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EXPERIMENTAL ASPECTS of CP VIOLATION*

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ABSTRACT

We review the experimental aspects of CP violation, from the past, present, and into the future, with particular emphasis on those experiments most likely to elucidate the mechanism of CP violation.

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1. Introduction and Overview

It is an honor to give lectures on the experimental aspects of CP violation at this year's SLAC Summer Institute, which includes a symposium that commemorates the discovery of the τ lepton. How are CP violation and the τ lepton related to one and other?

First, if you accept the most conventional explanation of CP violation, namely the Standard Model, the discovery of CP violation was the first, albeit *implicit*, evidence for a third generation of fermions. The quark mixing (Cabibbo-Kobayashi-Maskawa, or CKM) matrix cannot support a CP violating phase if there are only two generations, but has exactly one pigeon-hole for such a phase if there are three generations. Of course, the tau lepton was the first *explicit* evidence of a third generation. It is quite possible that study of the quark sibling of the tau in the third generation, the b-quark, will for once and for all elucidate the mechanism of CP violation.

Second, the discoveries of CP violation in the neutral kaon system and the τ lepton were made largely as a result of the experimenter's initiative. It seems today that the particle experimenter's role is to act out a script written by our theoretical colleagues. To be sure, the observation of the W, the characterization of the Z^0, and the probing of the symmetry breaking sector at the SSC, all involved, or will involve, impressive initiative, ingenuity, and originality by experimenters. But, the spark of initiative has come largely from the theoretical side in those cases. It is not always so, as CP violation and the τ lepton show. As I have struggled through a week of owl shifts, or made my way through the construction and calibration of hundreds of channels of detector, that thought has often kept me going! A quote, attributed to I.I.Rabi, puts it well:

> "The last person a young experimenter should ask, in my belief, about an experimental program is a theoretical physicist.
>
> "Not that theoretical physicists are stupid, but they have their ideas, and they want the answers to their own problems in their own terms. We would not have advanced very far in basic discoveries with a concentration on theory alone...
>
> "If you follow a theorist, you come in and say, 'Now what shall I do?' And then you do it and say, 'What have I done?' I don't know why people work that way.
>
> "My own view is that you take these things personally. You do an experiment because your own philosophy makes you want to know the result. It's too hard, and life is too short, to spend your life doing something because someone else has said it's important. You must feel the thing yourself—feel that it will change your outlook and your way of life."

However, failure never excuses ignorance, and although I am not a historian, I believe the discoverers of CP violation and the τ had a strong *logical*, if not *theoretical*, motivation for their work. In the case of the τ, perhaps the logic was, "Nature gave us the muon. Did Nature stop there?" Nowadays, when we know that the Z^0 decays into only three light neutrinos, initiation of a search for a fourth sequential lepton would be a bit ignorant. Perhaps for CP violation, the logic was "Nature disrespects parity (P). What about CP?" It is logical to expect P and C violation, as two experimenters, Purcell and Ramsey, noted in 1951, well before parity violation was observed[1]:

> "The argument against electric dipoles, in another form, raises directly the question of parity. A nucleon with an electric dipole moment would show an asymmetry between left- and right- handed coordinate systems; in one system the dipole moment would be parallel to the angular momentum and in the other, antiparallel. But there is no compelling reason for excluding this possibility. **It would not be the only asymmetry of particles of ordinary experience, which already exhibit conspicuous asymmetry in respect to electric charge.**"

Today, we view the predominance of positively charged protons in our galaxy as a logical *consequence* of CP violation, but the clever experimentalists of the past viewed that predominance as a logical *reason* to search for CP violation.

I would like to give two views of the 'logic' of today's experimental situation in CP violation. The first view is that the Standard Model is the source of CP violation. This view may seem like blind homage to the conventional, but really it isn't; the Standard Model has one huge success in describing CP violation: it gets the order of magnitude of $|\eta_{+-}|$ correct:

$$|\eta_{+-}| \equiv \left[\frac{\mathrm{B}(K_L^0 \to \pi^+\pi^-)}{\tau_L}\frac{\tau_S}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}\right]^{1/2} = (2.27 \pm 0.02)\times 10^{-3} \tag{1.1}$$

If CP violation is characterized by a simple Fermi-like coupling constant G_{CP} that describes the transition $K_L^0 \to \pi^+\pi^-$, then $G_{CP}/G_F \approx |\eta_{+-}|$, and naively, the gauge boson corresponding to G_{CP} would have mass $\approx M_W/\sqrt{|\eta_{+-}|} \approx 1.6\,\mathrm{TeV}$. From Yosef Nir's lectures, the Standard Model (through the second-order box diagrams, not first order Fermi-like coupling) gives:

$$|\eta_{+-}| \approx \frac{s_{23}s_{13}}{s_{12}}\sin\delta \approx 7\times 10^{-4}\sin\delta \tag{1.2}$$

where s_{23}, s_{13}, s_{12}, and δ are from the usual CKM parameterization (see Nir's lectures). This is the success. If it were to turn out that CP violation were caused by some 'new interaction' described by G_{CP}, one would view it as an incredible coincidence that G_{CP} happened to sit right in the range easily described by the Standard Model.

The next logical step, in this view, is to find the 'smoking gun' that *proves* that the Standard Model is the source of CP violation. Although $K_L^0 \to \pi^+\pi^-$ was first observed twenty-eight years ago, we still lack an incontrovertible second piece of evidence. The Standard Model was once thought to make the prediction[2]:

$$\frac{\mathrm{B}(K_L^0 \to \pi^0\pi^0)}{\mathrm{B}(K_L^0 \to \pi^+\pi^-)} \neq \frac{\mathrm{B}(K_S^0 \to \pi^0\pi^0)}{\mathrm{B}(K_S^0 \to \pi^+\pi^-)} \tag{1.3}$$

Indeed, the most experimental effort has gone in to trying to probe that inequality. The latest two experiments, NA31 and E731, each with a sensitivity to deviations from equality of the two sides of (1.3) at the 0.4% level, give a mild indication that the inequality is correct, but the significance is low. In the meantime, revision of the Standard Model prediction, mostly to account for the large top quark mass, has allowed equality in (1.3) within the Standard Model. A number of experimental groups are still pushing on to test (1.3) to even higher precision, and also to search for the 'smoking gun' in other kaon decay modes, such as $K_L^0 \to \pi^0 e^+e^-$, $K_L^0 \to \pi^0\nu\bar{\nu}$, or $K^+ \to \pi^+\pi^+\pi^-$. In every case the hope is to dredge up a small effect via use of huge statistics, or to see a small signal on a large background. If we get proof of the Standard Model from those experiments, that proof will not startle with its clarity.

The Standard Model makes a more startling prediction outside of the neutral kaon system, in the neutral B system.[3] A difference of order 10% is predicted between the probability that a B^0 will end up in a $J/\psi K_S^0$ final state, and the probability that a $\overline{B^0}$ will end up in the same final state. Observation of such a large asymmetry would, in this view, provide the crucial proof that CP violation is part of the Standard Model. If the asymmetry is large, a number of venues, including hadron colliders, symmetric e^+e^- colliders, and asymmetric e^+e^- colliders, all have a fair chance of seeing it first. For subsequent studies of other asymmetries in the B hadron system, studies that allow precision tests of the self-consistency of the pattern of CP violation predicted by the Standard Model, an asymmetric e^+e^- collider is probably the best bet.

The second view is contrarian. Here one dismisses the successful computation of $|\eta_{+-}|$ within the Standard Model as a coincidence; $\sin\delta$ might be very small, for example. Second, the origin of $K_L^0 \to 2\pi$ is the following process, or 'state mixing':

$$K_2^0 \to K_1^0 \to 2\pi, \tag{1.4}$$

and so a computation of G_{CP} could be modified to account for this second order process, where the CP violation occurs in the transition $K_2^0 \to K_1^0$. Under the assumption that a Fermi-type interaction produces CP violation in the $\Delta S = 2$ transition, $s\bar{d} \to \bar{s}d$, one can estimate

$$\langle K_1^0 | H | K_2^0 \rangle \approx G'_{CP} f_K^2 M_K$$

and

$$G'_{CP} \approx \frac{\sqrt{2}\Delta M_K}{f_K^2 M_K} |\eta_{+-}| = 7 \times 10^{-11} G_F \tag{1.5}$$

This is the superweak hypothesis of Wolfenstein.[4] The point is, $K_L^0 \to 2\pi$ renders up remarkable sensitivity to a new Fermi-type $\Delta S = 2$ interaction.

The superweak hypothesis predicts that CP-violating processes in the kaon system are caused solely by the transition $K_2^0 \to K_1^0$. All final states should show the same amount of CP violation, so equation (1.3) should be an equality, in agreement with contemporary Standard Model predictions. Rates for $K_L^0 \to \pi^0 e^+ e^-$ and $K_L^0 \to \pi^0 \nu\bar{\nu}$ are predictable from $K_S^0 \to \pi^0 e^+ e^-$ and $K_S^0 \to \pi^0 \nu\bar{\nu}$, and there would be no effects in $K^+ \to \pi^+\pi^+\pi^-$. The logic, then, in pursuing all of these modes of CP violation, is, *any one* deviation from superweak rejects the superweak hypothesis, independent of Standard Model considerations.

The pure superweak hypothesis makes no prediction about the magnitude of CP violation in the B system. A good benchmark is to assume universality between the $s\bar{d} \to \bar{s}d$, and $b\bar{d} \to \bar{b}d$ interactions, in which case the largest CP violating asymmetries will be of order of magnitude:

$$\approx \frac{\frac{\Delta M_K}{f_K^2 M_K}}{\frac{\Delta M_B}{f_B^2 M_B}} |\eta_{+-}| \approx \frac{1}{10} |\eta_{+-}| \approx 10^{-4} \tag{1.6}$$

or much smaller than the Standard Model predicts. If the superweak interaction is not of universal strength, asymmetries as large as those of the Standard Model might result; to do so, however, the superweak interaction must dominate the mass splitting ΔM_B in the B^0 system. One must dismiss as coincidence the easy accomodation of ΔM_B by the Standard Model as coincidence. The distinctive prediction of the superweak hypothesis is that the magnitude of all asymmetries in the B system will be identical, so to rule out superweak, *two* asymmetries must be measured.

I find the first-order superweak explanation of CP violation more elegant and simple than the rather convoluted, second order, CKM explanation. I also prefer to hope that there is new physics, not a desert, on the mass scale of $M_W/\sqrt{7 \times 10^{-11}} \approx 1000\,\text{TeV}$ implied by the superweak hypothesis. It makes good experimental sense to choose the most clear and startling phenomena predicted by the alternate hypothesis, namely the Standard Model, and pursue it. Observation of the large and varying asymmetries in the B hadron system that are predicted by the Standard Model would really change my outlook of particle physics.

The remainder of these lectures are split into four sections. First, I give a heuristic view of the phenomenology of CP violation. Second, I address why the *discovery* of CP violation was so easy, and why has the rest been so hard? Third, I discuss the contemporary experiments that seek CP violation in the kaon system, including E731 and NA31. Finally, I address in some detail the prospects for observation of CP violation in the B hadron system.

2. Phenomenology

Yosef Nir has given an excellent account of the theory of CP violation; I follow the same notation whenever possible. Here, I'd like to underscore some physical connections.[5]

2.1 Two Component Formalism, CPT and CP

The time development of an arbitrary K^0 state,

$$a|K^0\rangle + b|\overline{K^0}\rangle$$

is governed by the Schrödinger equation,

$$i\begin{bmatrix}\dot{a}\\ \dot{b}\end{bmatrix} = H\begin{bmatrix}a\\ b\end{bmatrix} = \begin{bmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M^*_{12} - \frac{i}{2}\Gamma^*_{12} & M_{22} - \frac{i}{2}\Gamma_{22}\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix} \tag{2.1}$$

Here M_{ij} are usually called the 'mass matrix' and Γ_{ij} are the 'decay matrix'. If there is CP violation in the time evolution governed by (2.1), that is called CP violation in state-mixing; it is not so easy to distinguish whether such CP violation originates in the mass matrix or the decay matrix.

It is a consequence of CPT invariance that separately $M_{11} = M_{22}$ and $\Gamma_{11} = \Gamma_{22}$. If we accept CPT, then the interesting physics of lifetime splitting, mixing, and CP violation comes from the off-diagonal terms. The only physics lost by dropping the diagonal terms of (2.1) is the common-mode lifetime, $|a|^2 \propto e^{-\Gamma_{11}t} \propto |b|^2$. The remaining description of the neutral kaon system is similar to the standard treatment of a spin-1/2 particle in a B field that has only x and y components.

How the remaining Hamiltonian transforms under CP depends on some phase conventions. The 'standard' convention is:

$$\begin{aligned} \mathrm{P}|K^0\rangle &= -|K^0\rangle, \quad \mathrm{P}|\overline{K^0}\rangle = -|\overline{K^0}\rangle \quad \text{(pseudoscalar mesons)} \\ \mathrm{C}|K^0\rangle &= -|\overline{K^0}\rangle, \quad \mathrm{C}|\overline{K^0}\rangle = -|K^0\rangle \\ \text{so, } \mathrm{CP}|K^0\rangle &= |\overline{K^0}\rangle, \quad \mathrm{CP}|\overline{K^0}\rangle = |K^0\rangle \end{aligned} \tag{2.2}$$

In the 2×2 subspace of (2.1), the combined operation CP is just:

$$\mathrm{CP} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = (\mathrm{CP})^\dagger$$

The test of CP invariance through state-mixing in (2.1) is whether:

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M^*_{12} - \frac{i}{2}\Gamma^*_{12} & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} =$$

$$\begin{bmatrix} 0 & M^*_{12} - \frac{i}{2}\Gamma^*_{12} \\ M_{12} - \frac{i}{2}\Gamma_{12} & 0 \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} 0 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M^*_{12} - \frac{i}{2}\Gamma^*_{12} & 0 \end{bmatrix}$$

or, simply,

$$M_{12} - \frac{i}{2}\Gamma_{12} \stackrel{?}{=} M^*_{12} - \frac{i}{2}\Gamma^*_{12} \tag{2.3}$$

According to (2.3), CP is conserved when both M_{12} and Γ_{12} are real. Actually, CP is

conserved whenever

$$|M_{12} - \frac{i}{2}\Gamma_{12}| = |M^*_{12} - \frac{i}{2}\Gamma^*_{12}| \tag{2.4}$$

The reason here is that no known physical process is sensitive to the phase change[6,2,7]

$$|K^0\rangle \to e^{-i\zeta}|K^0\rangle \ , \ |\overline{K^0}\rangle \to e^{+i\zeta}|\overline{K^0}\rangle \tag{2.5}$$

The choice of

$$\zeta = -\frac{1}{4}\left[\arg(M_{12} - \frac{i}{2}\Gamma_{12}) - \arg(M^*_{12} - \frac{i}{2}\Gamma^*_{12})\right]$$

causes (2.3) to follow from (2.4). then yields the equality in (2.3), and both M_{12} and Γ_{12} real. In the spin-1/2 analogy, this change just corresponds to use of a coordinate system rotated by 2ζ about the z axis from the original one. As Nir has described, it is also possible to 'rephase' the CP operator such that in the new basis, the standard convention (2.2) still holds.

For CP violation in state-mixing, it is necessary to have a difference in *magnitude* between the two off-diagonal elements of H. A nice physical picture of what this type of CP violation is a difference between the *rates* for $K^0 \to \overline{K^0}$ and $\overline{K^0} \to K^0$. The rate difference will be present when there is a phase difference between M_{12} and Γ_{12}. The standard kaon *convention* is to keep Γ_{12} real, and describe the observed CP violation by a small imaginary part in M_{12}.

2.2 Physical Features

2.2.1 Lifetime Difference, $\Delta I = 1/2$

In the neutral kaon system, the most striking physical feature is the difference in lifetime between K^0_{L} and K^0_{S}:

$$\frac{\tau_{\mathrm{L}}}{\tau_{\mathrm{S}}} = \frac{51.7\,\mathrm{ns}}{0.0892\,\mathrm{ns}} = 580 \tag{2.6}$$

If a K^0 is produced in some typical hadronization process, it appears to decay with two wildly different lifetimes! This is not due to CP violation. This difference is described in the phenomenology by the real part of Γ_{12}, which from Nir is given to second order:

$$\Gamma_{12} = 2\pi \sum_n \rho_n \langle K^0|H_{\Delta S=1}|n\rangle\langle n|H_{\Delta S=1}|\overline{K^0}\rangle = 2\pi \sum_n \rho_n A^*_n \overline{A}_n \tag{2.7}$$

where n are the final states accessible in the decays of *both* the K^0 and the $\overline{K^0}$, ρ_n is their density, and the decay amplitudes are $A_n = \langle n|H_{\Delta S=1}|K^0\rangle$, $\overline{A}_n = \langle n|H_{\Delta S=1}|\overline{K^0}\rangle$. The most important final states for the neutral kaon system are those of two pions with no relative orbital angular momentum, then: $\mathrm{CP}|2\pi\rangle = +|2\pi\rangle$. A state need not be a CP eigenstate to be included in the sum (2.7); an example is $\pi^+\pi^-\pi^0$. Only when all three pions have no relative angular momentum is $\pi^+\pi^-\pi^0$ an eigenstate of CP with $\mathrm{CP}|\pi^+\pi^-\pi^0\rangle = -|\pi^+\pi^-\pi^0\rangle$; we denote this $\pi^+\pi^-\pi^0$(S-wave). This final state is favored in K^0_{L} decay.

In the same formalism,

$$\Gamma_{11} = 2\pi \sum_m \rho_m |A_m|^2 = 2\pi \sum_m \rho_m |\overline{A}_m|^2 = \Gamma_{22}$$

The sum over m now encompasses *all* final states available to the K^0. In the kaon system, the 2π final states account for all but 1 part in 580 of Γ_{11}, which gives rise to (2.6).

The physical picture follows from first considering the limit of CP conservation. The CP eigenstates K_1^0 and K_2^0 are defined by:

$$\begin{aligned} |K_1^0\rangle &= \frac{1}{\sqrt{2}}\left[|K^0\rangle + |\overline{K^0}\rangle\right], \quad \mathrm{CP}|K_1^0\rangle = +|K_1^0\rangle \\ |K_2^0\rangle &= \frac{1}{\sqrt{2}}\left[|K^0\rangle - |\overline{K^0}\rangle\right], \quad \mathrm{CP}|K_2^0\rangle = -|K_2^0\rangle \end{aligned} \tag{2.8}$$

For $K_2^0 \to 2\pi$, $K^0 \to 2\pi$ and $\overline{K^0} \to 2\pi$ interfere destructively, and delete the 2π final state from the K_2^0's total width.

$$|\langle 2\pi|H_{\Delta S=1}|K_2^0\rangle|^2 = \frac{1}{2}\left[|A_{2\pi}|^2 + |\overline{A}_{2\pi}|^2 - 2\mathrm{Re}[A_{2\pi}\overline{A}^*_{2\pi}]\right] \tag{2.9}$$

When CP is conserved,

$$A_{2\pi} = \langle 2\pi|H_{\Delta S=1}|K^0\rangle = \langle 2\pi|(CP)^\dagger H_{\Delta S=1}(CP)|K^0\rangle = +\langle 2\pi|H_{\Delta S=1}|\overline{K^0}\rangle = \overline{A}_{2\pi}$$

and

$$A_{2\pi}\overline{A}^*_{2\pi} = |A_{2\pi}|^2 \tag{2.10}$$

So $|\langle 2\pi|H_{\Delta S=1}|K_2^0\rangle|^2$ vanishes. Similarly, $A_{3\pi} = -\overline{A}_{3\pi}$ results in the suppression through destructive interference of $K_1^0 \to \pi^+\pi^-\pi^0$(S-wave). These pieces of physics are described in the Schrödinger formalism by Γ_{12}.

In the limit of CP conservation, the K_1^0 has a total width of $\Gamma_{11} + \Gamma_{12} \approx \Gamma_S$, and the K_2^0 a width of $\Gamma_{11} - \Gamma_{12} = \Gamma_L$. The greater density of final states available to 2π compared to the other, mostly 3-body, final states is reason why the 2π states so dominate Γ_{11}. The dominance of the 2π final states is more surprising when stated as the following ratio, which has little correction for density of states:

$$\frac{\Gamma(K_S^0 \to 2\pi)}{\Gamma(K^+ \to 2\pi) + \Gamma(K^- \to 2\pi)} = 325 \tag{2.11}$$

This is one of the experimental underpinnings of the $\Delta I = 1/2$ rule. Two pion states with total isospin of both 0 and 2 are accessible from the K_S^0, which itself has isospin 1/2; two pion states with total isospin 1 are forbidden by generalized Bose symmetry. Only $(2\pi)_{I=2}$ is accessible from the K^+, which also has isospin 1/2. The $\pi^+\pi^0$ must have total isospin of at least 1, because the third component of isospin is 1. So, the ratio (2.11) can be described by a huge enhancement of the $\Delta I = 1/2$ transition relative

to the $\Delta I = 3/2$. This description predicts a value of 2 for what is known in neutral kaon physics as the 'single ratio':

$$\frac{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}{\mathrm{B}(K_S^0 \to \pi^0\pi^0)} = 2.19. \tag{2.12}$$

The prediction comes heuristically from noting that in the $(2\pi)_{I=0}$ state:

$$|(2\pi)_{I=0}\rangle = \sqrt{\frac{1}{3}}\left[|\pi_1^+\pi_2^-\rangle - |\pi^0\pi^0\rangle + |\pi_1^-\pi_2^+\rangle\right]. \tag{2.13}$$

there is, after Bose symmetrization, twice as much $\pi^+\pi^-$ as $\pi^0\pi^0$. The isospin 2,

$$|(2\pi)_{I=2}\rangle = \sqrt{\frac{1}{6}}\left[|\pi_1^+\pi_2^-\rangle + 2|\pi^0\pi^0\rangle + |\pi_1^-\pi_2^+\rangle\right]. \tag{2.14}$$

has two times as much $\pi^0\pi^0$ as $\pi^+\pi^-$, after Bose symmetrization. It is useful to remember this result when considering $\mathrm{Re}[\epsilon'/\epsilon]$.

One physical visualization of the $\Delta I = 1/2$ enhancement starts with the observation that the $I = 1$ π triplet is lower in mass than the $I = 0$ η singlet, largely because the η picks up rest energy from annihilation through gluons.[14] The $(2\pi)_{I=2}$ quintuplet should be degenerate at mass $\approx 2m_\pi$ with the $\pi^+\pi^+$, and cannot raise its mass through annihilation. Transitions to $(2\pi)_{I=2}$ are then unlikely to be enhanced from final state annihilations. The pions in $(2\pi)_{I=0}$ presumably do annihilate, so it is easy to visualize lots of resonant enhancement of this channel. The phase shift for $(2\pi)_{I=0}$ scattering at $\sqrt{s} = M_K$, $\delta_0 = 46° \pm 5°$, is large, while that for $(2\pi)_{I=2}$ scattering is small, $\delta_2 = -7.2 \pm 1.3$, supporting this simple physical picture. So final state rescattering probably contributes to the $\Delta I = 1/2$ enhancement. Other $\Delta I = 1/2$ enhancements, particularly from penguin diagrams,[2] are theoretically expected, but are a challenge to visualize.

No-one expects a dramatic lifetime split in the neutral B system. The reason is that the overwhelming majority of final states expected from B^0 decay are not accessible, in the Standard Model, from $\overline{B^0}$. A consequence for CP violation is that the condition of (2.4) is very nearly satisfied, from smallness of Γ_{12} On the empirical side, current experiments have not addressed Γ_{12}/Γ_{11}, but they probably would have noticed if it were much greater than 1/2.

