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Final Crystal Cooling to Reduce the Beam Current and Detector Backgrounds for a $\mu^+\mu^-$ Collider

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Abstract. There are a few major outstanding problems for a high luminosity $\mu^+\mu^-$ collider. Here we address two of them: the high current of $\mu^\pm$ in the storage ring and the backgrounds due to the decays of these $\mu^\pm$. Both problems may be solved if a final stage $\mu^\pm$ cooling employing a bent crystal cooling system, previously proposed by the authors, is used. We show how the transverse emittance can be reduced by a factor of at least 10^2 with such a cooling system. The cost of the collider would also be substantially reduced.

INTRODUCTION

Over the past four years the possibility of a real $\mu^+\mu^-$ collider has gained interest (1–3). With the proceedings of the recent San Francisco conference (4) and the "Snowmass Book" (5), the design goals have gained even more credibility. However, there are several serious problems still remaining, which are:

1) the high backgrounds in the detector from $\mu^\pm$ decay products (1–3),
2) the very high $\mu^\pm$ content in the final collider ($\sim 5 \times 10^{12}$ $\mu^\pm$ per bunch),
3) the relatively poor reduction of the phase-space using medium energy ionization cooling (6), and
4) the current high cost of the source due to the large yield of $\mu^\pm$ required to reach high luminosity.

Most of these problems can be partially cured if the number of $\mu^\pm$ required to produce high luminosity can be reduced.

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Over the past few years we have studied the use of crystal channels for cooling (7,8) and even for a $\mu^+\mu^-$ collider that would use bent crystal beam confinement. In this note we show how crystal cooling for the high energy $\mu^\pm$ beams could result in a dramatic decrease in the beam emittance and therefore provide high luminosity with a substantially reduced beam intensity and backgrounds of $\mu^\pm$ in the storage ring (1,2). Also, reducing the beam emittance at high energy helps the relatively poor low energy beam cooling (3). Finally, the lower yield of $\mu^\pm$ reduces the required proton current in the $\mu^\pm$ source, thus possibly reducing considerably the cost of the overall $\mu^+\mu^-$ collider (4).

We now turn to a discussion of the cooling concept. Our model calculation, presented here, shows that one can decrease the normalized emittance to less than $\varepsilon_N = 10^{-8}$ mrad by passing the muon beam through a cascade of many cooling modules.

CRYSTAL CHANNEL BEAM COOLING

We consider motion of planar channeled particles in a crystal, which is bent elasticity in a direction perpendicular to the particle velocity and to the channeling planes. The effect of bending introduces a centripetal force to the equation of transverse motion (9) (by adding a linear piece to the crystal potential), which is equivalent to lowering one side of the continuum potential well and raising the other. The equilibrium planar trajectory moves away from the midpoint of the planar channel toward the plane on the convex side of the curved planar channel. However, such shift would cause some fraction of the channeled particles to leave the potential well (dechannel) (10). The curvature at which no particle can remain channeled is reached when the equilibrium point of planar channeled motion is shifted to the position of the planar wall on the outside of the curve. This critical radius of curvature, known as the Tsyganov radius (11), ρ_T, is

$$\rho_T = \frac{2E_\mu}{\phi a}, \tag{1}$$

where $\phi = 6 \times 10^{12}$ GeV m^{-2} is a material constant, related to the curvature of the potential well (12), and "a" is the distance between adjacent atomic planes ($\approx$2 Å).

Using a simple formula linking equivalent magnetic bending field, B, with the trajectory's curvature, ρ, namely

$$B[\text{Tesla}] \times \rho_T[\text{m}] = 3.34 \times E_\mu[\text{GeV}], \tag{2}$$

one can calculate the maximum available equivalent bending field corresponding to the Tsyganov curvature. From Eqs. (1) and (2), this field is given by

$$B_T[\text{Tesla}] = 3.34 \times \frac{1}{2}\phi\, a. \tag{3}$$

Its numerical value for silicon is evaluated as: $B_T = 2 \times 10^3$ Tesla. We note in passing, that the maximum bending field is energy independent.

