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Comparisons of Multiparticle Distributions from $\bar{p}p, e^+e^-$, and NN Reactions with Exact Solutions from 1-d Ising Model

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ABSTRACT

We have calculated and derived explicit expressions in the 1-d Ising model for multiplicity distributions $P_{\delta\xi}(n)$ and factorial moments $F_q(\delta\xi)$. We identify the dynamical origins responsible for the form of $P_{\delta\xi}(n)$ and its important features that lead to universality $F_q(\delta\xi) = F_q[F_2(\delta\xi)]$. The predictions of the model compare well with the presently available data. We point out the important features that should be studied in future higher-energy experiments.

Introduction and Summary of Results

Multiparticle productions in hadronic interactions or in jet-hadronizations are effectively one-dimensional (1-d) distributions in the rapidity variable or its equivalence; thus we can study them in 1-d models. With the two interactions, nearest-neighbor (short-range) and an agitation, 1-d Ising provides the simplest dynamical model in which multiparticle distributions can be calculated. Indeed, we find that we can calculate explicitly all the 1-d multiparticle distributions. As reported in a previous publication[1],we found that the Ising model naturally gives universality $F_q(\delta\xi) = F_q[F_2(\delta\xi)]$ and derived their explicit expressions, where $\delta\xi$ is the interval of the 1-d variable (*e.g.* rapidity) and q positive integers.

Here we perform a different calculation in the 1-d Ising model and give explicit expressions for multiplicity distributions $P_{\delta\xi}(n)$, which provides the most basic information on the multiparticle distribution from the model. Other distributions like $F_q(\delta\xi)$ can be derived from it. Comparing $P_{\delta\xi}(n)$ with data gives more direct information then comparing its derived quantities with data. In addition, we identify the important characteristics of $P_{\delta\xi}(n)$ that can lead to the universality of $F_q[F_2(\delta\xi)]$ and show that there exit a large (actually infinite) number of functions for $P_{\delta\xi}(n)$ that can have universality and give similar numerical values. The advantages of our explicit Ising-model results are that we can identify and relate to the dynamical origins of the distributions explicitly.

After fixing the two parameters of the model by fitting with data $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$ at various values of $\delta\xi$, Figs.(1a) and (1b), the multiparticle distributions $P_{\delta\xi}(n)$ as functions of n are determined in the model and constitute as predictions of the model. They compare well with data, better for larger region of $\delta\xi$, Figs.(2a) and (2b). Interestingly the two dynamical parameters in the model become rather flat in their dependences on $\delta\xi$ for larger intervals of $\delta\xi \geq 3$. This implies that in the larger $\delta\xi$-intervals Ising model with constant interaction-parameters can also correctly describe the $\delta\xi$-dependences in $P_{\delta\xi}(n)$ and $F_q(\delta\xi)$; thus future data at higher energies, which make larger $\delta\xi$-intervals available, will provide further important information.

Our Ising results have no KNO scaling.[2] Negative-binomial (NB) distributions[3,4] have been popular in characterizing multiparticle distributions, but no explicit dynamical origin has been identified for them. Ising distributions $P_{\delta\xi}(n)$ are explicitly different functions from those of negative-binomials, though numerically they are similar in comparing with current data, see Figs.(2a) and (2b), and they share similar features that lead to universality in F_q.

From $P_{\delta\xi}(n)$, we can calculate factorial moments

$$F_q(\delta\xi) = [\langle n(n-1)\ldots(n-q+1)\rangle_{\delta\xi}]\ [\langle n\rangle_{\delta\xi}]^{-q}, \quad q = 1, 2, 3, \cdots, \tag{1}$$

where $\langle(\ )\rangle_{\delta\xi} \equiv \Sigma_{n=0}^{\infty}\ \{(\ )\ P_{\delta\xi}(n)\}$. These factorial moments F_q of multiparticle-productions were pointed out to be important to study in Ref.[5] and have been known in Refs. [6,7] to have the universal behavior: $F_q(\delta\xi) = F_q[F_2(\delta\xi)]$ independent of energies and reactions, though $F_2(\delta\xi)$'s are drastically different depending on reactions and energies. The data considered include $\bar{p}p(\sqrt{s} = 200$ to 900 GeV)[8], quark jets from $e^+e^-(\sqrt{s} = 91$ GeV)[9], as well as various nuclear reactions (p,^{16}O,^{32}S at 200 GeV/nucleon).[10] Here we compare these data with the parameter-free universal distributions $F_q[F_2(\delta\xi)]$ predicted from 1-d Ising model, see Figs.(3), (4a,b), (5a,b), and (6a,b). The agreement is excellent in the smaller $F_2(\delta\xi)$ region which comes from larger $\delta\xi$. We anticipate that this trend will continue at higher-energies where the available $\delta\xi$-interval will increase.