2.2.2 Flavor Oscillation and Mass Difference

The second distinctive physical feature of the neutral kaon system is that a K^0 eventually turns into a $\overline{K^0}$, through time evolution described by (2.1). The same mechanism produces the mass splitting between the K_S^0 and K_L^0, which was measured prior to the discovery CP violation. The physics of flavor oscillation and mass difference is the primary consequence of M_{12}. If CP is conserved, it is straightforward to obtain the probability of detecting a $\overline{K^0}$ at a later proper time t, if the initial state at $t = 0$

was a K^0, $P(\overline{K^0}, t; K^0)$ from (2.1):

$$\begin{aligned} P(\overline{K^0}, t; K^0) &= \frac{1}{4}\left[e^{-(\Gamma_{11}+\Gamma_{12})t} + e^{-(\Gamma_{11}-\Gamma_{12})t}\right] - 2\,e^{-\Gamma_{11}t}\cos(2M_{12}t) \\ &= \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t)\right] \\ &= P(K^0, t; \overline{K^0}) \end{aligned} \tag{2.15}$$

It is possible to directly 'sample' (2.15) through decays to final states that are not common to K^0 and $\overline{K^0}$, such as $\pi^{\mp}l^{\pm}\nu$.[15] Such data is shown in Fig. 1. An interesting feature is that ΔM_K is the 'beat frequency' in the interference between K^0_S and K^0_L. In the neutral kaon system ΔM_K is measured explicitly through that and other interferences, yielding

$$\Delta M_K \equiv M_L - M_S = -2M_{12} = (3.522 \pm 0.016)\,\mu\text{eV}.$$

Explicit interference has not yet been observed in the B system, but measurements of like and unlike sign dileptons from $\Upsilon(4S)$ decay can be related to the integral of $P(\overline{B^0}, t; B^0)$ over all time. The implication is:

$$|\Delta M_B| = (360 \pm 70)\,\mu\text{eV}. \tag{2.16}$$

This is for the B^0 system, and is usually quoted in a different notation:

$$x_d \equiv \frac{|\Delta M_B|}{\Gamma_b} = 0.71 \pm 0.14$$

The Standard Model predicts that for the B^0_s system, the mass splitting is expected to be about an order of magnitude higher than (2.16), while the lifetime is unchanged; so $x_s \approx 8$.

We have as yet no experimental measure phase of ΔM_B in the complex plane. It is possible in the Standard Model that ΔM_B is *purely* imaginary, a manifestation of the innately 'large' CP violation in the B system.

2.3 CP Violation

The first manifestation of CP violation was the observation of $K^0_L \to \pi^+\pi^-$. Terminology has changed here, Prior to the observation of $K^0_L \to \pi^+\pi^-$, one would have assumed that the long-lived kaon was the K^0_2; after, the more empirical K^0_L became appropriate. Mathematically, K^0_L denotes the longer-lived eigenstate of the Hamiltonian.

We will focus first on description of $K^0_L \to 2\pi$. The other important physical manifestation of CP violation is the distribution in time of decays, through which one 'samples' $P(\overline{K^0}, t; K^0)$ or $P(K^0, t; \overline{K^0})$; inequality between these is a clean proof of CP violation in state-mixing.

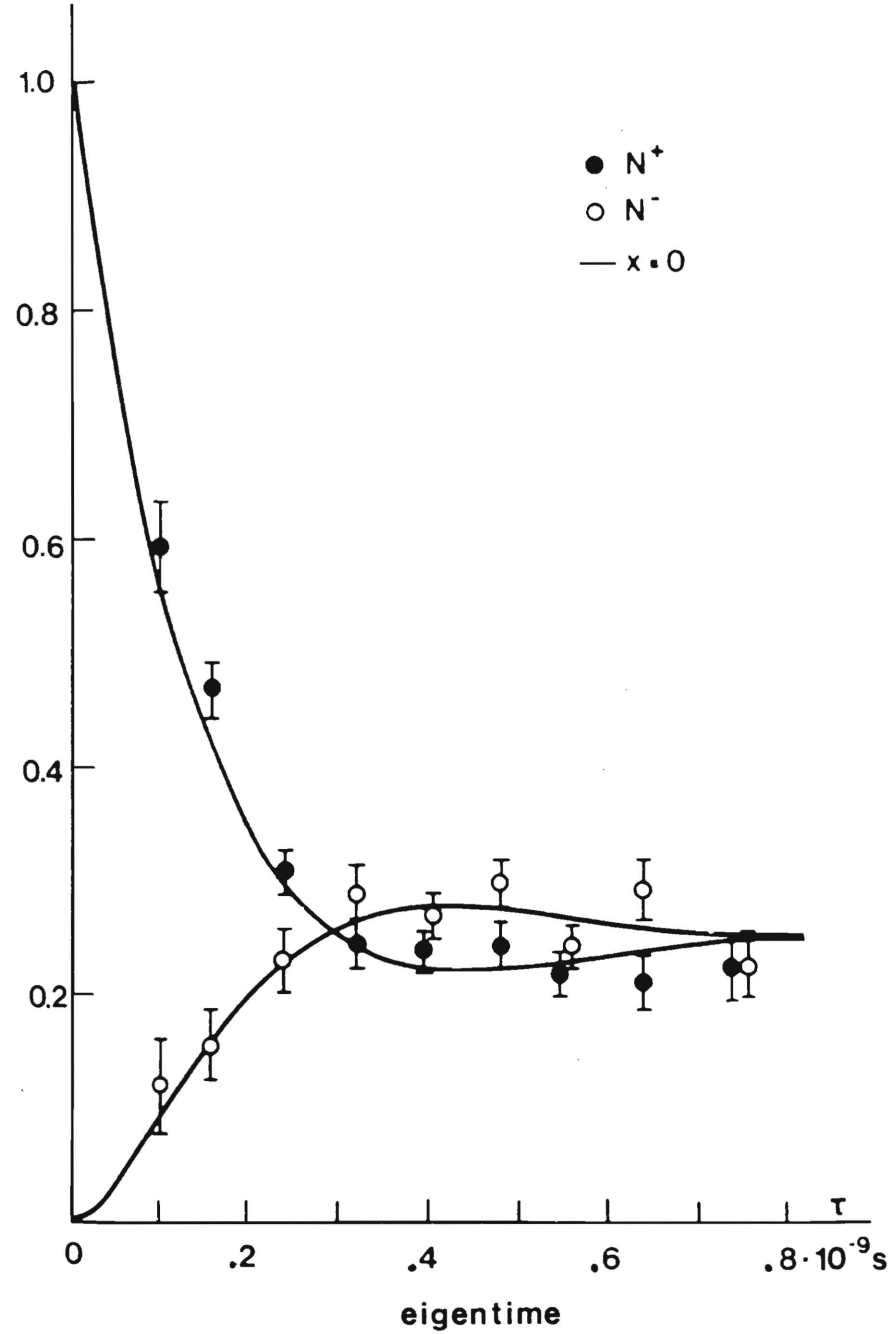

Figure 1. A K^0 was produced and tagged at $\tau = 0$; N^+ is the number of $\pi^-e^+\nu$ decays, N^- is $\pi^+e^-\nu$. A K^0_S lifetime is slightly less than one division.[15]

2.3.1 $K_L^0 \to 2\pi$

Consider state-mixing CP violation, that is, CP violation in the Hamiltonian; then through (2.4), we know that the off-diagonal elements of H differ in *magnitude*. The eigenvalues and eigenvectors of H, denoted by Nir $\Delta\mu/2$, are found from

$$\begin{vmatrix} -\Delta\mu/2 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & -\Delta\mu/2 \end{vmatrix} = 0 \quad \Rightarrow \Delta\mu/2 = \pm\left[(M_{12} - \frac{i}{2}\Gamma_{12})(M_{12}^* - \frac{i}{2}\Gamma_{12}^*)\right]^{1/2}$$

$$\begin{bmatrix} -\Delta\mu/2 & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & -\Delta\mu/2 \end{bmatrix} \begin{bmatrix} p \\ q \end{bmatrix} = 0 \quad \Rightarrow \frac{q}{p} = \pm\left[\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right]^{1/2} \neq e^{2i\zeta} \qquad (2.17)$$

The inequality at right is a reminder of (2.4); were that equality true, with (2.5) $p = q = \pm 1/\sqrt{2}$, (2.8) is recovered, and state-mixing CP violation is absent.

The usual overall phase convention is:

$$\begin{aligned} |K_S^0\rangle &= \left[p|K^0\rangle + q|\overline{K^0}\rangle\right], \quad \text{eigenvalue } \Delta\mu/2 \\ |K_L^0\rangle &= \left[p|K^0\rangle - q|\overline{K^0}\rangle\right], \quad \text{eigenvalue } -\Delta\mu/2 \end{aligned} \qquad (2.18)$$

Consider the rate for $K_L^0 \to 2\pi$, in analogy with (2.9):

$$|\langle 2\pi|H_{\Delta S=1}|K_S^0\rangle|^2 = |pA_{2\pi}|^2 + |q\overline{A}_{2\pi}|^2 - 2\mathrm{Re}(pq^* A_{2\pi}\overline{A}_{2\pi}^*) \qquad (2.19)$$

An important physical concept underlies (2.19): CP violation experiments always measure CP violation via decays, so the interpretation of their results always entangles state-mixing (p and q) with CP violation in the decay amplitudes (A and $\overline{A}$).

The usual supposition for the kaon system, which corresponds to Nir's case (*ii*) in his 'Manifestations of CP violation', is that the CP violation arises from state-mixing ($|p| \neq |q|$), then, by (2.10):

$$|\langle 2\pi|H_{\Delta S=1}|K_S^0\rangle|^2 = |A_{2\pi}|^2\,|p-q|^2 \qquad (2.20)$$

The factorization in (2.20) implies that *all* CP-forbidden decays rates will be in a universal ratio to their CP-allowed counterparts:

$$\begin{aligned} \left[|\eta_{+-}|^2 \equiv \frac{\Gamma(K_L^0 \to \pi^+\pi^-)}{\Gamma(K_S^0 \to \pi^+\pi^-)}\right] &= \left[|\eta_{00}|^2 \equiv \frac{\Gamma(K_L^0 \to \pi^0\pi^0)}{\Gamma(K_S^0 \to \pi^0\pi^0)}\right] = \\ \left[|\eta_{000}|^2 \equiv \frac{\Gamma(K_S^0 \to 3\pi^0)}{\Gamma(K_L^0 \to 3\pi^0)}\right] &= \left[\ldots\right] = \left|\frac{p-q}{p+q}\right|^2 \equiv |\epsilon|^2 \end{aligned} \qquad (2.21)$$

The parameter ϵ is introduced to quantify the deviation, small in the kaon system, of

q/p from unity.

$$\frac{q}{p} = \frac{1-\epsilon}{1+\epsilon} = \left[\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right]^{1/2}$$

The phase of ϵ is defined in harmony with the superweak hypothesis, by *the convention* (through use of (2.5)): $\mathrm{Im}[\Gamma_{12}] = 0$; then with

$$z \equiv i\mathrm{Re}M_{12} + \frac{1}{2}\Gamma_{12} = -\frac{i}{2}\Delta M_K + \frac{1}{4}(\Gamma_S - \Gamma_L) = (2.55\,\mu\mathrm{eV})e^{-43.7^\circ}$$

here, $\phi_\epsilon = \tan^{-1}(2\Delta M_K/(\Gamma_S - \Gamma_L)) = 43.7^\circ \pm 0.2^\circ$ is often called the 'superweak phase'; then

$$\frac{1-\epsilon}{1+\epsilon} = \left[\frac{1 + \mathrm{Im}[M_{12}]/z}{1 - \mathrm{Im}[M_{12}]/z}\right]^{1/2}$$

so approximately

$$\epsilon = -\left[\frac{\mathrm{Im}[M_{12}]}{5.09\,\mu\mathrm{eV}}\right] e^{43.7^\circ} \qquad (2.22)$$

From (1.1) and (2.21), $\mathrm{Im}[M_{12}] = -11.6\,\mathrm{neV}$. This one number best summarizes the hypothesis of state-mixing as the source of CP violation in the kaon system. Both the Standard Model and the superweak hypothesis are consistent with the value. It must be accidental that $\mathrm{Im}[M_{12}]$ is near the full width of the K_L^0, $\Gamma_L = 12.7\,\mathrm{neV}$.

One of the paradoxes of the neutral B system that although the Standard Model predicts $|\mathrm{Im}[M_{12}]| \approx |\Delta M_B| \approx 360\,\mu\mathrm{eV}$, four orders of magnitude larger than $\mathrm{Im}[M_{12}]$ in the kaon system, the observable consequences are less evident. One reason is that the lifetime splitting in the B system is expected to be only at the level of $|\Gamma_{11} - \Gamma_{12}|/|\Gamma_{11} + \Gamma_{12}| \approx 10^{-3} - 10^{-2}$, so the superweak phase is near 90°, and q/p is again unity.

For the kaon system, is the description that $\mathrm{Im}[M_{12}] \neq 0$ the complete story? What might come from $\mathrm{Im}[\Gamma_{12}]$, or $A \neq \overline{A}$ for some decay amplitude? Non-negligible contributions to Γ_{12} come from decay amplitudes to 2π and 3π final states. The contribution of the 3π states to Γ_{12} is at most:

$$\left|2\pi\rho_{3\pi}A_{3\pi}^*\overline{A}_{3\pi}\right| < \frac{1}{2}\frac{\mathrm{B}(K_L^0 \to 3\pi^0) + \mathrm{B}(K_L^0 \to \pi^+\pi^-\pi^0)}{\tau_L} = 2.2\,\mathrm{neV}$$

This is too small to give all of the observed CP violation. Only the 2π amplitudes have sufficient magnitude to vie with an explanation of pure $\mathrm{Im}[M_{12}]$. The usual phenomenology casts the amplitude for $K^0 \to \pi^+\pi^-$, A_{+-}, and that for $K^0 \to \pi^0\pi^0$, A_{00}, in terms of amplitudes to two pion states of total isospin 0, A_0, and total isospin 2, A_2:

$$\begin{aligned} A_{+-} &= \sqrt{\frac{2}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}}A_2 e^{i\delta_2} \\ A_{00} &= -\sqrt{\frac{1}{3}}A_0 e^{i\delta_0} + \sqrt{\frac{2}{3}}A_2 e^{i\delta_2} \end{aligned} \qquad (2.23)$$

it is a consequence of CPT invariance that the decay amplitudes of the $\overline{K^0}$ can be

$$\overline{A}_{+-} = \sqrt{\frac{2}{3}} A_0^* e^{i\delta_0} + \sqrt{\frac{1}{3}} A_2^* e^{i\delta_2}$$
$$\overline{A}_{00} = -\sqrt{\frac{1}{3}} A_0^* e^{i\delta_0} + \sqrt{\frac{2}{3}} A_2^* e^{i\delta_2} \tag{2.24}$$

That A_0 and A_2, but not the strong phase shifts, are complex conjugated is known as 'Watson's Theorem' and is subtle.[16]

From the $\Delta I = 1/2$ rule, we know $|A_2| << |A_0|$. Imagine $A_2 = 0$; then $|A_{+-}| = |\overline{A}_{+-}|$ and $|A_{00}| = |\overline{A}_{00}|$. It is part of the standard convention to *pick* A_0 as real, employing the mathematical artifice of (2.5) if necessary. **All** of the kaon CP violation could physically come through a phase in A_0, but the description is transferred to $\mathrm{Im}[M_{12}]$. In the Standard Model, for example, penguin diagrams do contribute to state-mixing CP violation through A_0.[2]

The only viable alternative source of kaon CP violation is a non-zero imaginary part of A_2 relative to A_0, which forces $|A_{+-}| \neq |\overline{A}_{+-}|$ *and* $|A_{00}| \neq |\overline{A}_{00}|$. Analysis of the ratio (2.11) indicates $|A_2/A_0| \approx 0.05$, so CP violation in A_2 is large enough to produce the entire $K_L^0 \to 2\pi$ rate. The prime physical effect of CP violation exclusively through A_2 would be a change the ratio of $K_L^0 \to \pi^0\pi^0$ to $K_L^0 \to \pi^+\pi^-$, relative to that for K_S^0. The contribution that is lowest order in A_2/A_0 (2.19) is, from (2.23) and (2.24):

$$\langle \pi^+\pi^- | H_{\Delta S=1} | K_L^0 \rangle = \frac{1}{\sqrt{2}} \left[\sqrt{\frac{1}{3}} (A_2 - A_2^*) e^{i\delta_2} \right] = \frac{1}{\sqrt{2}} \left[\sqrt{\frac{1}{3}}\, 2\, i \mathrm{Im} A_2 e^{i\delta_2} \right]$$
$$\langle \pi^+\pi^- | H_{\Delta S=1} | K_S^0 \rangle = \frac{1}{\sqrt{2}} \left[\sqrt{\frac{2}{3}}\, 2\, A_0 e^{i\delta_0} + \sqrt{\frac{1}{3}} (A_2 + A_2^*) e^{i\delta_2} \right]$$

If A_2 were to dominate, to lowest order in $|A_2/A_0|$:

$$\eta_{+-} = \frac{\langle \pi^+\pi^- | H_{\Delta S=1} | K_L^0 \rangle}{\langle \pi^+\pi^- | H_{\Delta S=1} | K_S^0 \rangle} = \frac{i}{\sqrt{2}} \mathrm{Im} \left[\frac{A_2}{A_0} \right] e^{i(\delta_2 - \delta_0)} \equiv \epsilon' \tag{2.25}$$

This second to last expression is the definition of ϵ'. For the transitions to $\pi^0\pi^0$:

$$\langle \pi^0\pi^0 | H_{\Delta S=1} | K_L^0 \rangle = \frac{1}{\sqrt{2}} \left[\sqrt{\frac{2}{3}} (A_2 - A_2^*) e^{i\delta_2} \right] = \frac{1}{\sqrt{2}} \left[\sqrt{\frac{2}{3}}\, 2\, i \mathrm{Im} A_2 e^{i\delta_2} \right]$$
$$\langle \pi^0\pi^0 | H_{\Delta S=1} | K_S^0 \rangle = \frac{1}{\sqrt{2}} \left[-\sqrt{\frac{1}{3}}\, 2\, A_0 e^{i\delta_0} + \sqrt{\frac{2}{3}} (A_2 + A_2^*) e^{i\delta_2} \right] \tag{2.26}$$
$$\eta_{00} = \frac{\langle \pi^0\pi^0 | H_{\Delta S=1} | K_L^0 \rangle}{\langle \pi^0\pi^0 | H_{\Delta S=1} | K_S^0 \rangle} = -2 \times \frac{i}{\sqrt{2}} \mathrm{Im} \left[\frac{A_2}{A_0} \right] e^{i(\delta_2 - \delta_0)} = -2\epsilon'$$

Were all kaon CP violation to come from A_2, the branching ratio for $K_L^0 \to \pi^0\pi^0$ would be twice as large as that for $K_L^0 \to \pi^+\pi^-$, as one would have expected from

(2.14), in contrast to (2.13). Experimentally, however, the latest experiments NA31 and E731 find nearly the same excess of charged pions as in (2.11). The dominant source of kaon CP violation is **not** A_2, but is state-mixing.

Nevertheless, the Standard Model predicts a small violation of CP through A_2; the amplitudes (2.25) and (2.26) add coherently to that of state mixing:

$$\eta_{+-} = \epsilon + \epsilon'$$
$$\eta_{00} = \epsilon - 2\epsilon' \tag{2.27}$$

The quantity that is accessible with small experimental systematic error is the 'double ratio', R:

$$R \equiv \frac{\mathrm{B}(K_L^0 \to \pi^0\pi^0)}{\mathrm{B}(K_L^0 \to \pi^+\pi^-)} \frac{\mathrm{B}(K_S^0 \to \pi^+\pi^-)}{\mathrm{B}(K_S^0 \to \pi^0\pi^0)} = \frac{|\eta_{00}|^2}{|\eta_{+-}|^2} \tag{2.28}$$

Very naively, one expects CP violation in A_2 will favor $R > 1$, because A_2 favors transitions of K_L^0 to neutral pions. However, the relative minus sign between the $|\pi^0\pi^0\rangle$ terms in (2.13) and (2.14) inverts this expectation, as first term in shows (2.26). Plugging (2.27) in, one gets to lowest order:

$$\mathrm{Re}(\epsilon'/\epsilon) = \frac{1}{6}[1 - R] \tag{2.29}$$

Accidental agreement of the phase of ϵ' (from strong interactions, $\pi/2 + \delta_2 - \delta_0 \approx 90° - (-7.2°) + 46° = 51° \pm 5°$) and that of ϵ means $\mathrm{Re}[\epsilon'/\epsilon] \approx \epsilon'/\epsilon$.

It is known as 'direct' CP violation when two decay amplitudes such as A_0 and A_2 interfere, and introduce a rate difference in decay. Direct CP violation corresponds to Nir's case (*i*) in his 'Manifestations of CP violation'.

The Standard Model should induce both state-mixing and direct CP violation in the B^0 system, but only at the same level as that observed in the kaon system, 10^{-3}.[17] The dominant feature in the experimentally attractive decay modes for detection of CP violation in the B^0 system, such as $B^0 \to J/\psi K_S^0$, is neither state-mixing nor direct CP violation: it is interference between the two, which corresponds to Nir's case (*iii*). For the B^0 system, $|q/p| \approx 1$, as discussed after (2.22), and as well $|\overline{A}_{J/\psi K_S^0}/A_{J/\psi K_S^0}| \approx 1$. However, the quantity

$$\lambda_{J/\psi K_S^0} = \frac{q}{p} \frac{\overline{A}_{J/\psi K_S^0}}{A_{J/\psi K_S^0}} = e^{i2\beta} \neq 1$$

Here, β is an angle of the most common 'unitary triangle', and can be of order unity. The point is, if one measures the asymmetry in decays to $J/\psi K_S^0$ between initial B^0 and $\overline{B^0}$, one sees an effect of order $\sin 2\beta$.

2.3.2 Asymmetries

If the probability of seeing a $\overline{K^0}$ at proper time t, given an initial K^0, $P(\overline{K^0}, t; K^0)$ differs from $P(K^0, t; \overline{K^0})$, that is evidence of CP violation. Experimentally, the first job is to tag the initial flavor; the second job is to detect a decay of the K^0 or $\overline{K^0}$ to a final state f, and record its time. A situation that cleanly selects only state-mixing CP violation arises when the decay f is not common to both K^0 and $\overline{K^0}$, such as $\pi^{\mp}e^{\pm}\nu$. If the final state f is common to both K^0 and $\overline{K^0}$, state-mixing CP violation, direct CP violation, or interference between the two will cause an interference pattern from the indistinguishable processes $K^0 \to \overline{K^0} \to f$ and $K^0 \to K^0 \to f$.

If state-mixing causes $K^0_L \to 2\pi$ one expects a rate asymmetry between $K^0_L \to \pi^+e^-\overline{\nu}$ and $K^0_L \to \pi^-e^+\nu$, and this is observed. Common final states in both the kaon and B systems are usually CP eigenstates, such as $\pi^+\pi^-$, $\pi^0\pi^0$, and $J/\psi K^0_S$, and it is by fitting the interference pattern that one can extract the phase of η_{+-} and η_{00}. The interference pattern is precisely how the asymmetry of order $\sin 2\beta$ is generated $B^0 \to J/\psi K^0_S$.

Call $\Psi(t)(\overline{\Psi}(t))$ the state that evolves from an initial $K^0(\overline{K^0})$. Decomposition into eigenstates of H yields:

$$\begin{aligned} |\Psi(t)\rangle &= \frac{1}{2}\left[(e^{-it\Delta\mu/2} + e^{it\Delta\mu/2})|K^0\rangle + \frac{q}{p}(e^{-it\Delta\mu/2} - e^{it\Delta\mu/2})|\overline{K^0}\rangle\right] \\ |\overline{\Psi}(t)\rangle &= \frac{1}{2}\left[(e^{-it\Delta\mu/2} + e^{it\Delta\mu/2})|\overline{K^0}\rangle + \frac{p}{q}(e^{-it\Delta\mu/2} - e^{it\Delta\mu/2})|K^0\rangle\right] \end{aligned} \tag{2.30}$$

where $\Delta\mu/2$ is the positive eigenvalue. It is evident from (2.30) that $P(K^0, t; K^0)$ and $P(\overline{K^0}, t; \overline{K^0})$ are both unaffected by CP violation, and remain as they were in its absence:

$$\begin{aligned} P(K^0, t; K^0) &= \frac{1}{4}\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} + 2\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t)\right] \\ &= P(\overline{K^0}, t; \overline{K^0}) \end{aligned}$$

To order ϵ, however,

$$\begin{aligned} P(\overline{K^0}, t; K^0) &= (\frac{1}{4} - \mathrm{Re}[\epsilon])\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t)\right] \\ \mathrm{P}(K^0, t; \overline{K^0}) &= (\frac{1}{4} + \mathrm{Re}[\epsilon])\left[e^{-\Gamma_S t} + e^{-\Gamma_L t} - 2\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t)\right] \end{aligned} \tag{2.31}$$

Plots of these expressions are shown in Figure 2. These equations express what state-mixing CP violation does; note their simple relationship to (2.15). One 'samples' (2.31) with distinct final states: only $K^0 \to \pi^-e^+\nu$ and only $\overline{K^0} \to \pi^+e^-\overline{\nu}_e$, for example. The form of (2.31) means *no tag is necessary*, so for an arbitrary initial superposition of K^0 and $\overline{K^0}$, the following asymmetry[18]:

$$\frac{N(\pi^-e^+\nu) - N(\pi^+e^-\overline{\nu})}{N(\pi^-e^+\nu) + N(\pi^+e^-\overline{\nu})} \xrightarrow{t\to\infty} \equiv \delta_e = 2\mathrm{Re}[\epsilon]. \tag{2.32}$$

The data used to measure (2.32) is shown in Fig. 3.

