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Slater Decomposition of Laughlin States*

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Abstract

The second-quantized form of the Laughlin states for the fractional quantum Hall effect is discussed by decomposing the Laughlin wavefunctions into the N-particle Slater basis. A general formula is given for the expansion coefficients in terms of the characters of the symmetric group, and the expansion coefficients are shown to possess numerous interesting symmetries. For expectation values of the density operator it is possible to identify individual dominant Slater states of the correct uniform bulk density and filling fraction in the physically relevant $N \to \infty$ limit.

1 Introduction

The variational trial wavefunctions introduced by Laughlin [1] form the basis for the theoretical understanding of the quantum Hall effect [2, 3]. The Laughlin wavefunctions describe especially stable strongly correlated states, known as *incompressible quantum fluids*, of a two dimensional electron gas in a strong magnetic field. They also form the foundation of the hierarchy structure of the *fractional* quantum Hall effect [4, 5].

In the extreme low temperature and strong magnetic field limit, the single particle electron states are restricted to the lowest Landau level. In the absence of interactions, this Landau level has a high degeneracy determined by the magnetic flux through the sample [6]. Much (but not all) of the physics of the quantum Hall effect may be understood in terms of

*UCONN-93-4

the restriction of the dynamics to fixed Landau levels [7, 3, 8]. The integer quantum Hall effect may be understood in terms of fully filled Landau levels, but the fractional quantum Hall effect involves fractionally filled Landau levels. Laughlin's (unnormalized) wavefunctions [1]

$$\Psi^m_{\text{Laughlin}}(z_1, \ldots, z_N) = \prod_{i<j}^{N} (z_i - z_j)^{2m+1} \; e^{-\frac{1}{2}\sum_{i=1}^{N} |z_i|^2}, \tag{1}$$

where m is an integer, correspond to states of fractional filling $\frac{1}{2m+1}$ in the lowest Landau level. By exploiting the connection with two dimensional one-component plasmas, it can be shown that these wavefunctions correspond to incompressible quantum fluids of uniform density $\frac{1}{2m+1}\left(\frac{eB}{hc}\right)$ [9]. Furthermore, quasiparticle excitations about these states have the appropriate fractional statistics [10, 3].

A convenient description of the macroscopic properties of quantum Hall samples is given by effective field theory techniques and the associated Chern-Simons formalism [11, 12, 3]. These effective field theories are in turn related to important *edge* effects in quantum Hall samples [13]. The second-quantized effective field theories provide a useful formalism for studying the large N limit. The relationship between the microscopic many-electron theory and these macroscopic effective field theories has been discussed in detail for fully filled states by Stone [14]. It is much more difficult to make precise this connection for the *fractional* quantum Hall effect.

In this paper I discuss the second quantized form of the fractional filling Laughlin wavefunctions (1) in their microscopic Fock space form. In practice, this involves expanding the Laughlin wavefunctions in terms of Slater wavefunctions. Since the Laughlin wavefunction has fixed angular momentum $J_{\text{Laughlin}} = (2m+1)\frac{1}{2}N(N-1)$, it may be expanded as a linear combination of Slater determinant wavefunctions each with this same angular momentum. The goal is to seek some structure in this expansion and use this to examine the $N \to \infty$ limit.

A related question has been addressed very recently by MacDonald and Mitra [15] who compute the angular momentum distribution function of the Laughlin wavefunctions - i.e. they compute the relative occupation numbers of the *single particle* angular momentum states in the one particle density matrix for the Laughlin states. Here, instead, I ask for the relative weights of the *multi-particle* Slater states which comprise the Laughlin state. This is a much more difficult question, as one sees simply by counting the states involved. It is often the case that one can compute matrix elements of second quantized operators in the Slater states with relative ease, so that if one knows the expansion of the Laughlin state in the Slater basis, one can then compute the expectation value in the Laughlin state without resorting to the first quantized technique of large multi-dimensional integrals.

There have been previous investigations of the expansion of the Laughlin wavefunctions in the Slater basis. Tao [16] has studied the projection of the Laughlin states on the ground state in the $N \to \infty$ limit. Datta and Ferrari [17] have discussed the large N limit by considering

an associated Langevin equation. As is perfectly clear, any *exact* analysis soon confronts the problem that it is very difficult to store, expand and manipulate large polyomials and determinants. Here I propose to make use of the equivalence of this expansion problem with a classic combinatorial problem in the theory of symmetric functions to give a formula for the coefficients in the expansion in terms of characters of the symmetric group. The intended advantage of this approach is that one does not need to refer to the wavefunctions at all in the computation. There are still significant combinatorial and computational difficulties, but this point of view brings to light many interesting symmetries of the expansion coefficients, some of which make it possible to address the $N \to \infty$ limit exactly in certain special cases. For example, somewhat surprisingly, for the expectation value of the density operator it is possible to identify a *single* Slater state which dominates the expansion in the $N \to \infty$ limit, and which has uniform bulk density $\frac{1}{2m+1}\left(\frac{eB}{hc}\right)$ and fractional filling $\frac{1}{2m+1}$.

This paper is organized as follows. In Section 2, I briefly review the second quantized formalism of N-particle quantum mechanics in the context of the Laughlin wavefunctions. The connection between the many body Slater wavefunctions and special symmetric polynomials known as "Schur functions" is discussed in Section 3, and this is used in Section 4 to obtain a general formula for the Slater expansion coefficients in terms of the characters of the symmetric group. Section 5 contains the results for some low N examples and in Section 6 I discuss some symmetries of the expansion coefficients for all N. These symmetries are used in Section 7 to analyze the density profiles and pair correlation functions of Laughlin states in the $N \to \infty$ limit. The paper concludes with some discussion and some suggestions for further investigation.

2 Second Quantized Form of Laughlin States

For very large numbers of particles ($N \to \infty$) it is often more convenient to use the second-quantized formulation of nonrelativistic quantum mechanics. In second quantization, one defines a field *operator* $\Phi(z)$ by the expansion

$$\Phi(z) = \sum_{k=0}^{\infty} a_k \phi_k(z) \tag{2}$$

where the $\phi_k(z)$ form a complete orthonormal set of single particle wavefunctions and the a_k are Grassmann annihilation operators satisfying the anticommutation relations

$$\left\{a_k, a_l^\dagger\right\} = \delta_{kl} \tag{3}$$

The Fock vacuum $|0>$ is defined as the state annihilated by all the a_k operators and an N-particle state $|[\lambda]>$ is defined by the action of N creation operators on the vacuum

$$|[\lambda]> \;\equiv\; |[\lambda_1, \lambda_2, \ldots, \lambda_N]>$$

$$\equiv \quad a^\dagger_{\lambda_1}\, a^\dagger_{\lambda_2} \ldots a^\dagger_{\lambda_N}\, |0> \tag{4}$$

The usual N-particle first-quantized wavefunction corresponding to this state is

$$\begin{aligned} \Psi^{[\lambda]}\,(z_1, \ldots, z_N) \quad &\equiv \quad \frac{1}{\sqrt{N!}} < 0|\Phi(z_1) \ldots \Phi(z_N)\, |[\lambda] > \\ &= \quad \frac{1}{\sqrt{N!}} \begin{vmatrix} \phi_{\lambda_1}(z_1) & \phi_{\lambda_2}(z_1) & \ldots & \phi_{\lambda_N}(z_1) \\ \phi_{\lambda_1}(z_2) & \phi_{\lambda_2}(z_2) & \ldots & \phi_{\lambda_N}(z_2) \\ \vdots & \vdots & & \vdots \\ \phi_{\lambda_1}(z_N) & \phi_{\lambda_2}(z_N) & \ldots & \phi_{\lambda_N}(z_N) \end{vmatrix} \end{aligned} \tag{5}$$

This is, of course, just the familiar Slater determinant for the N particle fermionic state in which the single particle states $\lambda_1, \lambda_2, \ldots, \lambda_N$ are occupied. By virtue of (3) and (4), these states are orthonormal

$$< [\lambda] \mid [\mu] >= \delta_{[\lambda]\,,\,[\mu]} \tag{6}$$

The second quantized density operator is

$$\rho(z) = \Phi^\dagger(z)\; \Phi(z) \tag{7}$$

and the (conserved) number operator is

$$N = \int d^2z\; \rho(z) \tag{8}$$

In the Fock state $|[\lambda] >$, the expectation value of the density operator is

$$< [\lambda] \mid \rho(z) \mid [\lambda] >= \sum_{i=1}^{N} |\phi_{\lambda_i}(z)|^2 \tag{9}$$

The expectation value of the pair correlation operator $\rho(z_1)\rho(z_2)$ is considered in Section 7. Notice that, in contrast to the first quantized approach, there are *no integrations* involved in computing these expectation values.

A general state is a linear combination of the Fock states $|[\lambda] >$

$$|state >= \sum_{[\lambda]} c_{\,[\lambda]} |[\lambda] > \tag{10}$$

and if the coefficients $c_{\,[\lambda]}$ are known then it is also straightforward to compute expectation values. For example, for the density operator only the diagonal matrix elements contribute and

$$\begin{aligned} < state|\rho(z)|state > \quad &= \quad \sum_{[\lambda]} |c_{\,[\lambda]}|^2 < [\lambda]|\rho(z)|[\lambda] > \\ &= \quad \sum_{[\lambda]} |c_{\,[\lambda]}|^2 \left(\sum_{i=1}^{N} |\phi_{\lambda_i}(z)|^2 \right) \end{aligned} \tag{11}$$

Note, of course, that the coefficients $c_{[\lambda]}$ uniquely determine the *state* and have nothing to do with the particular operator. Once the expansion coefficients are known for a given state, one can express the expectation value of any operator as a sum over its Slater state matrix elements. The advantage of the Slater basis is that these matrix elements are often easy to compute.

Since the first quantized Laughlin wavefunctions are of such fundamental importance in the study of the quantum Hall effect, it is natural to ask about their second quantized form. It is the purpose of this paper to address this question : namely, to determine the expansion coefficients $c_{[\lambda]}$ for the Laughlin state $| L >$

$$| L >= \sum_{[\lambda]} c_{[\lambda]} |[\lambda] > \tag{12}$$

The corresponding first quantized wavefunction is given by the relation (5), and so the $c_{[\lambda]}$ are determined by the decomposition of the Laughlin wavefunction (1) in terms of the Slater basis wavefunctions.

The relevant single particle wavefunctions are those corresponding to the lowest Landau level (we use the symmetric gauge for the vector potential $A_i = -\frac{B}{2}\epsilon_{ij}x^j$) :

$$\varphi_k(z) = \frac{1}{\sqrt{\pi k!}}\, z^k\, e^{-\frac{1}{2}|z|^2} \tag{13}$$

where $k = 0, 1, 2, \ldots$ is an angular momentum label, and where we have used $\sqrt{2}\ell$ as the unit of length, where $\ell = \sqrt{\frac{\hbar c}{eB}}$ is the magnetic length. With these single particle wavefunctions, the normalized Slater wavefunctions (5) are

$$\Psi^{[\lambda]}_{\text{Slater}}(z_1, z_2, \ldots z_N) = \frac{e^{-\frac{1}{2}\sum_{i=1}^{N}|z_i|^2}}{\sqrt{N!\pi^N \prod_{i=1}^{N} \lambda_i!}} \begin{vmatrix} z_1^{\lambda_1} & z_1^{\lambda_2} & \ldots & z_1^{\lambda_N} \\ z_2^{\lambda_1} & z_2^{\lambda_2} & \ldots & z_2^{\lambda_N} \\ \vdots & \vdots & & \vdots \\ z_N^{\lambda_1} & z_N^{\lambda_2} & \ldots & z_N^{\lambda_N} \end{vmatrix} \tag{14}$$

There are two convenient simplifications of these wavefunctions. First, it proves helpful to label the wavefunction not by the string of integers $[\lambda] = [\lambda_1, \lambda_2, \ldots \lambda_N]$, but by the string $\{\mu\} = \{\mu_1, \mu_2, \ldots \mu_N\}$ where

$$\lambda_i = \mu_i + N - i \tag{15}$$

This simply amounts to labelling relative to the *minimum* angular momentum wavefunction which has $[\lambda] = [N-1, N-2, \ldots, 2, 1, 0]$ and angular momentum $J = \frac{1}{2}N(N-1)$. This notation shall be used throughout the paper : square-bracketed integer strings $[\lambda]$ label the angular momenta of the occupied single particle states, while the curly-bracketed integer strings $\{\mu\}$ label states *relative* to the state $[\lambda] = [N-1, N-2, \ldots, 2, 1, 0]$.

