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A Bent Crystal Undulator to Provide Cold Positron Beams*

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Abstract

An effective method for high energy positron cooling via radiation damping in a novel bent crystal undulator is outlined here. Starting with an initially 'cool' positron beam (coming from a 1 GeV source, with the normalized emittance of $\varepsilon_N = 10^{-4}$ m rad) one can decrease the transverse emittance to less than $\varepsilon_N = 10^{-6}$ m rad by passing the positron beam through a sequence of bent crystal undulators followed by a high gradient re-acceleration. Required cooling length of 2.8 meters is relatively short. Dominant heating process (due to multiple scattering on electron gas inside a crystal channel) limits the minimum achievable emittance to about $\varepsilon_N = 10^{-7}$, while the characteristic undulator damping length is about 62.5 cm. Feasibility of both processes: fast undulator cooling and high gradient (a few GeV/m) re-acceleration is assured by ultra high fundamental crystal fields available in a solid state environment.

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1. Introduction – Crystal Cooling

The development of future $e^+ e^-$ colliders requires very cold e^+ beams. We outline here an effective method for high energy positron cooling via radiation damping in a novel bent crystal undulator. A model calculation, presented here, shows that starting with an initially 'cool' positron beam (coming from a 1 GeV source with the normalized emittance of $\varepsilon_N = 10^{-4}$) one can decrease emittance to less than $\varepsilon_N = 10^{-6}$ by passing the positron beam through a 4.2 meter long cascade of 28 cooling modules. As illustrated in Figures 1 and 2, each module consists of 10 cm long bent crystal undulators followed by a 5 cm high gradient re-acceleration (3 GeV/m).

Dominant heating process (due to multiple scattering on electron gas inside a crystal channel) is also taken into account. Derived transverse emittance 'cooling equation' shows that the minimum achievable emittance (equilibrium cooling limit) is of the order of $\varepsilon_N^{min} = 10^{-7}$, while the characteristic undulator damping length is about 62.5 cm.

To reach the final emittance of $\varepsilon_N = 10^{-6}$ one would have to pass positrons through total of 0.625 x 2 log10 meters (2.8 meters) of the bent crystal undulator. Both processes, undulator cooling and re-acceleration, require ultra high fundamental crystal fields available in a solid state environment[9,12].

2. Bent Crystal Channeling – Critical Curvature

We explore well known properties of relativistic channeling of charged particles in a bent Silicon crystal[9]. Most of available experimental data[15,16] concerns positrons or protons. Here, we project existing theory and experimental data to describe planar channeling of positrons in a crystal with a strain imposed curvature[9,11]. Particularly, we are interested in a relatively short crystal (10 cm) with periodically alternating (concave – convex) curvatures, which provides field configuration of a powerful electrostatic undulator for channeling particles (see Figure 2).

First, we describe beam dynamics in a straight (no bending) crystal. The equation of transverse motion for planar channeled relativistic ($\gamma = 2000$) positron of mas m is given by the following formula

$$m\gamma \frac{d^2x}{dt^2} + U'(x) = 0 \quad , \tag{2.1}$$

where x is the distance from the centerline between the atomic planes and U(x) is the averaged planar continuum electrostatic potential energy at the distance x. For a positive particle the continuum potential well is given to a good approximation by

$$U(x) = \frac{1}{2}\,\phi\, x^2. \tag{2.2}$$

The focusing strength of the crystal channel for [110] planar channeling in Silicon has been experimentally measured and has a value of[9]

$$\phi = 6 \times 10^{12}\ \mathrm{GeV\ m^{-2}}. \tag{2.3}$$

This corresponds to a transverse harmonic oscillator moving relativistically in the longitudinal direction – crystal channel plays the role a strongly focusing transfer line characterized by the betatron wavelength of

$$\beta_x = \sqrt{\frac{E_\mu}{\phi}} \quad . \tag{2.4}$$

Assuming 1 GeV positrons channeling in Silicon the corresponding beta function is evaluated as follows

$$\beta_x = 4 \times 10^{-7} \text{ m}. \tag{2.5}$$

Following Tsyganov[11], we consider the motion of planar channeled particles in a crystal bent elastically in a direction perpendicular to the particle velocity and to the channeling planes. The effect of bending introduces a centrifugal force to the equation of transverse motion, Eq.(2.1). The modification of the crystal continuum potential due to bending curvature, ρ, may be described as follows[9]

$$U(x) \rightarrow \frac{1}{2}\phi x^2 - \frac{m\gamma c^2}{\rho} x, \quad m\gamma c^2 = E \tag{2.6}$$