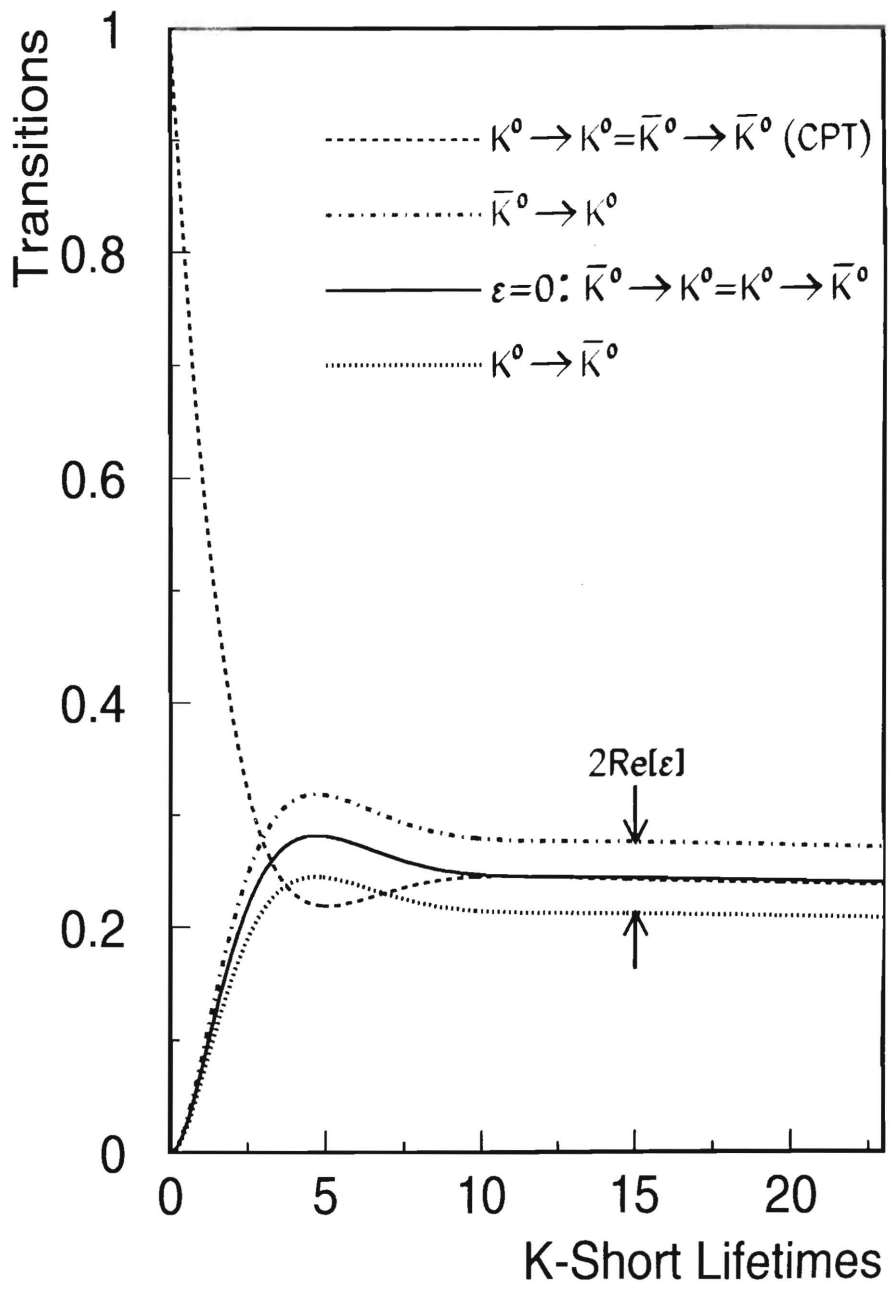

Figure 2. CP violation in state-mixing only modifies mixing.

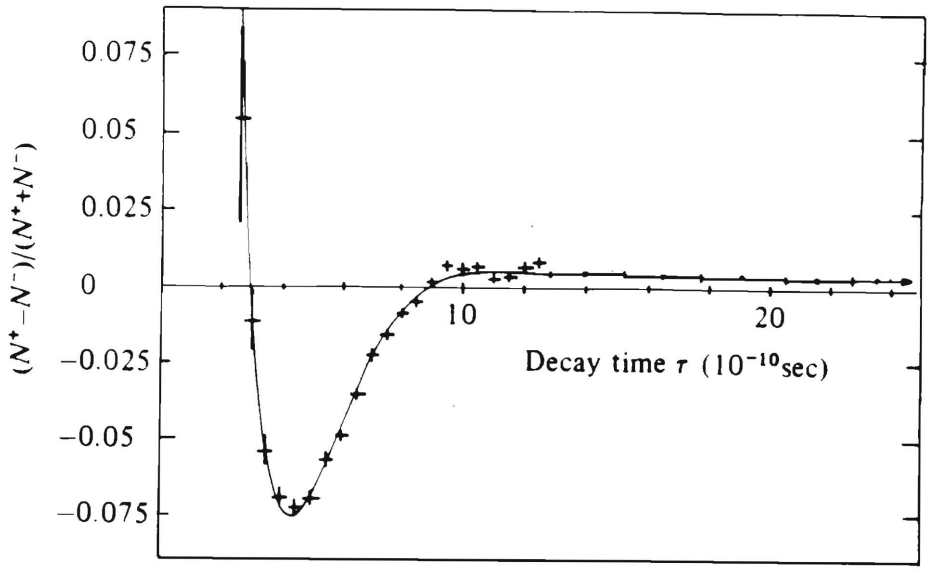

Figure 3. The charge asymmetry in the K^0 system. The asymmetry approaches a constant as $t \to \infty$, as the system becomes pure K^0_L.

The analogous situation in the B^0 system is simplified because the lifetime splitting is expected to be negligible, so the analog of (2.31) is:

$$\begin{aligned} P(\overline{B^0}, t; B^0) &= (1 - 4\mathrm{Re}[\epsilon_B])e^{-\Gamma_b t}\sin^2(\Delta M_B t/2) \\ P(B^0, t; \overline{B^0}) &= (1 + 4\mathrm{Re}[\epsilon_B])e^{-\Gamma_b t}\sin^2(\Delta M_B t/2) \\ P(B^0, t; B^0) &= e^{-\Gamma_b t}\cos^2(\Delta M_B t/2) \\ &= P(\overline{B^0}, t; \overline{B^0}) \end{aligned} \tag{2.33}$$

The experimental procedure to observe state-mixing in the B system would differ from that in the kaon system; one would look for evidence that $P(\overline{B^0}, t; B^0) \neq P(B^0, t; \overline{B^0})$ through unequal numbers of like-sign positively charged dilepton events and like-negatively charged dilepton events. If only incoherent $B^0\overline{B^0}$ pairs are made, for example,

$$\frac{N(l^+l^+) - N(l^-l^-)}{N(l^+l^-) + N(l^+l^+) + N(l^-l^+) - N(l^-l^-)} = 2\mathrm{Re}[\epsilon_B]\frac{x_d^2(2 + x_d^2)}{[1 + x_d^2]^2} \tag{2.34}$$

To compute the probability that an initially tagged K^0 decays to $\pi^+\pi^-$, the interference between $K^0 \to \pi^+\pi^-$ and $\overline{K^0} \to \pi^+\pi^-$ must be accounted for. The phase ϕ_{+-} defined through $\eta_{+-} = |\eta_{+-}|e^{i\phi_{+-}}$, is experimentally accessible through this interference. Calculation of the interference pattern is straightforward if (2.30) is rewritten

in terms of the eigenstates of the Hamiltonian:

$$\begin{aligned} \langle\pi^+\pi^-|H_{\Delta S=1}|\Psi(t)\rangle &= \langle\pi^+\pi^-|H_{\Delta S=1}\frac{1}{2p}\left[e^{-i\Delta\mu/2t}|K^0_S\rangle + e^{i\Delta\mu/2t}|K^0_L\rangle\right] \\ &= \frac{\langle\pi^+\pi^-|H_{\Delta S=1}|K^0_S\rangle}{2p}\left[e^{-it\Delta\mu/2} + \eta_{+-}\,e^{it\Delta\mu/2}\right] \end{aligned}$$

The probability that one observes a $\pi^+\pi^-$ from an initial K^0 or $\overline{K^0}$ is[19]

$$\begin{aligned} \mathrm{P}(\pi^+\pi^-, t; K^0) &\propto (\frac{1}{2} - \mathrm{Re}[\epsilon])\Big[e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} \\ &\quad + 2\,|\eta_{+-}|\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t - \phi_{+-})\Big] \\ \mathrm{P}(\pi^+\pi^-, t; \overline{K^0}) &\propto (\frac{1}{2} + \mathrm{Re}[\epsilon])\Big[e^{-\Gamma_S t} + |\eta_{+-}|^2 e^{-\Gamma_L t} \\ &\quad - 2\,|\eta_{+-}|\,e^{-(\Gamma_S+\Gamma_L)t/2}\cos(\Delta M_K t - \phi_{+-})\Big] \end{aligned} \tag{2.35}$$

The key point is that the interference term flips sign for initial antimatter. The expressions (2.35) are shown in Fig. 4, and experimental data is shown in Fig. 5. These distributions have been measured many times, most recently by NA31. The E731 experiment uses an initial state prepared by regeneration of a K^0_S beam from a K^0_L beam. This makes a *coherent* superposition of K^0 and $\overline{K^0}$, so adds a 'regeneration phase' to ϕ_{+-}, and modulates the interference term.

In the neutral B system, there is not expected to be a dramatic lifetime splitting, so the analog of (2.35) is a bit simpler. One obtains, for example,

$$\begin{aligned} \mathrm{P}(J/\psi K^0_S, t; B^0) &\propto (\frac{1}{4} - \mathrm{Re}[\epsilon_B])e^{-\Gamma_B t}\Big[\cos^2(\Delta M_B t/2) + |\lambda_{J/\psi K^0_S}|^2\sin^2(\Delta M_B t/2) \\ &\quad + \mathrm{Re}(i\lambda_{J/\psi K^0_S})\sin(\Delta M_B t)\Big] \\ &\propto e^{-\Gamma_B t}\Big[1 - \mathrm{Im}(\lambda_{J/\psi K^0_S})\sin(\Delta M_B t)\Big] \quad |\epsilon_B| \ll 1,\ |\lambda_{J/\psi K^0_S}| = 1 \\ \mathrm{P}(J/\psi K^0_S, t; \overline{B^0}) &\propto (\frac{1}{4} + \mathrm{Re}[\epsilon_B])e^{-\Gamma_B t}\Big[\cos^2(\Delta M_B t/2) + |\lambda_{J/\psi K^0_S}|^2\sin^2(\Delta M_B t/2) \\ &\quad - \mathrm{Re}(i\lambda_{J/\psi K^0_S})\sin(\Delta M_B t)\Big] \\ &\propto e^{-\Gamma_B t}\Big[1 + \mathrm{Im}(\lambda_{J/\psi K^0_S})\sin(\Delta M_B t)\Big] \quad |\epsilon_B| \ll 1,\ |\lambda_{J/\psi K^0_S}| = 1 \end{aligned} \tag{2.36}$$

When $\mathrm{Im}(\lambda_{J/\psi K^0_S})$ is near unity, as is possible in the Standard Model, a large asymmetry between an initial B^0 and a $\overline{B^0}$ should occur. The cost is a small branching ratio, $\mathrm{B}(B^0 \to J/\psi K^0_S) \approx 4 \times 10^{-4}$.

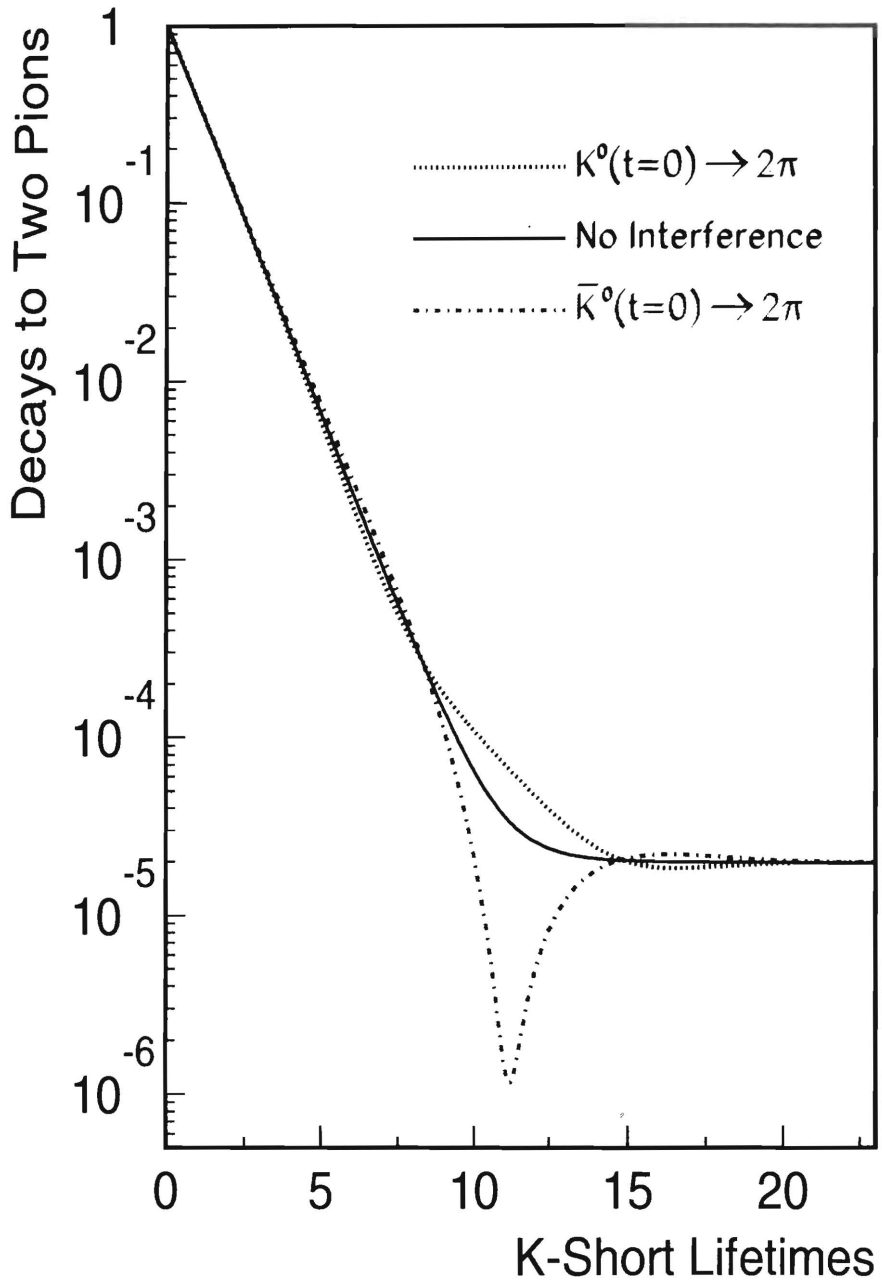

Figure 4. The distributions of 2π as a function of time, for an initial K^0, $\overline{K^0}$, and if there was no interference.

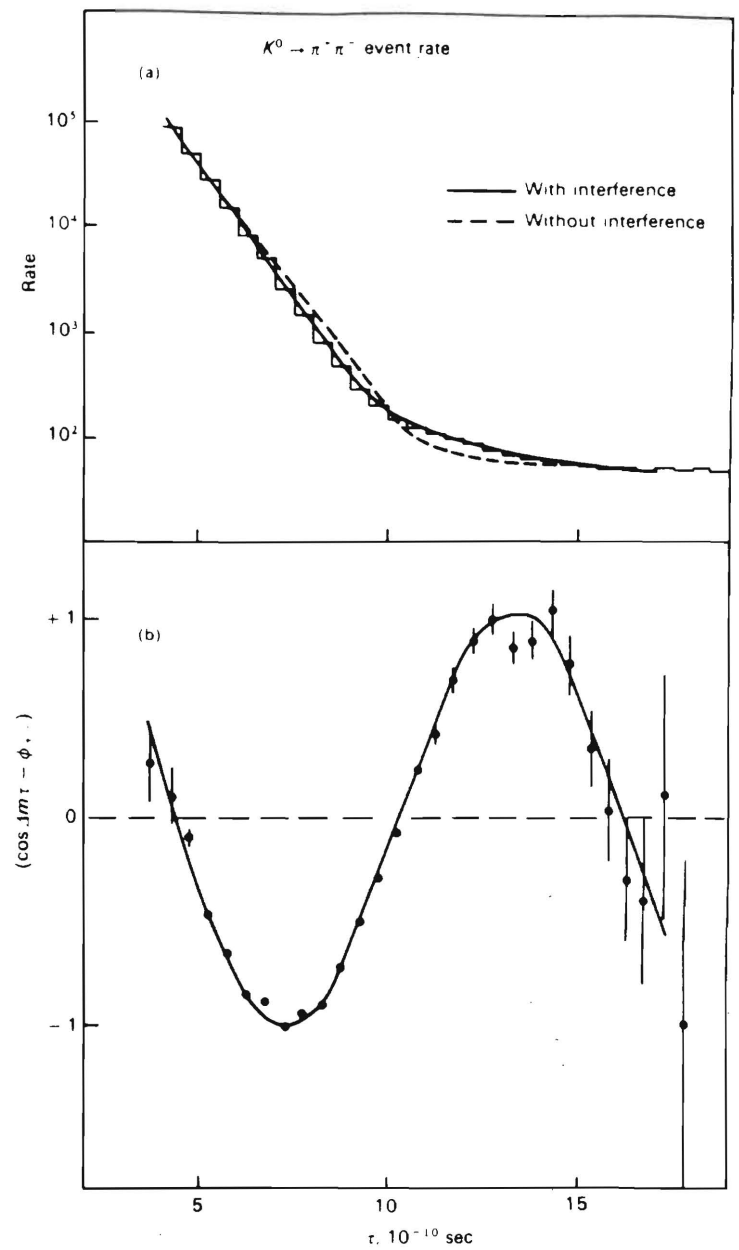

Figure 5. An experimental measure of the interference between $K^0 \to \pi^+\pi^-$ and $\overline{K^0} \to \pi^+\pi^-$. More K^0 are made at the target at $\tau = 0$, eliminating the need for tagging.[5]

For the B system, it is an experimental challenge to reconstruct the time evolution pattern in (2.36). Only recently have vertex detectors begun to see the exponential decay in the decay of B-hadrons. The time integrated asymmetry still shows evidence of CP violation:

$$A_{J/\psi K_S^0} \equiv \frac{\Gamma(B^0 \to J/\psi K_S^0) - \Gamma(\overline{B^0} \to J/\psi K_S^0)}{\Gamma(B^0 \to J/\psi K_S^0) + \Gamma(\overline{B^0} \to J/\psi K_S^0)} = -\frac{x_d}{1+x_d^2}\mathrm{Im}(\lambda_{J/\psi K_S^0}) \tag{2.37}$$

$$\approx -0.47\sin(2\beta)$$

2.3.3 Tagging

To measure an asymmetry, the initial state must be tagged as matter or antimatter. These are some of the methods:

1. Build the experiment out of matter, not equal parts matter and antimatter.
2. Reconstruct both flavors in the event.
3. Pair produce in a $C = +1$ state.

The recent kaon experiments, NA31 and E731, both used the first method. NA31 produced their kaons by steering a proton beam on to a target. Pairs of $s\bar{s}$ quarks are produced, but two to three times as many K^0 as $\overline{K^0}$ emerge from hadronization. The difference likely comes about because s quarks hadronize with the abundant u and d quarks to form baryons, whereas $\bar{u}$ and $\bar{d}$ quarks are less common. In E731 a $K^0/\overline{K^0}$ asymmetry is produced by a regenerator made of matter. Both experiments then see the interference term contained in (2.35), even though they make no effort, on an event-by-event basis, to determine the K^0 strangeness. This is an extremely important practical point, as it allows gathering of high statistics.

Recent data shows little, if any, matter/antimatter asymmetry in charm hadroproduction.[20] Probably any asymmetry in B hadroproduction is negligible. Very slight asymmetries have been discussed in considerations of SSC B production.

The fact that experiments are made of matter, and not equal parts matter and antimatter, can result in *fake* CP-like asymmetries. For example, the probability that a K^+ will fake a muon signal tends to be larger than the probability that a K^- will do so, simply because the $\bar{u}$ quark in the K^- causes the K^- to suffer a hadronic interaction more often than the K^+. Similarly, since matter is slightly richer in neutrons than protons, the π^- should fake a muon signal more often than a π^+. If CP violation is reported in the B system through observation of a muon charge asymmetry, *caveat emptor*.

The second method of tagging is to exploit flavor pair production. A clear example is the strategy of the CPLEAR experiment at CERN. They exploit the exclusive reactions:

$$p\bar{p} \to \begin{cases} \pi^- K^+ K^0 \\ \pi^+ K^- \overline{K^0} \end{cases} \tag{2.38}$$

The sign of the charged kaon tags whether the neutral kaon state was initially K^0 or $\overline{K^0}$. There is a loss of acceptance relative to NA31 or E731 due to the reconstruction of the $K^\pm\pi^\mp$, but background rejection is gained through kinematic constraints.

In most experimental proposals for the B system, only a partial reconstruction of the 'tag' hadron is made. For example, in a hadron collider, most proposals employ:

$$p\bar{p} \to \begin{cases} X\overline{B}_{\mathrm{tag}} B^0 \\ X B_{\mathrm{tag}} \overline{B^0} \end{cases}$$

Then the sign of the lepton in the semileptonic decay of the B_{tag} is used to get the flavor of the tagged B. Full kinematic reconstruction is not made, as in (2.38).

The process $e^+e^- \to \Upsilon(4s) \to B^0\overline{B^0}$ suffers a peculiar tagging deficiency. Even if CP violation is present in the B system, it cannot manifest itself with a *rate* asymmetry between initially tagged B^0 and $\overline{B^0}$, if the B production is from the $\Upsilon(4s)$. The reason is that the Υ has C=-1, and the strong decay to $B^0\overline{B^0}$ conserves C, so the $B^0\overline{B^0}$ product state ends up exclusively in

$$\frac{1}{\sqrt{2}}\left[|B^0\rangle|\overline{B^0}\rangle - |\overline{B^0}\rangle|B^0\rangle\right]$$

If an analogy with two spin-1/2 particles is made, this state is like the singlet. Then if the time evolution of the B^0 flavor is regarded as analogous to the precession of a spin 1/2 particle, then just as the singlet state of a pair of spin 1/2 particles cannot precess due to absence of a magnetic moment, it is difficult to see a time averaged CP violating effect.

In contrast, the C=+1 state is analogous to the spin triplet, and CP violating rate asymmetries are enhanced by a naive factor of 2 (spin-1/spin-1/2, the ratio of the 'precession frequencies') if $B^0\overline{B^0}$ can be produced in the C=+1 state. This is the third method of tagging, and can be achieved through the process $e^+e^- \to \overline{B}B^* \to \overline{B}B\gamma$. A careful evaluation includes the effect of mixing, and yields, for example, in place of (2.37),

$$A_{J/\psi K_S^0} \equiv \frac{\Gamma(B^0 \to J/\psi K_S^0) - \Gamma(\overline{B^0} \to J/\psi K_S^0)}{\Gamma(B^0 \to J/\psi K_S^0) + \Gamma(\overline{B^0} \to J/\psi K_S^0)} = -\left[\frac{\mathbf{2}}{1+x_d^2}\right]\frac{x_d}{1+x_d^2}\mathrm{Im}(\lambda_{J/\psi K_S^0})$$

$$\approx -0.63\sin(2\beta) \tag{2.39}$$

As we will discuss, the deficiency of the C=-1 $B\overline{B}$ is overcome by the introducing a beam energy asymmetry between the e^+ and e^- beams, thereby producing a moving $B\overline{B}$ system. Vertex reconstruction allows explicit reconstruction of the CP-violating interference as a function of time. The larger cross section of $e^+e^- \to \Upsilon(4S)$ ends up favoring the $\Upsilon(4S)$.

3. Easy, Hard, and Classic CP Violation Experiments

Why was the 'first' exploration of CP violation in kaon system so easy? Why has 'all the rest' been so hard?

