UCLA-92-TEP-10
LOCATION
Fermilab Library
0 1160 0016079 0
DUP

UCLA/92/TEP/10

March 1992

Singular Interactions in Two-Dimensional Hubbard Model

S.Yu. Khlebnikov *

Department of Physics, University of California, Los Angeles, CA 90024, USA

Abstract

We calculate quasiparticle interaction in two-dimensional Hubbard model by first finding the interaction vertex at small but non-zero temperature and then taking temperature to zero *after* the thermodynamic limit. The two limits prove to be non-interchangeable, signalling the presence of Kohn-Luttinger-type anomalous terms. These anomalous terms make the interaction singular for quasiparticles of opposite spins and equal momenta at the Fermi surface. This singularity is a specific feature of the lattice two-dimensional system as opposed to the continuum 2D Fermi gas with short-range repulsion.

PACS: 73.20.Dx - Electron states in low dimensional structures.

FERMILAB LIBRARY
MAY 1 1992

*On leave of absence from Institute for Nuclear Research of the Academy of Sciences, Moscow 117312 Russia.

Recently, there was much interest in the idea that the ground state of correlated two-dimensional electronic systems differs from the conventional Fermi liquid. Anderson [1, 2] has suggested that in two-dimensional (2D) Hubbard model the Landau function $f_{\mathbf{pp}'}$ describing interaction of quasiparticles in the Fermi-liquid state is singular for equal momenta $\mathbf{p} = \mathbf{p}'$, so that this interaction modifies the Fermi-liquid state in a singular way. Generally, the Landau function can be related to the real part of the effective particle-particle interaction vertex [3]. (The imaginary part of the vertex is irrelevant since it tends to zero more rapidly as external momenta $\mathbf{p}$ and $\mathbf{p}'$ approach the Fermi surface.) In Ref.[1] and subsequent works [4, 5, 6] the two-particle effective vertex in the 2D Hubbard model was studied at zero temperature within the ladder approximation [7], as expected to be appropriate at low electron density. In the ladder approximation only opposite spins interact and the real part of the vertex has the form [4, 5, 6]

$$\mathrm{Re}\Gamma(\mathbf{p}, \epsilon; \mathbf{p}', \epsilon') = U \left(1 + \frac{U}{V} \sum_{\mathbf{k}}{}' \frac{N(\mathbf{P}, \mathbf{k})}{\epsilon_{\mathbf{k}} + \epsilon_{\mathbf{P}-\mathbf{k}} - \epsilon - \epsilon'} \right)^{-1} . \tag{1}$$

Here $\mathbf{P} = \mathbf{p} + \mathbf{p}'$ is total momentum, U is on-site repulsion, V is total two-dimensional volume, $N(\mathbf{P}, \mathbf{k}) = 1 - n_{\mathbf{P}-\mathbf{k}} - n_{\mathbf{k}}$, $n_{\mathbf{k}}$ is the Fermi distribution, and $\epsilon_{\mathbf{k}}$ is the free-particle dispersion law. The prime on the sum sign indicates that momenta for which the energy denominator is zero are excluded from the sum. Engelbrecht, Randeria and Zhang [8] have attempted to calculate the Landau function using the zero-temperature ladder expression (1) and have found no singularity for equal momenta. However, it was already pointed out [2] that a singularity, if any, of the Landau function may be outside the conventional many-body perturbation theory and analogous to anomalous terms discovered by Kohn and Luttinger [9]. These terms can only be found by first doing a calculation at non-zero temperature and then taking temperature to zero *after* the thermodynamic limit is performed. As advocated in Ref.[9], this is the correct

order of limits when they are not interchangeable. In any calculation starting from the zero-temperature expression (1), the anomalous terms are absent from the very beginning.

It is the purpose of the present note to report an alternative calculation of the Landau function for 2D Hubbard model, using the correct order of the limiting procedures. We will see that anomalous terms are indeed present in this problem and render the Landau function singular for $\mathbf{p} = \mathbf{p}'$. Unlike the case discussed by Kohn and Luttinger, however, these anomalous terms have nothing to do with non-sphericity of Fermi surface which appears only in higher orders of the low density expansion. Thus, while the formal analogy with their considerations is complete, the underlying physics should be different. We do not discuss here what redefinitions of excitations at the Fermi surface are needed in the present case but our results certainly demonstrate the breakdown of Fermi-liquid behaviour for the 2D Hubbard model essentially in the way suggested in Refs.[1, 2].