Adding a linear (centrifugal) piece to the crystal potential is equivalent to lowering one side of the continuum potential well and raising the other. One can see from Eqs.(2.1) and (2.6) that the equilibrium planar trajectory to move away from the midpoint of the planar channel toward the plane on the convex side of the curved planar channel. Although, such shift would cause some fraction of the channeled particles to leave the potential well (dechannel). The curvature at which no particle can remain channeled is reached when the equilibrium point of planar channeled motion is shifted to the position of the planar wall on the outside of the curve. This critical radius of curvature, known as the Tsyganov radius[11] is defined by the following equilibrium condition, $U'(a/2) = 0$, where $a = 2 \times 10^{-10}$ m is the distance between adjacent atomic planes and $U(x)$ is given by Eq.(2.6). This translates into the following explicit expression for the Tsyganov radius, ρ_T

$$\rho_T = \frac{2E}{\phi a}. \tag{2.7}$$

Using simple formula linking equivalent magnetic bending field, B, with the trajectory's curvature, ρ, namely

$$B[\text{Tesla}] \times \rho_T[\text{m}] = 3.34 \times E[\text{GeV}] \,, \tag{2.8}$$

one can calculate the maximum available equivalent bending field corresponding to the Tsyganov curvature. From Eqs.(2.7)–(3.2) this field is given by

$$B_T[\text{Tesla}] = 3.34 \times \frac{1}{2}\, \phi\, a \quad . \tag{2.9}$$

Its numerical value for Silicon is evaluated as follows

$$B_T = 2 \times 10^3 \text{ Tesla} \,. \tag{2.10}$$

$$\rho_T = 4.2 \times 10^{-2} \text{ m} \,. \tag{2.13}$$

Assuming a realistic bending radius of $\rho_0 = 4 \times \rho_T$ one can use Eq.(2.8) to calculate the equivalent bending field for 1 GeV positrons as follows

$$B_0 = 500 \text{ Tesla} \,, \quad \rho_0 = 6.7 \text{ mm} \quad . \tag{2.14}$$

The above value of the equivalent magnetic field will be used for the proposed crystal undulator, which will be discussed in the next section. Furthermore, a choice of larger bending radius guarantees that the dechanneling effects due to the applied bending are negligible[16].

3. Bent Crystal Undulator – Radiation Damping

Applying classical theory of synchrotron radiation a relativistic (γ) charged particle passing through an undulator of length, ΔL, radiates total energy, ΔE, (integrated over all frequencies and full solid angle and summed over all the horizontal and vertical modes), which is given by the following formula[7,8]

$$\Delta E = \frac{e^2}{12\pi\varepsilon_0} \left(\frac{eB_0}{mc}\right)^2 \gamma^2 \Delta L \ , \tag{3.1}$$

or alternatively in practical units as

$$\Delta E[\mathrm{GeV}] = 6.4 \ \mathrm{x}\ 10^{-6}\ E^2[\mathrm{GeV}]\ \mathrm{x}\ B_0^2[\mathrm{Tesla}]\ \mathrm{x}\ \Delta L[\mathrm{m}] \ . \tag{3.2}$$

One can introduce the following useful quantity, Λ

$$\frac{1}{\Lambda} = \frac{1}{E}\frac{\Delta E}{\Delta L} \ , \tag{3.3}$$

a characteristic damping length – over which particle looses all its energy. Relativistic positrons passing though the undulator loose energy uniformly in both the transverse and longitudinal directions according to Eq.(3.1). After passing through a short undulator section ($\Delta L << \Lambda$) positrons are re-accelerated longitudinally to compensate for the radiated longitudinal energy. This leads to the transverse emittance shrinkage.