More specifically, why was it so easy to

1. Observe $K_L^0 \to \pi^+\pi^-$?
2. Measure the phase ϕ_{+-}?
3. Measure the semileptonic charge asymmetry?

I'm sure the people who did those experiments wouldn't call them 'easy', but the experiments were pretty much all wrapped up by the mid-seventies, with relatively primitive technology compared to what is available today. Contrast with the situation concerning $\mathrm{Re}[\epsilon'/\epsilon]$; now, in 1992, after a decade of intense experimentation, there is still controversy over whether $\mathrm{Re}[\epsilon'/\epsilon]$ is non-zero, and another decade-long round of experiments has been initiated. Contrast with the situation in the B system: despite the fact that Standard Model CP violating effects are in some sense 'large' in the B system (e.g., ΔM_B might be completely imaginary; $\epsilon_B \approx \epsilon$) more time has now elapsed between the discovery of the b quark and the present than elapsed between the discovery of strangeness and the first observation of CP violation.

Why ask why? Maybe there is an 'easy' method of making the next CP violation measurement that has not been elucidated.

The fundamental reasons that the early CP violation experiments were easy can be abstracted from the second section on phenomenology. They are:

1. The enormous lifetime splitting in the kaon system.
2. The dominant CP violation in the kaon system involves a *rate* difference between matter and antimatter transitions, so tagging is inessential.
3. Both K^0 production from a target and K_S^0 regeneration introduce an inclusive $K^0/\overline{K^0}$ asymmetry, allowing the measurement of *interference* effects, such as ϕ_{+-}.

To see how these fundamental aspects influence practical experimental considerations, let's look at the experiment that first observed $K_L^0 \to \pi^+\pi^-$, that of Christenson, Cronin, Fitch, and Turlay (CCFT).[21] Fig. 6 has a schematic of the experiment. After considering that experiment, we'll consider where one loses in the B system.

In the CCFT experiment, 30 GeV protons were incident on a beryllium target; a neutral beam was defined at an angle of 30° from the proton direction by collimators and sweeping magnets. At the target, more K^0's than $\overline{K^0}$'s were made, as discussed previously. Half of the K^0's and $\overline{K^0}$'s evolved into K_S^0 and decayed well before the region where K_L^0 decays were accepted by the detector, which started 17.4 meters downstream from the target, and extended for about 3 meters. The mean momentum of the K_L^0's that decayed in the acceptance was about measured with regeneration to be 1.1 GeV, so the distance from the target corresponded to only $\approx \gamma c\tau_L/2$. This mean momentum is *systematically lower* than that of the produced K^0 and $\overline{K^0}$, because, by time dilatation, fast K_L^0 systematically escape rather than decay. This effect is important in understanding contemporary $\mathrm{Re}[\epsilon'/\epsilon]$ experiments. Let's *guess* that the mean momentum of the initial K^0 and $\overline{K^0}$ was 2 GeV.

We can then estimate several interesting numbers. First, the distance from the target to the start of the K_L^0 acceptance was $\approx 160\gamma c\tau_S/2$. The possibility that a

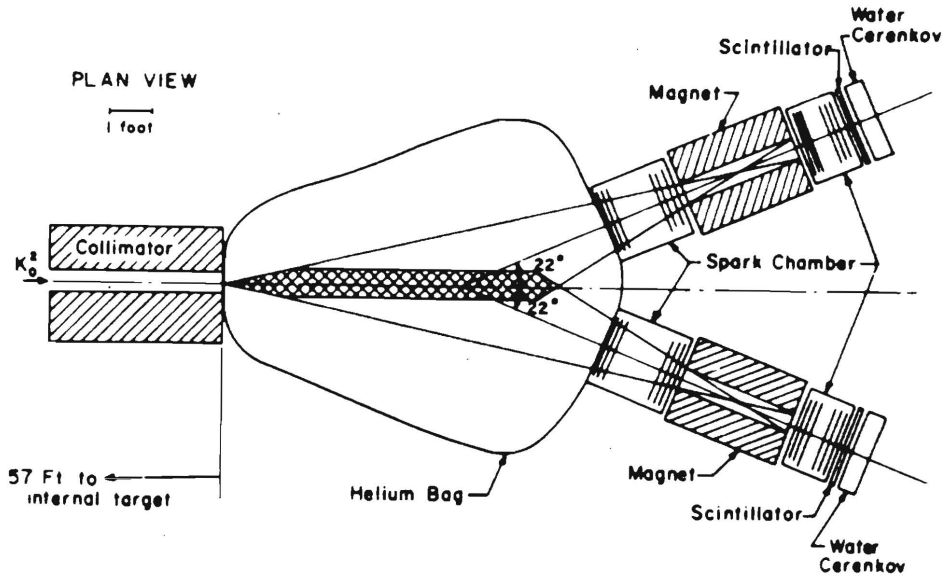

Figure 6. A schematic of the experiment that first observed $K_L^0 \to 2\pi$.

K_S^0 survived into the apparatus, and gave a false $K_L^0 \to \pi^+\pi^-$, was of order $e^{-160} \approx 10^{-70}$. This is the great practical benefit yielded by the lifetime splitting in the K^0 system; the CP+ component dies away, leaving a pure CP- beam. Second, only about $3/(2\gamma c\tau_L) \approx 1/40$ of the K^0 and $\overline{K^0}$ that were produced decayed in the acceptance.

About 5300 K_L^0 decays were reconstructed by CCFT, and most of those were from $K_L^0 \to \pi^\mp l^\pm \nu$ and $K_L^0 \to \pi^+\pi^-\pi^0$ decays, and so suffered apparent missing momentum and energy. About 60 lay within what would be termed today the 'search region' for $K_L^0 \to \pi^+\pi^-$, and the background from $K_L^0 \to 3$ body, extrapolated from the remaining 5240 decays, was 15. The resulting branching ratio for $K_L^0 \to \pi^+\pi^-$ of $(2.6 \pm 0.5)\times 10^{-3}$ is close to today's PDG value of $(2.03 \pm 0.04)\times 10^{-3}$.

To understand how point (2) helps make this observation 'easy'. The *magnitude* of the amplitude for $K^0 \to \overline{K^0} \to \pi^+\pi^-$ differs from that for $\overline{K^0} \to K^0 \to \pi^+\pi^-$. In absence of CP violation, the K_L^0 achieves its long life through the precise cancellation of $K^0 \to \pi^+\pi^-$ and $\overline{K^0} \to \pi^+\pi^-$ amplitudes; the amplitude difference slightly upsets this cancellation, and allows the K_L^0 to 'leak' slightly into $\pi^+\pi^-$. The rate at which it does so is independent of whether the initial state was K^0 or $\overline{K^0}$ (see (2.35)), so *no tag is necessary*.

Now, imagine the B^0 system; suppose one wants to see state mixing through the lepton asymmetry (2.34) of order of magnitude 10^{-3}. Where are the factors lost relative to the CCFT experiment?

The first price one must pay comes from not having large lifetime splitting. Whereas CCFT only needed to reconstruct about 1/40 of the K^0 and $\overline{K^0}$ they produced, and

let the rest decay either upstream or downstream of their apparatus, a B experiment must reconstruct all the B^0 and $\overline{B^0}$ it produces. Second, the natural division into short-lived CP+ state and long-lived CP- state that occurred in the K^0 system never happens in the B system. One is therefore obliged to tag the flavor of the initial B. Pair production of $b\bar{b}$ will most likely be exploited to do so; then the most realistic tag is the semileptonic decay of the $\bar{b}$; only 1/10 of the $\bar{b}$'s will undergo a semileptonic decay into the acceptance of a typical experiment. The same factor of 1/10 will be lost to obtain the flavor of the b.

The final factor that must be paid is the biggest. In the CCFT experiment they searched for a final state, $K^0_L \to \pi^+\pi^-$, which was rather distinct from its background. In a B lepton asymmetry experiment, one searches for a tiny systematic difference between two large numbers (the numbers of like-signed positive and negative lepton pairs, N^{++} and N^{--}, respectively). One must accumulate enough events to overcome statistical fluctuations, and that is a drastic penalty. If one demands that an asymmetry of 10^{-3} be observed above fluctuations with at least s standard deviations, then:

$$\frac{1}{\sqrt{N^{++}+N^{--}}} < \frac{1}{s}\times 10^{-3} \tag{3.1}$$

Putting the three factors together, one can estimate the number of $B^0\overline{B^0}$ events $N_{B^0\overline{B^0}}$ needed for three-standard observation of a 10^{-3} lepton asymmetry, relative to the number reconstructed in the CCFT experiment, $N_{\rm CCFT} = 5300$:

$$\begin{aligned} N_{B^0\overline{B^0}} &\approx [\,40\,] \times [\,10^2\,] \times [\,10^3\,] \times [\,s^2 \times 10^3 = 1.7 N_{\rm CCFT}\,] \\ &\approx 7 \times 10^6\, N_{\rm CCFT} \\ &\approx 4 \times 10^{10} \end{aligned} \tag{3.2}$$

That is more B hadrons than is reasonable to produce, so some trick must be employed to succeed in the B system. As we will discuss in the last section, that trick is to exchange the relatively high semileptonic branching ratio and small CP asymmetry for the tiny branching ratio but large CP asymmetry of $B^0 \to J/\psi K^0_S$.

Some good questions to ask at this point are:

A. To measure ϕ_{+-} in the kaon system, one is also *obliged* to tag the initial kaon flavor, (see (2.35)). Why then has it been straightforward to measure ϕ_{+-}?

B. Why has the measurement of the semileptonic charge asymmetry in the kaon system been possible?

C. One uses kaons to measure $\mathrm{Re}[\epsilon'/\epsilon]$, why has that been difficult?

The answer to (A) is point (3) above: fixed target experiments are constructed of matter, and there is an inclusive excess of K^0 when kaons are produced in manners such as used in the CCFT experiment. The 'price' of the tag has been avoided in kaon physics; however, the precision experiments of ϕ_{+-} have systematically started from K^0 initial states. The CPLEAR experiment will able to get a high statistics sample of initial $\overline{K^0}$. The current PDG value is:

$$\phi_{+-} = (46.6 \pm 1.2)^\circ$$

If all CP violation in the kaon system is state mixing, then from (2.27), it should be that $\phi_{+-} = \phi_\epsilon = (43.7 \pm 0.2)^\circ$. There is a mild discrepancy, of order 2 standard

deviations, but much too large to be accounted for by ϵ'; CPT violation in state mixing would be the culprit.[22,23] Recently, measurements of ϕ_{00} have improved dramatically, yielding the recent PDG value:

$$\phi_{00} = (46.6 \pm 2.0)^\circ$$

The kaon semileptonic charge asymmetry is peculiar and noteworthy. The asymmetry, as one gets far away from the kaon production point (see (2.32)), is independent of the initial superposition of K^0 and $\overline{K^0}$, just as the branching ratio for $K^0_L \to \pi^+\pi^-$ is (see (2.35)). Again, no tag is necessary. While the branching ratio for $K^0_L \to \pi^+\pi^-$ is $\propto |\epsilon|^2$, the semileptonic charge asymmetry is *linear* in ϵ. This has an interesting consequence; from the measured (PDG) value:

$$\frac{N(\pi^-e^+\nu) - N(\pi^+e^-\overline{\nu})}{N(\pi^-e^+\nu) + N(\pi^+e^-\overline{\nu})} \overset{t\to\infty}{\longrightarrow} \equiv \delta_e = 2\mathrm{Re}[\epsilon] = (3.27 \pm 0.12)\times 10^{-3}$$

one can conclude that there is systematically more K^0, which decays $\pi^-e^+\nu$, than $\overline{K^0}$, which decays $\pi^+e^-\overline{\nu}$, in the K^0_L. It immediately follows that the rate for $\overline{K^0} \to K^0$ must exceed that of $K^0 \to \overline{K^0}$. One could conclude that the *sign* of CP violation is such that particles with d quarks, like the K^0, *accumulate*. This is encouraging, if one hopes to explain the predominance of d quarks in the universe over $\bar{d}$ quarks through CP violation!

The difficulties in measuring $\mathrm{Re}[\epsilon'/\epsilon]$ arise for reasons peculiar to the measurement of $K^0_L \to \pi^0\pi^0$. The next lecture is largely about the details of $\mathrm{Re}[\epsilon'/\epsilon]$ measurement. It is left out of most discussions of $\mathrm{Re}[\epsilon'/\epsilon]$ that there was a time, in the late 1960's, when the data indicated that $\mathrm{Re}[\epsilon'/\epsilon]$ was large; so large, the situation seemed nearly that described by (2.25) and (2.26),

$$|\eta_{00}|^2 = 4|\eta_{+-}|^2.$$

In other words, A_2 seemed to dominate CP violation. A plot of

$$\left|\frac{\eta_{00}}{\eta_{+-}}\right|^2 = \frac{\mathrm{B}(K^0_L \to \pi^0\pi^0)}{\mathrm{B}(K^0_L \to \pi^+\pi^-)}$$

as a function of year is shown in Fig. 7. Note that a number of measurements seemed to indicate that state mixing CP violation, characterized by a value of $|\eta_{00}/\eta_{+-}|^2 \approx 1/2$, was ruled out; contemporary measurements have conclusively shown that

$$\frac{\mathrm{B}(K^0_L \to \pi^0\pi^0)}{\mathrm{B}(K^0_L \to \pi^+\pi^-)} \approx \frac{\mathrm{B}(K^0_S \to \pi^0\pi^0)}{\mathrm{B}(K^0_S \to \pi^+\pi^-)}$$

Why is there a checkered past?

The reason is background from the decay $K^0_L \to 3\pi^0$ is rather hard to reject. The decay $K^0_L \to 3\pi^0$ has a branching ratio 200 times higher than $K^0_L \to 2\pi^0$. In the sequence $K^0_L \to 3\pi^0 \to 6\gamma$, sometimes 2 of the final photons are lost, with no apparent momentum imbalance or loss of mass. When this happens, the apparent decay position and energy of the K^0_L are mismeasured, but the shifts go unnoticed, because photon directions are not usually reconstructed.

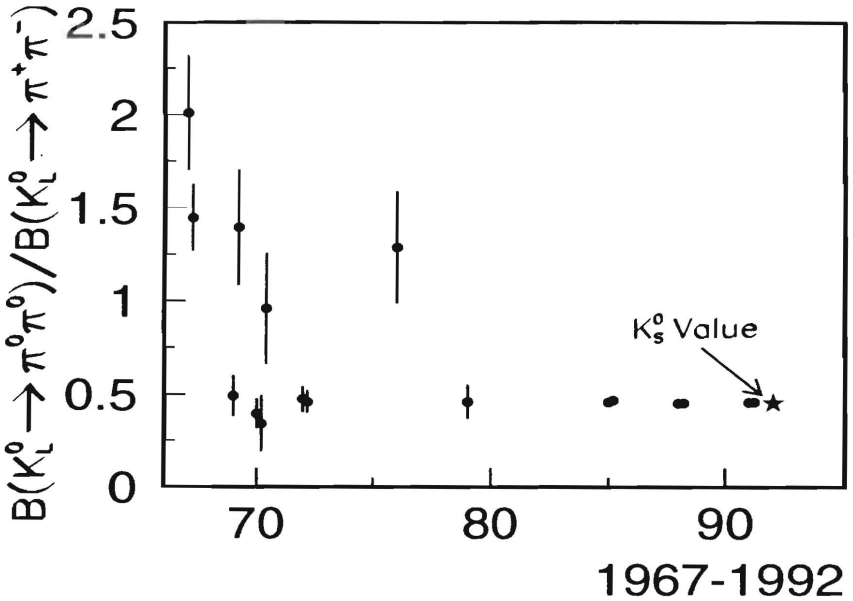

Figure 7. The ratio of branching ratios, $\frac{B(K_L^0 \to \pi^0\pi^0)}{B(K_L^0 \to \pi^+\pi^-)}$, as a function of year. The separation of $K_L^0 \to \pi^0\pi^0$ from $K_L^0 \to 3\pi^0$ is quite a challenge.

Contemporary $\mathrm{Re}[\epsilon'/\epsilon]$ experiments solved this problem by using electromagnetic calorimeters superior to earlier experiments, and also exploiting higher K^0 energies. The amount of $K_L^0 \to 3\pi^0$ background that overlaps the $K_L^0 \to \pi^0\pi^0$ signal region has been reduced to of order 1% of the $K_L^0 \to \pi^0\pi^0$ signal.

4. Contemporary CP Violation in s-quark Systems

All data concerning CP violation has, to date, come from the neutral kaon system, and with one exception, all data is consistent with a single number: that the kaon mass matrix has an imaginary part, $\mathrm{Im}[M_{12}] = -11.6\,\mathrm{neV}$. It is likely that the second manifestation of CP violation will be first seen in an s quark system. The possibilities I would like to address are:

1. $\mathrm{Re}[\epsilon'/\epsilon]$, pursued or under pursuit by:

   (a) NA31, who reported a significantly non-zero value[24]:

   $$\mathrm{Re}[\epsilon'/\epsilon] = (2.3 \pm 0.7)\times 10^{-3}$$

   They plan to continue refining their measurement of $\mathrm{Re}[\epsilon'/\epsilon]$ in a new experiment, NA48.

   (b) E731, who do not see a significantly non-zero value[25]:

   $$\mathrm{Re}[\epsilon'/\epsilon] = (0.74 \pm 0.60)\times 10^{-3}$$

   They plan to continue refining their measurement concerning $\mathrm{Re}[\epsilon'/\epsilon]$ in a new experiment, KTEV.

   (c) CPLEAR, an experiment under way at LEAR, the CERN low-energy antiproton facility.[26]

   (d) DAΦNE, an experiment planned at a ϕ factory under construction at Frascati.[27]

2. Rare decays, such as $K_L^0 \to \pi^0 e^+ e^-$ and $K_L^0 \to \pi^0 \nu\bar{\nu}$.
3. Asymmetries in Dalitz plots, such as in $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$.

A useful picture for understanding the $\mathrm{Re}[\epsilon'/\epsilon]$ experiments is to first imagine state-mixing CP violation as the process $K_L^0 \to K_S^0 \to 2\pi$, which is sufficiently true for useful approximation. If there is no direct CP violation, then all decays of the K_L^0 that violate CP will occur in the same relative proportions as the CP allowed decays of the K_S^0, as discussed in (2.20) and (2.21). A violation of this behavior is characterized, through (2.28) and (2.29), by non-zero $\mathrm{Re}[\epsilon'/\epsilon]$. The Standard Model predicts non-zero $\mathrm{Re}[\epsilon'/\epsilon] \approx 1 \times 10^{-3}$ due to penguin diagrams; a typical calculation is shown in Fig. 8. The simplest statement of the question addressed by $\mathrm{Re}[\epsilon'/\epsilon]$ experiments is then, 'Is a K_L^0, when it violates CP, any different than a K_S^0?'

This question guides one to a good viewpoint for the experiments that seek $\mathrm{Re}[\epsilon'/\epsilon] \neq 0$. A perfect experimental strategy would be to make a sample of K_L^0 that was as identical as possible to a sample of K_S^0. In practice, this is much harder than it sounds, because:

1. The K_L^0 lifetime is 580 times larger than the K_S^0 lifetime.
2. The 2π final states dominate the K_S^0 width, but are only about 1/300 of the K_L^0 width.

Additionally, in the end, one is comparing two final states that differ greatly in the way that they interact with the detector: $\pi^+\pi^-$ and $\pi^0\pi^0$. Such differences can amplify the differences inherent in points (1) and (2) above.

Each of the six experiments (NA31, NA48, E731, KTEV, CPLEAR, and DAΦNE) address the challenges of measuring $\mathrm{Re}[\epsilon'/\epsilon]$ in a different way. Broadly speaking, point (1) above is more important than (2) for first four, high-energy fixed target experiments. The background rejection required to address point (2) is more of a challenge at low energy; for CPLEAR and DAΦNE.

4.1 NA31 and E731

First, I will discuss NA31 and E731. One should note that their respective values for $\mathrm{Re}[\epsilon'/\epsilon]$ are not in serious disagreement: there is a 1.7σ discrepancy, an extremely common occurrence. Were not interest so strong in whether $\mathrm{Re}[\epsilon'/\epsilon]$ were significantly non-zero, the values would be termed 'consistent' and simply averaged. The fact that NA31 rules out $\mathrm{Re}[\epsilon'/\epsilon] = 0$ at a significant level, and E731 does not, creates an anxious situation.

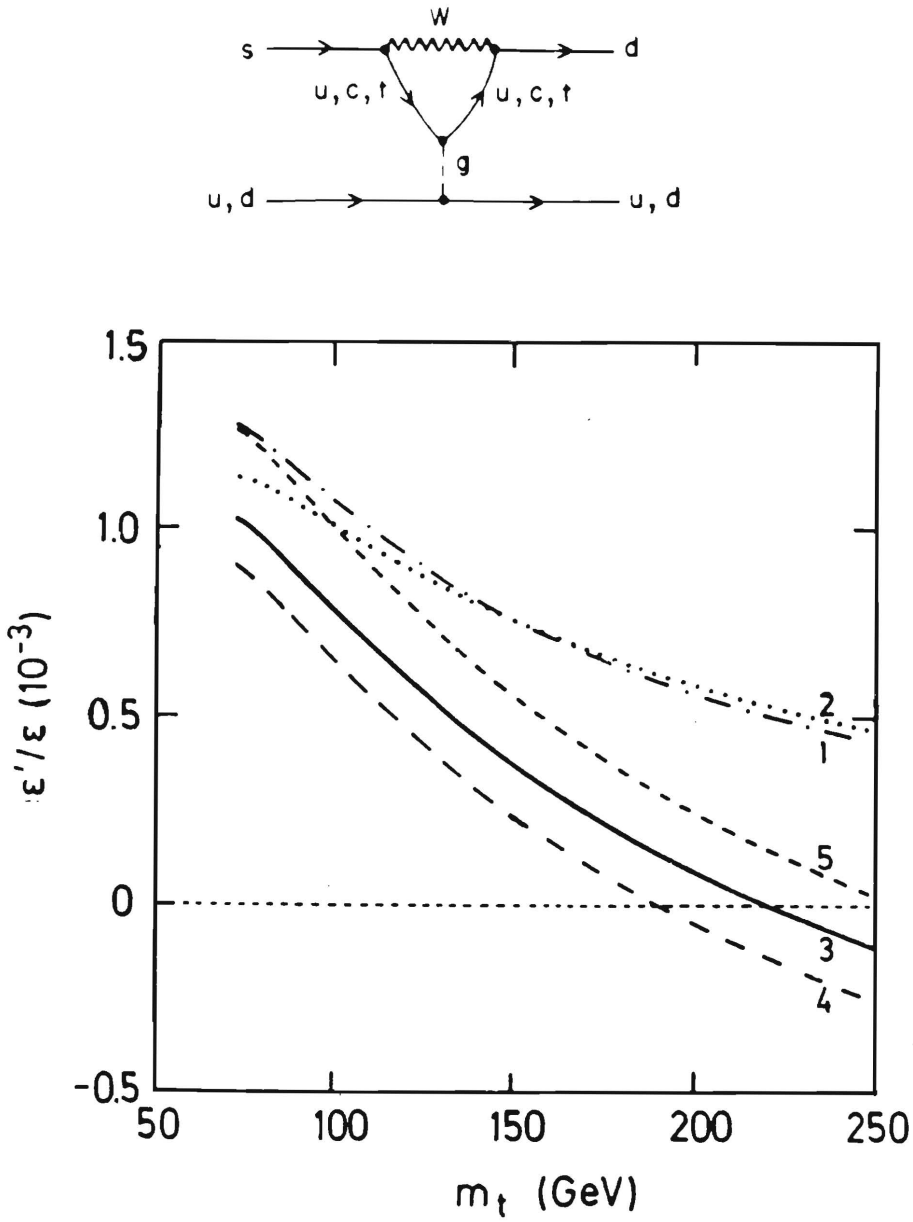

Figure 8. The penguin diagram that induces $\mathrm{Re}[\epsilon'/\epsilon]$ in the Standard Model; diagrams with γ or Z^0 instead of the gluon are now known to be important. The plot shows $\mathrm{Re}[\epsilon'/\epsilon]$ as a function of top quark mass; the curve labelled 3 is the complete calculation; its difference with 2 results from inclusion of the Z^0 penguin.[28]

Table 1

	$K_L^0 \to \pi^0\pi^0$	$K_L^0 \to \pi^+\pi^-$	$K_S^0 \to \pi^0\pi^0$	$K_S^0 \to \pi^+\pi^-$
	NA31			
events (10^3)	399	1055	2122	5120
background	3.1%	0.9%	< 0.1%	0.06%
sys. err. bkgd.	0.17%	0.15%	neglig.	neglig.
	E731			
events (10^3)	410	327	800	1061
background	5.16%	0.316%	2.84%	0.155%
sys. err. bkgd.	0.06%	0.014%	0.04%	0.011%

A diagram of the NA31 experiment is shown in Fig. NATO; the E731 experiment is shown in Fig. ESEV. It is always a good idea to dig out the raw numbers of the four types of events ($K_L^0 \to \pi^0\pi^0$, $K_L^0 \to \pi^+\pi^-$, $K_S^0 \to \pi^0\pi^0$, and $K_S^0 \to \pi^+\pi^-$), and their respective background levels. Here they are:

Neither experiment measures the four numbers independently. In the NA31 experiment, the ratio of $K_L^0 \to \pi^0\pi^0$ to $K_L^0 \to \pi^+\pi^-$ is measured for a day or so; then $K_S^0 \to \pi^0\pi^0$ and $K_S^0 \to \pi^+\pi^-$ are measured for about a day, and so on; a number of systematics, starting with livetime, cancel. In most of the E731 data, $K_L^0 \to \pi^0\pi^0$ and $K_S^0 \to \pi^0\pi^0$ are taken simultaneously for 10-20 days, then $K_L^0 \to \pi^+\pi^-$ and $K_S^0 \to \pi^+\pi^-$ are taken simultaneously for another similar period, and so on. These distinct strategies lead to different systematic errors.

