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OBSERVABLE EFFECTS OF ANOMALOUS HEAVY FERMIONS*

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ABSTRACT

We consider an anomaly-free chiral gauge theory, in which the anomaly cancellation occurs between fermions with very different masses. We show how this splitting of the anomaly cancellation can enhance the strength of certain amplitudes at low energies. As an example, we exhibit this enhancement in the magnetic moment of massive neutrinos to two loop order.

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1. Introduction

Chiral anomalies in gauged currents have long been known to lead to a breakdown of gauge invariance [1] and of renormalizability [2]. In consistent gauge theories, with chiral fermions, the sum of all chiral anomalies in gauged currents must cancel. To date, this argument provides perhaps one of the strongest reasons that the top quark should exist and have the same electric charge as the up and charm.

When fermions are rendered heavy, two types of situations may arise. When a gauge invariant Dirac mass term renders the fermions heavy, we are within the framework of the celebrated "decoupling theorem" [3]. To leading order, the heavy fermions decouple in the sense that we are left with a renormalizable gauge theory (at energies low compared to the mass of the heavy fermion) in which any reference to the heavy fermion has been deleted. The effect of the heavy fermions is to provide a contribution to the renormalizable coupling constants of this low energy theory, plus higher dimension operators, suppressed by negative powers of the large fermion mass. The renormalization effect of the low energy coupling constants is not physically observable, as can be seen by introducing a low-energy subtraction point, independent of the heavy fermion mass. Thus, fermions with large Dirac mass terms effectively decouple. Fermions for which a Dirac mass term can be written down, invariant under a gauge group G, are of course never anomalous with respect to the gauge group G.

When no gauge-invariant Dirac mass term can be written down, fermions may have anomalies. Gauge invariant fermion masses may then be generated either through a Yukawa coupling to a Higgs' field that acquires a vacuum expectation value, as in the standard model, or through a dynamical mechanism involving some further microscopic interactions, as in extended technicolor models [4]. The nature of the effects at low energy due to a heavy fermion of this type is much more complicated, and in general, heavy fermions do not "decouple"[5].

If the fermion mass is rendered large by keeping the Yukawa coupling fixed, but letting the vacuum expectation value of the Higgs field become large, then standard decoupling [3,6] again applies. Now all gauge bosons that acquired their masses through the Higgs mechanism must decouple as well, and anomalies in the corresponding gauge currents become inconsequential.

On the other hand, if the fermion acquires its large mass by keeping the Higgs expectation value fixed, but letting the Yukawa coupling grow large, (or through a dynamical scheme like extended technicolor) then we must deal with a strong coupling problem. The particle that becomes heavy also becomes more strongly coupled to the low energy physics. This circumstance puts us outside the framework of the standard coupling theorem, which assumes that all dimensionless couplings remain bounded, while masses are increased.

In the above cases, it may be very difficult to generate fermion masses much larger than the Higgs vacuum expectation value; we shall ignore such issues at present. Now, we do not expect the fermion to fully decouple.

There is in fact another, complementary reason, why chiral fermions cannot decouple in the above scenarios. To see this, let us consider a chiral gauge theory with (chiral) gauge group G. We shall assume that all masses are generated through the Higgs mechanism and Yukawa couplings. [The case of dynamical symmetry breaking is analogous]. We also assume that all anomalies with respect to the gauge group G cancel. Now, we render a fermion heavy by keeping the vacuum expectation value of the Higgs field fixed, and by increasing the value of the Yukawa coupling. If the fermion by itself produces an anomaly in gauged currents, then its removal would leave behind a gauge theory with an anomalous fermion content, and this theory would not make sense, lacking gauge invariance. In the next section we shall briefly review that instead of decoupling, a Wess-Zumino term is induced which maintains gauge invariance, but spoils renormalizability [7].

In the present talk, we shall examine the situation in which the fermion content of the theory is anomaly free, but with the anomaly cancellation occurring at widely different mass-scales. We shall argue that certain low energy amplitudes or operators must be enhanced by such splitting. Instead of presenting a general theory, we shall at this stage illustrate that such an enhancement is possible by presenting two separate examples. In the first model, the gauge group is Abelian $U(1)_V \times U(1)_A$ while in the second it is the $SU(2)_L \times U(1)_Y$ of the standard model. In the second model, we show how the magnetic moment of a massive neutrino is enhanced at two loops over the one loop contribution if the anomaly cancellation occurs at very different scales, while there is no enhancement when it occurs at comparable scales.