Introducing normalized transverse emittance, ε_N, given by

$$\varepsilon_N = \gamma\, \sigma_x\, \sigma_{x'} \ , \tag{3.4}$$

one can write down the normalized emittance budget in form of the following cooling/heating equation

$$\frac{d\varepsilon_N}{dL} = -\frac{\varepsilon_N}{\Lambda} + \left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{scatt} . \tag{3.5}$$

The last term in the above equation accounts for transverse heating processes contributing to the beam divergence increase according to the following relationship[15]

$$\left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{scatt} = \frac{1}{2}\gamma\,\beta_x \cdot \frac{\Delta\langle\theta\rangle_{scatt}}{\Delta L} \tag{3.6}$$

For positron channeling in a dielectric crystal the dominant scattering process comes from elastic (Rutherford) positron scattering off the valence electrons present in the channel. One can integrate the Rutherford cross section over the solid angle, which yields the following formula

$$\frac{\Delta\langle\theta\rangle_{scatt}}{\Delta L} = 16\,\pi\,n\,\frac{r^2}{\gamma^2}\,\log\left(\frac{\theta_{max}}{\theta_{min}}\right) , \tag{3.7}$$

where

$$\log\left(\frac{\theta_{max}}{\theta_{min}}\right) \approx 5 .$$

Here, $r = 2.8 \times 10^{-15}$ m, is the classical positron radius and n is the concentration of the valence electron gas in Silicon crystal, which can be estimated as follows

$$n = \frac{S}{\left(\frac{1}{2}a\right)^3} = 6 \times 10^{30}\ m^{-3} , \tag{3.8}$$

where S = 6 is the coordination number for the basic crystalografic cell for Silicon crystal and $a = 2 \times 10^{-10}$ m is the lattice constant.

Now, one can summarize balance between radiation damping (cooling) and multiple scattering (heating) in the derived cooling/heating equation, Eq.(3.5), by the following two quantities

$$\frac{1}{\Lambda} = 6.4 \ \text{x} \ 10^{-6} \ E \ B_0^{\ 2} \ , \tag{3.9}$$

and

$$\alpha = \left(\frac{\Delta\varepsilon_N}{\Delta L}\right)_{scatt} = 40 \ \pi \ n \ \frac{r^2}{\gamma} \ \beta_x \ , \quad \beta_x = 4 \ \text{x} \ 10^{-7} \ \text{m} \ . \tag{3.10}$$

Integrating the cooling equation, Eq.(3.5), one obtains the following compact solution in terms of the normalized transverse emittance evolution

$$\varepsilon_N = \varepsilon_N^{\ 0} \ e^{-\frac{L}{\Lambda}} + \Lambda\alpha \ (1 - e^{-\frac{L}{\Lambda}}) \ , \tag{3.11}$$

The last term in Eq.(3.11) sets the equilibrium cooling limit of

$$\varepsilon_N^{\ min} = \Lambda\alpha \quad , \quad L \rightarrow \infty \tag{3.12}$$

Assuming 1 GeV positrons one gets the equilibrium limit of the normalized emittance of

$$\varepsilon_N^{\ min} = 7.5 \ \text{x} \ 10^{-7} \ \text{m rad} \ . \tag{4.13}$$

Here the radiation damping length Λ is equal to 62.5 cm.