Here we propose a fast muon cooling scheme based on the ionization energy loss (13) experienced by high energy muons (25 GeV) channeling through a silicon crystal. Applying classical theory of ionization energy loss (14), a relativistic (γ) charged particle passing through a silicon crystal of length, ΔL, loses total energy of $\Delta E[\text{MeV}] = 4 \times 10^2 \times \Delta L[\text{m}]$. One can introduce a characteristic damping length, Λ,

$$\frac{1}{\Lambda} = \frac{1}{E_\mu}\frac{\Delta E}{\Delta L}, \tag{4}$$

over which the particle loses all its energy. Relativistic muons passing though the crystal lose energy uniformly in both the transverse and longitudinal directions according to Eq. (4). After passing through a short section of a crystal ($\Delta L << \Lambda$) muons are re-accelerated longitudinally to compensate for the lost longitudinal energy. This leads to the transverse emittance shrinkage.

Introducing normalized transverse emittance, $\varepsilon_N = \gamma\, \sigma_x\, \sigma_{x'}$, one can write the normalized emittance budget in the form of the following cooling/heating equation:

$$\frac{d\varepsilon_N}{dL} = -\frac{\varepsilon_N}{\Lambda} + \left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{scatt}. \tag{5}$$

The last term in the above equation accounts for transverse heating processes contributing to the beam divergence increase according to the following relationship (12):

$$\left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{scatt} = \frac{1}{2}\gamma\beta\,\frac{\Delta\langle\theta\rangle^2_{scatt}}{\Delta L}. \tag{6}$$

Here β is the beta function of a focusing crystal channel, which has an enormously small value ($\beta = 2 \times 10^{-6}$ m, for 25 GeV muons channeling through a silicon crystal).

For muon channeling in a dielectric crystal, the dominant scattering process comes from elastic (Rutherford) muon scattering off the conduction electrons present in the channel. One can integrate the Rutherford cross section over the solid angle, which yields the following formula:

$$\alpha = \left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{scatt} = 40\,\pi\, n\, \frac{r_\mu^2}{\gamma}\,\beta. \tag{7}$$

Here $r_\mu = 1.4 \times 10^{-17}$ m is the classical muon radius and $n = 6 \times 10^{29}$ m^{-3} is the average concentration of the conduction electron gas in silicon crystal.

Integrating the cooling equation, Eq. (5), one obtains the following compact solution in terms of the normalized transverse emittance evolution:

$$\varepsilon_N = \varepsilon_N^0\, e^{-(L/\Lambda)} + \Lambda\alpha\left[1 - e^{-(L/\Lambda)}\right]. \tag{8}$$

The last term in Eq. (8) sets the equilibrium cooling limit of

$$\varepsilon_N^{min} = \Lambda\alpha \quad , \quad L \rightarrow \infty. \tag{9}$$

Assuming 25 GeV muons, one gets $\Lambda = 62.5$ m and the equilibrium limit of the normalized emittance of

$$\varepsilon_N^{min} = 8 \times 10^{-9} \text{ mrad}. \tag{10}$$

This value of the normalized emittance will be used in our achievable luminosity estimate.

BENT-CRYSTAL COOLING RING

Here we employ previously discussed properties of the planar channeling of high energy muons in silicon to design components of a storage ring. Particularly, we are interested in a section of bent crystal followed by two straight pieces providing alternating horizontal–vertical focusing. A basic guiding cell is depicted schematically in Fig.1. One can notice that the induced configuration of guiding fields in this element is equivalent to a powerful alternating gradient achromat.