2. The Induced Wess-Zumino Term

To find the effect of heavy fermions on low energy physics, it is convenient to integrate out the heavy fermion fields, say in a functional integral formulation [6]. This requires a one-loop calculation of the effective action due to the heavy fermion field ψ, with Lagrangian

$$\mathcal{L}_\psi = \bar{\psi}_L\gamma^\mu(i\partial_\mu+g_L A_{L\mu})\psi_L+\bar{\psi}_R\gamma^\mu(i\partial_\mu+g_R A_{R\mu})\psi_R-\frac{M}{v}(\bar{\psi}_L\Phi\psi_R+\bar{\psi}_R\Phi^\dagger\psi_L) \quad (1)$$

Here A_L and A_R are the left and right gauge fields, and Φ is a Higgs field that acquires a vacuum expectation value $\Phi = \varphi U$, $U^\dagger U = 1$ $\langle 0|\varphi|0\rangle = v \neq 0$ where φ is a real scalar field. The effective action W is defined by integrating out ψ and

was calculated in [7], where it was found to be

$$W[A_L, A_R, \Phi] = \Gamma[A_L, A_R, U] + W_0[A_L, A_R, \Phi] \tag{2}$$

Here, the functional Γ is the so-called Wess-Zumino term [8] which captures precisely the chiral anomalies of the fermion functional integral (*i.e.* it obeys precisely the same gauge Ward identities as W). Its construction and explicit form may be found in [7]. The gauge invariant terms W_0 in (2) come in three types [7]. There are "renormalizable" terms like $\mathrm{tr}(D_\mu\Phi)^2$, $\mathrm{tr}\, F^2_{\mu\nu L}$, $\mathrm{tr}\, F^2_{\mu\nu R}$ etc., which one could view as resulting from "ordinary decoupling". Then, there are terms suppressed by negative powers of M, again analogous to what happens in standard decoupling, and finally, there are certain non-renormalizable terms that are not suppressed by the usual powers of M, like

$$\frac{1}{v^4}\ \mathrm{tr}[D_\mu\Phi^\dagger, D_\nu\Phi]^2, \frac{1}{v^4}\ \mathrm{tr}(D_\mu\Phi^\dagger D_\mu\Phi)^2 \quad \text{etc.} \tag{3}$$

All these terms are gauge invariant, and do not contribute to the anomaly structure of the theory.

The terms in the last series violates standard decoupling and contain interesting physics. As an example, we exhibit the dimension 6 terms

$$W_0 = \frac{i}{32\pi^2 v^2}\int d^4x[\mathrm{tr}\, g_2 F_{2\mu\nu} D^\mu\Phi^\dagger D^\nu\Phi + g_1 F_{1\mu\nu}\,\mathrm{tr}\,\tau^3 D^\mu\Phi^\dagger D^\nu\Phi + ...] \tag{4}$$

The simplest and perhaps most interesting physical effect of these contributions that does not seem to have been noticed so far is the anomalous magnetic moment of the $W^\pm$ particles, as produced by heavy fermions. We find

$$\Delta\mu_W = \frac{eg_2^2}{64\pi^2 M_W} + \mathcal{O}(M^{-2}) \tag{5}$$

per heavy fermion doublet. This type of corrections are somewhat analogous to oblique corrections (see e.g. [9]).

We now consider the problem posed in the Introduction : an anomaly-free chiral gauge theory in which an anomalous fermion is rendered heavy. We find that instead of leaving behind, at low energies, a theory with uncancelled gauge anomalies, the low energy theory has acquired a novel interaction - the Wess-Zumino term - that guarantees gauge invariance of the resulting low energy theory [7].

We are faced with the curious conclusion that the chiral anomaly by itself would prevent a heavy anomalous fermion from decoupling, even though the fermion also fails to decouple simply because a large mass for a chiral fermion necessitates a large Yukawa coupling.

In fact, the Wess-Zumino term - as well as the gauge invariant induced interactions - lead to non-renormalizability of the low energy theory. In view of gauge invariance alone, we might attempt to only retain the Wess-Zumino term in the low energy effective theory and omit the gauge invariant non-renormalizable terms; even so, renormalizability would be lost. This conclusion was implicit in [2], and was discussed in the special case of the $U(1)$ anomaly in [10]. Arguments for the general case were obtained in [11,12].

The apparent lack of renormalizability of the low energy theory due to the presence of the Wess-Zumino term (the infinite fermion mass limit), leads to an interesting question. If in the limit where an anomalous fermion is made infinitely heavy, the theory becomes non-renormalizable, then for finite but large mass, some amplitudes must grow with increasing fermion mass. This means that certain amplitudes must be enhanced as the mass of the fermion grown large. The chiral anomaly of the heavy fermion alone should be capable of introducing such an enhancement. This question goes back to the paper by Sterling and Veltman [10].

