UCONN-93-7
August, 1993

High Energy Elastic Scattering and Nucleon as a Topological Soliton*

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Abstract

The gauged nonlinear σ-model describes the nucleon as a topological soliton and introduces the ω meson as a gauge boson coupled to the topological baryonic charge. Validity of the model implies ω behaving as an elementary spin-1 particle. Such a behavior of ω has been observed previously in phenomenological analyses of high energy elastic scattering. In fact, from the ωNN form factor obtained in elastic scattering, it is possible to predict a pion field configuration for the nucleon, which compares reasonably with the pion field configuration predicted by the soliton model. To provide a realistic nucleon mass, the soliton model needs to be supplemented by a quark sector, where left and right quarks interact to form a chiral condensate. The nucleon emerges then as a topological soliton embedded in a chiral condensate—a physical picture that agrees fully with the high energy elastic scattering analyses.

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High energy elastic scattering is something we are all familiar with, and we have heard a great deal about it at this conference. So, I am going to start with the less familiar part of the title of my talk;namely, nucleon as a topological soliton. A soliton is a localized lump of energy which originates from a field with nonlinear interactions, and which is stable because of the conservation of a topological charge. A topological charge, unlike the usual Noether charge, is conserved independently of the equation of motion and has a geometrical origin connected with the field configuration.

To understand how the nucleon can be described as a topological soliton, let us start with a simple model—the linear σ-model with $SU(2)_L \times SU(2)_R$ symmetry and Higgs potential. The model is described by the Lagrangian [1]

$$\begin{aligned}\mathcal{L} &= \bar{\psi} i\gamma^\mu \partial_\mu \psi + \frac{1}{2}(\partial_\mu \sigma \partial^\mu \sigma + \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi}) \\ &\quad - G\bar{\psi}(\sigma + i\vec{\tau}\cdot\vec{\pi}\gamma^5)\psi - \lambda(\sigma^2 + \vec{\pi}^2 - f_\pi^2)^2, \end{aligned} \tag{1}$$

where ψ is the fermion or the quark field. We introduce the unitary field $U(x) = \exp(i\tau^a \Phi^a / f_\pi)$ and write $\sigma(x) + i\vec{\tau}\cdot\vec{\pi}(x) = \zeta(x)U(x)$. Here, $\Phi^a(x)$'s are the pions or

*Talk given at the International Conference (Vth Blois Workshop) on Elastic and Diffractive Scattering, Brown University, Providence, Rhode Island (June, 1993).

the Goldstone bosons, and $\zeta(x)$ is a scalar field. Eq.(1) can be rewritten as

$$\begin{aligned}\mathcal{L} &= \bar{\psi}_L i\gamma^\mu\partial_\mu\psi_L + \bar{\psi}_R i\gamma^\mu\partial_\mu\psi_R + \frac{1}{4}\zeta^2 tr[\partial_\mu U\partial^\mu U^\dagger] \\ &\quad + \frac{1}{2}\partial_\mu\zeta\partial^\mu\zeta - G\zeta(\bar{\psi}_L U\psi_R + \bar{\psi}_R U^\dagger\psi_L) - \lambda(\zeta^2 - f_\pi^2)^2; \end{aligned} \tag{2}$$

$\psi_L = \frac{1}{2}(1-\gamma^5)\psi$ and $\psi_R = \frac{1}{2}(1+\gamma^5)\psi$ are the left and the right chiral fields.

The model given by Eq.(2) can be easily generalized to $SU(3)_L \times SU(3)_R$ symmetry with local gauge invariance by considering the unitary field $U(x) = \exp(i\lambda^a\Phi^a/f_\pi)$ and by introducing the left gauge field $\mathcal{A}_\mu(x)$ and the right gauge field $\mathcal{B}_\mu(x)$. (λ_a's are the Gell-Mann matrices.). We want to explore the quantum aspects of this generalized model. So, we write a path-integral representation of its partition function:

$$\begin{aligned}Tr\left[e^{-iHT}\right] &= \frac{1}{N}\int DUD\zeta D\psi D\bar{\psi}\exp\Big[i\int d^4x\Big(\frac{1}{4}\zeta^2 tr[D_\mu U(D^\mu U)^\dagger] \\ &\quad + \frac{1}{2}\partial_\mu\zeta\partial^\mu\zeta - V(\zeta) + \bar{\psi}^{\mathcal{A}} i\,\not{D}(\mathcal{A})a_-\psi^{\mathcal{A}} + \bar{\psi}^{\mathcal{B}} i\,\not{D}(\mathcal{B})a_+\psi^{\mathcal{B}} \\ &\quad - G\zeta\bar{\psi}^{\mathcal{A}} U a_+\psi^{\mathcal{B}} - G\zeta\bar{\psi}^{\mathcal{B}} U^\dagger a_-\psi^{\mathcal{A}}\Big)\Big]; \end{aligned} \tag{3}$$

