MUON COOLING AND ACCELERATION EXPERIMENT AT TRIUMF*

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Abstract

In this paper we propose to demonstrate an effective method for cooling and accelerating muons by channeling them in a crystal structure. Leading schemes for future high energy $\mu^+\mu^-$ colliders[1,2] rely on fast cooling and high-gradient acceleration of short-lived muons ($\tau_{\text{int}}=2.1\times10^{-6}$ sec). This experiment aims to prove that both processes can be integrated and achieved in the ultra-strong focusing environment of a solid state system. Practical demonstration of transverse cooling in a modulated crystal channel and verification of theoretically predicted cooling efficiencies are the first crucial steps towards meeting the challenges of $\mu^+\mu^-$ colliders. Furthermore, experimental measurement of high-acceleration gradients on the order of GeV per meter, promised by the high fields in a crystal channel, would make $\mu^+\mu^-$ colliders a real possibility[3].

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1. Theoretical Overview

We explore a novel concept of cooling and accelerating muons by channeling them in a Silicon crystal. Leading schemes for future high energy $\mu^+\mu^-$ colliders rely on fast cooling and high-gradient acceleration of short-lived muons. Here, we propose a proof-of-principle experiment, which aims to demonstrate that both processes can be integrated and achieved in the ultra-strong focusing environment of a solid state system.

1.1 Cooling

Recent results on the radiation reaction of charged particles in a continuous focusing channel[4], indicate an efficient method to damp the transverse emittance of a muon beam. This could be done without diluting the longitudinal phase-space significantly. There is an excitation-free transverse ground state[4] to which a channeling particle will always decay, by emission of a photon. In addition, the continuous focusing environment in a crystal channel eliminates any quantum excitations from random photon emission, by constraining the photon recoil selection rules. A relativistic muon entering the crystal with a pitch angle of $\theta_p$ that is within the critical channeling angle[5] around $\theta_c=7$ mrad, will satisfy the "undulator" regime requirements given by the following inequality

$$\gamma \theta_p \ll 1.$$  \hspace{1cm} (1)

In this case, the particle will lose a negligible amount of the total energy while damping to the transverse ground state. Muons of the same energy but different $\theta_p$ will all end up in the same transverse ground state, limited by the uncertainty principle. Theoretically predicted ground state emittance is given by the following expression
\[ \gamma E_{\text{min}} = \frac{\lambda_{\mu}}{2}, \]  

where \( \lambda_{\mu} = \frac{h}{m_{\mu}c} \) is the Compton wavelength of a muon.

Following the solution of the Klein-Gordon equation\[4\], photons emitted in a "dipole regime", given by Eq.(1), obey the following selection rule

\[ \Delta n = n_i - n_f = 1, \]  

linking energies of the initial, \( E_i \), and the final, \( E_f \) state of a radiating particle according to the following formula

\[ E_f = \frac{E_i}{1 + \frac{1}{4}(\gamma \theta_p)^2} \approx E_i \left[ 1 - \frac{1}{2}(\gamma \theta_p)^2 \right], \]  

Therefore, the longitudinal energy spread will be very small, about \( \frac{1}{2}(\gamma \theta_p)^2 \).

This combination of both the transverse and the longitudinal phase-space features makes a radiation damping mechanism a very interesting candidate for transverse muon cooling in an ultra strong focusing environment inside a crystal.