3. The Abelian $U(1)_V \times U(1)_A$ Model

We first look at an Abelian Higgs model, with gauge group $U(1)_V \times U(1)_A$, and fermion of electric $U(1)_V$ charge e and axial $U(1)_A$ charges $\pm g$, which we denote by $\psi^{\pm}$. The Lagrangian is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}\, F_{\mu\nu}F^{\mu\nu} - \frac{1}{4} Z_{\mu\nu}Z^{\mu\nu} + \frac{1}{2}\left|i\partial_\mu\phi + gZ_\mu\phi\right|^2 - \frac{\lambda^2}{4}(|\phi|^2 - v^2)^2 \\ & \sum_{\pm} \bar{\psi}^{\pm}\gamma^\mu(i\partial_\mu + eA_\mu \pm gZ_\mu\gamma_5)\psi^{\pm} - \frac{m_{\pm}}{v}\, \bar{\psi}^{\pm}(v + \phi^1 \pm i\phi^2\gamma^5)\psi^{\pm} \end{aligned} \tag{6}$$

The Higgs field $\phi = v + \phi^1 + i\phi^2$ acquires a vacuum expectation value, so that $\langle 0|\phi^i|0\rangle = 0$ and we fix a standard 't Hooft Feynman $R_{\xi=1}$ gauge. Potential AAZ and ZZZ anomalies cancel between the two fermions.

We shall assume that $m_+ = M$, $m_- = m$ and $M \gg gv \gg m$ and $g \sim 1$, and perform a loop expansion in which we consider Green-functions of low energy fields only. Thus ψ^+ only propagates in loops, with low energy particle external legs. The fermion ψ^+ loop with two insertions obeys standard decoupling by renormalizing $F^2_{\mu\nu}, Z^2_{\mu\nu}$ and the Higgs quadratic terms. Such effects will be unobservable at low energy. The fermion ψ^+ loop with three insertions precisely contains the axial

anomaly graphs, and we need to find a process in which this effect may be seen. The lowest order graphs in which the effect can occur must be 2-loop, and we may retain the anomalous magnetic moment contribution to ψ^- as a viable candidate.

Tree level μ_o and one-loop μ_1 contributions to the magnetic moment of ψ^- are given by

$$\mu_o = \frac{2e}{m} \qquad\qquad \mu_1 = \frac{e\alpha}{m\pi} + \#\frac{em}{v^2} \tag{7}$$

To two loop level, we wish to find the dependence on the large mass M. The two point function leading to standard decoupling, its effects may be absorbed into charge and mass renormalization to this order. There are also M-dependent contribution from graphs other than the triangle, *e.g.* vacuum polarization. We shall not discuss them for this example, but postpone consideration to the next example of the standard model.

The two loop contribution from triangle graphs is exhibited in Fig. 1.

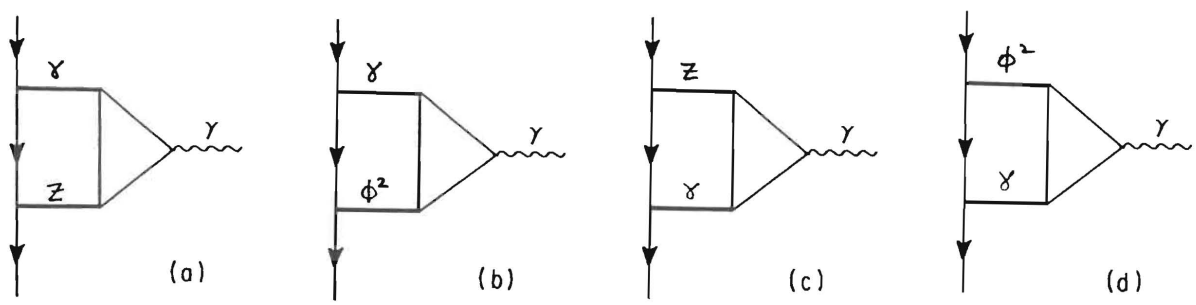

Fig. 1. Two loop contributions involving triangle graphs.

In all triangles, both ψ^+ and ψ^- propagate. In diagrams (a) and (c), only gauge couplings occur and no Yukawa couplings. As the couplings are M-independent, these diagrams by themselves obey standard decoupling. Since we are evaluating contributions to a dimension 5 operator, these graphs go like $1/M$ which we neglect. However, it should be pointed out that these graphs have logarithmic enhancement for small m, as the massless photon leads to an infrared divergence in the limit $m \mapsto 0$. This effects was discussed in [13]. In graphs (b) and (d), the coupling of ϕ^2 to the triangle is proportional to the masses m or M. The contribution of ψ^- is M-independent, and we neglect it consistently, thus we need to evaluate only (b) and (d) with ψ^+ in the triangle.