here, $D_\mu = \partial_\mu U + \mathcal{A}_\mu U - U\mathcal{B}_\mu$, $V(\zeta) = \lambda(\zeta^2 - f_\pi^2)^2$, $a_\pm = \frac{1}{2}(1\pm\gamma^5)$, $\not{D}(\mathcal{A}) = \gamma^\mu(\partial_\mu + \mathcal{A}_\mu)$, and $\not{D}(\mathcal{B}) = \gamma^\mu(\partial_\mu + \mathcal{B}_\mu)$. Something important happens at this point. The fermion measure in (3) is gauge dependent and can be written as the product of a Jacobian and an invariant measure: $D\psi D\bar{\psi} = JD\psi^0 D\bar{\psi}^0$. The Jacobian can be identified as $\exp(i\Gamma_{WZ})$, where Γ_{WZ} is the Wess-Zumino action. The net result, therefore, is the appearance of an additional piece of action in (3). It is not surprising that currents which were conserved before are no longer conserved, because of the contribution from Γ_{WZ} . This is known as the anomaly; namely, a current that is conserved at the classical level is no longer conserved when quantum effects are taken into account.

A simple model can now be built, if we: (i) replace the scalar field $\zeta(x)$ by its vacuum value f_π from the very beginning, (ii) keep the WZ action arising from the fermion measure, (iii)forget about the interaction in the quark sector. This is the gauged nonlinear σ model described by the partition function

$$Tr\left[e^{-iHT}\right] \simeq \frac{1}{N'}\int DU\exp\Big(i\int d^4x\frac{1}{4}f_\pi^2 tr[D_\mu U(D^\mu U)^\dagger] + i\Gamma_{WZ}\Big). \tag{4}$$

To describe the nucleon using this model, one further includes an additional $U(1)_V$ gauge invariance connected with baryonic charge conservation, and one introduces the vector meson ω as a gauge boson coupled to the baryonic charge. Limiting oneself to the $SU(2)_L \times SU(2)_R$ sector, the model now has ω, ρ, A_1 introduced as gauge bosons, and a WZ action that involves these vector mesons.[1]

The WZ action shows that ω couples to the pseudoscalar mesons via the interaction $g_\omega J_\mu\omega^\mu$, where $J^\mu = \epsilon^{\mu\nu\rho\sigma} tr[U^{-1}\partial_\nu U\ U^{-1}\partial_\rho U\ U^{-1}\partial_\sigma U]/24\pi^2$ is the baryonic

current. This current is conserved algebraically and can be identified as the topological current connected with the U-field configuration. The total baryonic charge $\int J_o d^3x$ then becomes the topological charge, or,the winding number of the unitary field. This model has been studied extensievly by three groups[2-4] in order to see whether the nucleon can be described as a topological soliton. To this end, one first determines all the unknown parameters in the WZ action by studying the pseudoscalar sector. One then proceeds to predict the low energy properties of the nucleon by making the ansatz $U(\vec{r}) = \exp(i\vec{\tau}\cdot\hat{r}\theta(r))$ $\left(i.e.\vec{\Phi}(\vec{r})/f_\pi = \hat{r}\theta(r)\right)$, known as the hedgehog solution. $\theta(r)$ is referred to as the pion profile function, or, the pion field configuration. The model has been found to describe low energy properties of the nucleon quite well, but fails in one respect; namely, the mass of the soliton turns out to be consistently too large ($\sim$ 1500 MeV)compared to the actual mass of the nucleon.

Before addressing this problem, let us at this point see how the topological soliton description gets linked with the high energy elastic scattering. High energy elastic scattering at the ISR and SPS Collider[5,6] have been analyzed by me and my collaborators over a number of years.[7-9] From our analyses, we have arrived at the following physical picture. The nucleon has an inner core and an outer cloud.In elastic scattering at small momentum transfer, the outer cloud of one nucleon interacts with that of the other giving rise to diffraction scattering. As the momentum transfer increases, one nucleon core begins to scatter off the other core via ω exchange, and this process starts to dominate. Fig.1 shows the fit obtained by us [9] at $\sqrt{s} = 53$ GeV.