For \( \mu^+ \) 's channeling in a Silicon crystal, the characteristic transverse damping time, \( \tau \), is given by the following formula

\[ \frac{1}{\tau} = 2 r_\mu \frac{e \Phi_1}{3 m_{\mu} c}, \]  

where \( r_\mu \) is the classical radius of a muon and \( e \Phi_1 = 6 \times 10^3 \text{ GeV/m}^2 \) is the focusing strength for Silicon crystal[15]. Although the characteristic damping time, \( \tau \approx 10^{-6} \text{ sec}, \)
for a spontaneous channeling radiation damping is rather long, one can enhance the lattice reaction[6] by using the crystal lattice as a microundulator (external strain modulation of the inter atomic spacing in the crystal lattice, e.g. an acoustic wave of wavelength \( l \)). As illustrated in Figure 1, if the acoustic wavelength, \( l \), matches the Doppler-shifted betatron oscillations of the beam, \( \gamma \lambda_\beta \), according to the following matching condition

\[
\gamma \lambda_\beta = \sqrt{2} l \quad \text{and} \quad \lambda_\beta = 2\pi \sqrt{\frac{m_\mu c^2}{e \phi}},
\]

(6)
a stimulated enhancement of the channeling radiation will occur – similar effect to the FEL amplification. In fact, if one could generate a standing acoustic wave of sizable amplitude in a crystal, then the relaxation time would shorten the damping time by more than three orders of magnitude.

1.2 Acceleration

According to previous calculations[7,8], one can achieve acceleration gradients on the order of GeV per meter in the high fields found in a crystal channel. The first paper[7] explores the idea of particle acceleration by self-coupling to plasma formed by the conduction electrons in a crystal. This can also be accentuated by resonant excitations of the plasma.

In another scheme[8], a high-gradient acceleration (GeV/m) is provided by inverse FEL coupling with a high-power optical driver. A strain-modulated Silicon crystal acts as a microundulator for a channeled muon beam. This crystal is then placed in an optical cavity – between two axicone mirrors powered by a GWatt laser at visible frequencies. A schematic setup of a proof-of-principle experiment is illustrated in Figure 4.
A beam of relativistic particles while channeling through the crystal follows a well-defined trajectory. Figure 1 depicts planar channeling paths for positive charged particles in [110] crystallographic direction. The center of the channeling axis is modulated by the acoustic wave periodicity to produce an undulator effect: i.e., the particles are periodically accelerated perpendicular to their flight path as they traverse the channel. The microundulator wavelength, $l$, (for a typical acoustic modulation) falls in the range 1000–5000Å, far shorter than those of any macroscopic undulator. Furthermore, the electrostatic crystal-fields involve the line-averaged nuclear field and can be two or more orders of magnitude larger than the equivalent fields of macroscopic magnetic undulators. Both of these factors hold the promise of greatly enhanced coupling between the beam and the accelerating electromagnetic wave.

The key to collective acceleration via inverse FEL mechanism is a spontaneous bunching of initially uniform beam channeling through a periodic crystal structure and interacting with the electromagnetic wave. Appropriate phase matching results in energy flow from the wave to the particle beam. This particular kind of particle density fluctuation has the form of a propagating density wave of the same frequency, $\omega$, as the emitted electromagnetic wave. The phase velocity of the moving bunch matches the velocity of particles in the beam. Therefore, the quantity $\gamma m \omega / \gamma p = k_b$ represents the wave vector of the propagating particle density bunch. Keeping in mind that the periodicity of the undulator represents a static wave with a wave vector $g = 2\pi / l$, and that $k$ is the wave vector of the electromagnetic wave, we can analyze our acceleration mechanism in the language of "three wave mixing" as illustrated in Figure 2. Here the "lower branch FEL" condition seems very appropriate for particle acceleration inside a crystal microundulator. It translates into the following acoustic – optical wavelength constraint
which in our numerical example fixes the accelerating optical wavelength at $\lambda=1000$ nm.