The triangle in question is the famous contributions to $\pi \longrightarrow \gamma\gamma$ decay. Including coupling constants, but retaining only linear order in photon momentum:

$$\frac{M}{v}\; T_{\mu\nu}(M,q,k) = e^2\, \epsilon_{\mu\nu\alpha\beta}\; \frac{q^\alpha k^\beta}{4\pi^2 v}\; T(M^2,k^2) \tag{8.a}$$

where the invariant amplitude is given by

$$T(M^2, k^2) = \int_0^1 d\alpha \ \frac{2M^2(1-\alpha)}{M^2 - \alpha(1-\alpha)k^2 - i\epsilon} \tag{8.b}$$

This triangle contribution is to be inserted into Figs. 1(b) and 1(d), and one may easily extract the leading magnetic moment contribution for large M:

$$\mu_2 = \frac{me^3}{4\pi^2 v^2} \int \frac{d^4k}{(2\pi)^4} \ \frac{1}{(k^2 - M_z^2)^2} \int_0^1 d\alpha \ \frac{2M^2(1-\alpha)}{M^2 - \alpha(1-\alpha)k^2 - i\epsilon} \tag{9}$$

Here, we have also neglected terms of higher order in $\frac{m}{v}$ The diagnosis of this integral is as follows: If we first let $M \longrightarrow \infty$ in the integrand on the right, then the triangle diagram reduces to the pointlike Wess-Zumino coupling, and the 2-loop contribution to the anomalous magnetic moment diverges logarithmically. For large M/v, the integral is finite. Combined with the contributions from graphs 1(a) and 1(c) for small m, as in [13], we find

$$\mu_2 = \frac{me^3}{64\pi^4 v^2} \ \ln \frac{M^2}{m^2}. \tag{10}$$

For fixed e and v, and large M^2/m^2, this contribution may in principle dominate the 1-loop Z-exchange. Unfortunately, the tree-level and one-loop contributions for charged particles are dominated by photon exchange, and the above effect may be difficult to observe.

When we let a pair of two fermions with opposite axial charges g become large, their logarithmic enhancements cancel. This can be seen by adding a fermion for which g, m and v would change sign; but this would change the sign of the above contribution and so they cancel. It is in this sense that we claim that the above enhancement effect is purely due to the anomaly splitting mechanism.

4. The $SU(2)_L \times U(1)_Y$ Model

To avoid the large tree and one-loop level contributions to the magnetic moment for charges particles, we now look for enhancements of the magnetic moment for neutral fermions. A natural situation is provided just by the standard model $SU(2)_L \times U(1)_Y$ gauge theory, but with a massive neutrino. We shall use a simplified version of the standard model, in which the anomaly cancellation occurs between a lepton doublet (ν ℓ^-) and another doublet (L^+ N) with left hypercharges $-\frac{1}{2}$ and $\frac{1}{2}$ and masses m and M respectively. We assume for simplicity

that the masses in the second doublet are identical, whereas in the first doublet the neutrino mass m_ν may be different from the lepton mass m. The second doublet replaces 3 colored quark doublets of left hypercharge 1/6, which would of course be the case in the standard model, with only one generation.

The Lagrangian for this model is that of the standard model with the above fermion content. We just recall here that $W^\pm$ couple purely lefthanded, and the couplings of the Z and γ to fermions are given by

$$\mathcal{L}_I = eQ\bar{\psi}\gamma^\mu\psi A_\mu + \frac{g_2}{4\cos\theta}\ Z_\mu\bar{\psi}\gamma^\mu\Big\{(1-\gamma_5)\tau^3 - 4\sin^2\theta\ Q\Big\}\psi \tag{9}$$

For further simplicity, we choose $4\sin^2\theta = 1$, so that the coupling of the Z to ℓ^- and L^+ is purely axial, whereas the coupling to the neutral particles is purely left-handed.

Again, we are interested in exhibiting a growing dependence on M in the anomalous magnetic moment, this time of the neutrino. Since the operator and amplitude only involve external low energy fields and particles, the M-dependence must come from an internal $(L^+\ N)$ loop. When this loop has two (three) low energy legs, the graphs that contribute to the magnetic moment are exhibited in Fig. 2(a), 2(b), 2(c) (Fig. 2(d). When the number of external legs is larger than 3, no 2-loop graph can occur for this process. So all relevant graphs are in Fig. 2.