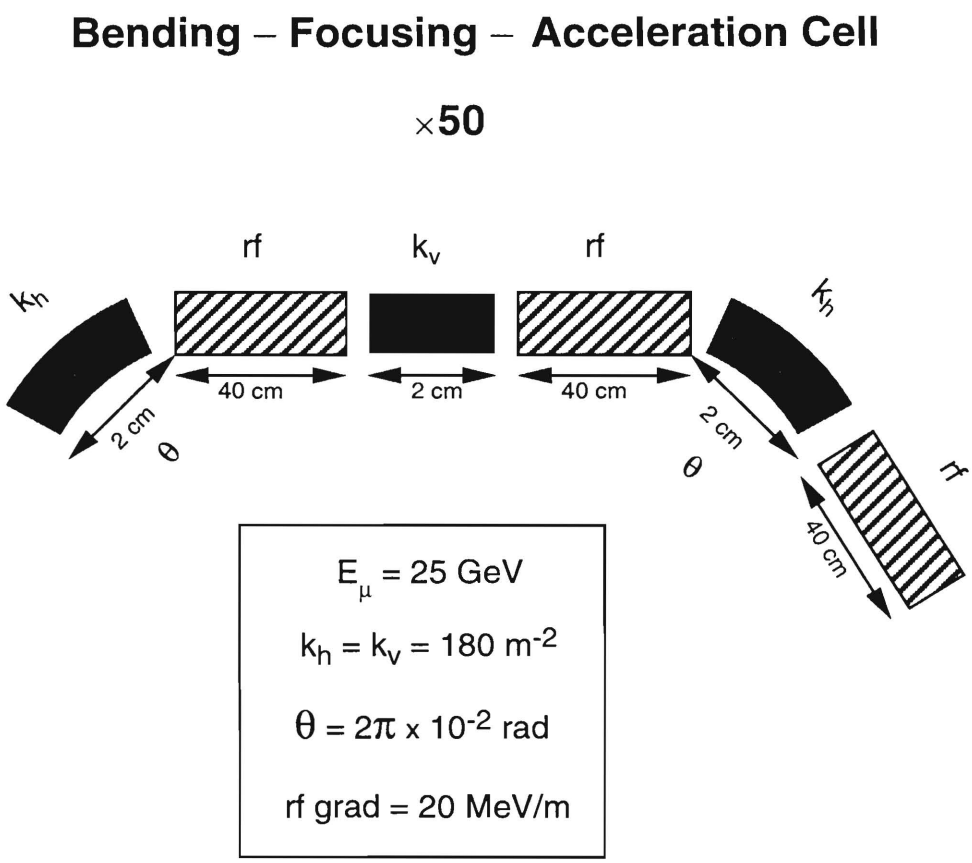

FIGURE 1. Layout of a 'cooling ring' consisting of 50 bending–focusing–acceleration multi-functional cells. A straight piece of silicon crystal rotated by 90° separating two sections of bent crystals provides vertical focusing, which maintains betatron phase stability in the proposed lattice. Conventional rf, 40-cm–long inserts (20 MeV/m) follow every 2-cm–long section of silicon crystal absorber.

Relativistic muons channeling through an Si crystal are confined between two neighboring atomic planes – they experience strong focusing electrostatic crystal-potential in the direction perpendicular to these planes, while there is virtually no confinement in the direction parallel to the planes (no focusing or defocusing). The focusing gradient $k = 1/\beta$ is equivalent to the magnetic quadrupole strength, k_1 (magnetic gradient), where

$$k_1 = \frac{1}{B\rho}\frac{\partial B_y}{\partial x}, \tag{11}$$

and

$$k = \frac{\phi}{E_\mu}. \tag{12}$$

E_μ is the total muon energy and $\phi = 6 \times 10^{12}$ GeV m^{-2} is a material constant, related to the curvature of the potential well (16). Assuming 25 GeV muons, crystal focusing gradient, k, yields an enormous value of 180 m^{-2} exceeding conventional quadrupole strength by four or five orders of magnitude.

As discussed in "Crystal Channel Beam Cooling" (Sec. 2), one can bend the crystal slightly, so that channeling muons follow the curvature of the guiding field, which results in bending of muon trajectories similar to the effect of a bending magnetic field. Projecting experimental results for proton channeling in a bent silicon crystal, one can assume that 25 GeV muons channeling through a 2-cm–long crystal should follow (without significant dechanneling effects) a bend of $\theta = 2\pi \times 10^{-2}$ rad (compared with the critical bending angle $\theta_T = 2 \times 10^{-1}$ rad). The lattice design presented here is based on these two numbers, k and θ.

