\tilde{f}_{stat}(\mu = M_P)}{\sqrt{M_P}} \left[1 - \frac{2}{3}\frac{\alpha_s(M_P)}{\pi}\right].$$

As a result of the summation of logarithms [17],[18] it appears that α_s should be taken at the scale of 1 GeV rather than at the scale of M_B, a result previously claimed by Broadhurst and Grozin [20]. This has the effect of increasing the size of the radiative correction in the sum rule for f_P^2 by roughly a factor of two. There is, of course, every reason to worry about the reliability of an estimate carrying a 100% two-loop correction. However, final judgement

can only be passed once the three-loop contribution is calculated (a monumental but long overdue calculation). In the meantime, taking the results of [17]-[18] at face value, and comparing them with relativistic estimates, one would conclude that

$$f_B/f_\pi \simeq 1-2\,,$$

which is an unfortunate, albeit realistic, summary of our present knowledge of this important parameter.

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References

[1] For a review of the method see e.g. M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. **B147** (1979) 385, 448; L.J. Reinders, H.R. Rubinstein, S. Yazaki, Phys. Rep. **C127** (1985) 1.

[2] D.J. Broadhurst, S.C. Generalis, Open University Report No. OUT-4102-8/R (1982); S.C. Generalis, Ph.D. Thesis, Open University Report No. OUT-4102-13 (1984). See also K. Schilcher, M.D. Tran, N.F. Nasrallah, Nucl. Phys. **B181** (1981) 91.

[3] C.A. Dominguez, N. Paver, Phys. Lett. **B197** (1987) 423.

[4] T.M. Aliev, V.L. Eletsky, Sov. J. Nucl. Phys. **38** (1983) 936.

[5] S. Narison, Phys. Lett. **B198** (1987) 104.

[6] L.J. Reinders, Phys. Rev. **D38** (1988) 423.

[7] K. Schilcher, Y.L. Wu, Z. Phys. **C54** (1992) 163..

[8] Argus Collaboration, H. Albrecht *et al*, Z. Phys. **C54** (1992) 1.

[9] S. Narison, CERN Report No. CERN-TH 6112/91 (1991) (Unpublished).

[10] S. Narison, Montpellier Report No. PM 91/32 (1991) (Unpublished).

[11] C.A. Dominguez, N. Paver, Phys. Lett. **B246** (1990) 493.

[12] S. Narison, Phys. Lett. **B279** (1992) 137.

[13] C.A. Dominguez, N. Paver, Phys. Lett. **B269** (1991) 169.

[14] C.A. Dominguez, N. Paver, Phys. Lett. **B276** (1992) 179.

[15] E. Shuryak, Nucl. Phys. **B198** (1982) 83.

[16] V. Eletsky, E. Shuryak, Phys. Lett. **B276** (1992) 191.

[17] E. Bagan *et al*, Phys. Lett. **B278** (1992) 457.

[18] M. Neubert, SLAC Report No. SLAC-PUB-5712 (1991).

[19] N.F. Nasrallah, K. Schilcher, Y.L. Wu, University of Mainz Report No. MZ-TH/91-02 (1991).

[20] D.J. Broadhurst, A.G. Grozin, Phys. Lett. **B274** (1992) 421.