Our goal is to start with the initial positron phase-space of the normalized emittance of 10^{-4} m rad and cool it down to the final emittance of 10^{-6} m rad. One can see from Eq.(3.11) that to achieve this goal positrons have to pass through the total undulator length of

$$L = 2 \ \log 10 \ \text{x} \ \Lambda = 2.8 \ \text{m} \ . \tag{3.14}$$

In the proposed cooling cell architecture the total undulator length of L = 2.8 m is divided into 28 of ΔL = 10 cm long sections, to comply with the adiabatic re-acceleration condition ($\Delta L << \Lambda$ = 62.5 cm). According to Eq.(3.2) the beam looses total energy of 1.6 GeV, after passing through 1 meter long section

of the proposed bent crystal undulator. This energy needs to be restored before the next undulator section is encountered. As illustrated in Figure 2, we suggest using a powerful crystal acceleration insert discussed elsewhere (5 cm long insert, providing a high acceleration gradient of 3.2 GeV/m can be achieved[13,14]).

4. Conclusions

We pointed out that initially cool positrons obtained from a 1 GeV production source could be used as a starting point for a high energy $e^+ e^-$ collider complex, providing that an effective positron cooling scheme is available. We suggest employing radiation damping in a bent crystal undulator as a cooling mechanism, since initially small positron phase space allows for efficient channeling through long sections of Silicon crystal. Ultra-strong focusing in a crystal channel combined with alternating bending makes it a powerful undulator. Derived here cooling equation shows that it is quite feasible to reduce transverse emittance by two orders of magnitude. Our model calculation done for 1 GeV positrons shows that final emittances as low as 7.5×10^{-7} m rad are readily achievable, limited only by multiple scattering off the valence electrons in the crystal.

We conclude our study with the following observation: the proposed bent crystal undulator cooling could be used at some later stages of the collider scheme, e.g. for the final cooling, due to 'favorable' energy scaling of the relevant cooling characteristics, ε_N^{min} and Λ. They can be summarized as follows

$$\varepsilon_N^{min} \sim \gamma^{-3/2}. \tag{4.1}$$

$$\Lambda \sim \gamma^{-1}. \tag{4.2}$$

Therefore, the proposed cooling mechanism scaled to higher energies looks even more attractive.

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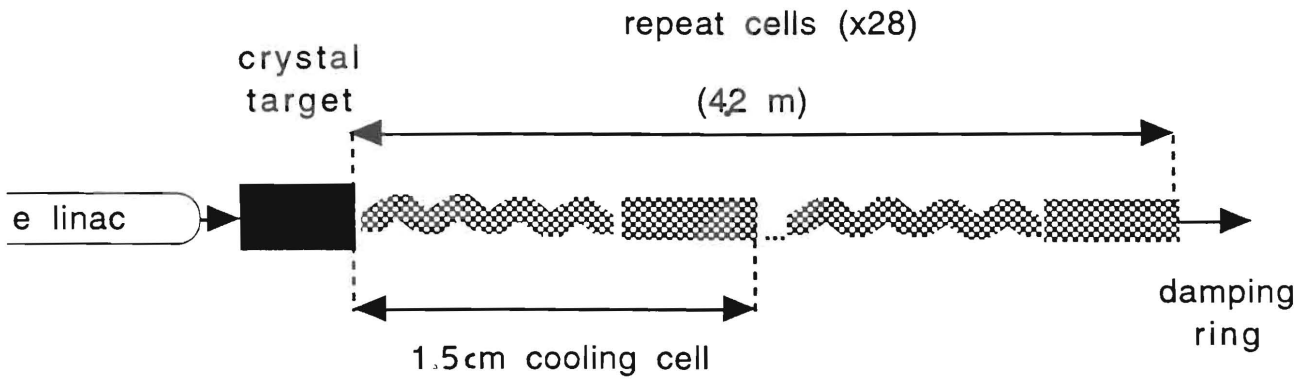
Figure Captions

Figure 1. Schematic of a 1 GeV positron source applied in conjunction with the bent crystal undulator to provide very cold positron beams for injection into a $e^+ e^-$ collider.