The second simplification arises because it is natural to consider the Gaussian exponential factor $e^{-\frac{1}{2}|z|^2}$ in the wavefunctions (13) as part of the Hilbert space measure, in which case the lowest Landau level wavefunctions form a basis for Bargman's Hilbert space of analytic functions [7, 18]. Thus, one may consider the simplified Slater wavefunctions (denoted by a lower case ψ)

$$\psi^{\{\mu\}}_{\text{Slater}}(z_1, \ldots, z_N) = \begin{vmatrix} z_1^{\mu_1+N-1} & z_1^{\mu_2+N-2} & \ldots & z_1^{\mu_N} \\ z_2^{\mu_1+N-1} & z_2^{\mu_2+N-2} & \ldots & z_2^{\mu_N} \\ \vdots & \vdots & & \vdots \\ z_N^{\mu_1+N-1} & z_N^{\mu_2+N-2} & \ldots & z_N^{\mu_N} \end{vmatrix} \tag{16}$$

The normalization factors in (14) are not relevant for the decomposition problem, and will be reintroduced when computing expectation values - see Section 7. In this notation, the (un-normalized) Laughlin wavefunction is

$$\psi_{\text{Laughlin}}(z_1, \ldots, z_N) = \left(\prod_{i<j}^{N} (z_i - z_j) \right)^{2m+1} \tag{17}$$

where m is an integer. The problem is to expand ψ_{Laughlin} in terms of the determinants $\psi^{\{\mu\}}_{\text{Slater}}$.

Since the Laughlin wavefunction has definite angular momentum

$$J_{\text{Laughlin}} = (2m+1)\frac{1}{2}N(N-1) \tag{18}$$

the expansion of ψ_{Laughlin} in terms of the $\psi^{\{\mu\}}_{\text{Slater}}$ will involve only Slater wavefunctions such that

$$\sum_{i=1}^{N} \mu_i = mN(N-1) \tag{19}$$

Thus, one can think of the label $\{\mu\}$ as a **partition** of $mN(N-1)$. The corresponding label $[\lambda]$, related to $\{\mu\}$ as in (15) is instead a partition of $(2m+1)\frac{1}{2}N(N-1)$.

It is instructive to consider some simple cases. When $N = 2$ and $m = 1$ (corresponding to the $\frac{1}{3}$ filled Laughlin state),

$$\begin{aligned} \psi_{\text{Laughlin}}(z_1, z_2) &= \left(\begin{vmatrix} z_1 & 1 \\ z_2 & 1 \end{vmatrix} \right)^3 \\ &= \begin{vmatrix} z_1^3 & 1 \\ z_2^3 & 1 \end{vmatrix} - 3 \begin{vmatrix} z_1^2 & z_1 \\ z_2^2 & z_2 \end{vmatrix} \\ &= \psi^{\{2\}}_{\text{Slater}}(z_1, z_2) - 3\, \psi^{\{1,1\}}_{\text{Slater}}(z_1, z_2) \end{aligned} \tag{20}$$

When $N = 2$ and $m = 2$ (corresponding to the $\frac{1}{5}$ filled Laughlin state),

$$\begin{aligned}
\psi_{\text{Laughlin}}(z_1, z_2) &= \left(\begin{vmatrix} z_1 & 1 \\ z_2 & 1 \end{vmatrix} \right)^5 \\
&= \begin{vmatrix} z_1^5 & 1 \\ z_2^5 & 1 \end{vmatrix} - 5 \begin{vmatrix} z_1^4 & z_1 \\ z_2^4 & z_2 \end{vmatrix} + 10 \begin{vmatrix} z_1^3 & z_1^2 \\ z_2^3 & z_2^2 \end{vmatrix} \\
&= \psi_{\text{Slater}}^{\{4\}}(z_1, z_2) - 5\, \psi_{\text{Slater}}^{\{3,1\}}(z_1, z_2) + 10\, \psi_{\text{Slater}}^{\{2,2\}}(z_1, z_2) \qquad (21)
\end{aligned}$$

When $N = 3$ and $m = 1$,

$$\begin{aligned}
\psi_{\text{Laughlin}}(z_1, z_2, z_3) &= \left(\begin{vmatrix} z_1^2 & z_1 & 1 \\ z_2^2 & z_2 & 1 \\ z_3^2 & z_3 & 1 \end{vmatrix} \right)^3 \\
&= \begin{vmatrix} z_1^6 & z_1^3 & 1 \\ z_2^6 & z_2^3 & 1 \\ z_3^6 & z_3^3 & 1 \end{vmatrix} - 3 \begin{vmatrix} z_1^6 & z_1^2 & z_1 \\ z_2^6 & z_2^2 & z_2 \\ z_3^6 & z_3^2 & z_3 \end{vmatrix} - 3 \begin{vmatrix} z_1^5 & z_1^4 & 1 \\ z_2^5 & z_2^4 & 1 \\ z_3^5 & z_3^4 & 1 \end{vmatrix} \\
&\quad + 6 \begin{vmatrix} z_1^5 & z_1^3 & z_1 \\ z_2^5 & z_2^3 & z_2 \\ z_3^5 & z_3^3 & z_3 \end{vmatrix} - 15 \begin{vmatrix} z_1^4 & z_1^3 & z_1^2 \\ z_2^4 & z_2^3 & z_2^2 \\ z_3^4 & z_3^3 & z_3^2 \end{vmatrix} \\
&= \psi_{\text{Slater}}^{\{4,2\}}(z_1, z_2, z_3) - 3\, \psi_{\text{Slater}}^{\{4,1,1\}}(z_1, z_2, z_3) - 3\, \psi_{\text{Slater}}^{\{3,3\}}(z_1, z_2, z_3) \\
&\quad + 6 \psi_{\text{Slater}}^{\{3,2,1\}}(z_1, z_2, z_3) - 15\, \psi_{\text{Slater}}^{\{2,2,2\}}(z_1, z_2, z_3) \qquad (22)
\end{aligned}$$

For $N = 2$ it is trivial to decompose ψ_{Laughlin} into Slater determinants, but when $N = 3$ even for the $m = 1$ Laughlin wavefunction it is not completely trivial to determine the decomposition (22). It is clear that to proceed to higher numbers of particles and higher m values a more systematic approach is needed.

In Section 4 a closed form expression is presented for the (integer) coefficients $a_{\{\mu\}}$ appearing in the expansion

$$\psi_{\text{Laughlin}} = \sum_{\{\mu\}} a_{\{\mu\}} \psi_{\text{Slater}}^{\{\mu\}} \qquad (23)$$

This expression involves characters of the symmetric group, which will be briefly reviewed in the next section.

3 Slater States and Schur Functions

It has already been noted that the Slater wavefunctions (16) which appear in the expansion (23) are labelled by partitions $\{\mu\}$ of $mN(N-1)$. Each of these Slater wavefunctions is a totally antisymmetric polynomial of homogeneous degree $(2m+1)\frac{1}{2}N(N-1)$. Each can therefore be written as the *product* of the Vandermonde determinant V and a totally *symmetric* polynomial of homogeneous degree $mN(N-1)$. Here V is given by

$$\begin{aligned} V &= \begin{vmatrix} z_1^{N-1} & z_1^{N-2} & \dots & 1 \\ z_2^{N-1} & z_2^{N-2} & \dots & 1 \\ \vdots & \vdots & & \vdots \\ z_N^{N-1} & z_N^{N-2} & \dots & 1 \end{vmatrix} \\ &= \prod_{i<j}^{N}(z_i - z_j) \end{aligned} \tag{24}$$

The corresponding symmetric polynomial is called a "Schur function" or "S-function" [19, 20].

$$\mathcal{S}^{\{\mu\}}(z_1,\dots,z_N) \equiv \frac{\begin{vmatrix} z_1^{\mu_1+N-1} & z_1^{\mu_2+N-2} & \dots & z_1^{\mu_N} \\ z_2^{\mu_1+N-1} & z_2^{\mu_2+N-2} & \dots & z_2^{\mu_N} \\ \vdots & \vdots & & \vdots \\ z_N^{\mu_1+N-1} & z_N^{\mu_2+N-2} & \dots & z_N^{\mu_N} \end{vmatrix}}{\begin{vmatrix} z_1^{N-1} & z_1^{N-2} & \dots & 1 \\ z_2^{N-1} & z_2^{N-2} & \dots & 1 \\ \vdots & \vdots & & \vdots \\ z_N^{N-1} & z_N^{N-2} & \dots & 1 \end{vmatrix}} \tag{25}$$

These symmetric polynomials play a key role in the theory of the symmetric group. This relationship between the Slater states and the Schur functions has previously been exploited for fully filled QHE states by Stone [14], while the suggestion of characterizing the quantum hall effect wavefunctions in terms of symmetric polynomials was originally made by Laughlin [1].

The unfamiliar reader should pause to note that it is by no means obvious at first sight that the ratio of determinants in (25) produces a polynomial!

For the purposes of this paper, the most important property of the Schur functions $\mathcal{S}^{\{\mu\}}$ is that they provide a linear basis for the space of symmetric polynomials of homogeneous degree $\sum_{i=1}^{N}\mu_i$. The Laughlin wavefunction (17) is just V^{2m+1} and so, dividing the expansion (23) through by the Vandermonde determinant V, one finds

$$\frac{\psi_{\text{Laughlin}}}{V} \equiv V^{2m}$$

$$= \sum_{\{\mu\}} a_{\{\mu\}} \mathcal{S}^{\{\mu\}} \tag{26}$$

Thus, to determine the coefficients $a_{\{\mu\}}$ of the expansion of $\psi_{\text{Laughlin}} \equiv V^{2m+1}$ in terms of the totally antisymmetric Slater wavefunctions $\psi^{\{\mu\}}_{\text{Slater}}$, equivalently one may consider the expansion of V^{2m} in terms of the totally symmetric Schur functions $\mathcal{S}^{\{\mu\}}$. From (23) and (26), the expansion coefficients are in 1-1 correspondence. This reduction of degree of the polynomial under consideration from V^{2m+1} to V^{2m} represents a significant computational simplification.

Another convenient basis for the space of totally symmetric polyomials of homogeneous degree $mN(N-1)$ is the "power sum basis". The power sums s_μ are defined for $\mu = 1 \ldots \infty$ as

$$s_\mu(z_1, \ldots, z_N) \equiv \sum_{i=1}^{N} z_i^\mu \tag{27}$$

Any totally symmetric polynomial may be expressed as a sum of products of the s_μ's, and so a basis for the totally symmetric polynomials of homogeneous degree $mN(N-1)$ is provided by the products

$$s_{\{\mu\}} \equiv s_{\mu_1} s_{\mu_2} \ldots s_{\mu_N} \tag{28}$$

where $\sum_{i=1}^{N} \mu_i = mN(N-1)$.