It is suprising, yet satisfying, that a simple model like Ising captures so many essential features of multiparticle distributions. Later we shall describe more specifically the important features to be studied in future higher-energy experiments.

Multiplicity Distributions from 1-d Ising Model

First we give a brief description of the 1-d Ising calculation. The Ising-model Hamiltonian is $H = -\varepsilon\Sigma_{\langle i,j\rangle}^{N} S_i \cdot S_j - b\Sigma_i^N S_i$, where $\langle i,j\rangle$ means $j = i \pm 1$ and i is summed over all lattice sites N; ε signifies the strength of next-neighbor interactions and b the strength of an agitating field. Conventionally S_i represents the spin values $\pm\frac{1}{2}$ at site i. Defining $n_i \equiv (\frac{1}{2} + S_i) = \{^1_0$ we can interpret n_i as one or no particle production at site i. In this interpretation, b represents the agitation that produces particles and ε the short-range next-neighbor interaction among the particles.

The multiplicity distribution in a sublattice N/M is $P_M(n) = (1/Z)\{\Sigma_{\{n_i|\Sigma n_i = n\}} e^{-\beta H}\}$ where $\{n_i|\Sigma n_i = n\}$ means summing over all possible configurations of n_i in the sublattice N/M with the constraint $\Sigma n_i = n$; and $Z \equiv \Sigma_{\{n_i\}} e^{-\beta H}$ is the partition function. Note that $\Sigma_{n=0}^{N/M} P_M(n) = 1$ and $Z = \Sigma_{n=0}^{N/M}\{\ \}$-part of $P_M(n)$. After some calculations, we obtain

$$P_M(n) = (1/z_M)\{\ \gamma^n\ \Sigma_{l=1}^{n}\ C_{l-1}^{n-1}\ [\ (N/M)\ C_{l-1}^{(N/M)-n-1}\ e^{-4\beta\varepsilon l}\ l^{-1}\ ]\ \}, \tag{2}$$

where $\gamma = e^{2\beta b}$; $z_M = [\Sigma_{n=0}^{N/M}\ \{\ \}$-part of Eq.(2)$] = \{(1+\gamma)/2 + [(1-\gamma)^4/4 + \gamma e^{-4\beta\varepsilon}]^{1/2}\}^{N/M}$ $+\{(1+\gamma)/2 - [(1-\gamma)^4/4 + \gamma e^{-4\beta\varepsilon}]^{1/2}\}^{N/M}$ the normalization factor, which is related to Z by $z_M = e^{-\beta(\varepsilon-b)N/M}\ Z$; and $C_k^j = j!/k!/(j-k)!$. Due to the γ^n-dependence of $\{\ \}$-part of $P_M(n)$

in Eq.(2), we can calculate $\langle n\rangle_M$ and $\langle n^2\rangle_M$ by $\partial\gamma$-differentiating the explicitly-given z_M: $\langle n\rangle_M = \Sigma_{n=0}^{N/M} nP_M(n) = (1/z_p)\partial z_M/\partial\gamma$ and $\langle n^2\rangle_M = \Sigma_{n=0}^{N/M} n^2 P_M(n) = (1/z_M)\partial^2 z_p/(\partial\gamma)^2 + (1/z_M)\partial z_p/\partial\gamma$, which are functions of $\beta\varepsilon, \beta b$ and N: $\langle n\rangle_M = (N/2M) + (N/2M)cos(2\phi)\ (1 - a^{N/M})\ (1+a^{N/M})^{-1}$ and $\langle n^2\rangle_M = \langle n\rangle_M^2 + (N/M)^2 cos^2(2\phi) a^{N/M}\ (1+a^{N/M})^{-2} + (N/4M)sin^2(2\phi)\ (1+a)(1-a)^{-1}\ (1-a^{N/M})\ (1+a^{N/M})^{-1}$ with $2\phi \equiv tan^{-1}[e^{-2\beta\epsilon}\ sinh^{-1}(\beta b)]$ and $a \equiv \{cosh(\beta b) - [sinh^2(\beta b) + e^{-4\beta\epsilon}]^{1/2}\}\ \{cosh(\beta b) + [sinh^2(\beta b) + e^{-4\beta\epsilon}]^{1/2}\}^{-1}$.