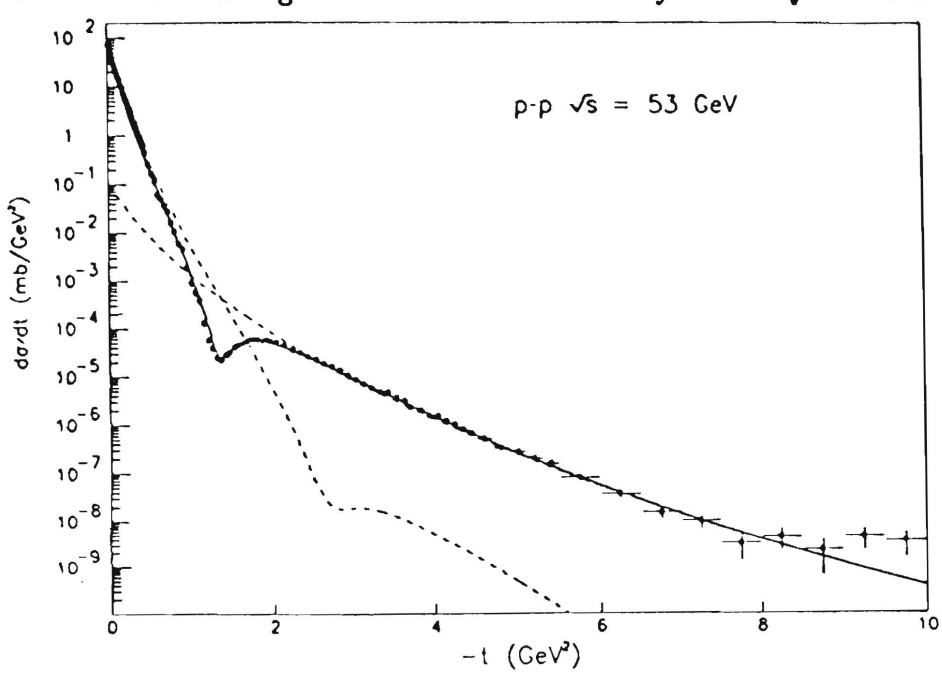

Fig.1. Solid curve represents calculated pp elastic differential cross section at $\sqrt{s} = 53$ GeV [Ref.9]. Differential cross section due to difraction alone (dot-dashed line) and due to ω-exchange alone (dashed line) are also shown. ISR data are from Nagy et al. [Ref.5].

In our analyses, ω behaves as an elementary spin-1 boson. This can be seen by noticing that the ω-exchange amplitude is dominantly of the form: $e^{i\chi_D(s,b=0)} s F^2(t)/(m_\omega^2 - t)$. The factor of s originates from spin 1 of ω. The t-dependence (product of two form factors and the ω propagator) shows that ω exchange corresponds to an elementary meson exchange between two sources. (The factor $e^{i\chi_D(s,b=0)}$ simply reflects the absorptoin effect due to diffraction scattering.) From the viewpoint of conventional high energy scattering models, this behavior of ω is hard to understand. In such models, ω is a composite object and, therefore, behaves as a Rgge pole. Its contribution away from the forward direction at high energy is then negligible. In contrast, the behavior of ω as a spin-1 boson even at high energy is easy to understand in the gauged nonlinear σ-model. The latter is an effective QCD model in which baryonic charge is topological, and ω is the gauge boson coupled to this charge.[1] As long as the model remains valid, baryonic charge continues to behave as topological charge, and the vector meson ω coupled to it continues to act as a gauge boson, i.e., as an elementary spin-1 particle. Obviously, this is the behavior observed in our phenomenological analyses of elastic scattering. An interesting quantitative comparison between our high energy analyses and the low energy soliton model calculations can be made at this point. The hedgehog model leads to the following expression for the baryonic charge density:

$$\tilde{\rho}(r) = -\frac{1}{2\pi^2}\frac{\sin^2\theta(r)}{r^2}\frac{d\theta(r)}{dr}, \tag{5}$$

where $\theta(r)$ is the pion field configuration introduced earlier. If we identify ω as the gauge boson coupled to the baryonic charge, then we can obtain $\tilde{\rho}(r)$ from the ωNN form factor found in elastic scattering,[7] and use Eq.(5) to derive the corresponding $\theta(r)$. This is shown in Fig.2 and compared with the $\theta(r)$ given by the minimal soliton model (only ω present) and the complete soliton model (equivalently, ω, ρ, A_1- all three present) calculations of Meissner et al.[2] The quantitative behavior from two totally different regimes of physics are not far apart.