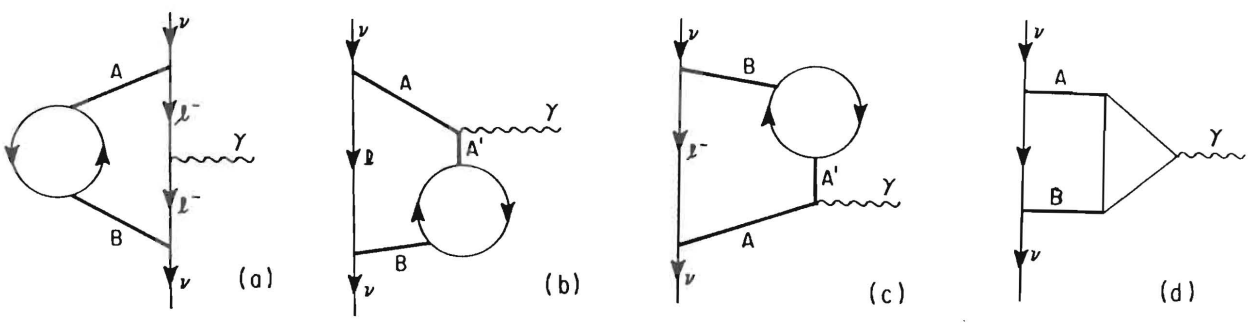

Fig. 2. Two loop anomalous magnetic moment contributions for large M.

In A and B, we may exchange all vector and Higgs bosons; however due to the fact that a neutrino is entering both in A and B vertices, both A and B photon contributions vanish. Since diagrams with $(A, B) = (W^+, W^-)$, (W^-, W^+), (Z, Z) involve only bounded couplings, standard decoupling occurs. Thus, these contributions will not lead to a growing M-dependence of the magnetic moment. Since W and Z are massive, there is also *no* logarithmic enhancement for small mass m from these graphs.

The graphs in Fig. 2(a,b,c) and the parity even part of Fig. 2(d) are M-dependent in view of the vacuum polarization insertions, and they are not related to the anomaly cancellation scheme. An involved argument, which will be presented elsewhere, shows that the dependence of these graphs is at most like ln M, *i.e.* of the same order as we shall find for the anomalous contributions from Fig. 2(d). Naive power of M contributions do not arise, because we shall always renormalize the theory perturbatively such that the physical masses of W and Z as well as the low energy fermions are kept fixed.

We now concentrate on the parity odd part of graphs Fig. 2(d). In view of the fact that this model is anomaly-free, the parity odd contribution in Fig. 2(d) are automatically transverse in the photon field, and $SU(2)_L \times U(1)_Y$ invariant. Also, it is clear that the contributions in Fig. 2(d) would cancel for fermions with opposite left hypercharges but with equal masses. Effects arising from Fig. 2(d) are thus characteristic of anomaly-induced effects, but with anomaly cancellation occurring at different mass-scales.

Finally, in view of our simplifying assumption that $4\sin^2\theta = 1$, a number of exchanges conveniently cancel by charge conjugation of the free fermion triangle (analogous to Furry's theorem). This leaves only a few graphs, with $(A, B) = (W^+, \phi^-)$, (W^-, ϕ^+), (ϕ^+, W^-), (ϕ^-, W^+) which indeed do not vanish.

Since we assumed identical masses of the two components in the $(L^+\ N)$ doublet, $\phi^\pm$ couples into the triangle as a pure pseudoscalar. By charge conjugation of the free fermion graph again, only the vector coupling of $W^\pm$ survives, and the triangle is again proportional to the $\pi^o \longrightarrow \gamma\gamma$ amplitude of (8).

This considerable simplification allows us to readily evaluate the parity odd part of graphs of Fig. 2(d), and we obtain their contribution to the neutrino magnetic moment, including one loop effects :

$$\mu = \frac{3eg_2^2\ m_\nu}{32\pi^2 M_W^2} + \frac{eg_2^4\ m_\nu}{1024\pi^4 M_W^2}\ \ln\frac{M}{M_W}$$

This is the proposed logarithmic enhancement.

5. Summary

We have presented examples in this paper of enhancements in low energy amplitudes due to fermions that produce a cancellation in gauge current anomalies at very different mass scales. Such enhancements would be absent if the anomaly cancellation took place between fermions of identical masses. There are also non-decoupling enhancements not related to the anomaly. To one loop these produce for example an enhancement of the W magnetic moment. To two loops, their contribution to the neutrino magnetic moment appears to be of the same order as the contribution from the anomalous diagrams.

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