To make the connection with multiparticle productions, we take the thermodynamical (continum) limit $N \to \infty$ with fixed $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$, which result in two finite-parameters $A_{\delta\xi} = Ne^{(-4\beta\epsilon)}$ and $B_{\delta\xi} = -\beta b$, (notice that $a < 1$ and all $a^{N/M} \to 0$).[1] Further we interprete[11] $M = \Delta\xi/\delta\xi$ and obtain

$$\langle n\rangle_{\delta\xi} = A_{\delta\xi}\ [2sinh(B_{\delta\xi})]^{-2}\ (\delta\xi/\Delta\xi) \quad , \quad \langle n^2\rangle_{\delta\xi} = \langle n\rangle_{\delta\xi}^2 + \langle n\rangle_{\delta\xi}\ coth(B_{\delta\xi})\ ; \tag{3}$$

and

$$P_{\delta\xi}(n) = (1/z_p)\ \{\ \gamma^n\ \Sigma_{l=1}^{n}\ C_{l-1}^{n-1}\ [\ c^l\ /\ l!\ ]\ \}\ , \tag{4}$$

$$z_p = exp[\gamma c/(1-\gamma)]\ , \tag{5}$$

where $z_p = z_M$ at $N \to \infty$; using Eq.(3) γ and c can be expressed in terms of $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$: $\gamma = e^{2\beta b} = [\langle n^2\rangle_{\delta\xi} - \langle n\rangle_{\delta\xi}^2 - \langle n\rangle_{\delta\xi}][\langle n^2\rangle_{\delta\xi} - \langle n\rangle_{\delta\xi}^2 + \langle n\rangle_{\delta\xi}]^{-1}$ and $c = 4\,\langle n\rangle_{\delta\xi}^3\,[\langle n^2\rangle_{\delta\xi}^2 - 2\,\langle n^2\rangle_{\delta\xi}\,\langle n\rangle_{\delta\xi}^2 + \langle n\rangle_{\delta\xi}^4 - \langle n\rangle_{\delta\xi}^2]^{-1}$.

Next we determine from data $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$, thus $A_{\delta\xi}$ and $B_{\delta\xi}$ in Eq.(3). Figs.(1a) and (1b) show the data $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$ from $\bar{p}p$ reactions[8]; Figs.(1c) and (1d) show the corresponding values of $A_{\delta\xi}$ and $B_{\delta\xi}$ obtained. For e^+e^- and NN reactions we can not make such detail analysis since only $\langle n\rangle_{\Delta\xi}$ and $\langle n^2\rangle_{\Delta\xi}$ are available, values shown in Fig.(3); thus we can only obtain $A_{\Delta\xi}$ and $B_{\Delta\xi}$. Fortunately, for analysing universality $F_q[F_2(\delta\xi)]$, we need only data of F_q and F_2, not those of $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$.

Predictions, Comparisons, and Outlook

Once the two parameters $A_{\delta\xi}$ and $B_{\delta\xi}$ are fixed via $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$, Eqs.(4) and (5) specify completely the multiparticle distributions from the 1-d Ising model; thus they constitute as predictions of the model. The n-distributions of $P_{\delta\xi}(n)$ given in Eq.(4) are shown in Figs.(2a), (2b) together with data from $\bar{p}p$ reactions. The results are good, better for larger $\delta\xi$.[13]

It is interesting to note in Figs.(1c), (1d) that $A_{\delta\xi}$ and $B_{\delta\xi}$ flaten out in the larger $\delta\xi$ region, $\delta\xi \geq 3$, which means that the Ising model actually can also correctly describe the

$\delta\xi$-distribution from its phase-space factor $M = \Delta\xi/\delta\xi$ alone. This feature is important to be checked out in future higher-energy data where larger $\delta\xi$ intervals will become available.