The linear transformation between the Schur function basis and the power sum basis is given by Frobenius' reciprocity formula [19, 20]:

$$s_{\{\mu\}} = \sum_{\{\lambda\}} \chi^{\{\lambda\}}_{\{\mu\}} \mathcal{S}_{\{\lambda\}} \tag{29}$$

where the $\chi^{\{\lambda\}}_{\{\mu\}}$ are characters of the representation $\{\lambda\}$ of the symmetric group of $\sum_{i=1}^{N} \mu_i$ symbols. For the application in this paper, $\sum_{i=1}^{N} \mu_i = mN(N-1)$, and so the relevant symmetric group is $S_{mN(N-1)}$.

It is important to note that the characters $\chi^{\{\lambda\}}_{\{\mu\}}$ are **integers**. Furthermore, these characters satisfy the following completeness relations [19, 20] which permit the inversion of Frobenius' formula (29). If the partition $\{\mu\}$ of $mN(N-1)$ is labelled with the Frobenius notation $(l_1, l_2, l_3, \ldots)$ where

$$l_1 + 2l_2 + 3l_3 + \ldots = mN(N-1) \tag{30}$$

then the characters $\chi^{\{\lambda\}}_{\{\mu\}} \equiv \chi^{\{\lambda\}}_{(l)}$ satisfy

$$\begin{aligned} \frac{1}{(mN(N-1))!} \sum_{(l)} g_{(l)} \chi^{\{\lambda\}}_{(l)} \chi^{\{\lambda'\}}_{(l)} &= \delta_{\{\lambda\},\{\lambda'\}} \\ \sum_{\{\lambda\}} \chi^{\{\lambda\}}_{(l)} \chi^{\{\lambda\}}_{(l')} &= \frac{(mN(N-1))!}{g_{(l)}} \delta_{(l),(l')} \end{aligned} \tag{31}$$

where

$$g_{(l)} \equiv \frac{(mN(N-1))!}{1^{l_1}2^{l_2}\ldots l_1!l_2!\ldots} \tag{32}$$

4 Formula for the Expansion Coefficients $a_{\{\mu\}}$

From Frobenius' formula (29), it is clear that if one can expand V^{2m} in the power sum basis, it is then straightforward to convert this exansion to the Schur function basis. This is a useful observation because it is relatively easy to express V^{2m} in the power sum basis since the "discriminant" V^2 may be expanded as

$$\begin{aligned} V^2 &= \begin{vmatrix} 1 & 1 & \ldots & 1 \\ z_1 & z_2 & \ldots & z_N \\ \vdots & \vdots & & \vdots \\ z_1^{N-1} & z_2^{N-1} & \ldots & z_N^{N-1} \end{vmatrix} \begin{vmatrix} 1 & z_1 & \ldots & z_1^{N-1} \\ 1 & z_2 & \ldots & z_2^{N-1} \\ \vdots & \vdots & & \vdots \\ 1 & z_N & \ldots & z_N^{N-1} \end{vmatrix} \\ &= \begin{vmatrix} N & s_1 & s_2 & \ldots & s_{N-1} \\ s_1 & s_2 & s_3 & \ldots & s_N \\ s_2 & s_3 & s_4 & \ldots & s_{N+1} \\ \vdots & \vdots & & \vdots & \\ s_{N-1} & s_N & s_{N+1} & \ldots & s_{2(N-1)} \end{vmatrix} \\ &= \sum_{perms.\ p} (-1)^p\, s_{p(1)-1} s_{p(2)} s_{p(3)+1} \cdots s_{p(N)+N-2} \end{aligned} \tag{33}$$

where in the last sum, the permutations p are permutations on N letters, and $s_0 \equiv N$.

Thus, equation (33) provides a simple decomposition of V^2 in the power sum basis

$$V^2 = \sum_{perms.\ p} (-1)^p s_{\{\mu_p\}} \tag{34}$$

where for each permutation p, $\{\mu_p\}$ is the (unordered) partition

$$\{\mu_p\} \equiv \{p(1)-1, p(2), p(3)+1, \ldots, p(N)+N-2\} \tag{35}$$

of $N(N-1)$. V^{2m} may similarly be expanded in terms of the power sum basis, and then in the Schur function basis by using Frobenius' formula (29). For definiteness and simplicity, we shall henceforth concentrate on the $m = 1$ case, which corresponds to the Laughlin wavefunction for fractional filling $\frac{1}{3}$.

In the definition (27) of the power sums, it is assumed that $\mu \geq 1$, so that the partitions in (28) are partitions into *nonzero parts*. Before applying Frobenius' formula to (34) it is necessary to separate the Laplace expansion (33) into permutations for which s_0 is a factor

(i.e those p for which $p(1) = 1$) and those for which s_0 is not a factor. The former correspond to partitions of $N(N-1)$ into exactly $N-1$ nonzero parts, while the latter correspond to partitions of $N(N-1)$ into exactly N nonzero parts. (This is a drastic reduction compared to *all possible* partitions of $N(N-1)$). Then, by Frobenius' formula, one finds the following expression for the expansion coefficients $a_{\{\mu\}}$:

$$a_{\{\mu\}} = N \sum_{p \text{ s.t. } p(1)=1} (-1)^p \chi^{\{\mu\}}_{\{\lambda_p\}} + \sum_{p \text{ s.t. } p(1)\neq 1} (-1)^p \chi^{\{\mu\}}_{\{\lambda_p\}} \tag{36}$$

It is important to stress a computational issue at this point. This formula allows one to compute the coefficient of any Schur function in the expansion of V^2 without ever having to make any reference to the *polynomial* V^2 itself. This is an enormous computational simplification, as the algebraic manipulation of large polynomials rapidly becomes prohibitively difficult. The characters $\chi^{\{\mu\}}_{\{\lambda\}}$ may be computed efficiently using (for example) the combinatorial package "combinat" on Maple V [21].

In the next section the results for $N = 2, 3, 4, 5, 6$ are presented. It is worth noting that although the expression (36) for $a_{\{\mu\}}$ is a (large) sum of integers, the final answers for the $a_{\{\mu\}}$ are particularly simple integers, with many interesting symmetry properties not evident from the expansion (36).

5 Examples

Before proceeding to the statement and discussion of some computer-generated results, it is instructive to consider in detail some low N examples to see how the formula (36) works. For $N = 2$,

$$\begin{aligned} V^2 &= \begin{vmatrix} 2 & s_1 \\ s_1 & s_2 \end{vmatrix} \\ &= 2s_2 - s_1^2 \\ &\equiv 2s_{\{2\}} - s_{\{1,1\}} \end{aligned} \tag{37}$$

The character table for the symmetric group S_2 is :

$\chi^{\{\lambda\}}_{\{\mu\}}$	{ 1,1 }	{ 2 }
{ 1,1 }	1	-1
{ 2 }	1	1

Table 1: character table for symmetric group S_2

Frobenius' formula (29) says

$$
\begin{aligned}
s_{\{2\}} &= \mathcal{S}_{\{2\}} - \mathcal{S}_{\{1,1\}} \\
s_{\{1,1\}} &= \mathcal{S}_{\{1,1\}} + \mathcal{S}_{\{2\}}
\end{aligned}
\tag{38}
$$

which one may easily check since the explicit $N = 2$ power sums and Schur functions are

$$
\begin{aligned}
s_{\{2\}} &= z_1^2 + z_2^2 \\
s_{\{1,1\}} &= (z_1 + z_2)^2 \\
\mathcal{S}_{\{2\}} &\equiv \begin{vmatrix} z_1^3 & 1 \\ z_2^3 & 1 \end{vmatrix} \Big/ \begin{vmatrix} z_1 & 1 \\ z_2 & 1 \end{vmatrix} \\
&= z_1^2 + z_1 z_2 + z_2^2 \\
\mathcal{S}_{\{1,1\}} &\equiv \begin{vmatrix} z_1^2 & z_1 \\ z_2^2 & z_2 \end{vmatrix} \Big/ \begin{vmatrix} z_1 & 1 \\ z_2 & 1 \end{vmatrix} \\
&= z_1 z_2
\end{aligned}
\tag{39}
$$

Inserting these expansions into (37) yields

$$
V^2 = \mathcal{S}_{\{2\}} - 3\mathcal{S}_{\{1,1\}} \tag{40}
$$

which should be compared with the Slater expansion of V^3 in (20).

For $N = 3$,

$$
\begin{aligned}
V^2 &= \begin{vmatrix} 3 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix} \\
&= -s_2^3 + 2s_1 s_2 s_3 - 3s_3^2 - s_1^2 s_4 + 3s_2 s_4 \\
&\equiv -s_{\{2,2,2\}} + 2s_{\{3,2,1\}} - 3s_{\{3,3\}} - s_{\{4,1,1\}} + 3s_{\{4,2\}}
\end{aligned}
\tag{41}
$$

It is clear that this decomposition of V^2 into the power sum basis is labelled by partitions of $N(N-1) = 6$ into 2 or 3 parts. The character table for the symmetric group S_6 is 10x10, but in fact only a 5x5 subblock contributes, corresponding to the 5 partitions appearing in the expansion (41).

Then, from equation (36),

$$
a_{\{\mu\}} = 3\left(\chi^{\{\mu\}}_{\{4,2\}} - \chi^{\{\mu\}}_{\{3,3\}}\right) + 2\chi^{\{\mu\}}_{\{3,2,1\}} - \chi^{\{\mu\}}_{\{2,2,2\}} - \chi^{\{\mu\}}_{\{4,1,1\}} \tag{42}
$$

Then, using the character table for S_6 in Table 2 one finds the expansion

$$
V^2 = \mathcal{S}_{\{4,2\}} - 3\mathcal{S}_{\{4,1,1\}} - 3\mathcal{S}_{\{3,3\}} + 6\mathcal{S}_{\{3,2,1\}} - 15\mathcal{S}_{\{2,2,2\}} \tag{43}
$$

$\chi^{\{\lambda\}}_{\{\mu\}}$	{ 2,2,2 }	{ 3,2,1 }	{3,3}	{4,1,1}	{4,2}
{2,2,2 }	3	-1	2	1	-1
{3,2,1 }	1	1	-2	0	0
{3,3}	-3	1	2	-1	-1
{4,1,1}	-2	-1	1	0	0
{4,2}	3	0	0	-1	1

Table 2: partial character table for symmetric group S_6

which once again should be compared with the explicit expansion (22) of V^3 into Slater determinants.