Let us return to the problem of nonlinear σ-model predicting too large a soliton mass. This problem now becomes of greater concern to us, since the high energy elastic scattering analyses appear to indicate a topological baryonic core of the nucleon. The problem can be resolved in the following way.[10] In the conventional nonlinear σ-model, one completely ignores the quark sector. The implicit assumption is that quarks are there, but their interactions are not important compared to the interactions in the pseudoscalar meson sector. This implies that the nucleon described as a soliton is surrounded by an ordinary noninteracting Dirac sea of quarks and antiquarks. On the other hand, let us consider the linear σ-model, where there is the scalar field $\zeta(x)$, that mediates an interaction between the left and the right quarks. In this case, the soliton is surrounded by an interacting Dirac sea. If the scalar field $\zeta(x)$ has a critical behavior (zero at small distance, rising sharply to vacuum value f_π at some distance r=R), then the total energy of the interacting Dirac sea plus the

energy of the scalar field can be less than the energy of the noninteracting Dirac sea.

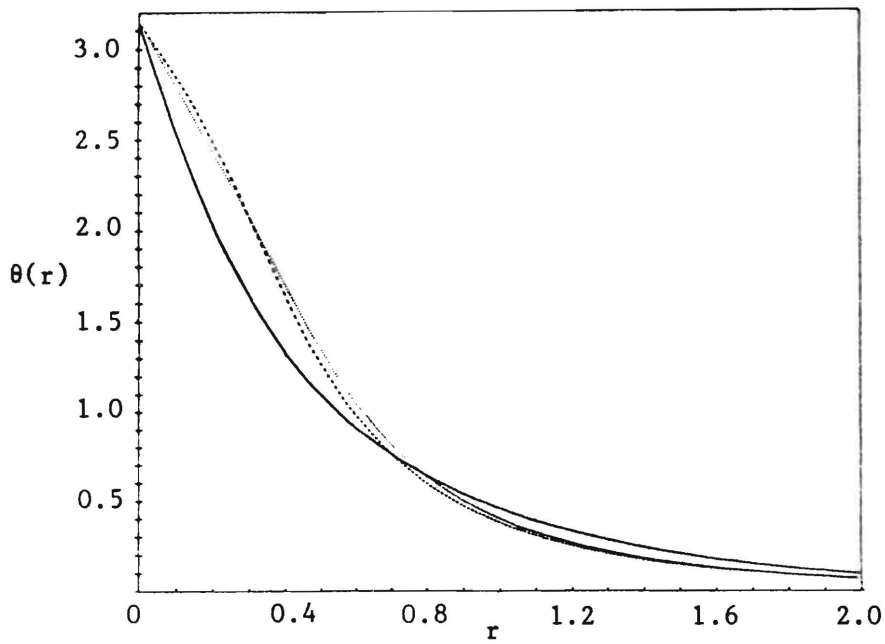

Fig.2. The pion field configuration $\theta(r)$ as a function of r in Fermi. The continuous curve represents $\theta(r)$ obtained from high energy elastic scattering. The dotted and the dashed curves represent $\theta(r)$ calculated from low energy in the minimal and in the complete soliton model by Meissner et al. [Ref.2].

The system can then make a transition to the lower ground state and can reduce its energy or mass by the condensation energy. The new ground state will be a condensed ground state of chiral quarks (i.e. left-handed and right-handed quarks.) analogous to a superconducting state. These developments[10] indicate that the topological soliton can not only have a substantially lower mass by condensation, but will also be embedded in a chiral quark condensate (Fig.3). The physical picture of the nucleon that emerges now is exactly the same as the phenomenological picture obtained by us from the analyses of high energy elastic scattering.[11]

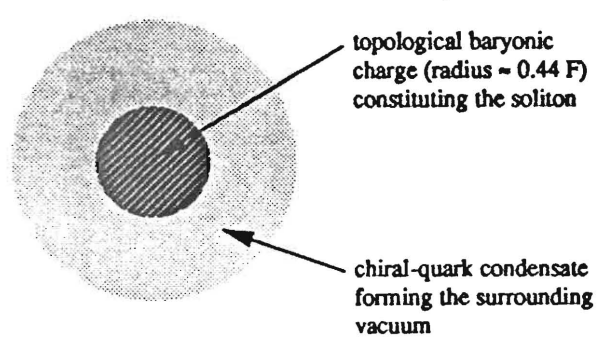

Fig.3. Structure of the nucleon as a topological soliton embedded in a chiral condensate.

One final remark: Many models of the nucleon have been proposed—such as the quark constituent model, the MIT Bag model, the chiral bag model, the topological soliton model, the nontopological soliton model, the color dielectric model.[12–14] Low energy properties of the nucleon and low energy nucleon-nucleon interaction have not been able to single out one model incorporating the key features. High energy elastic scattering, on the other hand, appears to provide strong evidence in favor of the topological soliton model with the soliton embedded in a chiral condensate. If the nonperturbative dynamics underlying our considerations are borne out by future quantitative calculations, then we would conclude that the elastic data from the CERN ISR and SPS Collider have played a vital role in determining the composite structure of the nucleon.

This work is supported in part by the U.S.Department of Energy.

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