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On the 545 keV Line in the Spectrum of the Crab Nebula

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The GRANAT group (R. Sunyaev *et al.*, Central Bureau of Astronomical Telegrams, International Astronomical Union, Circular No. 5481) recently reported the observation of a (545 ± 11) keV line in the spectrum of the Crab Nebula. It is tempting to associate this with the positron-electron annihilation line at 511 keV. If this line originates from some transition at the surface of the neutron star in the Crab, we expect a red shift of around 10% rather then the observed blue shift of ~ 35 keV; in fact, this gravitational red shift implies that the natural frequency of the observed line is also 10% higher, or around 600 keV. In this article we shall present a mechanism for generating such a split from 511 keV. For this mechanism to be successful we require: (a) surface magnetic fields, in the annihilation region, to be $(3-8) \times 10^{12}$ G and to point in our direction, (b) surface densities of $(10^5 - 10^6)$ g/cc. Both of these

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assumptions are consistent with pulsar models.

Models of pulsating neutron stars [1, 2] require surface magnetic fields $B = (10^{12} - 10^{13})$ G. Electrons in such fields arrange themselves into degenerate Landau levels [3] with quantum numbers n, m (m is the orbital angular momentum along the magnetic field) and energies

$$E_{n,m,s} = \sqrt{M^2 + (2n + 1 + s)|eB|}\,. \tag{1}$$

M is the mass of the electron and $s = \pm 1$ depending on whether the electron spin is along or opposite to the magnetic field. In the above, and subsequently, we shall ignore motion along the field lines. The energies of positrons in such a field are obtained from Eq. (1) by letting $s \to -s$ in the right hand side of that equation. For electrons the lowest energy state is the one with $n = 0,\ s = -1$ while the next highest energy state has quantum numbers $n = 0,\ s = +1$ or $n = 1,\ s = -1$;

$$\begin{aligned} E_{0,m,-1} &= M \\ E_{0,m,+1} &= \sqrt{M^2 + 2|eB|} \\ E_{1,m,-1} &= \sqrt{M^2 + 2|eB|}\,. \end{aligned} \tag{2}$$

The energies of the lowest positron levels are obtained by changing the signs of the spins. We propose that the observed line is due to the annihilation of a positron in the $(0, m, +1)$ state with an electron in the $(0, m, +1)$ state. (It will turn out that only states with $m = 0$ or $m = 1$ contribute significantly to this annihilation.) For this mechanism to be valid we require magnetic fields $B = 3 \times 10^{12}$ G, for no gravitational red shift or $B = 8.4 \times 10^{12}$ G for a 10% red shift. Consistent with magnitudes postulated to exist on the surface of pulsating neutron stars.

We take the positrons to be in the ground state and their positron density is assumed not to be high enough to fill more than one Landau level; any positrons in

higher energy levels will very rapidly decay into the ground state. How many levels the electrons fill up depends on the density. As no peak corresponding to annihilation with electrons in levels with energies $\sqrt{M^2 + 2|eB|}$ are seen this level must be empty. This forces the electron Fermi level, ϵ_F, to satisfy $M < \epsilon_F \leq \sqrt{M^2 + 2|eB|}$ and in turn the density, ρ, in units of g/cc, to be bounded by $10^5 \leq \rho \leq 3 \times 10^5$ for the case of no gravitational red shift and $5 \times 10^5 \leq \rho \leq 1.5 \times 10^6$ for a 10% red shift. These are densities conjectured to exist on the magnetic surface next to the outer crust of a neutron star. It still remains to be shown why no annihilation with electrons in the lowest state is observed. To answer this question we must examine some of the details of the annihilation process.

To lowest order in eB/M^2 the annihilation amplitude is [4]

$$A(n_p, m_p, s_p; n_e, m_e, s_e) = \int d^2P_e d^2P_p \; u^\dagger(s_p)\psi^*_{n_p,m_p}(P_p)\mathcal{M}(P_p, P_e)\psi_{n_e,m_e}(P_e)u(s_e)\,; \tag{3}$$

the u's are two component spinors, $\psi_{n,m}(P)$ is the momentum space wave function for the Landau level (n, m) and

$$\mathcal{M}(P_p, P_e) = \frac{\sigma_y}{2M^2}\Big[\sigma \cdot \epsilon_1 \; (\Pi_e - \Pi_p) \cdot \epsilon_2 + \sigma \cdot \epsilon_2 \; (\Pi_e - \Pi_p) \cdot \epsilon_1\Big]\,; \tag{4}$$

$\Pi_e = P_e - eA$, $\Pi_p = P_p + eA$, where A is the vector potential responsible for the magnetic field. Looking at the rotation (around the B axis) properties of the wave functions one notes that the terms proportional to Π_e connect a positron with $m = 0$ to an electron with $m = 1$ and the terms involving Π_p connect a positron with $m = 1$ to electrons with $m = 0$. Thus in the annihilation process the orbital angular momentum about the magnetic field changes by one unit, $\Delta l = \pm 1$. For the electron and positron spins aligned the change in the total angular momentum component along the magnetic field, j, satisfies $\Delta j = 0, \pm 2$ while for the spins opposite to each

other $\Delta j = \pm 1$. In the fist case both photons can emerge along the field direction, while in the second there must be a photon momentum component transverse to the field. Eq. (3) and Eq. (4) give explicit distributions consistent with these arguments. The photon angular distributions are:

$$d\Gamma(0,0,+1;0,1,-1) = 2C\cos^2(\theta)\sin^2(\theta)d\cos(\theta)\,;$$
$$d\Gamma(0,0,+1;0,1,+1) = C\cos^4(\theta)d\cos(\theta)\,; \tag{5}$$

the first set of indices refer to the quantum numbers of the positron while the second to those of the electron. θ is the angle of the emitted photon with respect to the magnetic field while C is a constant. The distribution for an annihilation of positrons with $m = 1$ and electrons with $m = 0$ has the same form. For directions along the magnetic field photons from annihilations with electrons whose spin quantum number $s = +1$ will predominate (in a 30° cone around the magnetic field the intensity of this line will be a factor of 14 larger than that for the line coming from annihilations with electrons in the lowest state). Thus if the magnetic field responsible for the splitting points in our direction, lines from annihilation of electrons and positrons with parallel spins will be much stronger than those from antiparallel spins and we will not see transitions with electrons in the ground state. As this annihilation occurs at the outer crust it may be susceptible to surface glitches and thus be episodic. Transitions between Landau levels should occur. These are the same as the cyclotron radiation observed for other systems [5]. For the present case these shoul be at 35 keV for the no red shift case and 80 keV for a 10% red shift. A simultaneous measurement of annnihilation lines and cyclotron lines will determine both the magnetic field and M_P/R_P of the pulsar.

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