We note from Eq.(4) that Ising $P_{\Delta\xi}(n)$ has no KNO scaling,[2] *i.e.* $[\langle n\rangle_{\Delta\xi} P_{\Delta\xi}(n)]$ is not a scaling function of $[n/\langle n\rangle_{\Delta\xi}]$ though approximate KNO scaling can be true for some limited energy-range. Clearly the function forms of our Ising $P_{\delta\xi}(n)$ in Eqs.(4) and (5) are different from those of negative-binomial[14], though numerical comparison at present energies are quite similar, see dashed lines in Figs.(2a) and (2b).

From the $P_{\delta\xi}(n)$'s of Eq.(4) we can calculate $F_q(\delta\xi)$. Due to the γ^n-dependence of the $\{\ \}$-part in $P_{\delta\xi}(n)$ and $z_p = \Sigma_{n=0}^{\infty}\ \{\ \}$-part of Eq.(4), we obtain the neat results $\langle n(n-1)\cdots(n-q+1)\rangle_{\delta\xi} = (1/z_p)\gamma^q[\partial^q z_p/(\partial\gamma)^q]$; together with the nice characteristics of z_p in Eq.(5), satisfying $\partial z_p/\partial\gamma = g\ z_p$, we have

$$F_q(\delta\xi) = [\frac{\gamma^q}{z_p}\frac{\partial^q z_p}{(\partial\gamma)^q}][\frac{\gamma}{z_p}\frac{\partial z_p}{\partial\gamma}]^{-q} = [z_p\ g^q]^{-1}[\frac{\partial^q z_p}{(\partial\gamma)^q}]\ , \tag{6}$$

which can be easily calculated to be

$$F_q[F_2(\delta\xi)] = 1 + \Sigma_{l=1}^{q-1}\frac{(q-1)!q!}{l!(q-l-1)!(q-l)!2^l}[F_2(\delta\xi)-1]^l\ ,$$
$$[F_2(\delta\xi)-1] = 2\frac{1-e^{-2B_{\delta\xi}}}{A_{\delta\xi}}\frac{\Delta\xi}{\delta\xi}\ , \tag{7}$$

i.e. all the dependences on the three parameters of the model, $A_{\delta\xi}, B_{\delta\xi}$, and $\delta\xi/\Delta\xi$, are absorbed in F_2. That is why to analyze universality we do not need explicit information on $A_{\delta\xi}$ and $B_{\delta\xi}$.

Since $\langle n(n-1)\cdots(n-q+1)\rangle_{\delta\xi}$ is also the fully-integrated q-particle inclusive crosection in the interval $\delta\xi$, the universality $F_q[F_2(\delta\xi)]$ implies that all fully-integrated multiparticle inclusive crosections are functions of fully-integrated 2-particle inclusive crosection in the interval $\delta\xi$.

This universality, Eq.(7), was obtained in Ref.[1] from a different calculation, bypassing the calculation of $P_{\delta\xi}(n)$. In the present calculation, we first obtain $P_{\delta\xi}(n)$ and identify the characteristics in $P_{\delta\xi}(n)$ that leads to universality in F_q. Once the final expression in Eq.(6) is reached, we can easily show that universality is true as long as $[g^{-3}\partial^2 g/(\partial\gamma)^2]$ is an arbitry function of $[g^{-2}\partial g/\partial\gamma]$. Ising model gives explicitly $g = c/(1-\gamma)^2$ which gives a particular form of that function

$$[g^{-3}\partial^2 g/(\partial\gamma)^2] = (3/2)[g^{-2}\partial g/\partial\gamma]^2. \tag{8}$$

Negative-binomial distributions satisfy a corresponding equation like Eq.(8) but with 3/2 on the right-hand-side replaced by 2.[14] Therefore we see that mathematically it is rather easy

to obtain universality, as long as we have the following three ingredients: γ^n-dependence in $P_{\delta\xi}(n)$, $\partial z_p/\partial\gamma = g\ z_p$, and $[g^{-3}\ \partial^2 g/(\partial\gamma)^2]$ = arbitrary function of $[g^{-2}\ \partial g/\partial\gamma]$; but the Ising dynamics picks a particular way. The agitation force b is responsible for the particle production and the form of the interactions in productions and among the particles are responsible for the shape of the distributions and universality.