As an exercise let's just compute the double ratio for the two data sample, after subtracting the backgrounds:

$$R_{\mathrm{NA31}} = 0.892$$
$$R_{\mathrm{E731}} = 1.625$$

In neither case is the double ratio near to unity. At this point you should be mighty suspicious! What is going on?

Both discrepancies have their origins in point (1) above, and in the strategy each experiment has used to overcome the lifetime difference. In the NA31 experiment, the origin is the fact that the energies of K_L^0 decays are systematically lower than the energies of K_S^0 decays, as shown in Fig. 11. The reason is simple: the acceptance region in NA31 is 50 meters, or many K_S^0 mean flight-paths. All K_S^0 produced by the proton beam incident on the K_S^0 target in the apparatus decay. The acceptance is a small fraction of the a typical K_L^0 flight-path, so the soft K_L^0 systematically decay. The probability of decay folds the K_L^0 energy spectrum with $1/p$ to obtain the energy spectrum of observed decays.

The NA31 collaboration simply computes the double ratio in bins of energy, so similar K_L^0 and K_S^0 energies are compared. The double ratio R is shown as a function

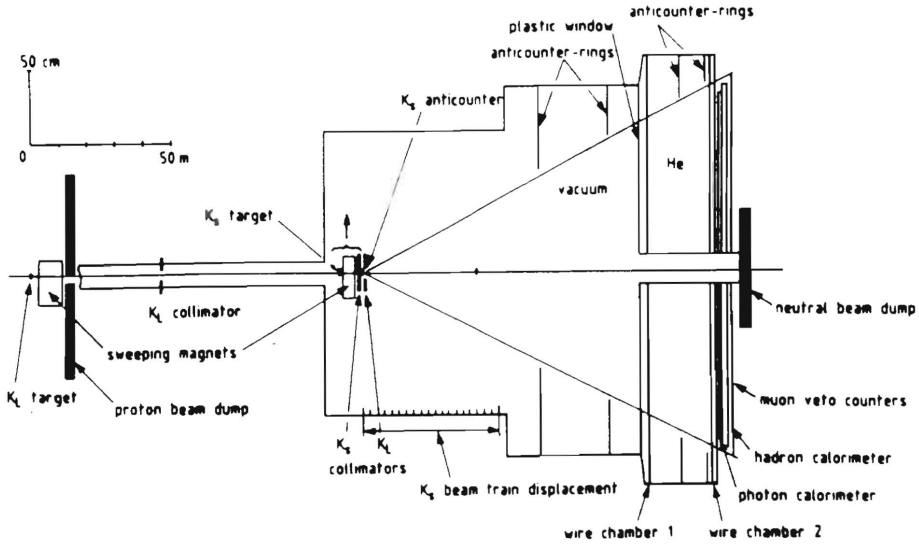

Figure 9. The NA31 Detector. The K_S^0 target is on a train, and can be moved throughout the fiducial acceptance.

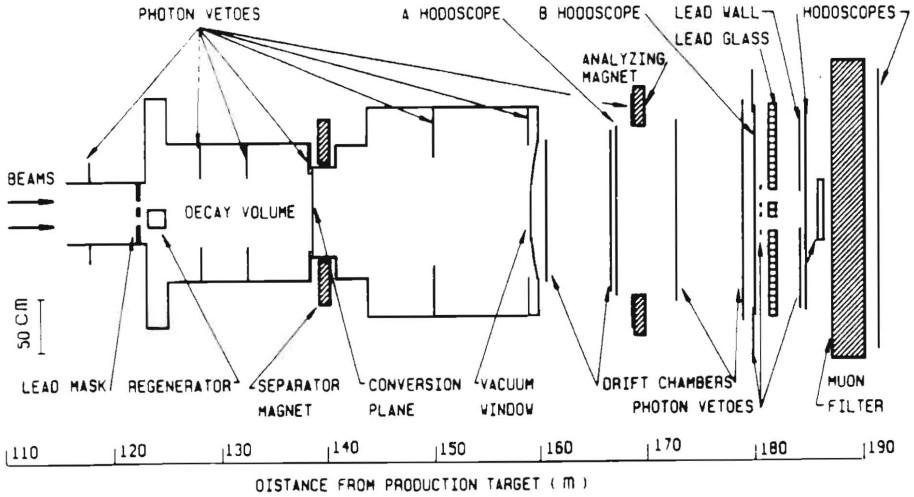

Figure 10. E731 Experiment. Twin K_L^0 beams enter; a regenerator that alternates between those beams produces K_S^0.

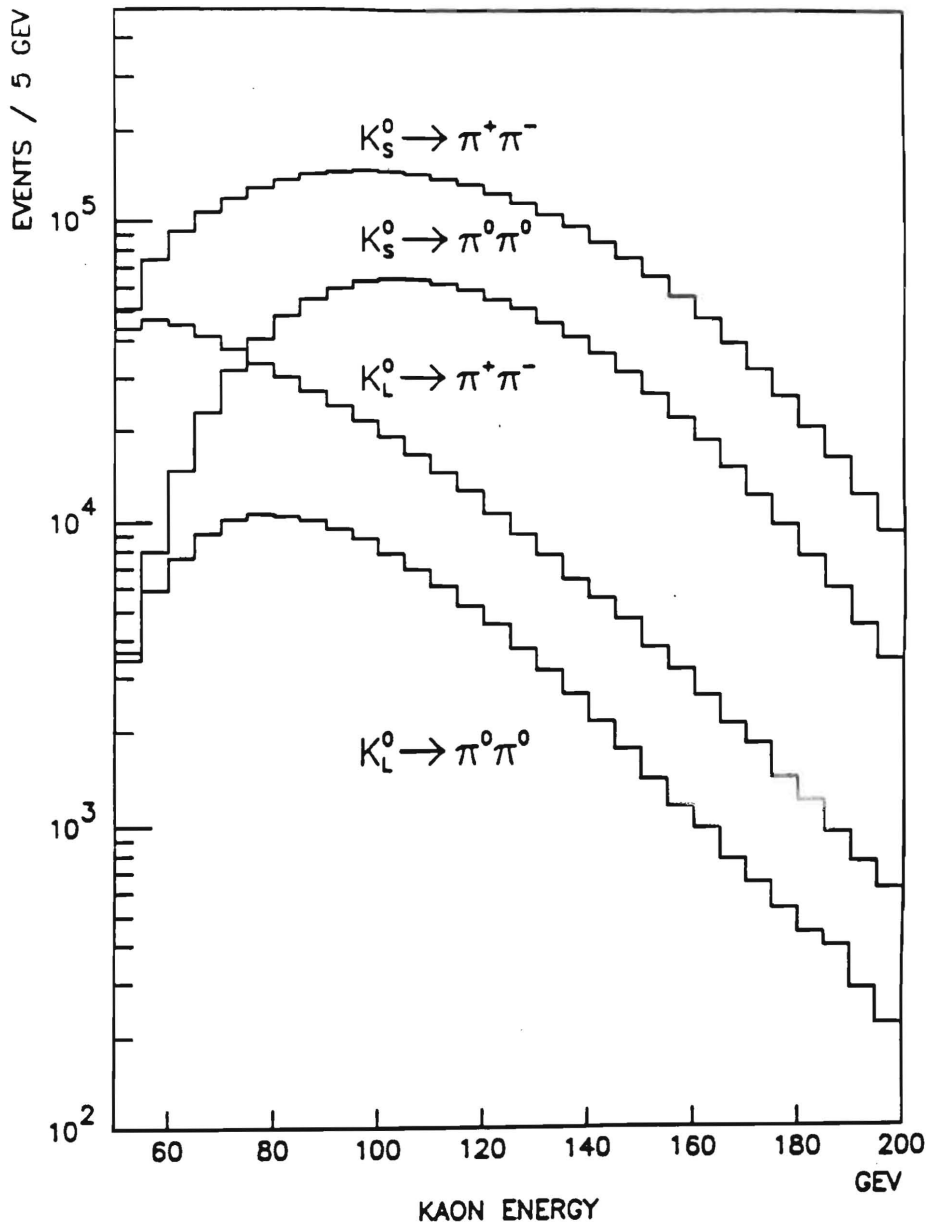

Figure 11. NA31 Energy Spectra. Because these are not identical, R must be computed in bins of energy.

of energy in Fig. 12. Upon doing that binning, the ratio is within 0.3% of its final value.

A good question to ask is: what is the effect of the different *spatial* distributions of K_S^0 and K_L^0 decays in the NA31 apparatus? At energies of about 100 GeV (see Fig 11), the K_S^0 mean flight-path is only about 5 meters. The K_L^0 mean flight-path at these energies is more like 3 kilometers. There is an acceptance difference between the K_S^0 that decay near their target, and the K_L^0 that decay uniformly throughout the acceptance. To address this point, NA31 built their K_S^0 target on a train, that could be moved through the acceptance. With that train, they make sure that the initial distribution of K_S^0 decays is essentially identical to that of K_L^0, as manifest in Fig. 13. For technical reasons, they actually bin their data in both energy and spatial bins (usually referred to as z-bins). The fact that the initial K_S^0 and K_L^0 spatial distributions are identical makes this procedure robust. A drawback of the NA31 K_S^0/K_L^0 beam alternation is that some effects related to the beam and the data acquisition do not necessarily drop out of the double ratio determination.

In the E731 experiment, most correction of the double ratio from 1.625 down to unity comes from consideration of the effects of distinct spatial distributions among the four decay modes. The simultaneous K_S^0 and K_L^0 beams are achieved by starting from twin K_L^0 beams, and then placing a regenerator into one of them. The regenerator alternates between the two beams, but is at a fixed location along the beam axis. The K_S^0 decays, then, fall mostly in a region of several meters immediately downstream of the regenerator, while K_L^0 decays extend roughly from a few meters upstream of the regenerator to 30 meters downstream, as shown in Fig. 14. Correction for the relative spatial acceptance among the four modes, obtained by Monte Carlo simulation, brings the double ratio to near unity. As discussed below, the quoted systematic error on the acceptance is 0.064%; in other words, the correction of 1.625 to near unity is known to 1 part in 1000. To achieve such a small error, Monte Carlo simulation was extensively checked with the high statistics decays $K_L^0 \to 3\pi^0$ and $K_L^0 \to \pi^\mp e^\pm \nu$.

E731 is more free from corrections due to a difference between K_L^0 and K_S^0 energy spectra than NA31. The reason is, the probability to regenerate a K_S^0 from a K_L^0 beam goes like $1/p^{0.60}$. The K_S^0 spectrum is similar to that of the K_L^0, as shown in Fig. 15.

The extraction of $\mathrm{Re}[\epsilon'/\epsilon]$ from the raw data of E731 involves a fit to the data, binned in energy and z bins. The fitting function takes into account both the time evolution, (2.35), and interference effects with the regenerated K_S^0.

The NA31 and E731 results, with statistical errors separated, are:

$$\begin{aligned} \mathrm{Re}[\epsilon'/\epsilon] &= (2.3 \quad \pm 0.3 \quad \pm 0.7) \quad \times 10^{-3} \quad \text{NA31} \\ &= (0.74 \quad \pm 0.52 \quad \pm 0.29) \quad \times 10^{-3} \quad \text{E731} \end{aligned}$$

In both cases, the statistical error is close to that one obtains simply from the event numbers in Table 1. I summarize in Table 2 the sources of systematic error in each case:

It is useful to comment on the systematic errors category by category:

1. **Inefficiencies.** Here, NA31 must employ different trigger conditions for the two modes they accumulate simultaneously ($\pi^+\pi^-$ and $\pi^0\pi^0$). Distinct scintillator planes provide the pre-trigger for $\pi^+\pi^-$ and $\pi^0\pi^0$, for example. If one scintillator paddle in the $\pi^0\pi^0$ hodoscope becomes intermittent in K_L^0 running only, a false

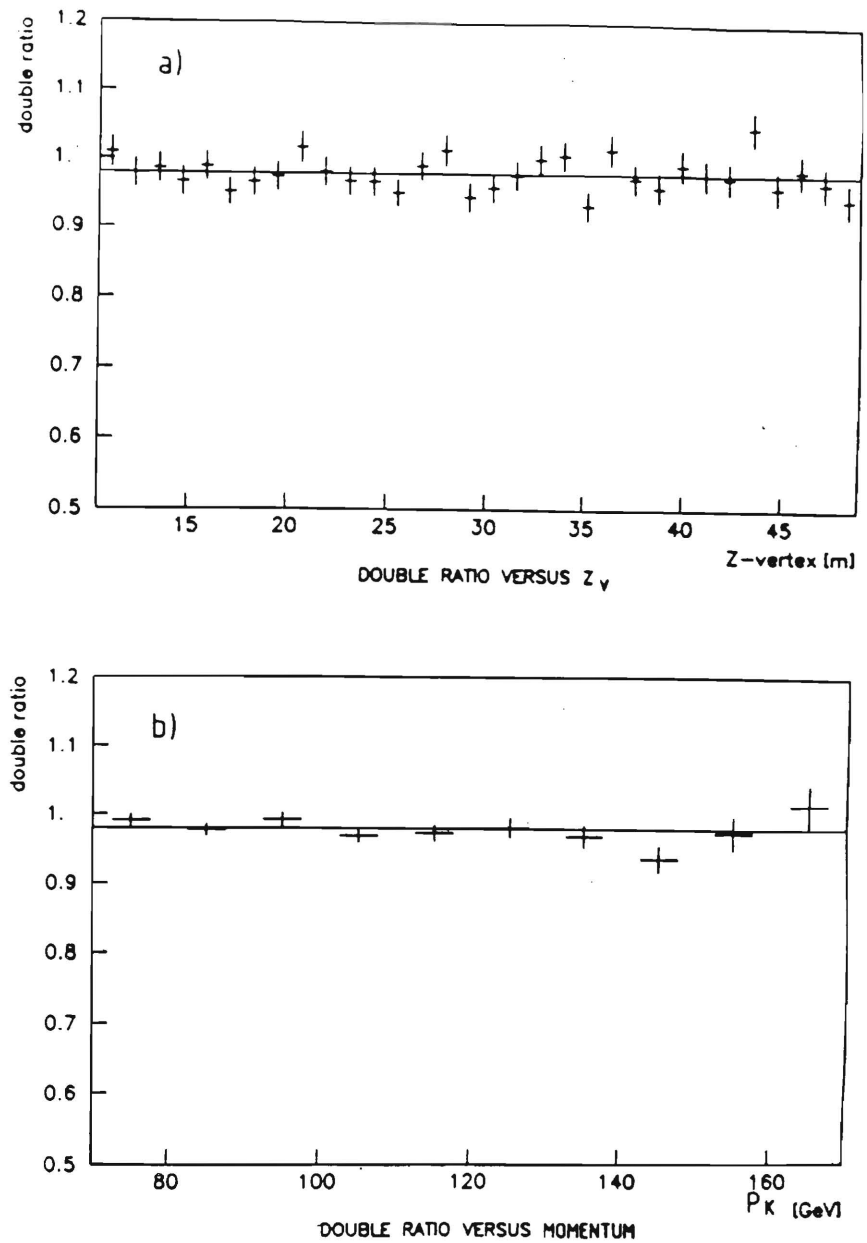

Figure 12. NA31 double ratio as a function of position along beamline (z), and energy.[30]

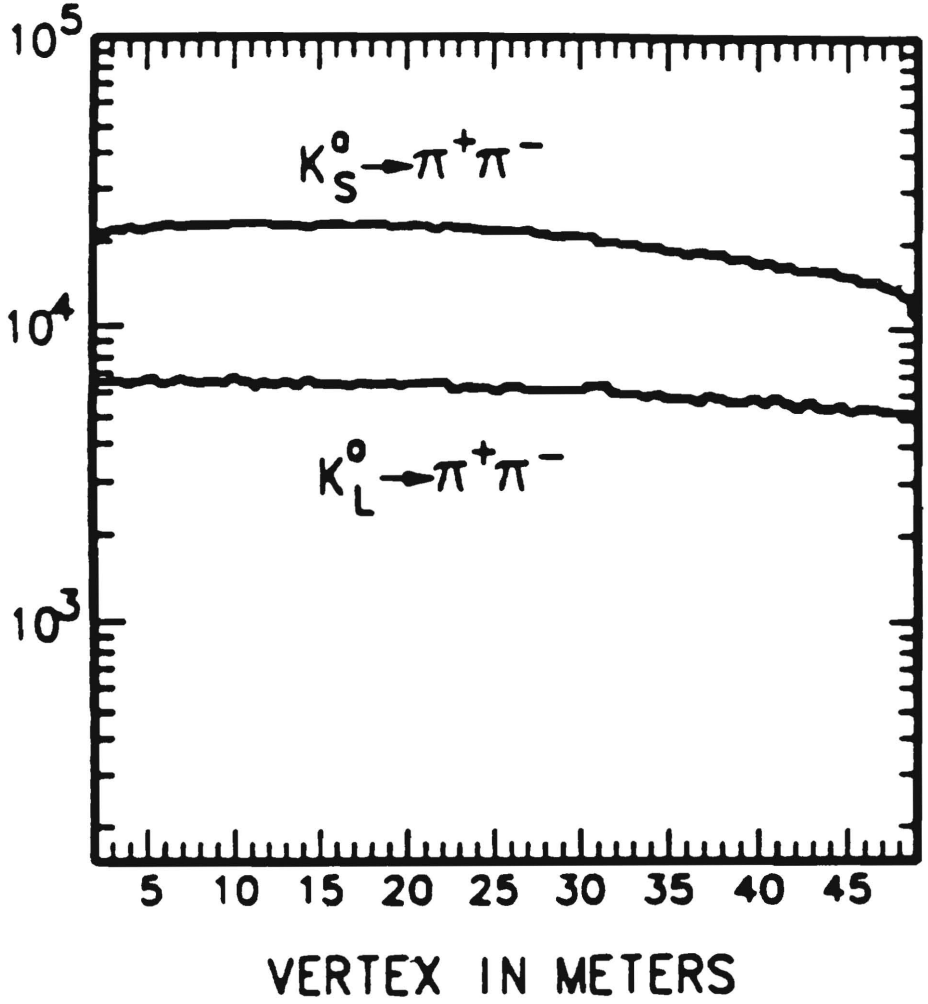

Figure 13. The distribution of decay locations of $\pi^+\pi^-$ events in NA31. The slight difference between K^0_S and K^0_L is a remnant of the different energy spectra.

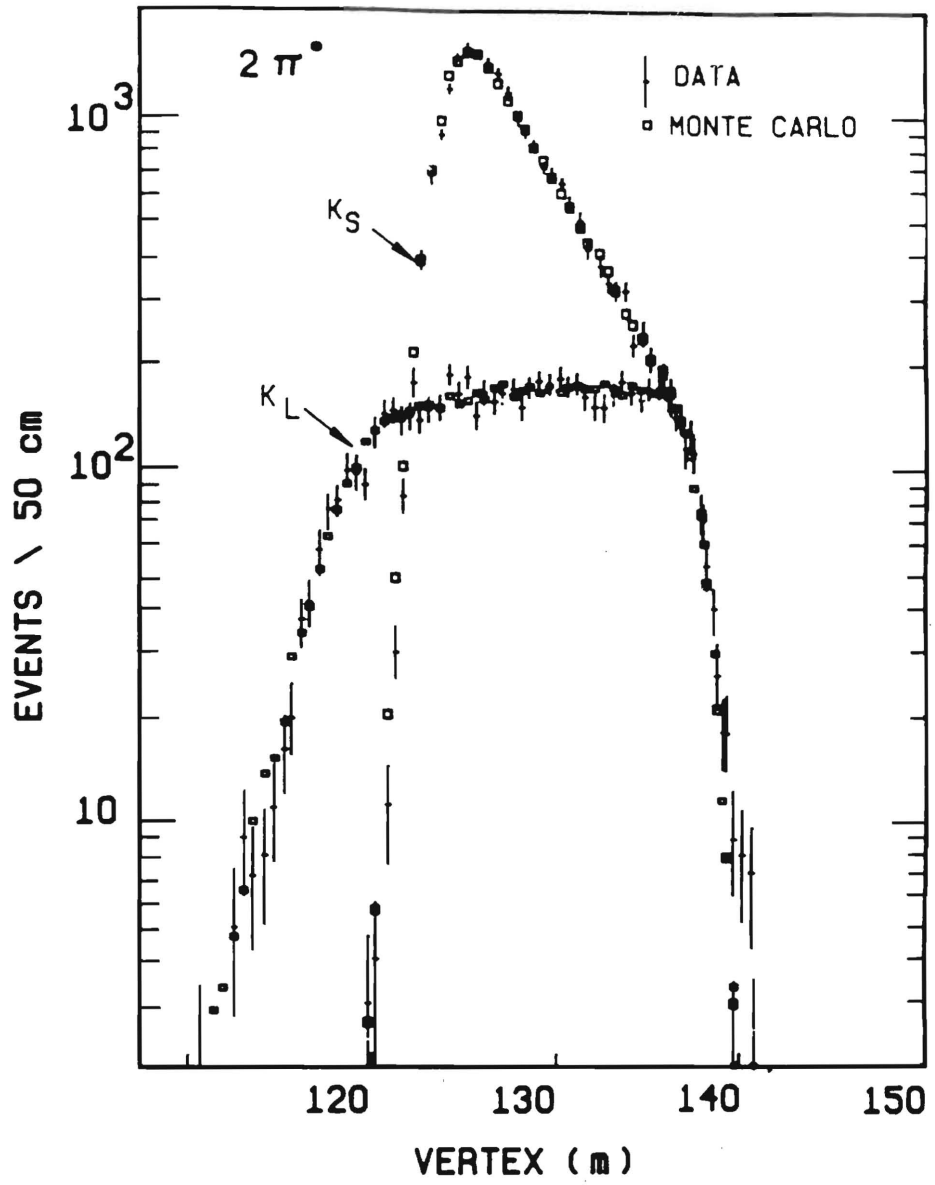

Figure 14. Distribution of $K^0 \to \pi^0\pi^0$ decays in E731. The smooth K^0_S distribution shows exponential decay downstream of the regenerator, whereas the K^0_L distribution is long and flat. The final E731 data sample includes events that extend to approximately 152 meters downstream of the target.[31]

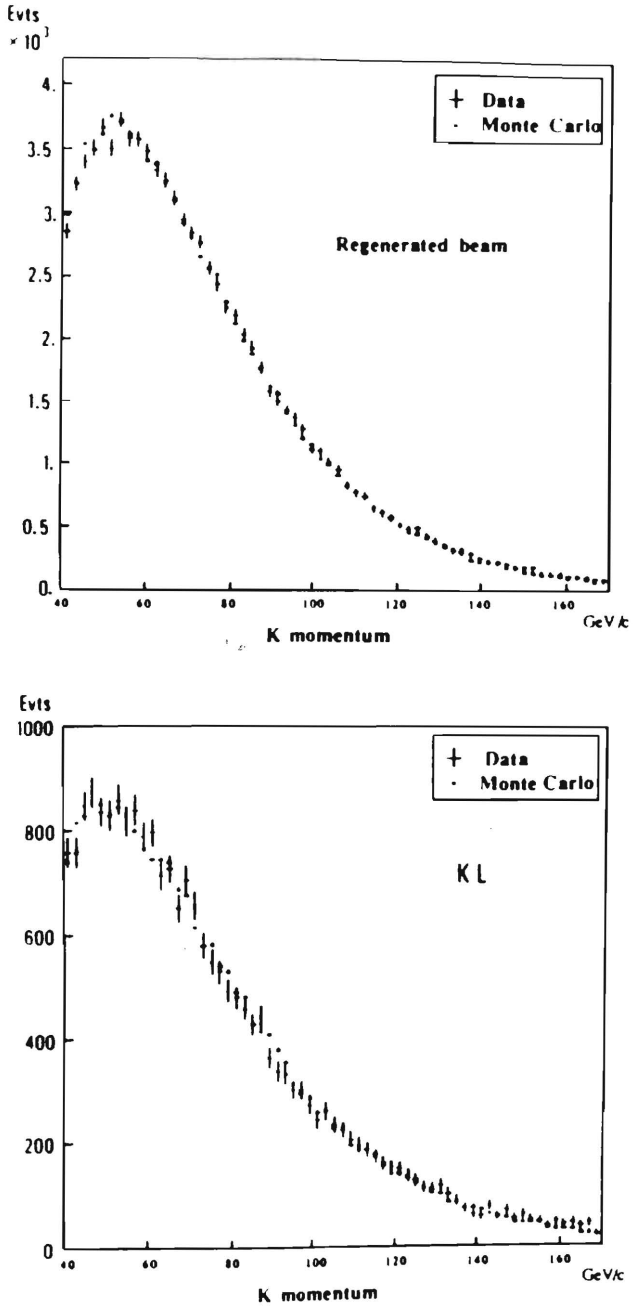

Figure 15. Energy spectra of K_L^0 and K_S^0 decays in E731. Use of a regenerator to produce the K_S^0 makes these spectra almost identical.[32]

Table 2

Systematic Errors on $\mathrm{Re}[\epsilon'/\epsilon](10^{-3})$		
	NA31	E731
Inefficiencies (Mostly Trigger)	0.38	0.12
Accidentals	0.33	0.11
$\pi^0\pi^0$ background	0.28	0.12
$\pi^+\pi^-$ background	0.25	0.029
Energy Calibration	0.23	0.16
Monte Carlo/Acceptance	0.17	0.12
total	0.69	0.29

$\mathrm{Re}[\epsilon'/\epsilon]$ could result. They use pre-scaled events not subject to trigger conditions to directly limit the relative inefficiencies, and the quoted systematic error is actually statistically limited by the number of those events. The E731 experiment has in principle less sensitivity to trigger inefficiencies, because similar final states are accumulated simultaneously. The effect of intermittent channels cancels in the ratio K_S^0/K_L^0. However, the unlike decay vertex distributions do cause unlike illumination of the trigger elements for K_S^0 and K_L^0; E731 addresses these differences through comparison of their high statistics samples and Monte Carlo, and put the systematic error under 'Acceptance'. The number in Table 2 for E731 is arrived at by consideration of the likelihood that a $\pi^0\pi^0$ event would self-veto in their trigger plane.