For $N = 4$ one needs the characters of S_{12} [22]. Then Equation (36) leads to an expansion of V^2 in terms of 16 different Schur functions:

$$\begin{aligned} V^2 \;=\; & \mathcal{S}_{\{6,4,2\}} - 3\mathcal{S}_{\{5,5,2\}} - 3\mathcal{S}_{\{6,3,3\}} + 6\mathcal{S}_{\{5,4,3\}} \\ & -15\mathcal{S}_{\{4,4,3\}} - 3\mathcal{S}_{\{6,4,1,1\}} + 9\mathcal{S}_{\{5,5,1,1\}} + 6\mathcal{S}_{\{6,3,2,1\}} \\ & -12\mathcal{S}_{\{5,4,2,1\}} - 9\mathcal{S}_{\{5,3,3,1\}} + 27\mathcal{S}_{\{4,4,3,1\}} - 15\mathcal{S}_{\{6,2,2,2\}} \\ & +27\mathcal{S}_{\{5,3,2,2\}} - 6\mathcal{S}_{\{4,4,2,2\}} - 45\mathcal{S}_{\{4,3,3,2\}} + 105\mathcal{S}_{\{3,3,3,3\}} \end{aligned} \tag{44}$$

For $N = 5$ there are 59 Schur functions in V^2 and for $N = 6$ there are 247. The results for the $N = 5$ expansion are presented in Table 3. The $N = 6$ expansion is tabulated in the Appendix. These expansions have been checked by explicit expansion of the polynomial V^3 in terms of Slater determinants, using Mathematica [23], although that technique deals directly with the polynomials themselves and so is slower. Notice that the coefficients are 'simple' integers, and there are clear symmetries and recursive patterns - these will be discussed in more detail in Section 6.

6 Symmetry Properties of Expansion Coefficients

The most striking symmetry property of the expansion coefficients is the following *exact* symmetry. For a given partition $\{\mu\}$, define the "reversed" partition $\{\tilde{\mu}\}$ by

$$\{\tilde{\mu}\} = \{2(N-1) - \mu_N, 2(N-1) - \mu_{N-1}, \ldots, 2(N-1) - \mu_1\} \tag{45}$$

Then the expansion coefficients of $\{\mu\}$ and $\{\tilde{\mu}\}$ are equal:

$$a_{\{\mu\}} = a_{\{\tilde{\mu}\}} \tag{46}$$

This result has direct physical significance because the integers μ_i correspond to the angular momenta $\lambda_i = \mu_i + N - i$ of single particle states and these states are strongly localized at radius $\sqrt{\lambda_i}$. In terms of the single particle state labels $[\lambda]$ in (15), the reversal operation is

$$[\tilde{\lambda}] = [3(N-1) - \lambda_N, 3(N-1) - \lambda_{N-1}, \ldots, 3(N-1) - \lambda_1] \tag{47}$$

Thus, the relation (46) relates the expansion coefficients of states peaked at one point in the droplet with those for states peaked at other points. In particular, this may be used to relate states localized near the edge of the droplet with those localized near the center.

While this symmetry is clear from the examples presented in the previous section, it is not at all clear from the expansion coefficient formula (36). There is, however, an easy proof[1] of (46). $V^2(z_1, \ldots, z_N)$ is a homogeneous polynomial of degree $N(N-1)$ and it satisfies

$$\begin{aligned} V^2(z_1, \ldots, z_N) &= \left(\prod_{i=1}^{N} z_i^{2(N-1)}\right) V^2\left(\frac{1}{z_1}, \ldots, \frac{1}{z_N}\right) \\ &= \left(\prod_{i=1}^{N} z_i^{2(N-1)}\right) \sum_{\{\mu\}} a_{\{\mu\}} \mathcal{S}_{\{\mu\}}\left(\frac{1}{z_1}, \ldots, \frac{1}{z_N}\right) \end{aligned} \tag{48}$$

But, as can be seen from the definition (25),

$$\left(\prod_{i=1}^{N} z_i^{2(N-1)}\right) \mathcal{S}_{\{\mu\}}\left(\frac{1}{z_1}, \ldots, \frac{1}{z_N}\right) = \mathcal{S}_{\{\tilde{\mu}\}}(z_1, \ldots, z_N) \tag{49}$$

Hence,

$$\begin{aligned} V^2(z_1, \ldots, z_N) &= \sum_{\{\mu\}} a_{\{\mu\}} \mathcal{S}_{\{\tilde{\mu}\}}(z_1, \ldots, z_N) \\ &= \sum_{\{\mu\}} a_{\{\tilde{\mu}\}} \mathcal{S}_{\{\mu\}}(z_1, \ldots, z_N) \end{aligned} \tag{50}$$

In addition to this exact symmetry of the Slater expansion coefficients, it is possible to deduce simple combinatorial formulas (for *any* N) for the coefficients of certain special Slater states. These are highly nontrivial results, and somewhat fortuitously correspond to physically important Slater states in the $N \to \infty$ limit (see Section (7)).

The simplest such example is the state

$$\{\mu\} = \{2(N-1), 2(N-2), \ldots, 4, 2\} \tag{51}$$

for which

$$a_{\{\mu\}} = 1. \tag{52}$$

[1]Thanks to D. Jackson for suggesting this approach

This corresponds to the Slater state

$$|[\lambda]>=|[3(N-1),3(N-2),\ldots,6,3,0]> \tag{53}$$

in which every third single particle angular momentum state is filled. It is also the most uniformly distributed (in the bulk) of the Slater states (see Section 7). The other extreme is the Slater state in which the angular momentum levels are most closely bunched

$$\{\mu\}=\{N-1,N-1,\ldots,N-1\} \tag{54}$$

for which

$$a_{\{\mu\}}=(-1)^{[N/2]}\,(2N-1)!! \tag{55}$$

where $[N/2]$ means the integer part of $N/2$. Furthermore, **all** the expansion coefficients $a_{\{\mu\}}$ lie between these extremes (in magnitude)

$$1\leq|a_{\{\mu\}}|\leq(2N-1)!! \qquad \text{, for all } \{\mu\}. \tag{56}$$

Each of these states (51) and (54) is invariant under the "reversal" operation defined above (45). However, another important state (which is not reversal invariant) is that for which one electron is in the 0 angular momentum state and the remaining $N-1$ electrons are bunched together

$$\{\mu\}=\{\overbrace{N,N,\ldots,N}^{N-1}\} \tag{57}$$

for which

$$a_{\{\mu\}}=(-1)^{[N/2]}\,(2N-3)!! \tag{58}$$

The reversed partition is

$$\{\tilde{\mu}\}=\{2(N-1),N-2,\ldots,N-2\}, \tag{59}$$

for which, by (46), the coefficient is also given by (58).

Other closed formulas exist for the reversal-invariant states that begin with the "maximally bunched" state (54) and successively move the extreme inner and outer electrons in and out (respectively) by one step :

$$\begin{aligned}
a_{\{2N,2N-2,2N-3,\ldots,N+1,N,N-2\}} &= (-1)^{[N/2]+1}\,(N-1)(2N-3)!! \\
a_{\{2N+1,2N-2,2N-3,\ldots,N+1,N,N-3\}} &= (-1)^{[N/2]+1}\,N(N-1)(2N-5)!! \\
&\vdots \\
a_{\{3(N-1),2N-2,2N-3,\ldots,N+1,N,0\}} &= (-1)^{[N/2]+1}\,(2N-5)!!
\end{aligned} \tag{60}$$

These all correspond to states highly localized in the region $\sqrt{N}\leq|z|\leq\sqrt{2(N-1)}$.

Similarly, one may begin with the maximally distributed state (51) and make local shifts of electrons between angular momentum levels. The simplest such shifts just involve one pair of electrons, in which one electron is raised by one angular momentum step and another is lowered (remember that the *total* angular momentum must not change). If these two electrons were initially separated by 3 units of angular momentum, then after such a shift they will be separated by just 1 unit of angular momentum. Consulting the results for the coefficients $a_{\{\mu\}}$ we note the remarkable fact that such an operation always changes the expansion coefficient $a_{\{\mu\}}$ by a factor of -3. If the two electrons were initially separated by 6 units (with another electron midway in between) then the change in the coefficient $a_{\{\mu\}}$ is $+6$. In fact, in an outer sub-block of M uniformly spaced electrons (spaced by 3 units of angular momentum, as in the state (51)), if the outermost and innermost electrons are lowered and raised (respectively) by one unit each, then the only change in the expansion coefficient is multiplication by a factor

$$(-1)^{M-1} 3 \,.\, 2^{M-2} \tag{61}$$

For example, the state with

$$\{\mu\} = \{2N-3, 2N-4, 2N-6, \ldots, 4, 2, 1\} \tag{62}$$

has angular momentum labels

$$[\lambda] = [3N-4, 3N-6, 3N-9, \ldots, 6, 3, 1] \tag{63}$$

and differs from the maximally distributed state by the bringing together (by one unit each of angular momentum) of the outermost and innermost electrons, and its expansion coefficient is

$$a_{\{\mu\}} = (-1)^{N-1} 3 \,.\, 2^{N-2} \tag{64}$$

There are also clear *recursive* properties of the expansion coefficients. For example, the N particle states in which one particle is in the "innermost" 0 level are in one-to-one correspondence with all the $(N-1)$ particle states. Further, by the reversal symmetry property (46), this also applies to the N particle states for which one particle is in the "outermost" $3(N-1)$ level. This property is a simple consequence of the fact that one may expand the N particle Vandermonde determinant in powers of any given z_i, say z_1,as

$$\begin{aligned} V_N(z_1, z_2, \ldots, z_N) &= \prod_{1 \le i < j}^{N} (z_i - z_j) \\ &= V_{N-1}(z_2, \ldots, z_N) \prod_{j=2}^{N} (z_1 - z_j) \\ &= V_{N-1}(z_2, \ldots, z_N) \\ &\quad (z_1^{N-1} - e_1 {z_1}^{N-2} + e_2 {z_1}^{N-3} - \ldots + (-1)^{N-1} e_{N-1}) \end{aligned} \tag{65}$$

where $e_\mu = e_\mu(z_2, \ldots, z_N)$ is the μth elementary symmetric polynomial in the $N-1$ variables $z_2, \ldots, z_N$. (e_μ is defined to be the sum of all products of μ of the variables; e.g. $e_1 = z_2 + \ldots + z_N$, and $e_{N-1} = z_2 z_3 \ldots z_N$). From the expansion (66) it is clear that the only way to obtain the term $z_1^{3(N-1)}$ (this corresponds to putting particle 1 in the outermost level) in V_N^3 is by accompanying it by $(V_{N-1}(z_2, \ldots, z_N))^3$, and so one is left with the decomposition problem for $N-1$ particles.

Similar reasoning leads to further, deeper, recursive patterns. Define $\#(N)$ to be the number of Schur functions appearing in the expansion of V_N^2 (equivalently, the number of Slater determinants appearing in the expansion of V_N^3). Then, with a consistent ordering of the coefficients (see tables) one sees that the first $\#(N-1)$ coefficients for N particles coincide with all the coefficients for $N-1$ particles. Further, the next $\#(N-2)$ coefficients of the N particle problem are given by -3 times the $\#(N-2)$ coefficients of the $N-2$ particle problem; the next $\#(N-3)$ coefficients of the N particle problem are given by 6 times the $\#(N-3)$ coefficients of the $N-3$ particle problem; the next $\#(N-4)$ coefficients of the N particle problem are given by -12 times the $\#(N-4)$ coefficients of the $N-4$ particle problem.etc.... For example, for the $N=5$ case, the first $16+5+2=23$ coefficients are determined by this recursive pattern from the $N=2,3,4$ results. When combined with the other symmetry properties mentioned thus far, a total of 35 of the 59 coefficients are determined from prior knowledge.

Of course, ultimately one would like a simple combinatorial formula (of the form of (60) for example) for *all* the coefficients in the expansion for *any* N. No such formula is currently known, although the results presented here suggest that such a simple formula may exist. Indeed, the reversal symmetry and recursive properties of the coefficients, together with their simple nature suggest that such a formula would be a product of ratios of factorials. However, any such formula would necessarily involve in general all the parameters $\{\mu\}$ which specify the partition in question - the simple closed formulas given above are for especially symmetric classes of partitions.