The universal curves of $F_q[F_2(\delta\xi)]$ given in Eq.(7) for $q = 3, 4, 5$ are plotted in solid lines in Figs.(4a), (5a), and (6a). The agreement with tha data is suprisingly good (excellent for smaller $F_2(\delta\xi)$ region), considering they being parameter-free predictions. From data-points of F_2 in Fig.(3) we can also obtain the Ising predictions of $F_q(\delta\xi) = F_q[F_2(\delta\xi)]$ of Eq.(7). They are shown as solid lines in Figs. (4b), (5b), and (6b).

Finally we make a few concluding remarks. The important future tests on the 1-d Ising results are in the larger $\delta\xi$ regions. If the trend of constancy of $A_{\delta\xi}$ and $B_{\delta\xi}$ continues as available $\delta\xi$-interval increases at higher-energies, the $\delta\xi$-dependence in $F_q(\delta\xi)$ can also be anticipated : as $\delta\xi \to \infty$, from Eq.(3) $\langle n\rangle_{\delta\xi} \sim \delta\xi$; from Eq.(7) $[F_2(\delta\xi) - 1] \sim (\delta\xi)^{-1}$ and all $F_q(\delta\xi) \to 1$; they are shown as dash-dot lines in Figs.(3), (4b), (5b), and (6b) for the $\bar{p}p$ reactions, in particular they are straight lines in Fig.(3) showing $ln[F_2(\delta\xi) - 1]$ vs $ln\delta\xi$. As F_2 decreases for larger $\delta\xi$, there will also be more region in F_2 to check universality $F_q[F_2]$. We hope that soon $\langle n\rangle_{\delta\xi}$, $\langle n^2\rangle_{\delta\xi}$, and $P_{\delta\xi}(n)$ will also be provided from other reactions besides $\bar{p}p$, so full analysis as shown in Figs.(1) and (2) can also be done. It will be interesting to see if the gluon jets will have the same universal $F_q[F_2(\delta\xi)]$ as the quark jets. We need better measurements of F_q in high energy nuclear reactions, which can be made in future relativistic heavy-ion experiments.

Indeed, we look forward to more higher-energy multiparticle-production data in different reactions to be further studied and compared with these results from 1-d Ising model.

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Figure Captions

Fig.(1a) $\langle n \rangle_{\delta\xi}$ vs rapidity-interval $\delta\xi$; data are from Ref.[8].

Fig.(1b) $\langle n^2 \rangle_{\delta\xi}$ vs rapidity-interval $\delta\xi$; same data as in Fig.(1a).

Figs.(1c), (1d) $A_{\delta\xi}$ and $B_{\delta\xi}$ of Eq.(3) determined from data $\langle n \rangle_{\delta\xi}$ and $\langle n^2 \rangle_{\delta\xi}$ of $\bar{p}p$ shown in Figs.(1a) and (1b).

Figs.(2a), (2b) $P_{\delta\xi}(n)$ as function of n : Data are from Ref.[8]. Solid lines are predictions of Ising model; and the dashed lines are from the NB distributions.

Fig.(3) $[F_2(\delta\xi) - 1]$ as function of $\delta\xi$; respectively $\delta\xi$ represents rapidity-interval δy for e^+e^- quark-jet data; pseudorapidity-interval $\delta\eta$ for $\bar{p}p$ data; or $\delta\eta \cdot \delta\phi$ for nuclear data, where ϕ is the azimuthal angle. Data are from Refs. [8-10]. The dash-dot lines, which are straight lines in this $ln[F_2(\delta\xi) - 1]$ vs $ln\delta\xi$ plot, are anticipated Ising results for $\delta\xi > 10$ assuming that $A_{\delta\xi}, B_{\delta\xi}$ will stay constant at their present values at $\delta\xi = 10$ as shown in Figs.(1c) and (1d).

Fig.(4a) Universal curve of $F_3[F_2]$ as function of F_2 : Data are from Refs. [8-10], denoted the same way as specified in Fig.(3). Notice that all data fall so closely on a universal curve that in order to see clearly the data from $\bar{p}p$, e^+e^- quark-jets, and nuclear reactions, we need to display them separatly. The solid lines are Ising predictions Eq.(7), with no free parameters. As stated in the text, important future tests of the model lie in the smaller F_2 and larger $\delta\xi$ region.