2. **Accidentals.** This category describes the consequences of the arrival of two or more events nearly simultaneously, referred to in simple experiments as 'pileup'. Both experiments suffer from accidental effects, because their K_S^0 and K_L^0 beams are not identical: NA31 accumulates these data at different times, while in E731 the K_S^0 and K_L^0 beams are distinct in space. Of the two, NA31 sees a larger effect, again because they accumulate unlike final states ($\pi^+\pi^-$ and $\pi^0\pi^0$) simultaneously. For example, if the rate of extra hits in their drift chambers is larger in K_L^0 running than in K_S^0 running, NA31 might systematically lower their observed $\mathrm{B}(K_L^0 \to \pi^+\pi^-)/\mathrm{B}(K_S^0 \to \pi^+\pi^-)$, but leave the equivalent ratio for neutral decays unchanged. They tune their K_L^0 and K_S^0 beams to equalize the rate of extra hits. They 'symmetrize' their selection criteria between $\pi^+\pi^-$ and $\pi^0\pi^0$ final states: allowance of one *additional* drift chamber hit in each case tends to make accidental effects cancel in the ratio $\pi^+\pi^-/\pi^0\pi^0$. Approximately $2-3\%$ of each of the four event types is lost. Their systematic error estimate results from study of data events upon which random trigger events have been overlaid in software. In E731, losses of events due to pileup tend to be nearer to 10%, but simultaneous accumulation of like final states reduces the sensitivity of $\mathrm{Re}[\epsilon'/\epsilon]$ to these losses. They too quote an error based upon the overlay technique.

3. $\pi^0\pi^0$ **background** Both experiments extrapolate from background regions under their signal. The systematic error represents their estimate on how well the extrapolation can be trusted; E731 asserts closer understanding of the shape

of the extrapolation, because of their extensive data/Monte Carlo comparisons. Both experiments are capable of lowering this systematic error by removing the events with decay vertices closest to the detector, thereby suppressing $K_L^0 \to 3\pi^0$ background. The $\pi^0\pi^0$ statistics are so valuable that they compromise.

4. $\pi^+\pi^-$ **background** Here, NA31 sees a larger systematic error because they do not employ a magnetic spectrometer.
5. **Energy Calibration.** This systematic error arises because different detectors are used to reconstruct $\pi^0\pi^0$ events and $\pi^+\pi^-$ events, allowing the possibility of systematically different reconstructed energies. In the NA31 experiment, the liquid argon calorimeter is used to determine the energy of $K^0 \to \pi^0\pi^0$ events, and track opening angle is used to determine the energy of $K^0 \to \pi^+\pi^-$ events, with a small correction of magnitude less than a few percent from the hadron calorimeter. In the E731 experiment, the lead glass calorimeter is used to reconstruct the energies of $K^0 \to \pi^0\pi^0$ events, and the magnetic spectrometer for $K^0 \to \pi^+\pi^-$ energies. If the $\pi^0\pi^0$ and $\pi^+\pi^-$ energy scales differ, then binning described earlier can then be incorrect, and compare kaons of systematically different energies. In the NA31 experiment, calibration is made using the K_S^0 from the target that moved throughout the apparatus, a relative energy scale accurate to 0.05% is quoted. In E731, the position of their regenerator is similarly exploited, and a relative energy scale accurate to 0.03% is reported.
6. **Monte Carlo/Acceptance** Corrections based on Monte Carlo simulation are small, by experimental design, in NA31. Some things must be studied, such as small acceptance shifts from K_S^0 scattering in the K_S^0 target, and the unequal beam divergences of K_S^0 and K_L^0. The E731 Monte Carlo simulation is more fundamental to the analysis, and has been extensively compared with high statistics samples, resulting in the systematic error quoted.

We will use the systematic error table as a guide to understanding the descendant experiments, NA48 and KTEV. First, I would like to comment that if two experiments have ever had uncorrelated systematic errors, they are NA31 and E731. It is reasonable to make a combined average:

$$\mathrm{Re}[\epsilon'/\epsilon] = (1.33 \pm 0.26 \pm 0.50) \times 10^{-3} \tag{4.1}$$

This is 2.4σ off zero, weak evidence of direct CP violation, and completely consistent with the Standard Model.

4.2 NA48 AND KTEV

Tables 1 and 2 can be used as a guide to understanding the experiments descended from NA31 and E731, NA48[33] and KTEV,[33] respectively, plan to push the measurement of $\mathrm{Re}[\epsilon'/\epsilon]$ to the $0.1 - 0.2\times10^{-3}$ level:

1. **Increase $K_L^0 \to \pi^0\pi^0$ statistics by an order of magnitude.** The NA48 experiment can obtain an order of magnitude more protons on target than NA31 by simply by moving the experiment from upstairs in the North Area at CERN to the shielded area downstairs. The KTEV experiment can obtain a factor of four more protons on target than E731. Both NA48 and KTEV could also use K_L^0 beams of greater solid angle than their predecessors, at the price of more background.

Given that the instantaneous rates must be an order of magnitude higher, it is clear that both experiments must improve their treatment of accidentals, or else systematic errors will skyrocket to 3×10^{-3} for NA48 and 0.4×10^{-3} for KTEV. Solutions are:

2. **Speed up detector elements, in particular the electromagnetic calorimeter.** For NA31, the slowest detector element was the liquid argon calorimeter, which would show pileup effects for events separated by as much as several microseconds. For NA48, a completely sampling liquid krypton calorimeter will be used, and should not show pileup effects for events separated by more than 80 ns. The KTEV experiment will employ an undoped CsI electromagnetic calorimeter.
3. **Take like final states simultaneously.** A substantial part of NA31's systematic error from accidentals arose because $K_S^0 \to \pi^+\pi^-$ and $K_L^0 \to \pi^+\pi^-$ were not accumulated simultaneously. In the NA48 experiment, all both modes of K_L^0 and K_S^0 decays will be taken simultaneously. A result will be suppression of the systematic error due to trigger efficiency. The E731 experiment, of course, already took data in this manner, with twin K_L^0 beams and an alternating regenerator, and will continue to do so in KTEV.

Accidentals can still induce a false $\mathrm{Re}[\epsilon'/\epsilon]$ if the K_S^0 and K_L^0 illuminate the detector elements differently. To minimize this effect:

4. **Make the K_S^0 and K_L^0 beams as similar as possible.** In the NA48 design, twin K_L^0 and K_S^0 beams are both produced by protons impinging on targets. The setup is shown in Fig. 16. A bent crystal is used to simultaneously attenuate and redirect protons that survive traversal of the K_L^0 target, and send them toward the K_S^0 target. In this manner, the K_S^0 beam is 7.2 cm above the K_L^0 beam, at a distance of some 90 meters from the first detector element. The K_S^0 decays are distinguished from the K_L^0 decays by the presence of a pulse in a tagging counter just upstream of the K_S^0 target, which indicates a proton was incident on there simultaneous with the decay. The NA48 detector is shown in Fig. 17. In the KTEV experiment, shown in Fig. 18, the twin beams are about twice as far apart, about 15 cm, at a distance of 40 meters from the first detector element.

The remaining systematic errors are from backgrounds, energy scale, and acceptance. To address these:

5. **For $\pi^0\pi^0$, minimize the length of the K_L^0 acceptance region.** The background from $K_L^0 \to 3\pi^0$ tends to show an apparent shift toward the detector. The NA48 experiment plans to accept K_L^0 decays in the first 10 meters past the final collimator; based on extrapolation from NA31, they expect a background an order of magnitude lower than in Table 1, and the systematic error with it. The KTEV experiment will use a longer fiducial region, of 20-30 meters.
6. **For $\pi^+\pi^-$, use a magnetic spectrometer.**
7. **Improve the electromagnetic calorimeter.**
8. **Make the K_L^0 decay vertex distribution as much like the K_S^0 as possible.** The plan in NA48 is first, to use only the first two K_S^0 lifetimes (approximately 10 meters) past the final collimator; then those K_L^0 decays will be de-weighted to match the K_S^0 decay distribution. This procedure suppresses systematic errors in acceptance, but dispenses with some of the statistical weight of the K_L^0 sample. In KTEV, such a procedure will not be used; as in E731, extensive Monte Carlo simulation will be used to correct the raw data.

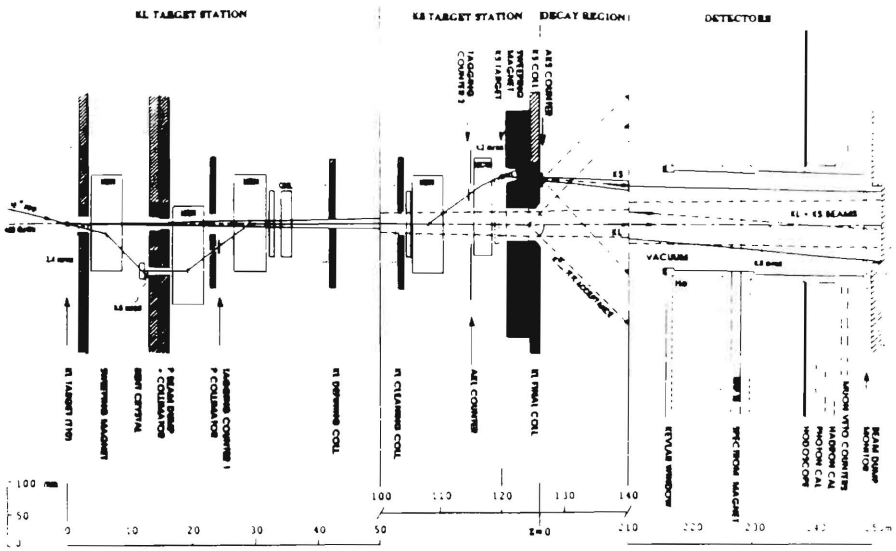

Figure 16. Beamline for the planned NA48 experiment.[33]

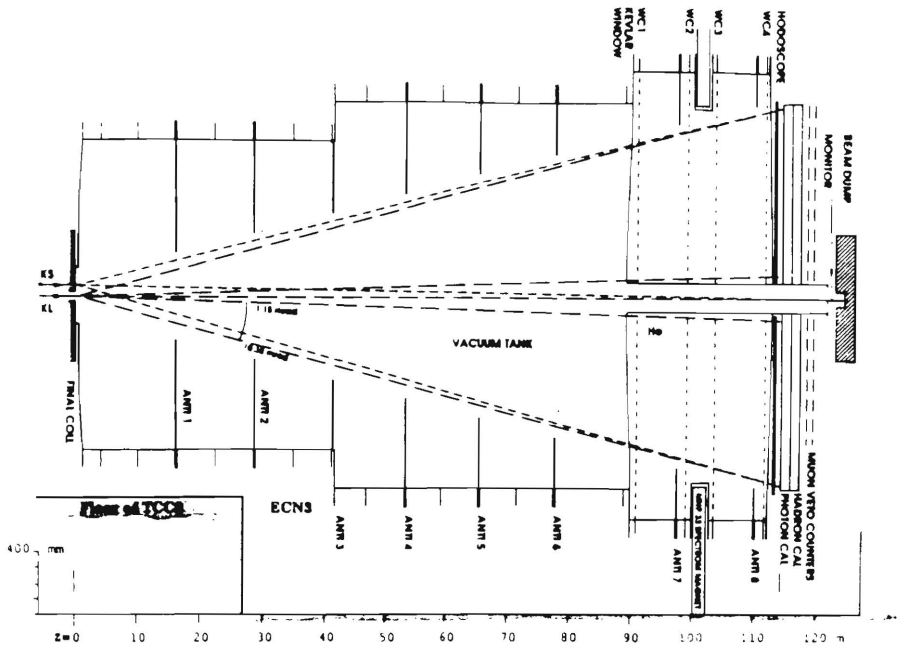

Figure 17. Detector for the planned NA48 experiment.[33]

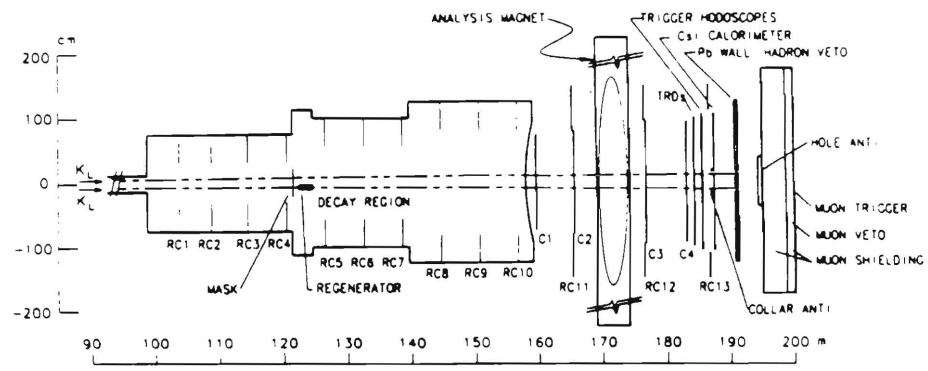

Figure 18. Detector for the planned KTEV (now E832) experiment.[34]

4.3 CPLEAR AND DAΦNE

Two experiments will try to achieve measurements of $\mathrm{Re}[\epsilon'/\epsilon]$ outside of the fixed-target environment. These are the CPLEAR experiment that is operating at CERN, and the DAΦNE program at Frascati, scheduled to being commissioning in late 1995.

The method of CPLEAR was mentioned earlier, and consists first of reconstruction of the processes:

$$p\overline{p} \rightarrow \begin{cases} \pi^- K^+ 2\pi \\ \pi^+ K^- 2\pi \end{cases} \tag{4.2}$$

The $\overline{p}$ have momenta of about 200 MeV. A view of the detector is shown in Figure 19. In events tagged with $K^-(K^+)$ it is known that a $K^0(\overline{K^0})$ was present at $t = 0$, so the decay vertex positions of 2π states should be described by an expression of the form (2.35). Due to interactions in the detector, made of matter only, they identify about 20% more K^+ than K^-; that is not CP violation.

While the high energy fixed target experiments devoted to $\mathrm{Re}[\epsilon'/\epsilon]$ suffer systematic effects due to the difference in properties between K^0_S and K^0_L, the difference between K^0 and $\overline{K^0}$ properties is so small that CPLEAR is essentially immune to this type of error. For example, if they reconstruct the time dependent asymmetries to 2π decays, A_{+-} and A_{00}:

$$A_{+-}(t) = \frac{\mathrm{P}(\pi^+\pi^-, t; K^0) - \mathrm{P}(\pi^+\pi^-, t; \overline{K^0})}{\mathrm{P}(\pi^+\pi^-, t; K^0) + \mathrm{P}(\pi^+\pi^-, t; \overline{K^0})}$$

$$A_{00}(t) = \frac{\mathrm{P}(\pi^0\pi^0, t; K^0) - \mathrm{P}(\pi^0\pi^0, t; \overline{K^0})}{\mathrm{P}(\pi^0\pi^0, t; K^0) + \mathrm{P}(\pi^0\pi^0, t; \overline{K^0})}$$

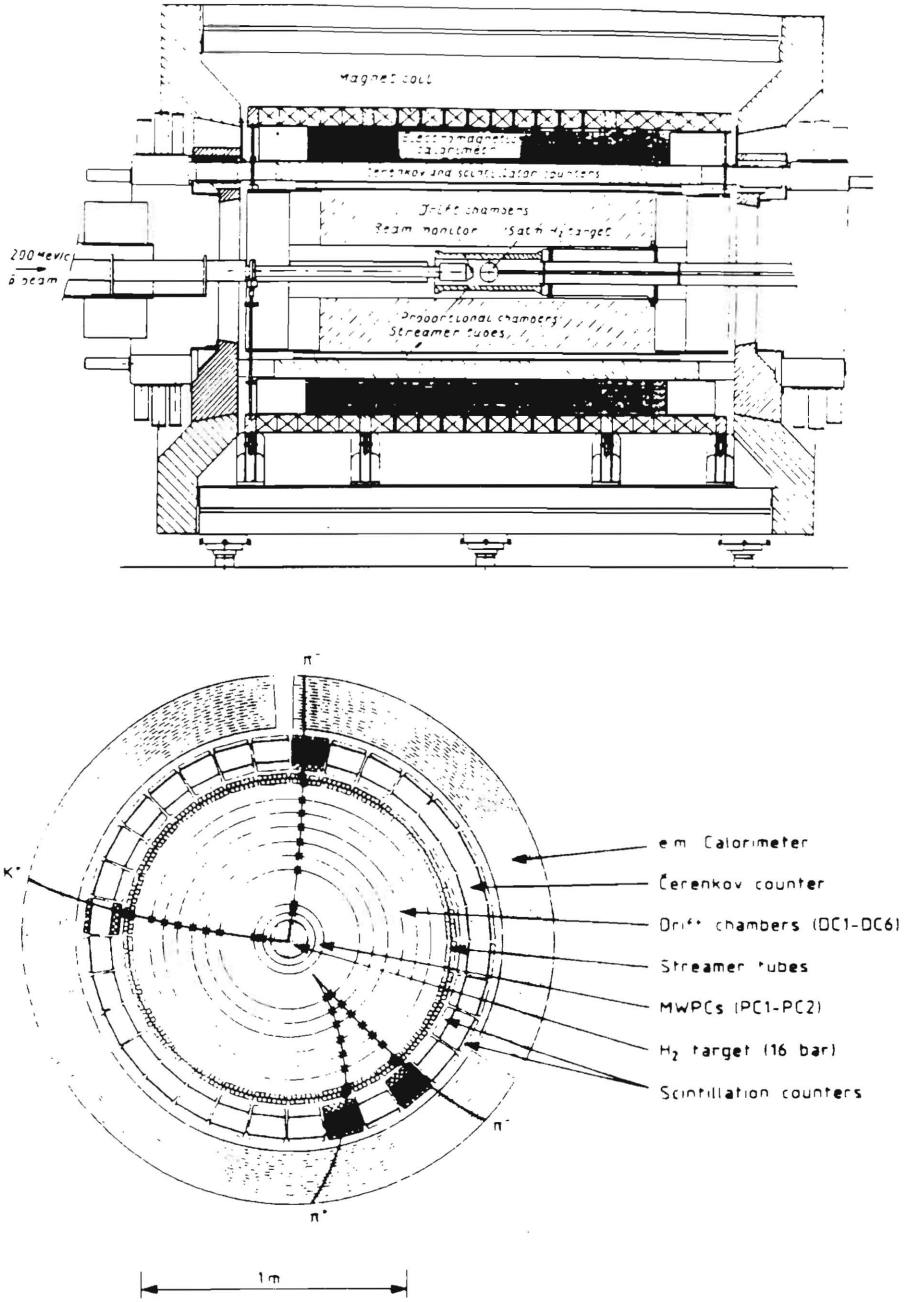

Figure 19. The CPLEAR Detector.

then effects from both acceptance, and from the migration of events between different t bins, cancel.

The asymmetries $A_{+-}(t)$ and $A_{00}(t)$ contain information on $\mathrm{Re}[\epsilon'/\epsilon]$ in a different manner rather than the simple double ratio, (2.28). As $t \to 0$, $A_{+-}(t)(A_{00}(t)) \to -2\mathrm{Re}(\epsilon - \eta_{+-}(\eta_{00}))$; *only* direct CP violation is initially present. Then as $t \to \infty$, both asymmetries go to $-2\mathrm{Re}(\epsilon)$; only state-mixing CP violation survives. A plot of the asymmetries for all times is shown in Fig. 20; the largest asymmetry is found at about 11 K_S^0 lifetimes from the interaction. Although the asymmetry is largest there, the statistics is low. To see where the information on ϵ'/ϵ comes from, I have plotted a 'sensitivity' s:

$$s \propto |A_{+-}(t) - A_{00}(t)| \sqrt{\mathrm{P}(2\pi, t; K^0) + \mathrm{P}(2\pi, t; \overline{K^0})}$$

in Fig. 20 as well. Most of the sensitivity to $\mathrm{Re}[\epsilon'/\epsilon]$ comes from the region near ≈ 5 K_S^0 lifetimes from the interaction.

At the current time, CPLEAR has not begun to operate with their electromagnetic calorimeter, so background dominates their $K_L^0 \to \pi^+\pi^-$ signal by about a factor of five. Nevertheless, because most information comes from the region around 5 K_S^0 lifetimes, the influence of the background is suppressed by about a factor of $|\epsilon|^2/e^{-5} \approx 1000$ over what would be the case in NA31 or E731. As a result, CPLEAR has already measured $|\eta_{+-}|$ with to about six percent ($|\eta_{+-}| = (2.32 \pm 0.14 \pm 0.03)\times 10^{-3}$), based on the data shown in Fig. CPLasym, even though they formally have not seen a clear $K_L^0 \to \pi^+\pi^-$ decay. That measurement is based on about 10^5 reconstructed events; with an increase of a factor of 100 in their data, and the commissioning of their electromagnetic calorimeter, CPLEAR will pull close to the accuracy of the NA31 and E731 results.

In the DAΦNE program, the process:

$$e^+e^- \to \phi \to K\overline{K} \tag{4.3}$$

will be exploited for study of CP violation. This K^0 production mechanism is particularly beautiful, both from a physics viewpoint, and from the viewpoint of elimination of some systematic errors.

The ϕ is $J^{PC} = 1^{--}$, and kaons are spin-0, so the $K\overline{K}$ state must be in a P-wave, with parity +1. The strong decay $\phi \to K\overline{K}$, populates only C=-1 final states, of which that involving neutral kaons is:

$$=[|K^0, \vec{p}\rangle|\overline{K^0}, -\vec{p}\rangle - |\overline{K^0}, -\vec{p}\rangle|K^0, \vec{p}\rangle]/\sqrt{2}$$
$$=[|K_2^0, \vec{p}\rangle|K_1^0, -\vec{p}\rangle - |K_1^0, -\vec{p}\rangle|K_2^0, \vec{p}\rangle]/\sqrt{2}$$

The time evolution of this state is a bit complicated, because of all the correlations. Here are some qualitative observations[35]:

1. Were CP conserved, one would never see a final state where both neutral kaons decayed to CP eigenstates with the same CP eigenvalue, for example $\pi^+\pi^-\pi^0\pi^0$. When spatial parity is included, the CP of such a state is -1, while that of the ϕ is +1.