In the absence of such a general formula for the coefficients $a_{\{\mu\}}$ for arbitrary N, one may still discuss the question of whether the Slater decomposition (23) is in some sense "dominated" by certain special states. This is not yet a well posed question, because the notion of which states dominate depends on which expectation value is being considered, and so this is no longer a property of the Laughlin state itself. However, it is still possible to identify important representative Slater states from the decomposition (23) and estimate their relative weights exactly as $N \to \infty$. Remember, of course, that in order to address such questions, it is necessary to reintroduce the correct normalization factors (which were previously dropped for convenience) so that the expansion of the Laughlin state is in terms of *normalized* Slater states, rather than just in terms of the bare Slater determinants themselves. These issues will be discussed further in the next Section.

7 Density Profiles

In this Section, the expansion coefficients for $N = 2, 3, 4, 5, 6$ are used to plot the expectation value of the particle density and the pair correlation function in the $\frac{1}{3}$ filled Laughlin state. Allowing for the change in the labelling convention from $[\lambda] \to \{\mu\}$ as given by (15), the expectation value of the density operator is

$$< L|\rho(z)|L >= \frac{1}{\mathcal{N}} \sum_{\{\mu\}} |c_{\{\mu\}}|^2 \sum_{i=1}^{N} |\phi_{\mu_i+N-i}(z)|^2 \tag{66}$$

where the normalized single particle states are given by (13), $\frac{1}{\mathcal{N}}$ is an overall normalization factor, and the coefficients $c_{\{\mu\}}$ are related to the coefficients $a_{\{\mu\}}$ of (23) by the reintroduction of the Slater normalization factors in (14)

$$c_{\{\mu\}} = a_{\{\mu\}} \sqrt{\prod_{i=1}^{N} (\mu_i + N - i)!} \tag{67}$$

The common factors of $\sqrt{N!\pi^N}$ have been absorbed into the overall normalization which is fixed by demanding that the integral of the expectation value of the density operator is just the total particle number

$$\int d^2z < L|\rho(z)|L >= N \tag{68}$$

This is achieved by taking

$$\mathcal{N} = \frac{1}{\sqrt{N!\pi^N}} \sum_{\{\mu\}} |a_{\{\mu\}}|^2 \left(\prod_{i=1}^{N} (\mu_i + N - i)! \right) \tag{69}$$

Since the size of the $\frac{1}{3}$ filled droplet increases with N, in order to compare the density profile for different N it is appropriate to rescale the length by a factor $\sqrt{3N}$ (for the general Laughlin state of filling fraction $\frac{1}{2m+1}$, this rescaling factor would be $\sqrt{(2m+1)N}$)

$$r \equiv \frac{1}{\sqrt{3N}} |z| \tag{70}$$

Then one can plot

$$\hat{\rho}(r) \equiv \pi < L|\rho(\sqrt{3N}r)|L > \tag{71}$$

This rescaling is chosen so that the **edge** of the droplet is at $r = 1$ **for all N**. The normalization is chosen such that for a *perfectly* uniform $\frac{1}{3}$ filled state $\hat{\rho}(r)$ would be a step function

$$\hat{\rho}_{\text{uniform}}(r) = \begin{cases} \frac{1}{3} & 0 \leq r < 1 \\ 0 & r > 1 \end{cases} \tag{72}$$

In Figure 1 is plotted the exact expectation value of the density in the $\frac{1}{3}$ filled Laughlin state for $N = 2, 3, 4, 5, 6$ particles. These plots show a clear tendency towards the form (72). For these small values of N there is also evidence of the characteristic "boundary hump" studied by Datta and Ferrari [17] (see also Figure 3).

The density profiles (71) are *weighted* sums of the density profiles of the individual Slater states which make up the Laughlin state.

$$\hat{\rho}(r) = \left(\frac{\pi}{\sum_{\{\mu\}} |c_{\{\mu\}}|^2}\right) \sum_{\{\mu\}} |c_{\{\mu\}}|^2 \hat{\rho}_{\{\mu\}}(r) \tag{73}$$

where the density profile of the individual Slater state $\{\mu\}$ is

$$\hat{\rho}_{\{\mu\}}(r) \equiv e^{-3Nr^2} \sum_{i=1}^{N} \frac{(3Nr^2)^{\mu_i+N-i}}{(\mu_i + N - i)!} \tag{74}$$

The *relative* weighting factors are just given by the squares of the expansion coefficients $c_{\{\mu\}}$, with the overall normalization fixed by (68). Note that each density function $\hat{\rho}_{\{\mu\}}(r)$ is bounded $0 \leq \hat{\rho}_{\{\mu\}}(r) \leq 1$ for all r. This means that, given information about the expansion coefficients, one can ask which (if any) Slater states dominate this weighted sum in the $N \to \infty$ limit.

Consider, for example, the Slater density for the maximally distributed state, $[\lambda] = |[3(N-1), 3(N-2), \ldots, 3, 0] >$, for which the $\{\mu\}$ partition is $\{\mu\} = \{2(N-1), 2(N-2), \ldots, 2, 0\}$ and for which the expansion coefficient is $a_{\{\mu\}} = 1$. Then the weighted expansion coefficient in the density sum is

$$|c_{\{2(N-1),2(N-2),\ldots,2,0\}}|^2 = \prod_{k=0}^{N-1} (3k)! \tag{75}$$

The density profile of this individual Slater state is given by

$$\hat{\rho}_{\{2(N-1),\ldots,2,0\}}(r) = e^{-3Nr^2} \sum_{k=0}^{N-1} \frac{(3Nr^2)^{3k}}{(3k)!} \tag{76}$$

This is plotted in Figure 2 for $N = 10, 100, 100$, and one sees that in the bulk of the droplet, this Slater state (by itself!) represents a uniform density of fractional filling $\frac{1}{3}$ with remarkable accuracy even for such low values of N. (The analogous result is true for all fractional fillings $\frac{1}{2m+1}$). The failure at $r = 0$ and $r = 1$ is analogous to the breakdown of the WKB approximation near classical turning points (see [24, 25] for a discussion of the importance of the large N and semiclassical limits at the edge of quantum Hall samples). In applications to the quantum Hall effect one is actually more interested in *annular* samples [9, 26] (rather than disc samples) and so $r = 0$ is replaced by an edge at some small nonzero radius.

This is, however, just one term of many contributing to the Laughlin density expectation value. One should compare it with the other terms in the expansion. The states with the largest $a_{\{\mu\}}$ expansion coefficients are the maximally bunched states (54) with $\{\mu\} = \{N-1, N-1, \ldots, N-1\}$, for which $|a_{\{\mu\}}| = (2N-1)!!$. These states are highly localized near central values of the radius. These states have Slater occupation numbers given by (54) and (15)

$$|[\lambda]> = |[2N-2, 2N-3, \ldots, N, N-1]> \tag{77}$$

and, relative to (75) their weighting in the expansion (73) is

$$\frac{|c_{\{\mu\}}|^2}{\prod_{k=0}^{N-1}(3k)!} = \frac{((2N-1)!!)^2\,((2N-2)!(2N-3)!\ldots N!(N-1)!)}{\prod_{k=0}^{N-1}(3k)!} \tag{78}$$

For small N this increases as the numerator dominates. For $N = 12$ the ratio is $\sim 2.10^6$, but after $N = 12$ the denominator begins to dominate and the ratio decreases rapidly. For $N = 20$ the ratio is ~ 0.2, and by the time $N = 50$ it has dropped dramatically to $\sim 10^{-134}$. The relative magnitudes of the $c_{\{\mu\}}$ involves a *competition* between the $a_{\{\mu\}}$ factors and the factorial normalization factors $\sqrt{\prod_{i=1}^{N}(\mu_i + N - i)!}$. From the results in the previous section, the $a_{\{\mu\}}$ factors tend to increase as the electrons become more closely bunched together. But the factorial factors tend to increase with the angular momentum levels being as high as possible. One might therefore expect that the states of the form (57) which have $N-1$ electrons bunched together at high angular momentum and just one electron in the $k = 0$ level to be a dominant state in the density expansion. From (57) and (58) its relative wieght is

$$\frac{|c_{\{\mu\}}|^2}{\prod_{k=0}^{N-1}(3k)!} = \frac{((2N-3)!!)^2\,((2N-1)!(2N-2)!\ldots(N+2)!(N+1)!)}{\prod_{k=0}^{N-1}(3k)!} \tag{79}$$

This ratio also increases with small N , but peaks at $N = 15$ at a value of $\sim 2.10^9$, and then decreases rapidly. For $N = 20$ it is still $\sim 10^6$, but for $N = 50$ it is $\sim 10^{-109}$. For low N these are the *most dominant* states in the density expansion, and from Figures 1 and 3, in which the density profiles are plotted for $N = 2, 3, 4, 5, 6$, one sees their contribution to the characteristic "boundary hump" discussed previously in [17].

In fact, the factorial factors in (67) eventually dominate in the $N \to \infty$ limit, since there is an increasing number of *factorials* of numbers which are themselves increasing. This grows much more quickly than the $a_{\{\mu\}}$ factors, which are bounded in magnitude by $(2N-1)!!$. This tends to favor the evenly distributed states. Indeed, the dominant states are those which are "close to" the maximally distributed state (51). By "close to" is meant states which are related to (51) by a shift of just two electrons. Shifts of more than two electrons tend to be suppressed by the factorial factors in the $N \to \infty$ limit. Of these states, the ones with the largest weighting coefficients $c_{\{\mu\}}$ are those states (62) in which the innermost electron is raised from the 0 level to the $k = 0$ level and the outermost electron is lowered

from the $k = 3(N-1)$ level to the $k = 3N-4$ level. For these states,

$$\begin{aligned}\frac{|c_{\{\mu\}}|^2}{\prod_{k=0}^{N-1}(3k)!} &= 9\,.\,2^{2(N-2)}\frac{(3N-4)!(3N-6)!\ldots 3!1!}{(3N-3)!(3N-6)!\ldots 3!0!} \\ &= 3\frac{2^{2(N-2)}}{N-1} \end{aligned} \tag{80}$$

This ratio tends to infinity as $N \to \infty$. The density profile of this dominant class of states is given, for all N, by

$$\hat{\rho}_{\text{dom}}(r) = e^{-3Nr^2}\left(3Nr^2 + \sum_{k=1}^{N-2}\frac{(3Nr^2)^{3k}}{(3k)!} + \frac{(3Nr^2)^{3N-4}}{(3N-4)!}\right) \tag{81}$$

These density profiles are plotted in Figure 4 for $N = 10, 100, 1000$. These plots show that in the bulk of the droplet the uniform density of the $\frac{1}{3}$ filled Laughlin state is well approximated by the density profile of this individual Slater state. Also, apart from the behavior near $r = 0$ these plots are indistinguishable from those of the maximally distributed state (51) in Figure 2.