The acceleration efficiency,[8] $\alpha$, is described as the rate of optical amplitude depletion per one particle. Assuming a muon beam of initial energy 250 MeV, a typical value of the beam concentration, $n=10^{16}$ cm$^{-2}$, acoustic wavelength of $l=5000$ Å and the initial longitudinal momentum spread of $\Delta p/p = 10^{-2}$ one gets the following value of the acceleration rate[8]

$$\alpha = 2 \times 10^{-3} \text{ cm}^{-1}. \tag{8}$$

The nominal acceleration efficiency in units of MeV/m will, obviously, depend on the energy density of the actual optical cavity. The recent advances in high power laser technology based upon diode laser pumped solid state lasers[9] promise a power of a few MWatts, optically focused to provide energy densities of $E_{\text{max}} = 10^{10}$ V/m, where $E_{\text{max}}$ is the electric field amplitude of the standing cavity mode. This combined with Eq.(7) sets the final accelerating efficiency equivalent to an accelerating gradient of 2 GeV/m.

One can also test the inverse Cerenkov acceleration mechanism[10], since the index of refraction for Silicon is very large, $n=1.5$. Matching the phase velocity of the optical mode to the muon velocity requires a relatively large crossing angle (between the beam and the laser pulse). This enhances the longitudinal projection of the radial component of the electric field, which in turn yields a high accelerating gradient.
2. Experimental Overview

The experiment will be performed in three phases. The first phase, called the transmission experiment, will show channeling of muons in a 3 mm long Silicon crystal. The second phase will measure the cooling of channeled muons in a long, 4 cm Silicon crystal. Finally, the third phase will incorporate acceleration.

2.1 Phase I – Transmission

Channeling of muons has been observed at 4 MeV. However, channeling of higher energy muons has not been measured conclusively. We require measurements of the critical angle for channeling muons and the ionization energy loss for channeled versus unchanneled muons. We will use the 35 MeV/c momentum surface muons provided by the M13 beam-line at TRIUMF, which was selected based on our assessment of the quality of the muon beam. At this energy, the stopping power of amorphous Silicon is high, about 4 mm. In this case, only channeled muons will survive the crystal. Unchanneled particles are subject to typical energy loss mechanisms of ionization and bremsstrahlung. Whereas, the energy loss of channeled particles is severely reduced. A schematic of the proposed setup for the transmission experiment is illustrated in Figure 3. The beam, incident from the left, passes through a trigger and veto formed by two scintillation counters. The veto rejects muons that do not enter the geometric dimensions of the crystal. The muons then enter the crystal at some incident angle, \( \theta \), with respect to the axis of the crystal. The crystal's orientation is controlled remotely by a goniometer. Just downstream of the crystal, the exiting muon energy and flux will be measured with a surface-barrier detector. Such detectors are capable of about 20 keV energy resolution at 35 MeV/c. The data will indicate the yield as a function of \( \theta \), allowing extraction of the
critical angle within which muons are effectively channeled. Furthermore, we can measure the energy spread of the exiting beam. This will indicate the degree of energy straggling we should expect when compared to the incident energy distribution provided by the beam-line. It is important to align the crystal with the beam. A rough alignment will be performed with X-rays. Then we will use the tracking information and a goniometer to maximize the number of channeled muons exiting the crystal. TRIUMF's M13 beam-line is the best choice for effective channeling through a 4 cm sample of Silicon crystal. It provides surface muons at high intensity – about $1.2 \times 10^6$ per second\cite{12}. They carry momentum of 35 MeV/c with a longitudinal spread of about 4% FWHM. Assuming optimum tuning of the final focus quadrupole doublet in M13, one could achieve a spot size of 2 cm x 2 cm with horizontal and vertical divergence of 25 mrad and 65 mrad, respectively. The critical planar channeling angle in a Silicon crystal is about 12 mrad for 35 MeV/c positive muons, if one extrapolates critical angle measurements from proton channeling\cite{5}. In this case, a sizable fraction of the incident muons will channel through the crystal.