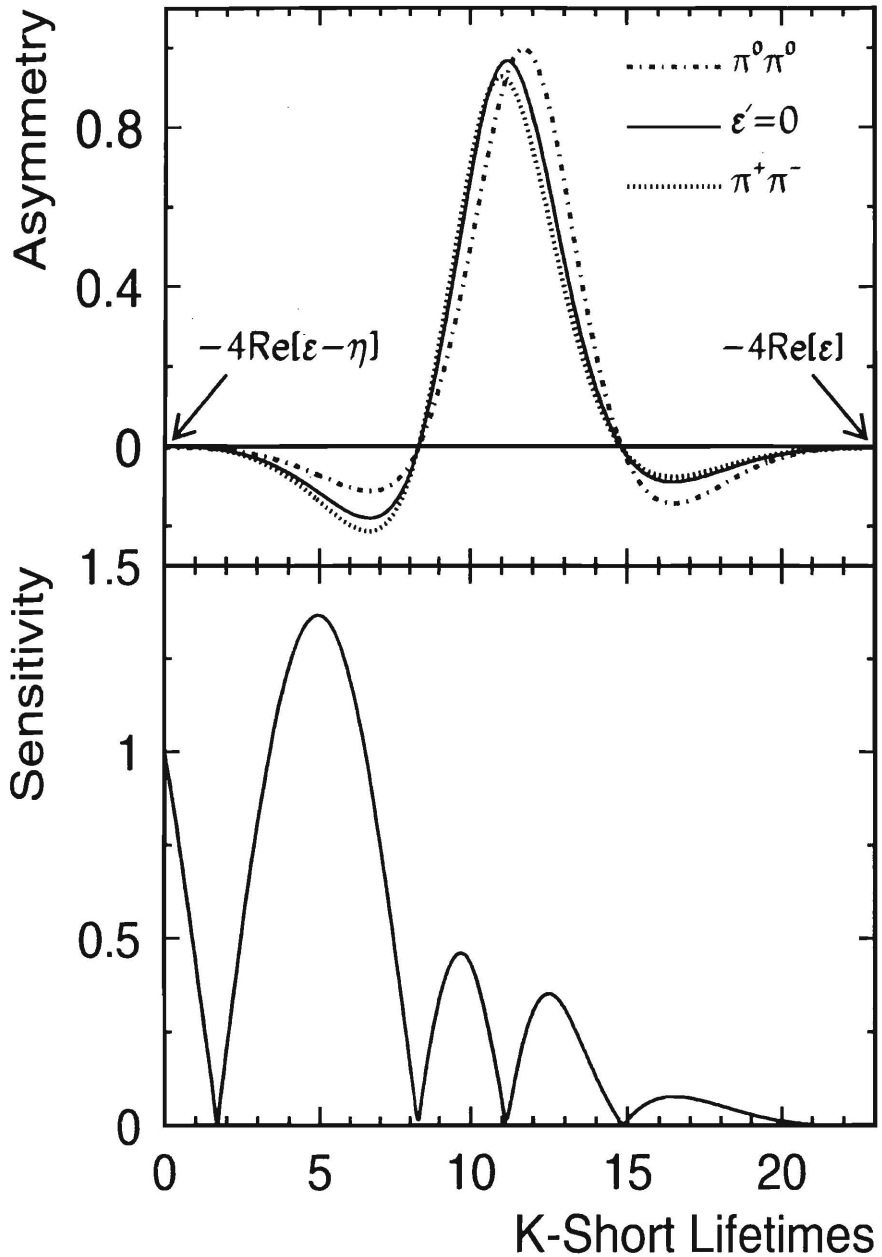

Figure 20. Asymmetries between tagged K^0 and $\overline{K^0}$ decays to two pions. For the top curves, $\mathrm{Re}[\epsilon'/\epsilon]=0.2$ is taken to exaggerate the effect. The sensitivity used in the bottom is defined in the text.

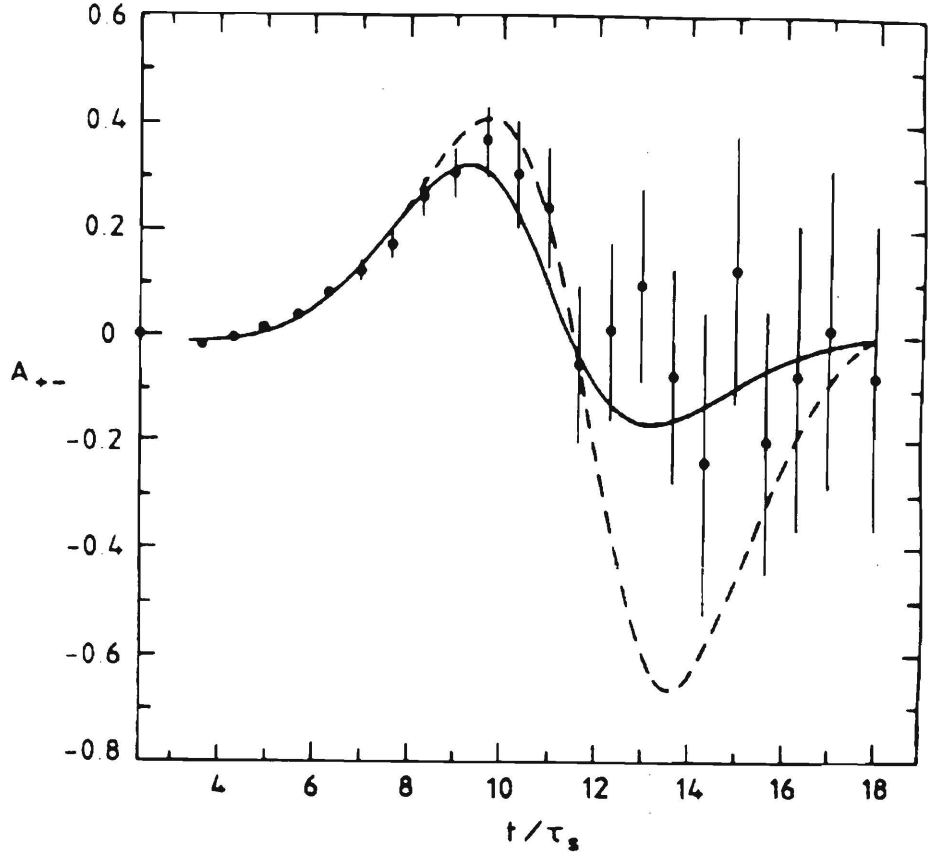

Figure 21. Asymmetry between K^0 and $\overline{K^0}$ tags in CPLEAR. They actually show the *integrated* rate,

$$A_{+-}(t) = \frac{\int_0^t dt' \mathrm{P}(\pi^+\pi^-, t'; K^0) - \int_0^t dt' \mathrm{P}(\pi^+\pi^-, t'; \overline{K^0})}{\int_0^t dt' \mathrm{P}(\pi^+\pi^-, t'; K^0) + \int_0^t dt' \mathrm{P}(\pi^+\pi^-, t'; \overline{K^0})}$$

in order to reduce sensitivity to vertex resolution.

2. If the kaon that first decays goes to a final state that is a CP eigenstate, for example $\pi^+\pi^-$, then it is known with certainty at that instant that the other kaon is in an eigenstate of CP with opposite parity: in this case a K_2^0. The amplitude for that kaon to decay simultaneously to the same CP eigenstate, $\pi^+\pi^-$, is zero, even if there is CP violation present. Simultaneous decay to distinct CP eigenstates, such as $\pi^+\pi^-\pi^0\pi^0$, is an unambiguous signature of direct CP violation.
3. If the kaon that first decays goes to a final state that is not common to K^0 and $\overline{K^0}$, for example $\pi^-e^+\nu$, at that instant one knows with certainty that the other kaon has opposite flavor, in this case a K^0. Its subsequent decay distribution to CP eigenstates is just that as given in equation (2.35).
4. The complete expression for the correlated decays of the two kaons to $\pi^+\pi^-\pi^0\pi^0$ includes interference terms, so that presumably $\mathrm{Re}[\epsilon'/\epsilon]$ can be measured with interference, in the high statistics regime of $K_S^0 \to 2\pi$, in a manner similar to CPLEAR.

Several of the points above are examples of the EPR paradox. Perhaps DAΦNE will make observations more fundamental than CP violation.

The primary decays DAΦNE will exploit to get $\mathrm{Re}[\epsilon'/\epsilon]$ will be to those from $\phi \to K\overline{K} \to \pi^+\pi^-\pi^0\pi^0$. The kaons are monochromatic, so the proper times t_{+-} and t_{00} of the decays are easily reconstructed from the small beam position and the vertex positions of the decays; they can reconstruct the $\pi^0\pi^0$ vertex from both kinematic reconstruction and time of flight. The joint decay probability density will then be[36]:

$$\begin{aligned}\Gamma(t_{+-}, t_{00}) \propto\ & e^{-(\Gamma_S+\Gamma_L)(t_{+-}+t_{00})/2} \times \\ & \Big[\big|(\eta_{+-}+\eta_{00})\sin[\Delta t\Delta\mu/2]\big|^2 + \big|(\eta_{+-}-\eta_{00})\cos[\Delta t\Delta\mu/2]\big|^2 \\ & - 2\mathrm{Im}\big[(\eta_{+-}+\eta_{00})(\eta_{+-}^*-\eta_{00}^*)\sin[\Delta t\Delta\mu/2](\cos[\Delta t\Delta\mu/2])^*\big]\Big]\end{aligned} \tag{4.4}$$

Here $\Delta\mu/2$ is the positive eigenvalue defined in Section 2.3.1, and $\Delta t = t_{+-} - t_{00}$. It is probable that a maximum likelihood fit to (4.4) will be used to determine $\mathrm{Re}[\epsilon'/\epsilon]$ from the data.

Most published work states the statistical error on $\mathrm{Re}[\epsilon'/\epsilon]$ will come primarily from the number of $K_L^0 \to 2\pi$ decays that will be reconstructed. In light of the discussion of CPLEAR, and Fig. 20, that is not completely obvious. From decays of the ϕ, $\gamma\beta c\tau_S = 0.592\,\mathrm{cm}$ and $\gamma\beta c\tau_L = 343\,\mathrm{cm}$. If K_L^0 decays are the statistical limitation, there is a premium on making the sensitive volume of the detector extend to as large a radius as possible; DAΦNE hopes to cover 1.5 meters from the interaction point; about 1/4 of K_L^0 decay in that volume. About 10^7 $\pi^+\pi^-\pi^0\pi^0$ decays are required for a statistical error of 10^{-4} on $\mathrm{Re}[\epsilon'/\epsilon]$; that corresponds to about 2×10^{10} ϕ's produced, which has a cross section of 4μb. An integrated luminosity of $5\times10^{39}\,1/\mathrm{cm}^2$ is required.

The extraction of $\mathrm{Re}[\epsilon'/\epsilon]$ at DAΦNE requires exceedingly accurate knowledge of the detector's ability to reconstruct t_{+-} and t_{00}, much as E731 has done. Generally, they will exploit the high statistics samples, $K_L^0 \to \pi^\mp l^\pm\nu$, $K_L^0 \to 3\pi^0$, and $K_L^0 \to \pi^+\pi^-\pi^0$, to map their acceptance and efficiency. In order to control systematic errors from background from these high statistics modes must be suppressed by about 10^5, which has been shown possible if all kinematic features of the ϕ decay are exploited.

4.4 The rare decays $K_L^0 \to \pi^0e^+e^-$ and $K_L^0 \to \pi^0\nu\overline{\nu}$

Just as observation of $K_L^0 \to 2\pi$ showed that CP was violated, the observation of $K_L^0 \to \pi^0e^+e^-$ or $K_L^0 \to \pi^0\nu\overline{\nu}$ might show that direct CP violation occurs.[37–38] Should these processes be observed, their rates could be *cleanly* related to the CKM parameters of the Standard Model, because hadronic parameters can be extrapolated from measurements of the kaon semileptonic decay. Unfortunately, both processes suffer background from processes other than direct CP violation.

The decay processes $K_L^0 \to \pi^0e^+e^-$ and $K_L^0 \to \pi^0\nu\overline{\nu}$ violate CP *if* a single gauge boson produces the e^+e^- or the $\nu\overline{\nu}$. Exactly these types of processes proceed in the Standard Model by the diagrams shown in Fig. 22. The Standard Model prediction is:

$$\mathrm{B}(K_L^0 \to \pi^0e^+e^-)_{\mathrm{directCP}} \approx 1\times10^{-5}\left[\frac{s_{23}s_{13}}{s_{12}}\sin\delta\right]^2\left\{|\tilde{C_{7V}}|^2 + |\tilde{C_{7A}}|^2\right\} \tag{4.5}$$

From (1.2), $s_{23}s_{13}/s_{12} \approx 7\times10^{-4}$; the expression $|\tilde{C_{7V}}|^2 + |\tilde{C_{7A}}|^2$ can be determined from parameters from the $K_L^0 \to \pi^\mp e^\pm\nu$ decay and the mass of the top quark, and is ≈ 0.6 for $m_t = 150\,\mathrm{GeV}$. So, one expects

$$\mathrm{B}(K_L^0 \to \pi^0e^+e^-)_{\mathrm{directCP}} \approx 3\times10^{-12}\sin^2\delta \tag{4.6}$$

Processes that contribute to $K_L^0 \to \pi^0e^+e^-$, but that do not involve CP violation in the decay amplitude, are:

1. $K_L^0 \to \pi^0(\gamma\gamma \to e^+e^-)$, which involves two intermediate photons. This process is calculable because of recent measurements[39,40] of the decay $K_L^0 \to \pi^0\gamma\gamma$. Broadly speaking, if the two photons are in a relative S-wave, the contribution to $K_L^0 \to \pi^0e^+e^-$ process is helicity suppressed, but is not if the two photons are in a relative D-wave. The observation of $K_L^0 \to \pi^0\gamma\gamma$ shows that the $\gamma\gamma$ peak at high mass, which supports the dominance of the S-wave, and a limit of 3×10^{-14} on the contribution to $K_L^0 \to \pi^0e^+e^-$. Some D-wave contribution could still be present, and use of chiral perturbation theory allows one to limit the D-wave contribution to:
$$\mathrm{B}(K_L^0 \to \pi^0(\gamma\gamma \to e^+e^-)) < 5\times10^{-12}.$$
Eventually, when more $K_L^0 \to \pi^0\gamma\gamma$ decays have been collected by experiments, the Dalitz plot can be analyzed to remove the model dependence.
2. The process $K_2^0 \to K_1^0 \to \pi^0e^+e^-$, which proceeds by state-mixing. Were one to measure $\mathrm{B}(K_S^0 \to \pi^0e^+e^-)$, this contribution to $\mathrm{B}(K_L^0 \to \pi^0e^+e^-)$ would be:
$$\mathrm{B}(K_L^0 \to \pi^0e^+e^-)_{\mathrm{state-mixing}} = \frac{\Gamma_S}{\Gamma_L}|\epsilon|^2\mathrm{B}(K_S^0 \to \pi^0e^+e^-)$$
If one assumes that the partial rate for $K_S^0 \to \pi^0e^+e^-$ is equal to that of $\Gamma(K^+ \to \pi^+e^+e^-) + \Gamma(K^- \to \pi^-e^+e^-)$, one estimates
$$\mathrm{B}(K_L^0 \to \pi^0e^+e^-)_{\mathrm{state-mixing}} \approx 1.2\times10^{-11}.$$
It is logical that this is an overestimate, however, simply because the K_S^0 is neutral. Recent data from the $K^+ \to \pi^+e^+e^-$ Dalitz plot[41] combined with a

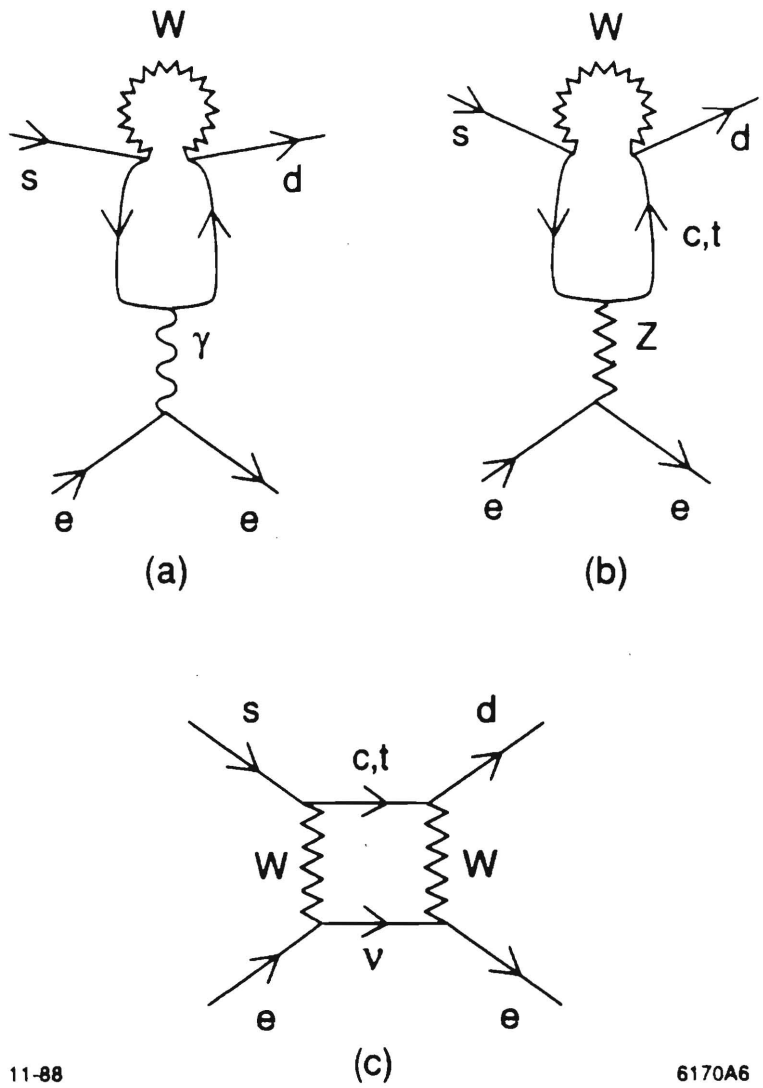

Figure 22. Penguin-like processes that contribute to $K_L^0 \to \pi^0 l\bar{l}$ in the Standard Model.

rather sensitive model-dependent extrapolation indicates that:

$$B(K_L^0 \to \pi^0 e^+ e^-)_{\text{state-mixing}} < 1.6\times10^{-12}.$$

3. The radiative Dalitz decay $K_L^0 \to \gamma\gamma e^+ e^-$ can mimic $K_L^0 \to \pi^0 e^+ e^-$, if the $\gamma\gamma$ falls near the mass of the π^0. This background contributes an effective branching ratio of approximately 3×10^{-11}, much larger than (4.6); however, given sufficient statistics, this background can be removed by extrapolation under the π^0 peak.[42] A more clever way to eliminate this background would be to follow CPLEAR, and detect $K_L^0 \to \pi^0 e^+ e^-$ through its *interference* with $K_S^0 \to \pi^0 e^+ e^-$, in the $K_S^0 \to \pi^0 e^+ e^-$ time evolution.

As this look at the hard numbers shows, detection of direct CP violation through $K_L^0 \to \pi^0 e^+ e^-$ will be no cake-walk. The current limit[43] is $B(K_L^0 \to \pi^0 e^+ e^-) < 5.5\times10^{-9}(90\%)$. A new round of experimentation is underway, with preliminary results expected soon from E-799 at FNAL, and further runs expected both there and at KEK.

In contrast, the decay $K_L^0 \to \pi^0 \nu\bar{\nu}$ will not suffer background from state-mixing CP violation, simply because the partial rate for $K_S^0 \to \pi^0 \nu\bar{\nu}$ is similar to that for $K_L^0 \to \pi^0 \nu\bar{\nu}$. The computation of the branching ratio for $K_L^0 \to \pi^0 \nu\bar{\nu}$ is similar to (4.5), however, the $\nu\bar{\nu}$ final state is favored over e^+e^-, because the Z^0 couples more strongly to $\nu\bar{\nu}$, and because all three neutrino flavors contribute; $B(K_L^0 \to \pi^0 \nu\bar{\nu}) \approx 2\times10^{-11}$ is expected.

The experimental signature of $K_L^0 \to \pi^0 \nu\bar{\nu}$ is a lone π^0 observed in the detector. Backgrounds are many to this signal; a good example is $K_L^0 \to \pi^0\pi^0$, with the two photons from one π^0 lost. The most hermetic veto system possible must be built to suppress this background. Even then, should a signal be seen, the cross checks necessary for proof that the signal is not induced from neutrons in the K_L^0 beam, for example, will be onerous. The current limit, from an interesting analysis of E731[44] which exploited Dalitz decay of the lone π^0, is $B(K_L^0 \to \pi^0\pi^0) < 2.2 \times 10^{-4}$.

4.5 Asymmetries in Dalitz plots, such as in $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$.

The square of the matrix element for $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ has a term linear in the energy of the pion with charge opposite to that of the kaon; the slope is denoted $g_\pm$. Should $g_+ \neq g_-$, that would be a manifestation of direct CP violation. Older estimates indicated that the asymmetry should be only of order $|\epsilon'|$, or of order 10^{-6}.[45] Some newer estimates[46] suggest the asymmetry will be nearer 5×10^{-4}, as shown in Figure 23.

Exploitation of this method of observation of direct CP violation has a great attribute that only *charged* particles need be detected. If the asymmetry really is large, then either DAΦNE or the AGS at Brookhaven can get the statistics necessary to observe this effect. The experimental challenge is really to suppress systematic errors.

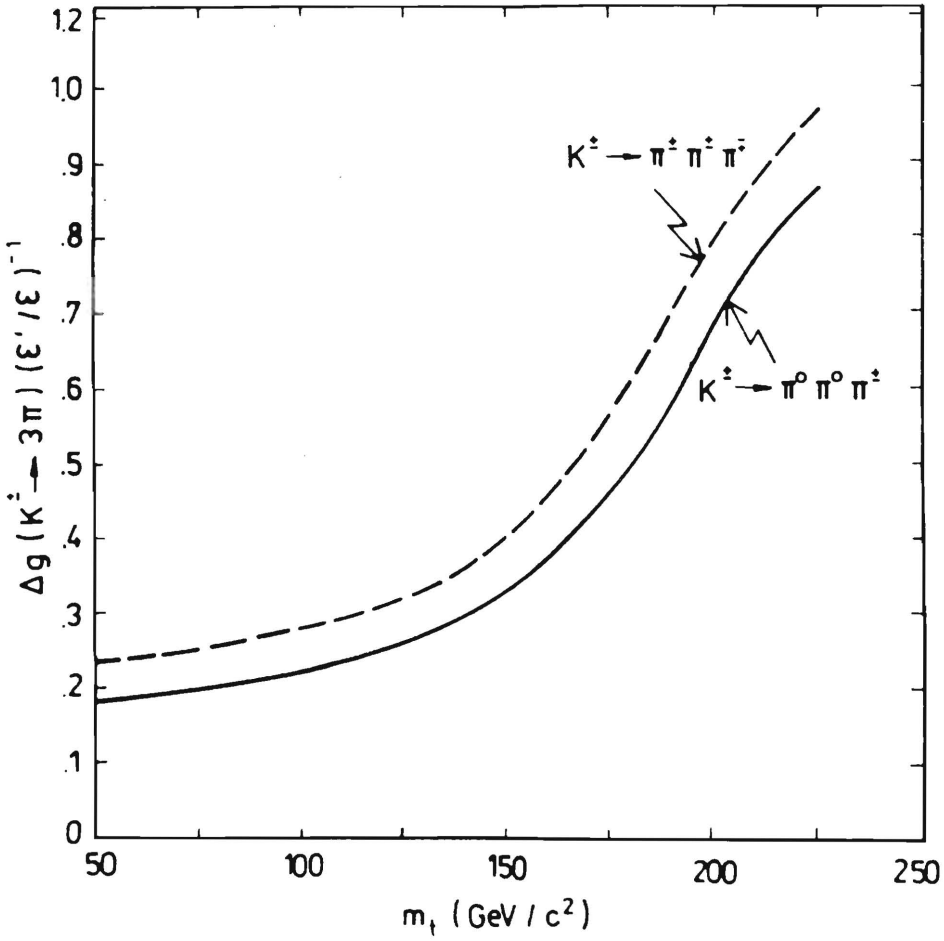

Figure 23. The asymmetry in charged kaon decay as a function of top quark mass is now claimed to be large.[46]

5. How and Where CP Violation will Appear in the B System

The dominant source of CP violation in the kaon system, state-mixing, causes the rate for $K^0 \to \overline{K^0}$ to differ from that of $\overline{K^0} \to K^0$, at the level of 1 part in 1000; the culprit can be taken to be a small imaginary part in the mass matrix, $\mathrm{Im}[M_{12}] = -11.6\,\mathrm{neV}$.

If CP violation is superweak in origin, and if the interaction that causes CP violation is universal among the quarks, then $\mathrm{Im}[M_{12}]$ will be about an order of magnitude lower in the B^0 system than in the kaon system; CP violation would be in practice unobservable in the B system. The only hope to see CP violation would be if the superweak interaction *heavily* favored the b-quark.