The results for the expansion coefficients found for $N = 2, 3, 4, 5, 6$ may also be used to plot the pair correlation function $g(z_1, z_2)$. Conventionally [27, 9, 28], one considers the radial pair correlation

$$\begin{aligned} g(|z|) &\equiv g(|z|, 0) \\ &= \frac{< L|\rho(|z|)\rho(0)|L >}{< L|\rho(|z|)|L >< L|\rho(0)|L >} \end{aligned} \tag{82}$$

near the center of the disc-like sample, to minimize edge effects. Since the density operator (7) at the center has the simple form $\rho(0) = a_0^\dagger a_0$, it is easy to see that $g(|z|)$ is given in terms of the expansion coefficients $c_{\{\mu\}}$ and the single particle states (13) by

$$g(|z|) = \frac{\sum_{\{\mu\}\ s.t.\ \mu_N \neq 0}|c_{\{\mu\}}|^2 \sum_{i=1}^{N-2}|\phi_{\mu_i+N-i}(|z|)|^2}{\left(\sum_{\{\mu\}}|c_{\{\mu\}}|^2|\phi_{\mu_i+N-i}(|z|)|^2\right)\left(\sum_{\{\mu\}\ s.t.\ \mu_N\neq 0}|c_{\{\mu\}}|^2\right)} \tag{83}$$

In order to compare with previous computations using the first quantized approach [27, 28, 29], this has been plotted in Figure 5 for the $N = 6$ case in terms of the variable $x = \frac{1}{\sqrt{2}}|z|$ (i.e. in units of the magnetic length, rather than the scaling (70) used previously for the density plots). The exact result from Equation (83) and the coefficients (67) computed here for the $N = 6$ $\frac{1}{3}$ filled Laughlin state agrees (as it must) with the exact result of MacDonald and Murray [29], which as they point out is very close to the $N \to \infty$ Monte Carlo result [30]. In Figure 5, I have also plotted the pair correlation function computed from (83) in the

$N \to \infty$ limit by just retaining a *single* dominant Slater state in each of the sums appearing in the expression (83). This leads to

$$g_{\text{approx}}(x) = \frac{(x^2/2)^4 + \sum_{j=2}^{N-2} \frac{(x^2/2)^{3j}}{(3j)!} + \frac{(x^2/2)^{3N-4}}{(3N-4)!}}{x^2/2 + \sum_{j=1}^{N-2} \frac{(x^2/2)^{3j}}{(3j)!} + \frac{(x^2/2)^{3N-4}}{(3N-4)!}} \tag{84}$$

This function (84) at $N = 6$ is also virtually indistinguishable from its $N \to \infty$ limit. From the Figure 5 one sees that this truncated pair correlation function has the correct tendency, but slightly overestimates $g(x)$ at distances less than ~ 3 magnetic lengths. This may be understood since the use of expression (82) for the pair correlation function assumes uniformity near the origin $x = 0$, which is not true for the individual $N \to \infty$ 'dominant' Slater states (see Figures 2,4).

8 Conclusions

To conclude, the re-interpretation of the expansion of the Laughlin wavefunctions in Slater wavefunctions as the expansion of V^{2m} in terms of Schur functions leads to an expression for the coefficients of this expansion in terms of the characters of the symmetric group $S_{mN(N-1)}$. This may be used to peform the Slater decomposition of the Laughlin states without explicitly expanding the polynomial V^{2m+1}. Some low dimensional examples are presented here (as is seen from the Appendix, even the *tabulation* of results for higher N becomes difficult) and from these it has been possible to glean some important symmetry properties of the expansion coefficients. In many cases this means a simple combinatorial formula for the expansion coefficient. While a simple product formula has not been found for all states for all N, it is still possible to show that for the expectation value of the density operator there is a single dominant Slater state which has the correct uniform bulk density and filling fraction. It is natural and consistent to identify this with the leading term in the large N expansion of the density, which is intricately linked with the semiclassical limit of the two dimensional electron gas [25].

However, the question of the 'dominance' of the Slater expansion of the Laughlin state is specific to the operator in question. Thus, the dominant state for the expectation value of the density operator is in no way necessarily dominant when considering the expectation values of other operators, such as the pair correlation operator or the energy operator. Indeed, the state (62) resembles the superlattice state, considered originally by Tao and Thouless [31], which was unable to explain all features of the fractional quantum Hall effect. The question of dominance for the density is simplified by the fact that the individual density profiles $\hat{\rho}$ are bounded between 0 and 1. It would be important to understand this issue for other operators, especially the energy operator.

It would also be interesting to explore the relationship with the approach of MacDonald and Mitra [15] which concentrates on the one particle density matrix. They compute the angular distribution function $< n_m >$ which is related to the expansion coefficients $c_{[\lambda]}$ discussed here by $< n_m >= \sum_{[\lambda]_m} (c_{[\lambda]})^2$, where the sum $\sum_{[\lambda]_m}$ is the sum over all $[\lambda]$ such that the angular momentum index $m \in [\lambda]$.

Acknowledgements

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Note Added

After completion of this work I became aware of a package called "SF" written for Maple V by J. Stembridge [32]. which includes procedures for manipulating and analyzing symmetric functions. One of its procedures converts a symmetric function expressed in the power sum basis into an expansion in the Schur function basis. For low N, using the analysis of Section 4, this gives a very efficient way to compute the Slater decomposition of the Laughlin states. It would be interesting to test the efficiency of this package for larger N.

$\{\mu\}$	$a_{\{\mu\}}$	$\{\mu\}$	$a_{\{\mu\}}$	$\{\mu\}$	$a_{\{\mu\}}$
{ 8, 6, 4, 2, 0}	1	{ 6, 6, 6, 1, 1}	45	{ 8, 4, 4, 2, 2}	-6
{ 7, 7, 4, 2, 0}	-3	{ 8, 6, 3, 2, 1}	6	{ 7, 5, 4, 2, 2}	-27
{ 8, 5, 5, 2, 0}	-3	{ 7, 7, 3, 2, 1}	-18	{ 6, 6, 4, 2, 2}	111
{ 7, 6, 5, 2, 0}	6	{ 8, 5, 4, 2, 1}	-12	{ 6, 5, 5, 2, 2}	-18
{ 6, 6, 6, 2, 0}	-15	{ 7, 6, 4, 2, 1}	24	{ 8, 4, 3, 3, 2}	-45
{ 8, 6, 3, 3, 0}	-3	{ 7, 5, 5, 2, 1}	18	{ 7, 5, 3, 3, 2}	81
{ 7, 7, 3, 3, 0}	9	{ 6, 6, 5, 2, 1}	-54	{ 6, 6, 3, 3, 2}	-18
{ 8, 5, 4, 3, 0}	6	{ 8, 5, 3, 3, 1}	-9	{ 7, 4, 4, 3, 2}	72
{ 7, 6, 4, 3, 0}	-12	{ 7, 6, 3, 3, 1}	18	{ 6, 5, 4, 3, 2}	-144
{ 7, 5, 5, 3, 0}	-9	{ 8, 4, 4, 3, 1}	27	{ 5, 5, 5, 3, 2}	45
{ 6, 6, 5, 3, 0}	27	{ 7, 5, 4, 3, 1}	-36	{ 6, 4, 4, 4, 2}	-90
{ 8, 4, 4, 4, 0}	-15	{ 6, 6, 4, 3, 1}	-27	{ 5, 5, 4, 4, 2}	270
{ 7, 5, 4, 4, 0}	27	{ 6, 5, 5, 3, 1}	81	{ 8, 3, 3, 3, 3}	105
{ 6, 6, 4, 4, 0}	-6	{ 7, 4, 4, 4, 1}	-36	{ 7, 4, 3, 3, 3}	-180
{ 6, 5, 5, 4, 0}	-45	{ 6, 5, 4, 4, 1}	72	{ 6, 5, 3, 3, 3}	45
{ 5, 5, 5, 5, 0}	105	{ 5, 5, 5, 4, 1}	-180	{ 6, 4, 4, 3, 3}	270
{ 8, 6, 4, 1, 1}	-3	{ 8, 6, 2, 2, 2}	-15	{ 5, 5, 4, 3, 3}	-75
{ 7, 7, 4, 1, 1}	9	{ 7, 7, 2, 2, 2}	45	{ 5, 4, 4, 4, 3}	-420
{ 8, 5, 5, 1, 1}	9	{ 8, 5, 3, 2, 2}	27	{ 4, 4, 4, 4, 4}	945
{ 7, 6, 5, 1, 1}	-18	{ 7, 6, 3, 2, 2}	-54		

Table 3: Schur function decomposition for N=5

Appendix : N=6 Schur function decomposition

$\{\mu\}$	$a_{\{\mu\}}$	$\{\mu\}$	$a_{\{\mu\}}$	$\{\mu\}$	$a_{\{\mu\}}$
{10, 8, 6, 4, 2, 0}	1	{9, 8, 5, 5, 3, 0}	18	{7, 7, 6, 5, 5, 0}	-75
{9, 9, 6, 4, 2, 0}	-3	{10, 6, 6, 5, 3, 0}	27	{7, 6, 6, 6, 5, 0}	-420
{10, 7, 7, 4, 2, 0}	-3	{9, 7, 6, 5, 3, 0}	-36	{6, 6, 6, 6, 6, 0}	945
{9, 8, 7, 4, 2, 0}	6	{8, 8, 6, 5, 3, 0}	-27	{10, 8, 6, 4, 1, 1}	-3
{8, 8, 8, 4, 2, 0}	-15	{8, 7, 7, 5, 3, 0}	81	{9, 9, 6, 4, 1, 1}	9
{10, 8, 5, 5, 2, 0}	-3	{9, 6, 6, 6, 3, 0}	-36	{10, 7, 7, 4, 1, 1}	9
{9, 9, 5, 5, 2, 0}	9	{8, 7, 6, 6, 3, 0}	72	{9, 8, 7, 4, 1, 1}	-18
{10, 7, 6, 5, 2, 0}	6	{7, 7, 7, 6, 3, 0}	-180	{8, 8, 8, 4, 1, 1}	45
{9, 8, 6, 5, 2, 0}	-12	{10, 8, 4, 4, 4, 0}	-15	{10, 8, 5, 5, 1, 1}	9
{9, 7, 7, 5, 2, 0}	-9	{9, 9, 4, 4, 4, 0}	45	{9, 9, 5, 5, 1, 1}	-27
{8, 8, 7, 5, 2, 0}	27	{10, 7, 5, 4, 4, 0}	27	{10, 7, 6, 5, 1, 1}	-18
{10, 6, 6, 6, 2, 0}	-15	{9, 8, 5, 4, 4, 0}	-54	{9, 8, 6, 5, 1, 1}	36
{9, 7, 6, 6, 2, 0}	27	{10, 6, 6, 4, 4, 0}	-6	{9, 7, 7, 5, 1, 1}	27
{8, 8, 6, 6, 2, 0}	-6	{9, 7, 6, 4, 4, 0}	-27	{8, 8, 7, 5, 1, 1}	-81
{8, 7, 7, 6, 2, 0}	-45	{8, 8, 6, 4, 4, 0}	111	{10, 6, 6, 6, 1, 1}	45
{7, 7, 7, 7, 2, 0}	105	{8, 7, 7, 4, 4, 0}	-18	{9, 7, 6, 6, 1, 1}	-81
{10, 8, 6, 3, 3, 0}	-3	{10, 6, 5, 5, 4, 0}	-45	{8, 8, 6, 6, 1, 1}	18
{9, 9, 6, 3, 3, 0}	9	{9, 7, 5, 5, 4, 0}	81	{8, 7, 7, 6, 1, 1}	135
{10, 7, 7, 3, 3, 0}	9	{8, 8, 5, 5, 4, 0}	-18	{7, 7, 7, 7, 1, 1}	-315
{9, 8, 7, 3, 3, 0}	-18	{9, 6, 6, 5, 4, 0}	72	{10, 8, 6, 3, 2, 1}	6
{8, 8, 8, 3, 3, 0}	45	{8, 7, 6, 5, 4, 0}	-144	{9, 9, 6, 3, 2, 1}	-18
{10, 8, 5, 4, 3, 0}	6	{7, 7, 7, 5, 4, 0}	45	{10, 7, 7, 3, 2, 1}	-18
{9, 9, 5, 4, 3, 0}	-18	{8, 6, 6, 6, 4, 0}	-90	{9, 8, 7, 3, 2, 1}	36
{10, 7, 6, 4, 3, 0}	-12	{7, 7, 6, 6, 4, 0}	270	{8, 8, 8, 3, 2, 1}	-90
{9, 8, 6, 4, 3, 0}	24	{10, 5, 5, 5, 5, 0}	105	{10, 8, 5, 4, 2, 1}	-12
{9, 7, 7, 4, 3, 0}	18	{9, 6, 5, 5, 5, 0}	-180	{9, 9, 5, 4, 2, 1}	36
{8, 8, 7, 4, 3, 0}	-54	{8, 7, 5, 5, 5, 0}	45	{10, 7, 6, 4, 2, 1}	24
{10, 7, 5, 5, 3, 0}	-9	{8, 6, 6, 5, 5, 0}	270	{9, 8, 6, 4, 2, 1}	-48