Let us emphasize that the singularity we obtain is a distinct feature of the 2D lattice system. It does not occur for the continuum 2D Fermi gas with short range repulsion. The presence of singularity is closely related to the logarithmic divergency of the sum in eq.(1) near the excluded point $\mathbf{k} = \mathbf{p}$, when $\mathbf{p} = \mathbf{p}'$, $\epsilon = \epsilon'$ is on the energy shell and the size L of the system goes to infinity. For the continuum Fermi gas, the corresponding sum contains a vacuum two-particle scattering amplitude $f(\Delta\mathbf{p}, \Delta\mathbf{k})$ [7], where $\Delta\mathbf{p} = (\mathbf{p} - \mathbf{p}')/2$, $\Delta\mathbf{k} = \mathbf{k} - \mathbf{P}/2$. In two dimensions, for $|\Delta\mathbf{p}|, |\Delta\mathbf{k}| \to 0$, the amplitude goes to zero as inverse logarithm of the second argument [10],

$$f(\Delta\mathbf{p}, \Delta\mathbf{k}) \sim 1/\log|\Delta\mathbf{k}| \ , \tag{2}$$

so that the integral

$$\int d\mathbf{k} f(0, \mathbf{k} - \mathbf{p}) f(0, \mathbf{k} - \mathbf{p}) \frac{N_{\mathbf{k}}}{\epsilon_{\mathbf{k}} + \epsilon_{2\mathbf{p}-\mathbf{k}} - 2\epsilon_{\mathbf{p}}} \ , \tag{3}$$

is convergent at $\mathbf{k} = \mathbf{p}$, being $\int KdK(K\log K)^{-2} < \infty$ [6]. As a consequence, the singularity of the Landau function will not appear in the continuum.

The conclusion of instability of the Fermi-liquid state of the 2D Hubbard model was drawn by us previously [6] from the presence of an *off-shell* isolated pole of the zero-temperature effective vertex Γ below the occupied states [4, 5, 6]. This pole is also specific for the lattice system as opposed to the continuum Fermi gas [6]. Though the interpretation of the pole as an instability still seems most plausible to us, another interpretation [4] considers it as a bosonic excitation of a stable Fermi-liquid state. Let us note that the argument for the breakdown of Fermi-liquid behaviour in lattice 2D systems, based on the singularity of Landau function, is free from any such interpretation problem.

The effective interaction at finite temperature T is obtained as usual by analytical continuation in ϵ, ϵ' of the thermodynamic vertex part defined on a discrete set of imaginary frequencies. This results in the following replacement in eq.(1),

$$N(\mathbf{P}, \mathbf{k}) = \frac{1}{2}\left(\tanh\frac{\epsilon_{\mathbf{P}-\mathbf{k}}}{2T} + \tanh\frac{\epsilon_{\mathbf{k}}}{2T}\right) , \tag{4}$$

where the dispersion law is counted from the chemical potential μ. We use eq.(1) with this replacement in the general relation between the Landau function $f_{\mathbf{p}\mathbf{p}'}$ and the particle-particle interaction vertex [3],

$$f_{\mathbf{p}\mathbf{p}'} = F(\mathbf{p}, \epsilon_{\mathbf{p}}; \mathbf{p}', \epsilon_{\mathbf{p}'}) , \tag{5}$$

where function F is the solution of equation,

$$F(\mathbf{p}, \epsilon; \mathbf{p}', \epsilon') = \Gamma(\mathbf{p}, \epsilon; \mathbf{p}', \omega) - \frac{i}{(2\pi)^3}\int \Gamma(\mathbf{p}, \epsilon; \mathbf{q}, \omega)\phi(\mathbf{q}, \omega)F(\mathbf{q}, \omega; \mathbf{p}', \epsilon')d^2q d\omega . \tag{6}$$

Here $\phi(\mathbf{q}, \omega)$ is the regular part of the square of the single-particle Green function of momentum $\mathbf{q}$ and frequency ω, and we omit the spin indices. In the second term

on the r.h.s. of eq.(6), the singularity of F at $\mathbf{p} = \mathbf{p}'$, $\epsilon = \epsilon' = \epsilon_{\mathbf{p}}$, which we obtain below, is integrated over, so the singular part of the Landau function coincides with that of the first term.

The logarithmic divergency at $\mathbf{k}$ near $\mathbf{p}$ at $\mathbf{p} = \mathbf{p}'$ and $\epsilon = \epsilon' = \epsilon_{\mathbf{p}}$ tends to make the sum in eq.(1) infinite as $\log L$. Therefore, at strictly zero temperature eq.(1) turns to zero in the thermodynamic limit. The main idea behind our analysis is that at *any* finite temperature, for $|\mathbf{p}|$ close to the Fermi surface with the $1/\log L$ accuracy, the smallness of tanh functions in eq.(4) can compensate the logarithmic growth of the sum and give rise to a pole in eq.(1). This pole is in the *on-shell* effective interaction and leads, through eq.(6), to a singular Landau function. The singular terms are precisely of the Kohn-Luttinger type and would be switched off if one takes T to zero before the thermodynamic limit, so that $N(\mathbf{P}, \mathbf{k})$ becomes a step-function and can provide no small factors.