In contrast, the Standard Model suggests that $\mathrm{Im}[M_{12}]$ in the B^0 system is four orders of magnitude larger than in the K^0 system. The difference between the rates for $B^0 \to \overline{B^0}$ and $\overline{B^0} \to B^0$ is still predicted to be at the level of 1 part in a 1000, due to the suppression from the superweak phase. As emphasized in the second section, something like $4{\times}10^{10}$ events must be reconstructed to observe state-mixing in the B system.

Fortunately, there is a clever trick available, that saves roughly three orders of magnitude in events. The Standard Model allows the possibility that branching ratios for $B^0 \to J/\psi K^0_S$ and for $\overline{B^0} \to J/\psi K^0_S$ will be different, as expressed in the asymmetry $A_{J/\psi K^0_S}$:

$$A_{J/\psi K^0_S} \equiv \frac{\Gamma(B^0 \to J/\psi K^0_S) - \Gamma(\overline{B^0} \to J/\psi K^0_S)}{\Gamma(B^0 \to J/\psi K^0_S) + \Gamma(\overline{B^0} \to J/\psi K^0_S)} = -\frac{x_d}{1+x_d^2}\mathrm{Im}(\lambda_{J/\psi K^0_S})$$

$$\approx -0.47\sin(2\beta)$$

Here β is one angle in the so-called 'Unitarity Triangle'. The branching ratio for $B^0 \to J/\psi K^0_S$ is low, only 4×10^{-4} or so. You should wonder, is it really an advantage, to obtain a large asymmetry at the expense of a low branching ratio. The answer is **yes**, simply because the number of events required to observe an asymmetry is reduced as the *square* of that asymmetry. To scale the earlier result by the ratio of asymmetries $(10^{-3}/0.5)$ and the ratio of semileptonic to $J/\psi K^0_S$ branching ratios, $(0.1/4{\times}10^{-4})$,

$$N_{B^0\overline{B^0}} \approx 4{\times}10^{10}\left[\frac{10^{-3}}{0.5}\right]^2\left[\frac{0.1}{4\times 10^{-4}}\right] \approx 4{\times}10^7$$

The Standard Model calculation of $A_{J/\psi K^0_S}$ is particularly easy because this asymmetry arises from purely from phase differences. The calculation is well outlined in Nir's lectures. I will point out that β above is one of the angles in the triangle formed in the complex plane by the CKM matrix elements, when one of the unitarity constraints is invoked:

$$V^*_{ub} + V_{td} = -V_{cd}V_{cb} \tag{5.1}$$

(V_{ud} and V_{tb} have been neglected). The magnitudes of these quantities are experimentally determined, respectively, by: the rate of $b \to u$ transitions ($|V^*_{ub}|$); the cross section for charm production from neutrino interactions ($|V_{cd}|$); the b-lifetime ($|V_{cb}|$);

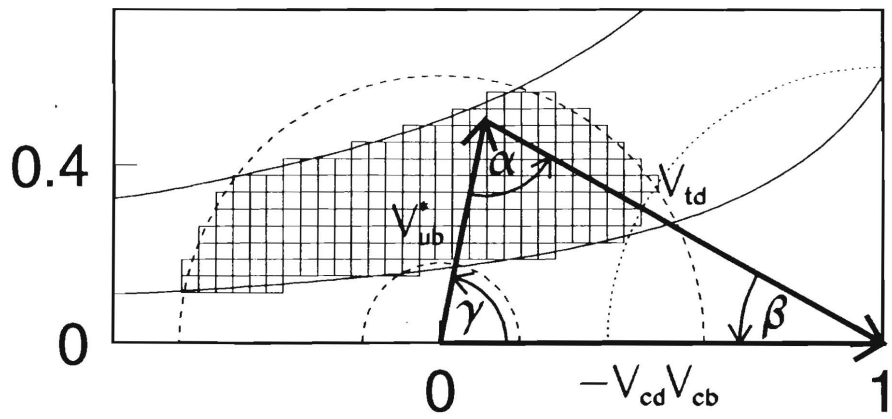

Figure 24. The unitary triangle; the usual convention is to place $-V_{cd}V_{cb}$ along the horizontal axis, and normalize to it. The semiannular region about $(0,0)$ comes from $|V_{ub}/V_{cb}| = 0.09 \pm 0.05$; the swath from lower left to upper right comes from ϵ, and the circle centered on $(1,0)$ comes from ΔM_B. The region with the grid is allowed.

and $|\Delta M_B|$ ($|V_{td}|$). A diagram of this unitarity triangle, along with its current experimentally allowed region, is shown in Fig. 24. The angle β is that between V_{td} and the negative real axis. There is considerable theoretical uncertainty in extracting the CKM matrix elements from the data, nonetheless, Nir concludes:

$$\begin{aligned} &\sin 2\beta > 0.15 \\ \text{so} \quad &-0.47 < A_{J/\psi K^0_S} < -0.07 \end{aligned} \tag{5.2}$$

One would need a factor of 50 more $B^0\overline{B^0}$ events, should Nature unkindly place $\sin 2\beta$ near its lower limit. The *sign* of (5.2) deserves note: $\sin 2\beta$ is positive, in essence because $|V_{ub}/V_{cb}| < |V_{cd}|$, and so the asymmetry $A_{J/\psi K^0_S}$ is *negative*. In the neutral kaon system, the rate asymmetry $A_{2\pi} \approx 6\mathrm{Re}[\epsilon]$, and is thus *positive*. The CP of $J/\psi K^0_S$ is negative, while that of two pions is positive, so these asymmetries make intuitive sense. Should CP violation prove not to be from the Standard Model, but superweak in origin, $A_{2\pi}$ and $A_{J/\psi K^0_S}$ would probably have relative sign opposite to $\mathrm{Re}[\epsilon]$ and $\mathrm{Re}[\epsilon_B]$.

The other two angles of the unitarity triangle (5.1), α (between V_{td} and V^*_{ub}) and γ (between the positive real axis and V^*_{ub}) are related, respectively to the asymmetries in $B \to \pi^+\pi^-$ and $B \to \rho K^0_S$. Nir has given a nice discussion of the allowed values of these asymmetries in the Standard Model: I would like to point out that the Standard Model allows the $B \to \pi^+\pi^-$ asymmetry to have the *same* sign as $A_{J/\psi K^0_S}$, which is forbidden in the simplest superweak scenario.[47] It is γ that is the hardest to measure: to get it, one needs to both start from a B^0_s initial state, and observe and a $b \to u$ transition.

In principle, measurement of all three of the asymmetries to $J/\psi K^0_S$, $\pi^+\pi^-$, and ρK^0_S allows one to overconstrain the unitarity triangle, and test the Standard Model. It is my opinion that this is the second task one should pursue in the B^0 system; the first task is to find one asymmetry that is much larger than the simplest superweak scenario, (1.6), would predict.[48] With luck, that will be $A_{J/\psi K^0_S}$, because a variety of experimental programs have sensitivity to it. I will focus the remainder of this lecture on those experimental programs. It is clear that only one of these programs, that of an asymmetric B-factory, has the capability to probe well beyond this one asymmetry.

I would like to consider four possible accelerators where one could pursue observation of the asymmetry in $B^0 \to J/\psi K^0_S$:

1. An asymmetric B-factory (ABF); when considering observation of the asymmetry, I will separate SLAC and Cornell (CESR).[51,52]
2. The upgraded LEP, LEP-II.[53]
3. The upgraded Tevatron, TeV-II, using the D0 detector, for pseudorapidity $|\eta| < 3.1$, which is roughly $\theta > 5°$.[52]
4. The SSC at small angle, between $1° - 10°$ from the beam axis, or $2.4 < |\eta| < 4.7$. The $b\bar{b}$ that are produced in this angular region have a hard energy spectrum, with mean value 300 GeV, which facilitates triggering and lepton ID. The geometric acceptance is low, and the mean number of charged particles in $b\bar{b}$ events that populate this region is about 30.[54]
5. The SSC in the central region, for $b\bar{b}$ produced at angle greater than 10° from the beam axis, or $|\eta| < 2.4$. There is high geometric acceptance, but the mean energy is low, about 24 GeV, and the charged multiplicity in $b\bar{b}$ events is high, about 50.[54]

I summarize the production information in Table 3. Much of the table is taken from Natalie Roe's lectures on B physics, and is straightforward.

Table 3

	$\sqrt{s}$ (GeV)	$\mathcal{L}$, 10^{33} cm^{-2}s^{-1}	$\sigma_{b\bar{b}}$ (μb)	$\frac{(B^0+\overline{B^0})}{b\bar{b}}$	Geom. Acc.	Trig. Eff.	$N_{B^0+\overline{B^0}}$ (Year)
LEP-II	90	0.1	0.005	3/4	1	1	3.8×10^6
ABF	10	3	0.001	1	1	1	30×10^6
TeV-II $\lvert\eta\rvert < 3.1$	1.8×10^3	0.05	40	3/4	0.45	0.1	0.68×10^9
SSC $1°-10°$	40×10^3	0.1	500	3/4	0.038	0.22	3.1×10^9
SSC $\lvert\eta\rvert < 2.4$	40×10^3	0.1	500	3/4	0.31	0.064	7.5×10^9

The column labelled $(B^0 + \overline{B^0})/b\bar{b}$ warrants discussion. I assume that in the continuum, the b quark hadronizes to $\overline{B^0} : B^- : \overline{B^0_s}$: (B-Baryons) with relative frequency [3/8] : [3/8] : [0.1] : [0.15]. Then it is possible to compute:

$b\bar{b}$	$0B^0$	$1B^0$ or $1\overline{B^0}$	B^0 and $\overline{B^0}$	Mean $\#(B^0+\overline{B^0})$
Continuum	$[5/8]^2 = 0.39$	$2[3/8][5/8] = 0.47$	$[3/8]^2 = 0.14$	$0\times0.39 + 1\times0.47 + 2\times0.14 = 3/4$
$\Upsilon(4S)$	0.5	0	0.5	$2\times0.5 = 1$

Note that in the continuum $(B^0+\overline{B^0})/b\bar{b}$ corresponds to independent hadronization of the two sides: on gets just twice the probability of hadronization to B^0 or $\overline{B^0}$: 3/4. The explicit accounting is useful for calculation of the dilution factor.

For hadron colliders, the geometric acceptance is that for all charged particles to enter the angular bite; for e^+e^-, this factor is nearer unity, and incorporated later. The small acceptance of the $1^\circ - 10^\circ$ SSC experiment is apparent, as is the compensation in trigger efficiency, from the harder $b\bar{b}$ production. The trigger requirements are:

$$\text{TeV-II: D0, } |\eta|<3.1 \quad \begin{cases} 2\mu \\ p_T > \text{ several GeV} \end{cases}$$

$$\text{SSC: } 1^\circ - 10^\circ \quad \begin{cases} 3\mu \\ p > 8 \quad \text{GeV} \\ p_T > 1.5 \quad \text{GeV} \end{cases}$$

$$\text{SSC: } |\eta| < 2.4 \quad \begin{cases} 3\mu \\ p > 5 \quad \text{GeV} \\ p_T > 1.5 \quad \text{GeV} \end{cases}$$

Consider now the efficiency of the tag, and of the $J/\psi K^0_S$ reconstruction, and the effects of mis-tags and background. The numbers are summarized in Table 4.

Note that all six experiments use the sign of the muon semimuonic B decay as tag. The asymmetric B factories claim a substantial increase in tagging efficiency, primarily from use of the sign of the lepton l from the cascade $b-c-l$, the strangeness of the soft charged kaon from the cascade $b-c-s$, and even from reconstruction of some $Dn(\pi)$ from the c hadronization. The result is an increase in tagging efficiency of about a factor of 5 over the other experiments! This is a bold and crucial increase: up until now, the B flavor tagging that has been used, for example to measure $B-\overline{B}$ mixing, has been primarily from the semileptonic tag.

Incorrect tags reduce the observed CP asymmetry by a factor termed the 'dilution factor'. Generally, the value of the dilution factor can be empirically deduced from study of tagged $B^- \to J/\psi K^-$. More relevant is that more data is required to observe a significant effect, if the CP asymmetry is diluted. Broadly speaking, dilution comes from four sources:

1. Oscillation of the tag B^0 or B^0_s to $\overline{B^0}$ or $\overline{B^0_s}$ prior to decay.
2. Attribution of a tag lepton from $b-c-l$ to $b-l$, or similar misattribution of a kaon from hadronization to $b-c-s$.
3. Fake leptons or kaons.
4. Correlated evolution of the initial $C=-1$ $B^0\overline{B^0}$ state from the $\Upsilon(4S)$: here the dilution factor is 0, unless an Asymmetric B Factory is used.

Table 4

	Tag BR×ϵ	Tag Dilu-tion	$J/\psi K^0_S$: ϵ ×BR(J/ψ) ×BR(K^0_S)	$\frac{S}{S+B}$	Tagged $J/\psi K^0_S$ (Year)	Observed Asymmetry $\sin 2\beta = 0.43$	Years 3σ
LEP-II	0.050 $e+\mu$	0.75 vtx	0.080	0.98	6	-0.15	68
ABF (SLAC)	0.41 $e+\mu$ K	0.82 vtx	0.081	0.98	400	-0.16	0.86
ABF (CESR)	0.61 $e+\mu$ $K+Dn(\pi)$	0.84 vtx	0.041	0.98	300	-0.16	1.1
TeV-II $\lvert\eta\rvert < 3.1$	0.086 μ	0.44	0.015	0.50	350	-0.044	6.6
SSC $1^\circ - 10^\circ$	0.10 μ	0.28	0.030	0.23	3700	-0.013	3.4
SSC $\lvert\eta\rvert < 2.4$	0.066 μ	0.28	0.010	0.60	2000	-0.034	2.3

The expressions (2.15) can be used to show that $x_d^2/[2(1+x_d^2)] = 0.16$ of B^0 tags actually were $\overline{B^0}$ at production. This is an insidious effect, because the flavor is reconstructed to be exactly opposite of what it should be; the dilution factor for such events is 1–2×0.16. For B^0_s tags, $x_s^2/[2(1+x_s^2)] = 0.492$, and so the dilution is almost complete. The charged B and baryon tags do not dilute, so the average continuum dilution factor from tag mixing, D_{mix}, is:

$$D_{\text{mix}} = 0.10+0.375+0.375\times(1-2\times0.16)+0.15\times(1-2\times 0.492) = 0.73$$

The cascade sequence b–c–l produces a lepton exactly opposite in sign to that from $b-l$, which can double the dilution, as in mixing. The leptons from $b-c-l$ are softer than those from $b-l$ both in momentum and momentum transverse to the b direction. These properties are exploited by the B-factory proposals to suppress dilution from this source, and even, for the CESR proposal, to enhance the tagging efficiency. The TeV-II and SSC have smaller dilution factors because they make more modest claims on the ability to distinguish cascades.

The rate of fake leptons and/or kaons are the hardest to estimate without a specific detector design, and a detailed Monte-Carlo; probably the best estimate has been made for the D0 detector at TeV-II. Fake rates are not included in the SSC estimates.

The last source of dilution is peculiar to $B^0\overline{B^0}$ production from the $\Upsilon(4S)$. As mentioned, the dilution factor is exactly zero, heuristically because the $C=-1$ $B^0\overline{B^0}$ state is like a flavor singlet, of from direct application of (4.4) to the B case.

Salvation of the $\Upsilon(4S)$ arrives **if** it is possible to determine which happened first in time: the decay of the tag B^0 or $\overline{B^0}$, or the decay into $J/\psi K^0_S$; denote the earlier

decay by < and the later by >. Then one obtains for the pair of asymmetries:

$$A_> = \frac{\Gamma([\overline{B^0}]_<[J/\psi K^0_S]_>) - \Gamma([B^0]_<[J/\psi K^0_S]_>)}{\Gamma([\overline{B^0}]_<[J/\psi K^0_S]_>) + \Gamma([B^0]_<[J/\psi K^0_S]_>)} = -\frac{1}{2}\frac{x_d}{1+x_d^2}\sin 2\beta$$

$$A_< = \frac{\Gamma([B^0]_>[J/\psi K^0_S]_<) - \Gamma([\overline{B^0}]_>[J/\psi K^0_S]_<)}{\Gamma([B^0]_>[J/\psi K^0_S]_<) + \Gamma([\overline{B^0}]_>[J/\psi K^0_S]_<)} = -\frac{1}{2}\frac{x_d}{1+x_d^2}\sin 2\beta$$

So, an effective asymmetry $A'_{J/\psi K^0_S}$ is related to $\sin 2\beta$ just as (2.37):

$$A'_{J/\psi K^0_S} = A_> + A_< = -\frac{x_d}{1+x_d^2}\sin 2\beta$$

It is essentially impossible to achieve measurement of the time ordering of the tag and the $J/\psi K^0_S$ at an e^+e^- collider with two beams of equal energy, such as CESR. The momentum of a B^0 from $\Upsilon(4S)$ decay is only about 330 MeV, so $\gamma\beta c\tau_b \approx 25\,\mu$m; the current generation of silicon vertex detectors cannot measure such a small flight-path. In addition, two out of three of the sizes a typical e^+e^- annihilation region are usually far larger than $25\,\mu$m.

The solution is to exploit e^- and e^+ beams of unequal energy. Although $\sqrt{s}$ is not changed, the center of mass system is given a boost down the beam axis. As long as:

1. The momentum of the $B^0\overline{B^0}$ from the boost is >> 330 MeV, the value obtained simply from $\Upsilon(4S)$ decay, and
2. The flight-path of the B^0 is several times longer then the vertex resolution of the tracking system,

then it is possible to perform the time-ordering of tag and decay to $J/\psi K^0_S$ by simply reconstructing the projections of the tag and $J/\psi K^0_S$ decay along the beam axis. It is unnecessary to know the production point. Optimization studies, including acceptance considerations, have shown that an electron beam of 9 GeV and a positron beam of 3.1 GeV are near the optimum; the center of mass speed is then $\beta_{\rm cm} \approx 0.5$, and $\gamma\beta c\tau_b \approx 215\,\mu$m. This is so much greater than the intrinsic resolution of a state-of-the-art vertex detector that not only is the dilution of $A_>$ and $A_<$ from mistakes in time ordering negligible, but it should be possible to study the detailed time evolution of the > and < $B^0\overline{B^0}$ pairs, and so extract a better statistical precision. Figure 25 shows a time difference histogram; a simple way to express the statistical precision afforded by a fit to this distribution is to quote a 'dilution' $D_{\rm fit} = 1.2$, greater than unity. Inclusion of this factor is marked by 'vtx' in Table 4.

The branching ratio for $B^0 \to J/\psi K^0_S$ is assumed to be 4×10^{-4}, and is omitted from the efficiency in Table 4; however, the branching ratios used to reconstruct the J/ψ and the K^0_S are included. Most of the dispersion among the values arises from use of just $J/\psi \to \mu^+\mu^-$ or additional inclusion of $J/\psi \to e^+e^-$, and similarly, use of just $K^0_S \to \pi^+\pi^-$ or inclusion of $K^0_S \to \pi^0\pi^0$.

Background to $J/\psi K^0_S$ should show no asymmetry from CP violation, and contribute a dilution factor of $S/(S+B)$. More events are also present to measure the smaller asymmetry, which I will include in the calculation of the time it takes to observe the CP asymmetry.

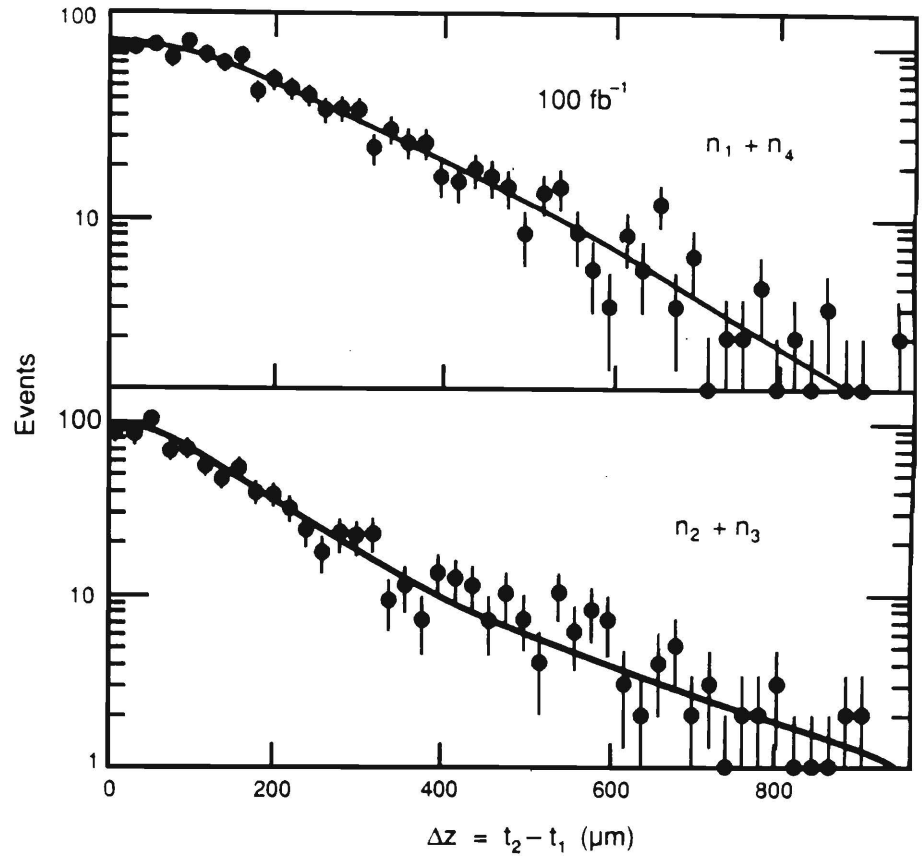

Figure 25. Simulated exploitation of vertex resolution in the measurement of $\sin 2\beta$. The upper plot has two types of events: those where the $\overline{B^0}$ decay precedes the $J/\psi K^0_S$ decay ($[\overline{B^0}]_<[J/\psi K^0_S]_>$) and $J/\psi K^0_S$ decay occurs prior to the B^0 decay ($[B^0]_>[J/\psi K^0_S]_<$); the lower plot has the other two combinations. Note the similarity to Fig. 4. These simulate a $\sin 2\beta = -0.4$, which is actually ruled out already.

From (5.2), a value of $\sin 2\beta = 0.43$ is in the middle of the allowed range; I will estimate the time it takes the six experiments to achieve an error one-third of this value. It is straightforward to estimate how many tagged $J/\psi K_S^0$ events per year, $N_{J/\psi K_S^0}$. The observed CP asymmetry will be:

$$A_{\rm obs} = -\frac{S}{S+B} D \frac{x_d^2}{1+x_d^2} \sin 2\beta$$

The observed asymmetry provides a feeling for the susceptibility of the measurement to systematic errors.

To estimate the number of years, n_y, to achieve 3σ sensitivity to $\sin 2\beta = 0.43$, one must account for the increased statistics provided by the background:

$$n_y = \frac{1}{3}\sqrt{\frac{S}{S+B}} \frac{(1/A_{\rm obs})}{\sqrt{N_{J/\psi K_S^0}}}$$

The strongest conclusion one can draw from the last column of Table 4 is that LEP-II is unlikely to be a contender in B CP violation, unless one of you conceives of a clever way to do better.[56] The other experiments are surprisingly close. You should draw your own conclusions, but I would advise you to keep Tables 3 and 4 in your pocket in case anybody tries to sell you a CP violation experiment!

6. Conclusions

The mystery of CP violation remains unsolved, despite twenty-eight years of intense and clever experimentation, phenomenology and theory. The prime suspect is the Standard Model, and the case is compelling, but the jury is still out. I hope I have supplied your toolbox with some of the tools one of you will need to succeed in executing the crucial experiment that will either remove all reasonable doubt, or discover the guilty party.

I regret that I did not find time to put in a good discussion of electric dipole moments in these lectures; I can suggest a recent review article.[57]

My own prediction is that after another decade of intense experimentation at several accelerators, we will still be like the ostrich in Fabergé's famous cartoon,[58] which I have modified in Fig. 26!

J. Fabergé. *CERN Courier*. **6**, No. 10, 193 (October 1966). [Courtesy of Madame Fabergé.]

Figure 26. The situation in 2001? Apologies to Fabergé.

7. Acknowledgements

As usual, the Twentieth SLAC Summer Institute has been marvelous fun and physics. Like many SLAC alumni, it was actually my first chance to systematically attend the whole thing. My thanks to the organizers, not just for this year, but for those years when I would duck in for a few lectures (but *never* a free meal).

Without the assistance and tolerance of David Leith and David Burke, this writeup never would have been completed.

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