Appendix : N=6 Schur function decomposition (continued)

$\{\mu\}$	$a_{\{\mu\}}$	$\{\mu\}$	$a_{\{\mu\}}$	$\{\mu\}$	$a_{\{\mu\}}$
{9, 7, 7, 4, 2, 1}	-36	{9, 6, 6, 5, 3, 1}	-81	{8, 8, 8, 2, 2, 2}	225
{8, 8, 7, 4, 2, 1}	108	{8, 7, 6, 5, 3, 1}	162	{10, 8, 5, 3, 2, 2}	27
{10, 7, 5, 5, 2, 1}	18	{7, 7, 7, 5, 3, 1}	162	{9, 9, 5, 3, 2, 2}	-81
{9, 8, 5, 5, 2, 1}	-36	{8, 6, 6, 6, 3, 1}	162	{10, 7, 6, 3, 2, 2}	-54
{10, 6, 6, 5, 2, 1}	-54	{7, 7, 6, 6, 3, 1}	-486	{9, 8, 6, 3, 2, 2}	108
{9, 7, 6, 5, 2, 1}	72	{10, 7, 4, 4, 4, 1}	-36	{9, 7, 7, 3, 2, 2}	81
{8, 8, 6, 5, 2, 1}	54	{9, 8, 4, 4, 4, 1}	72	{8, 8, 7, 3, 2, 2}	-243
{8, 7, 7, 5, 2, 1}	-162	{10, 6, 5, 4, 4, 1}	72	{10, 8, 4, 4, 2, 2}	-6
{9, 6, 6, 6, 2, 1}	72	{9, 7, 5, 4, 4, 1}	-81	{9, 9, 4, 4, 2, 2}	18
{8, 7, 6, 6, 2, 1}	-144	{8, 8, 5, 4, 4, 1}	-117	{10, 7, 5, 4, 2, 2}	-27
{7, 7, 7, 6, 2, 1}	360	{9, 6, 6, 4, 4, 1}	-117	{9, 8, 5, 4, 2, 2}	54
{10, 8, 5, 3, 3, 1}	-9	{8, 7, 6, 4, 4, 1}	234	{10, 6, 6, 4, 2, 2}	111
{9, 9, 5, 3, 3, 1}	27	{7, 7, 7, 4, 4, 1}	-81	{9, 7, 6, 4, 2, 2}	-162
{10, 7, 6, 3, 3, 1}	18	{10, 5, 5, 5, 4, 1}	-180	{8, 8, 6, 4, 2, 2}	-69
{9, 8, 6, 3, 3, 1}	-36	{9, 6, 5, 5, 4, 1}	216	{8, 7, 7, 4, 2, 2}	333
{9, 7, 7, 3, 3, 1}	-27	{8, 7, 5, 5, 4, 1}	108	{10, 6, 5, 5, 2, 2}	-18
{8, 8, 7, 3, 3, 1}	81	{8, 6, 6, 5, 4, 1}	-324	{9, 7, 5, 5, 2, 2}	81
{10, 8, 4, 4, 3, 1}	27	{7, 7, 6, 5, 4, 1}	-288	{8, 8, 5, 5, 2, 2}	-153
{9, 9, 4, 4, 3, 1}	-81	{7, 6, 6, 6, 4, 1}	720	{9, 6, 6, 5, 2, 2}	-117
{10, 7, 5, 4, 3, 1}	-36	{9, 5, 5, 5, 5, 1}	225	{8, 7, 6, 5, 2, 2}	234
{9, 8, 5, 4, 3, 1}	72	{8, 6, 5, 5, 5, 1}	-405	{7, 7, 7, 5, 2, 2}	-711
{10, 6, 6, 4, 3, 1}	-27	{7, 7, 5, 5, 5, 1}	90	{8, 6, 6, 6, 2, 2}	-36
{9, 7, 6, 4, 3, 1}	99	{7, 6, 6, 5, 5, 1}	675	{7, 7, 6, 6, 2, 2}	108
{8, 8, 6, 4, 3, 1}	-162	{6, 6, 6, 6, 5, 1}	-1575	{10, 8, 4, 3, 3, 2}	-45
{8, 7, 7, 4, 3, 1}	-81	{10, 8, 6, 2, 2, 2}	-15	{9, 9, 4, 3, 3, 2}	135
{10, 6, 5, 5, 3, 1}	81	{9, 9, 6, 2, 2, 2}	45	{10, 7, 5, 3, 3, 2}	81
{9, 7, 5, 5, 3, 1}	-162	{10, 7, 7, 2, 2, 2}	45	{9, 8, 5, 3, 3, 2}	-162
{8, 8, 5, 5, 3, 1}	81	{9, 8, 7, 2, 2, 2}	-90	{10, 6, 6, 3, 3, 2}	-18

Appendix : N=6 Schur function decomposition (continued)

$\{\mu\}$	$a_{\{\mu\}}$	$\{\mu\}$	$a_{\{\mu\}}$	$\{\mu\}$	$a_{\{\mu\}}$
{9, 7, 6, 3, 3, 2}	-81	{7, 6, 6, 5, 4, 2}	-1215	{10, 5, 4, 4, 4, 3}	-420
{8, 8, 6, 3, 3, 2}	333	{6, 6, 6, 6, 4, 2}	405	{9, 6, 4, 4, 4, 3}	720
{8, 7, 7, 3, 3, 2}	-54	{8, 5, 5, 5, 5, 2}	450	{8, 7, 4, 4, 4, 3}	-180
{10, 7, 4, 4, 3, 2}	72	{7, 6, 5, 5, 5, 2}	-900	{9, 5, 5, 4, 4, 3}	675
{9, 8, 4, 4, 3, 2}	-144	{6, 6, 6, 5, 5, 2}	2250	{8, 6, 5, 4, 4, 3}	-1215
{10, 6, 5, 4, 3, 2}	-144	{10, 8, 3, 3, 3, 3}	105	{7, 7, 5, 4, 4, 3}	270
{9, 7, 5, 4, 3, 2}	162	{9, 9, 3, 3, 3, 3}	-315	{7, 6, 6, 4, 4, 3}	345
{8, 8, 5, 4, 3, 2}	234	{10, 7, 4, 3, 3, 3}	-180	{8, 5, 5, 5, 4, 3}	-900
{9, 6, 6, 4, 3, 2}	234	{9, 8, 4, 3, 3, 3}	360	{7, 6, 5, 5, 4, 3}	1800
{8, 7, 6, 4, 3, 2}	-468	{10, 6, 5, 3, 3, 3}	45	{6, 6, 6, 5, 4, 3}	-720
{7, 7, 7, 4, 3, 2}	162	{9, 7, 5, 3, 3, 3}	162	{7, 5, 5, 5, 5, 3}	1050
{10, 5, 5, 5, 3, 2}	45	{8, 8, 5, 3, 3, 3}	-711	{6, 6, 5, 5, 5, 3}	-3150
{9, 6, 5, 5, 3, 2}	108	{9, 6, 6, 3, 3, 3}	-81	{10, 4, 4, 4, 4, 4}	945
{8, 7, 5, 5, 3, 2}	-351	{8, 7, 6, 3, 3, 3}	162	{9, 5, 4, 4, 4, 4}	-1575
{8, 6, 6, 5, 3, 2}	-162	{7, 7, 7, 3, 3, 3}	-90	{8, 6, 4, 4, 4, 4}	405
{7, 7, 6, 5, 3, 2}	801	{10, 6, 4, 4, 3, 3}	270	{7, 7, 4, 4, 4, 4}	45
{7, 6, 6, 6, 3, 2}	-180	{9, 7, 4, 4, 3, 3}	-486	{8, 5, 5, 4, 4, 4}	2250
{10, 6, 4, 4, 4, 2}	-90	{8, 8, 4, 4, 3, 3}	108	{7, 6, 5, 4, 4, 4}	-720
{9, 7, 4, 4, 4, 2}	162	{10, 5, 5, 4, 3, 3}	-75	{6, 6, 6, 4, 4, 4}	225
{8, 8, 4, 4, 4, 2}	-36	{9, 6, 5, 4, 3, 3}	-288	{7, 5, 5, 5, 4, 4}	-3150
{10, 5, 5, 4, 4, 2}	270	{8, 7, 5, 4, 3, 3}	801	{6, 6, 5, 5, 4, 4}	945
{9, 6, 5, 4, 4, 2}	-324	{8, 6, 6, 4, 3, 3}	-108	{6, 5, 5, 5, 5, 4}	4725
{8, 7, 5, 4, 4, 2}	-162	{7, 7, 6, 4, 3, 3}	-201	{5, 5, 5, 5, 5, 5}	-10395
{8, 6, 6, 4, 4, 2}	666	{9, 5, 5, 5, 3, 3}	90		
{7, 7, 6, 4, 4, 2}	-108	{8, 6, 5, 5, 3, 3}	324		
{9, 5, 5, 5, 4, 2}	-405	{7, 7, 5, 5, 3, 3}	-1422		
{8, 6, 5, 5, 4, 2}	567	{7, 6, 6, 5, 3, 3}	270		
{7, 7, 5, 5, 4, 2}	324	{6, 6, 6, 6, 3, 3}	45		

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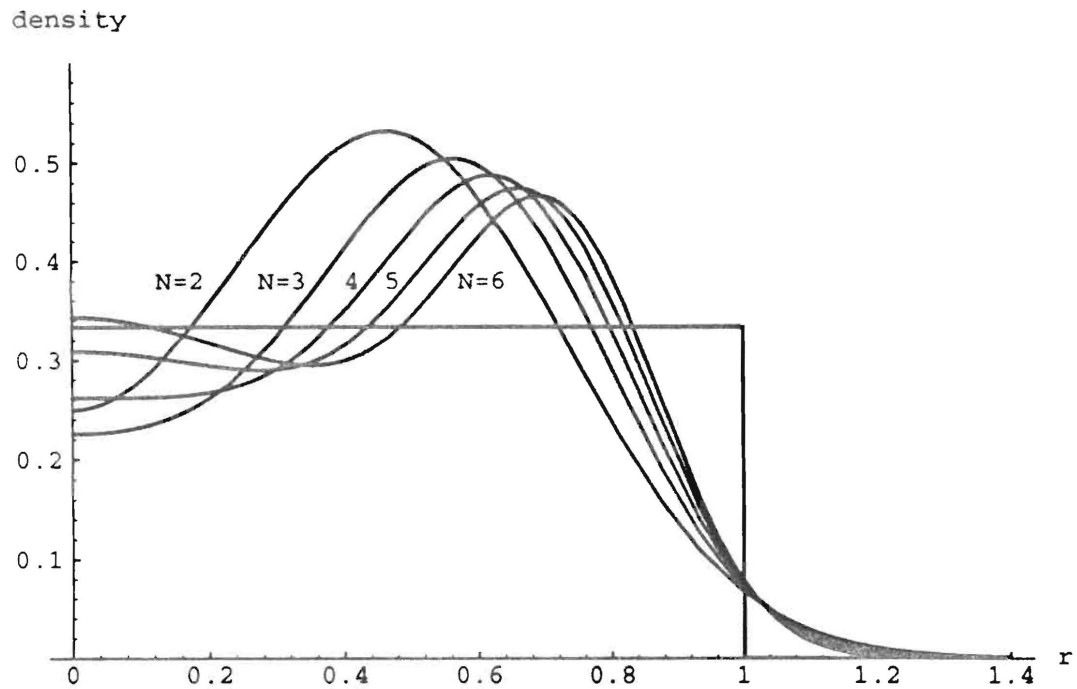

Figure 1: The exact density profiles $\hat{\rho}(r)$, scaled as in (71) so that the sample edge is at $r = 1$ for all N, of the $\frac{1}{3}$ filled Laughlin state for N=2,3,4,5,6, together with the perfectly uniform profile.