2.2 Phase II – Cooling

This phase of the experiment will test the cooling mechanisms summarized in the theory section above. The beam momentum will be about 250 MeV/c as provided by forward decay muons\cite{13} in M11. In this case, both channeled and unchanneled muons will penetrate the 4 cm crystal and the cooling process can be compared for the two. In addition, a higher energy beam tests cooling at the energies considered for realistic collider schemes. The first step of Phase II is to measure initial and final emittances of an unmodified crystal. We will track each muon individually using five sets of drift chambers. This way we can identify channeled and unchanneled particles on an event-by-
event basis. We can also obtain the exact initial and final emittances for channeled and unchanneled muons separately. It is also possible to separate channeled and unchanneled muons by plotting their energy loss. The trigger will be provided by scintillation counters upstream, combined with a veto counter that rejects muons which do not intersect the crystal. A time-of-flight counter will be placed in a downstream position, to be used in concert with the 1 picosecond timing pulse provided by the M11 beam-line. This will aid in particle identification since some positrons and pions will likely contaminate the beam. To enhance the cooling, we will generate a strain modulation of the planar channels. An acoustic wave of 1 GHz is excited via a piezoelectric transducer. We will also detect predicted channeling radiation by surrounding the crystal with CsI scintillation detectors, which are sensitive to X-rays. The M11 beamline is presently a source of high energy pions[13]. Straightforward modification of the beamline will provide a collimated beam of forward-decay muons at high intensity[14] – about $10^6$ per second at 250 MeV/c. The longitudinal momentum spread is about 2% FWHM. Assuming optimum tuning of the final focus quadrupole doublet in M11, we can achieve a spot size of 3 cm x 2 cm with horizontal and vertical divergence of 10 mrad and 16 mrad respectively. The critical angle for planar channeling of $\mu^+$ at 250 MeV/c in Silicon is about 7 mrad, extrapolating from proton channeling data. A sizable fraction of the muons should channel through a few centimeters of the crystal.

### 2.3 Phase III – Acceleration

A high gradient acceleration will be tested in two steps. Initially, an unmodified crystal will be used to test plasma acceleration[7]. Then we will apply a laser to test inverse FEL[8] and inverse Cerenkov[10] acceleration of muons. The optical setup, illustrated in Figure 4, is analogous to the Inverse Cerenkov Accelerator Experiment at
Brookhaven. It provides a pulse of radially polarized light, which couples energy to the muon beam channeling through a crystal via the inverse FEL mechanism. Here a strain modulation in the crystal imposed by an acoustic wave plays the role of an ultra-short wave undulator. Optical energy will be transferred to the muon beam with an efficiency on the order of GeV per meter. A 4 cm Silicon crystal would provide a 40 MeV energy burst. Using a bending magnet between drift chambers, we will measure the final energy of muons channeling through the crystal. The initial energy of each muon is provided by a spectrometer in the beamline.
References


Figure Captions:

Figure 1: Schematic of the flight path of particles, accelerated perpendicularly, as they traverse the crystal channel.

Figure 2: Matching conditions, of the phase and group velocities of a moving “bunch” of particles in the beam, for the upper and lower FEL branches.

Figure 3: The schematic of the proposed setup for the transmission experiment at TRIUMF.

Figure 4: Schematic of proof-of principle experiment of a superlattice crystal accelerator.
lower branch FEL

\[ \nu^- = 0 \]
\[ \frac{\omega}{\nu_z} + k - g = 0 \]

\[ \lambda = (\beta^{-1} + 1) \, \ell \]

\[ \beta^{-1} \approx 1 + \frac{1}{2\gamma^2} \]

\[ \lambda \approx 2\ell \]

upper branch FEL

\[ \mu^+ = 0 \]
\[ - \frac{\omega}{\nu_z} + k - g = 0 \]

\[ \lambda = (\beta^{-1} - 1) \, \ell \]

\[ \lambda \approx \frac{\ell}{2\gamma^2} \]
Helium - Isobutane bag

MHz muon beam
Vacuum

collimator
scintillator
drift wires

Silicon crystal

window

Silicon barrier detector
Superlattice Crystal Accelerator
(proof-of-principle experiment)

\[ \lambda = (\beta^{-1} + 1) \ell \]