With the same definitions of the relative momenta $\Delta\mathbf{p}$ and $\Delta\mathbf{k}$ as above, the on-shell effective interaction vertex has the form

$$\Gamma_{\mathbf{p}\mathbf{p}'} = \Gamma(\mathbf{p}, \epsilon_{\mathbf{p}}; \mathbf{p}', \epsilon_{\mathbf{p}'}) \sim U \left(1 + \frac{U}{2V} {\sum_{\mathbf{k}}}' \frac{1}{|\Delta\mathbf{k}|^2 - |\Delta\mathbf{p}|^2} \left(\tanh\frac{\epsilon_{\mathbf{P}-\mathbf{k}}}{2T} + \tanh\frac{\epsilon_{\mathbf{k}}}{2T}\right)\right)^{-1} . \tag{7}$$

Here we have used the quadratic approximation $\epsilon_{\mathbf{p}} = \mathbf{p}^2/2 - \mu$ for both external and intermediate states, so that the sum has to be cut at momenta of order of inverse lattice spacing and will be calculated with logarithmic accuracy. When the external momenta are not equal, $|\Delta\mathbf{p}| \neq 0$ in the thermodynamic limit, the sum in eq.(7) is of order $\log(1/k_F a)$, where a is the lattice spacing, and the Landau function is not singular.

Let us now consider the case of almost equal momenta, $|\Delta\mathbf{p}| \sim 1/L$. The main

contribution to the sum in this case comes from the vicinity of $\Delta\mathbf{k} = 0$, and

$$\Gamma_{\mathbf{pp}'} \sim U \left(1 + \text{const. } U \tanh\frac{\epsilon_{\mathbf{p}}}{2T} \log\frac{L}{a}\right)^{-1} , \tag{8}$$

where const. is *positive*. Therefore, for any small but non-zero temperature, there are states within $1/\log L$ below the Fermi surface ($\epsilon_{\mathbf{p}} < 0$), for which the expression in brackets becomes zero, so that the r.h.s of eq.(8) becomes infinite. When L is taken to infinity *before* temperature is turned off completely, these states produce the singularity of Landau function for equal momenta right at the Fermi surface.

In conclusion, we have proven the conjecture of Refs.[1, 2] that the Landau interaction function in the 2D Hubbard model is singular for quasiparticles with opposite spins and equal momenta at the Fermi surface. The singular terms are anomalous in the Kohn-Luttinger sense, they can only be recovered by starting from the finite-temperature interaction vertex and taking temperature to zero after the thermodynamic limit. The singularity results in the breakdown of Fermi-liquid behaviour and is characteristic of lattice two-dimensional systems as opposed to continuum ones. Let us note in closing that this distinction between lattice and continuum is compatible with the idea of a fractional exclusion principle for opposite spins [2]. Indeed, the Hilbert space of single-particle states in a lattice system, but not in a continuum one, is "finite and extensive", in Haldane's [11] terminology, that is the number of states is proportional to the volume. Excitations in such systems can possess fractional statistics in arbitrary dimensions [11]. In three-dimensional Hubbard model, there are no anomalous terms in quasiparticle interaction to order $k_F a$, as may be seen from the explicit mapping of this problem on the continuum Fermi gas (with the scattering length of 0.315 of the lattice spacing for large U) [6]. However, we expect such terms to appear in higher orders of the low-density expansion, where deviations from spherical Fermi surface will come into play.

The author is grateful to J. Engelbrecht for sending a copy of Ref.[8] prior to publication. The author is supported by the Julian Schwinger Postdoctoral Fellowship at UCLA.

References

[1] P. W. Anderson, Phys. Rev. Lett. **65**, 2306 (1990).

[2] P. W. Anderson, Phys. Rev. Lett. **66**, 3226 (1990) and preprint.

[3] L. D. Landau, Sov. Phys. JETP, **8**, 70 (1959); A. A. Abrikosov, L. P. Gorkov and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics*, Prentice, 1963, Sect.18.

[4] J. R. Engelbrecht and M. Randeria, Phys. Rev. Lett. **65**, 1032 (1990); **66**, 3225 (1991) and preprint.

[5] H. Fukuyama, O. Narikiyo and Y. Hasegawa, J. Phys. Soc. Japan **60**, 372 (1991).

[6] S. Yu. Khlebnikov, *Hubbard model at low density in two and three dimensions*, preprint UCLA/91/TEP/50.

[7] V. M. Galitskii, Sov. Phys. JETP **7**, 104 (1958).

[8] J. R. Engelbrecht, M. Randeria and L. Zhang, *Landau f-function for the dilute Fermi gas in two dimensions*, preprint.

[9] W. Kohn and J. M. Luttinger, Phys. Rev. **118**, 41 (1960).

[10] P. Bloom, Phys. Rev. B **12**, 125 (1975).

[11] F. D. M. Haldane, Phys. Rev. Lett. **67**, 937 (1991).