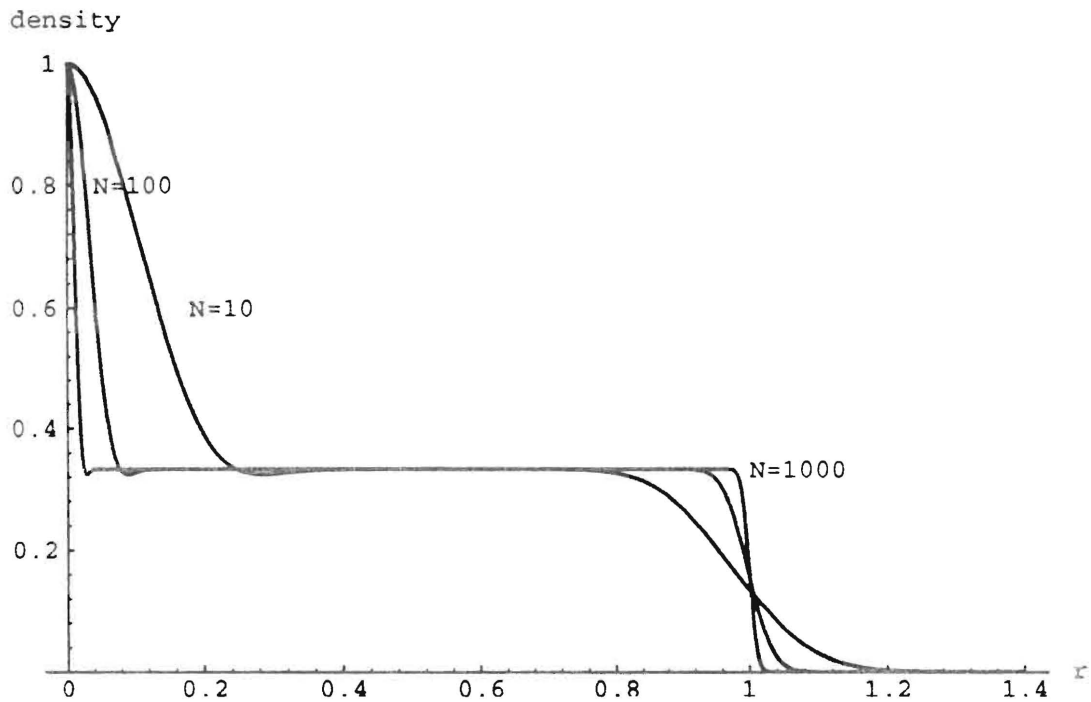

Figure 2: The density profiles of the maximally distributed states (51) for $N = 10, 100, 1000$ electrons. The plateau is *exactly* at density $\frac{1}{3}$.

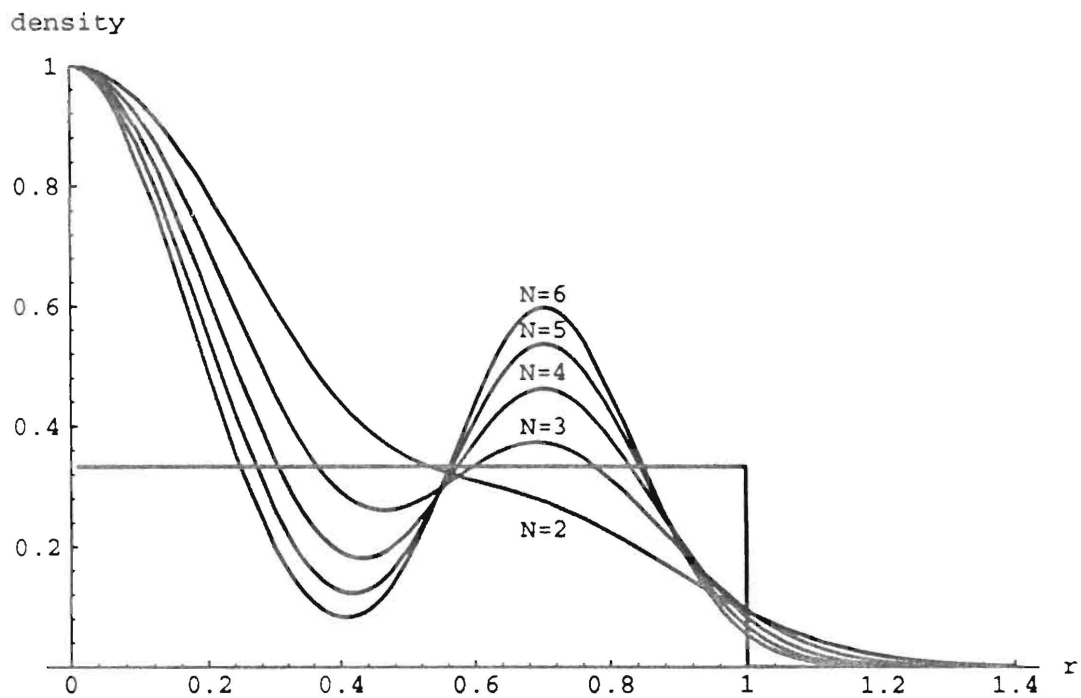

Figure 3: The density profiles of the states (57), showing their conribution to the 'boundary hump' for the $N = 2, 3, 4, 5, 6$ electron cases.

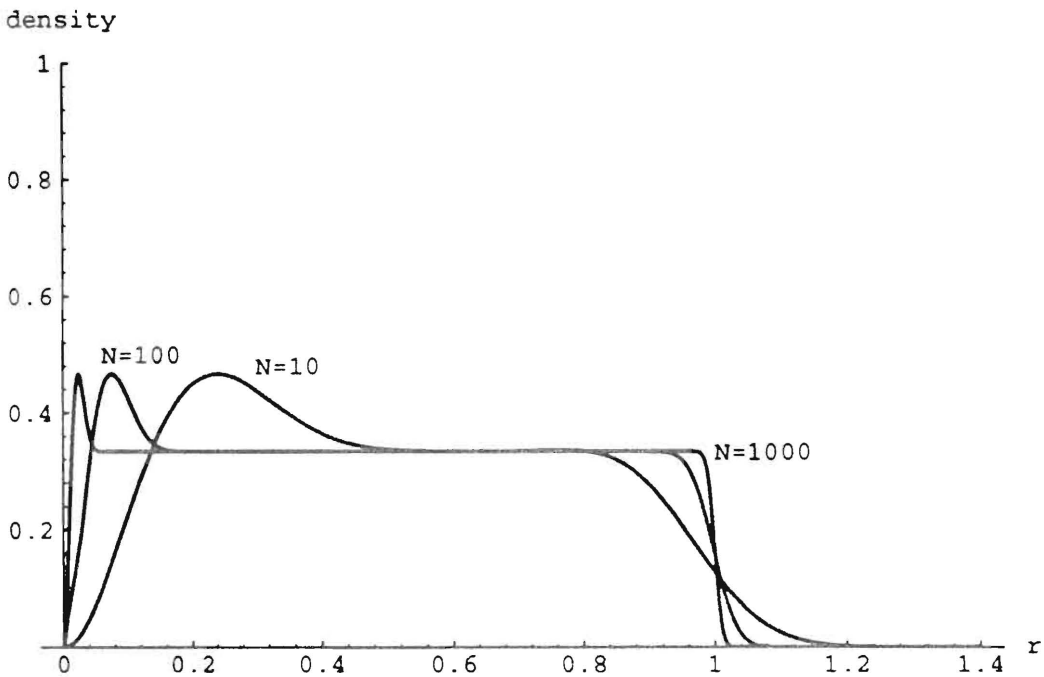

Figure 4: The density profiles of the dominant states (62) for $N = 10, 100, 1000$ electrons. As in Figure 2, the plateau is exactly at density $\frac{1}{3}$.

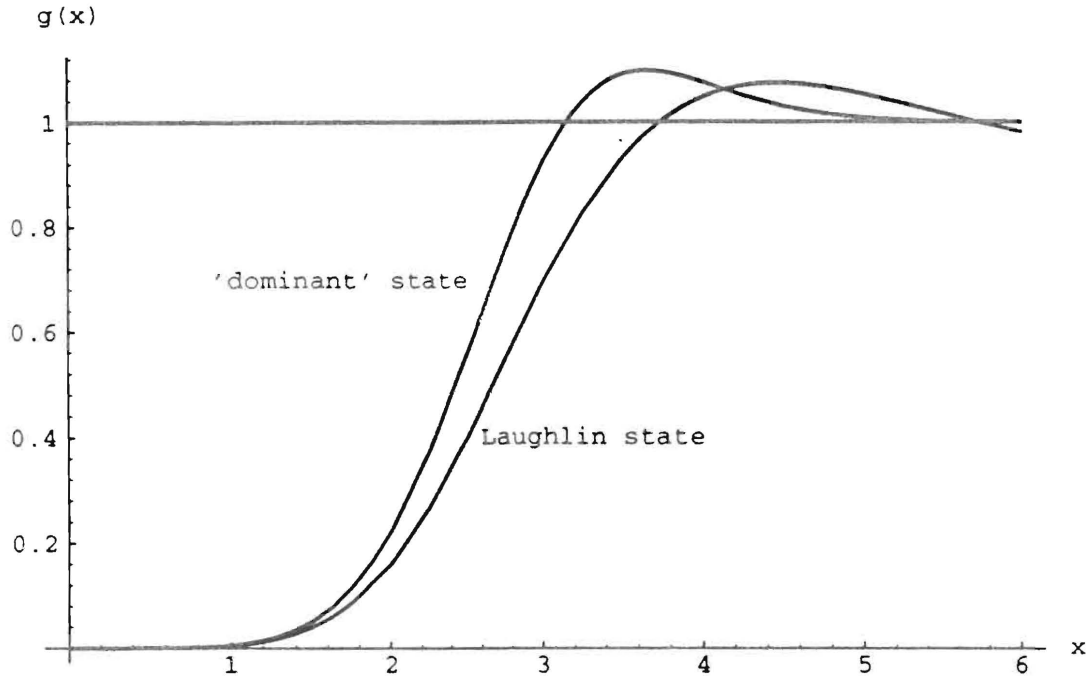

Figure 5: The pair correlation function for the $\frac{1}{3}$ filled $N = 6$ Laughlin state, as computed in equation (83), together with the approximate correlation function (84) obtained by retaining a dominant Slater state in (83).