Figure 1 illustrates a functional bending–focusing cell, where alternating sections of the horizontal and vertical continuous focusing channels are combined with sections of horizontally bent Si crystals. The following sequence of crystal elements: a horizontally focusing bent crystal (2-cm long) – a short drift space – a conventional rf re-acceleration section (40-cm long) – a short (2-cm long) vertically focusing straight crystal – a short drift space – another conventional rf re-acceleration section (40-cm long) – finally, a horizontally focusing bent crystal (2-cm long) followed by a conventional rf re-acceleration section (40-cm long) completes the proposed elementary cell. At 25 GeV, one could close the entire collider ring using 50 of the above $F_hOF_vOF_hO$ cells. For a sequence of the above described cells, one can find periodic betatron trajectories in both the horizontal and vertical planes – the betatron phase stability is provided by the proposed lattice configuration (alternating horizontal/vertical focusing). By virtue of [110] planar channeling, discussed in detail in the previous section, a crystal channel provides an ultra-strong electrostatic focusing gradient in the [110] direction with practically no confinement or defocusing in the plane perpendicular to the [110] direction. This fact guarantees both local and global decoupling of the horizontal and vertical betatron motions for the proposed collider lattice.

Practical realization of muon cooling at 25 GeV could be done in a compact 'cooling ring'. Assuming characteristic damping length, Λ, of 62.5 meters, the energy loss suffered by the muon beam after passing through a 2-cm–long section of a silicon crystal, is equal to 8 MeV. In principle, a conventional high gradient (20 MeV/m) acceleration inserts (40-cm long rf insert following every 2-cm–long crystal absorber) could be used to replenish the suffered energy loss (0.4 m $\times$ 20 MeV/m = 8 MeV). The proposed cooling ring of 50-fold symmetry would have a nominal circumference of 63 meters!

Our goal is to start with the initial muon phase-space of the normalized emittance of 2.5×10^{-7} mrad and cool it down to the final emittance of 2.5×10^{-9} mrad. One can see from Eq. (8) that to achieve this goal, muons have to pass through the total silicon crystal length of

$$L = 2 \log 10 \times \Lambda = 280 \text{ m}. \tag{13}$$

In the proposed cooling cell architecture, the total cooling medium (silicon) length of L = 280 m is equivalent to about 90 turns of the beam circulation in the ring. The lost energy is replenished every $\Delta L = 2$ cm, which satisfies the adiabatic re-acceleration condition ($\Delta L << \Lambda = 62.5$ m).

To go beyond the above simple analytic calculation, we are planning to carry out realistic computer simulations of planar channeling in bent crystals. One should track a charged particle through the distorted crystal lattice with the use of a realistic continuous-potential approximation and take into account the processes of both single and multiple scattering of electrons and nuclei, as well as on various defects and imperfections of the crystal lattice.

APPLICATION TO HIGH ENERGY COOLING: CONCLUSIONS

In this paper we show how crystal cooling for the high energy $\mu^{\pm}$ beams could result in a dramatic decrease in the beam emittance and therefore would provide high luminosity with a substantially reduced beam intensity and backgrounds of $\mu^{\pm}$ in the storage ring. Also, reducing the beam emittance at high energy helps the relatively poor low energy beam cooling. Finally, the lower yield of $\mu^{\pm}$ reduces the required proton current in the $\mu^{\pm}$ source, thus possibly considerably reducing the cost of the overall $\mu^{+}\mu^{-}$ collider.

We suggest employing ionization energy loss in an alternating focusing crystal channel as a cooling mechanism, since initially small muon phase space allows for efficient channeling through long sections of silicon crystal. Ultra-strong focusing in a crystal channel combined with alternating bending makes it a powerful focusing cell with ultra-small beta function. The cooling equation derived here shows that it is quite feasible to reduce transverse emittance by two orders of magnitude. Our model calculation done for 25 GeV muons shows that final emittances as low as 10^{-9} mrad are readily achievable, limited only by multiple scattering off the valence electrons in the crystal.

We conclude our study with the following observation: The proposed ionization crystal cooling could be used at some later stages of the collider scheme (e.g., for the final cooling), because of 'favorable' energy scaling of the relevant cooling characteristics, α, Λ, and ε^{N}_{min}. They can be summarized as follows:

$$\alpha \sim \gamma^{-3/2}, \tag{14}$$

$$\Lambda \sim \log \gamma, \tag{15}$$

and

$$\varepsilon_N^{min} = \Lambda\alpha \sim (\gamma^{-3/2}) \log \gamma. \tag{16}$$

Therefore, the proposed cooling mechanism scaled to higher energies looks even more attractive.

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