Figure 2. Schematic of the bent crystal undulator and re-accelerator cell. A 1 meter long Silicon crystal with alternating (concave-convex) curvature radius, ρ, imposed is followed by a 0.5 meter long high gradient (8 GeV/m) re-acceleration insert.


repeat cells (x28)
crystal
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1.5cm cooling cell

Figure 1

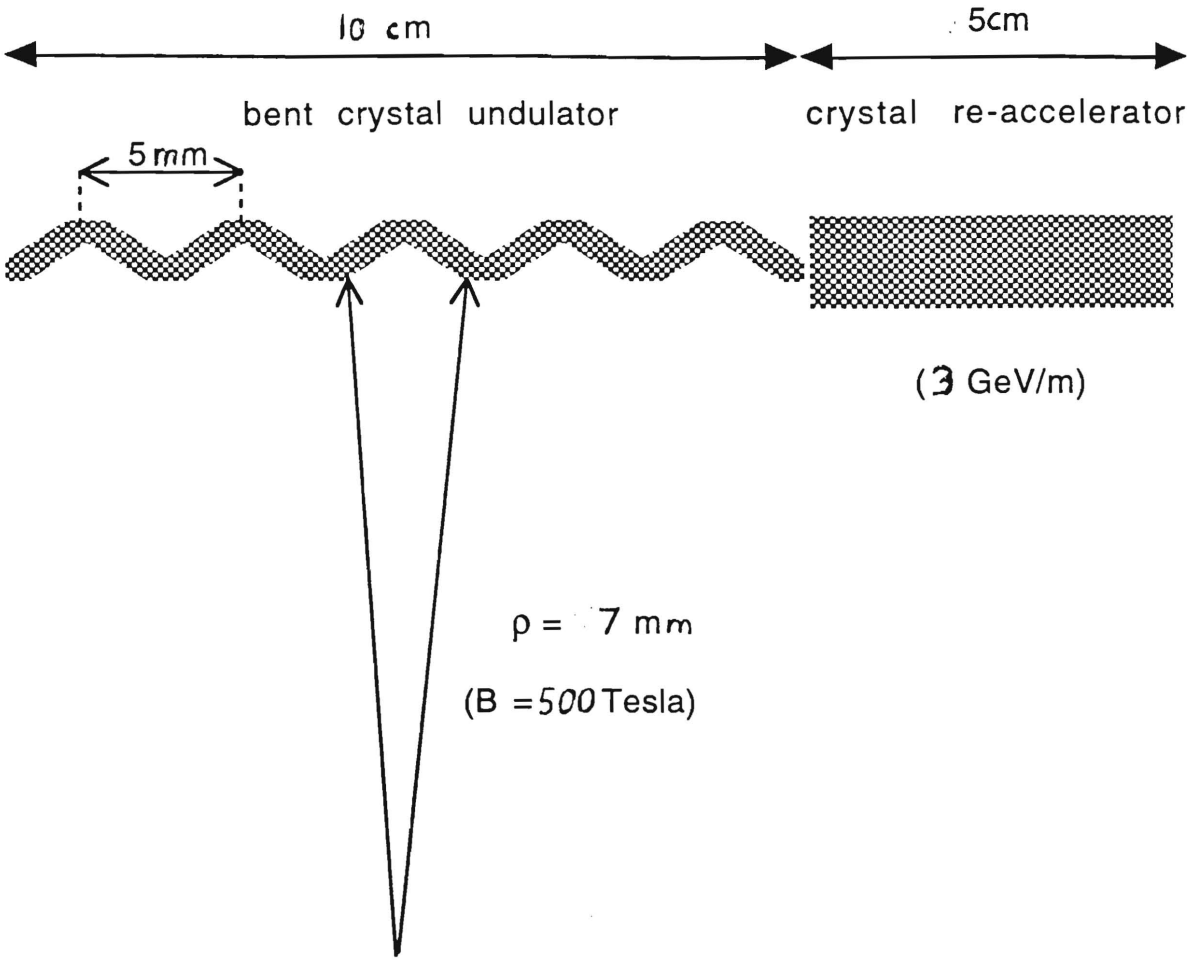
10 cm
5cm
bent crystal undulator
crystal re-accelerator
5 mm
(3 GeV/m)
ρ = 7 mm
(B = 500 Tesla)

Figure 2