Fig.(4b) $F_3[\delta\xi]$ from the same data as indicated in Fig.(3) for $F_2(\delta\xi)$. The solid lines are predictions from Ising universality Eq.(7) for $F_q[F_2(\delta\xi)]$ and data points of $F_2(\delta\xi)$ shown in Fig.(3). To avoid cluttering the figure with more points, we show in solid lines $F_q[F_2(\delta\xi)]$ as function of $\delta\xi$ obtained by connecting the central points of the data $F_2(\delta\xi)$ shown in Fig.(3) and substituting them in Eq.(7). As stated in the text, important tests of the model lie in the larger $\delta\xi$ region which will be available as energy increases. The dash-dot lines are anticipated Ising results for $\delta\xi > 10$ assuming that $A_{\delta\xi}, B_{\delta\xi}$ will stay constant at their values at $\delta\xi = 10$ as shown in Figs.(1c) and (1d).

Figs.(5a), (6a) Same as Fig.(4a), except for $F_4[F_2], F_5[F_2]$, respectively.

Figs.(5b), (6b) Same as Fig.(4b), except for $F_4[\delta\xi], F_5[\delta\xi]$, respectively.

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11. Note that if $A_{\delta\xi}$ and $B_{\delta\xi}$ are constant in $\delta\xi$, especially in the small $\delta\xi$ regions, Ising results do give intermittent behavior $F_q(\delta\xi) \to (1/\delta\xi)^{q-1}$ as $\delta\xi \to 0$, see Eq.(7). In Ref.[1] we tried to obtain the $\delta\xi$ dependence by interpreting $M = (\Delta\xi/\delta\xi)^{1/\alpha}$ and fitted data to find out the value of α. Here we have changed the strategy. Rather than presuppose that all the $\delta\xi$-dependence is in M, we leave it open and find out from data what are the dependence on $\delta\xi$ in the two parameters of the model $A_{\delta\xi}$ and

$B_{\delta\xi}$ when we take the most natural interpretation $M = (\Delta\xi/\delta\xi)$. In doing so we find, see Figs.(1c) and (1d), that $A_{\delta\xi}$ and $B_{\delta\xi}$ have rather strong dependence on $\delta\xi$ in the small $\delta\xi$ region; thus in this approach, we can not discuss the intermittent behaviors. In any case, the existence of intermittency in multiparticle-particle-production is hard to establish due to the following intrinsic difficulties: the error in data increases as $\delta\xi$ decreases; as $\delta\xi$ decreases to a value such that the average number of particles produced in that width of $\delta\xi$ become comparable or smaller than q, *i.e.* $\langle n\rangle_{\Delta\xi}\cdot(\delta\xi/\Delta\xi) \lesssim q$, the factorial moments $F_q(\delta\xi)$ is no more a good quantity to measure fractal behavior. (This is the same situation as measuring coast line with a ruler smaller than the wavelength of the waves at the shore.) Such difficulties will not be overcome even if the energies of reactions increase since that will only increase the range of rapidity but not improving the errors in small-$\delta\xi$ measurements of F_q nor increasing the number of particles in given $\delta\xi$. That is why there are many different opinions on intermittence studies;[1,12] thus here we focus and emphasize the larger $\delta\xi$ region.

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13. It is forseeable that the agreement of the 1-d Ising results with data can be improved by fine-tuning the model, *e.g.* including the clustering effects; varing ε to include site-dependence in ε; or to introduce some next-nearest neighbor interactions.

14. The negative binomial distribution[3] is given by

$$P^{NB}_{\delta\xi}(n) = (1/z^{NB}_p)\{(\gamma^{NB})^n[(n+k_{\delta\xi}-1)!][n!(k_{\delta\xi}-1)!]^{-1}\},$$

where $\gamma^{NB} = \langle n\rangle_{\delta\xi}/(k_{\delta\xi}+\langle n\rangle_{\delta\xi})$, $(k_{\delta\xi})^{-1} = \langle n^2\rangle_{\delta\xi}/\langle n\rangle^2_{\delta\xi} - 1/\langle n\rangle_{\delta\xi} - 1$ and $z^{NB}_p = (1-\gamma^{NB})^{-k_{\delta\xi}}$, where $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$ are supposed to be determined from experiments. Our Ising formulation has exactly the same degrees of freedom determined from data, but the dependences of $P_{\delta\xi}(n)$ on $\langle n\rangle_{\delta\xi}$ and $\langle n^2\rangle_{\delta\xi}$ are very different. Following similar discussions as for the Ising case, one can easily derive the factorial moments $F^{NB}_q(\delta\xi)$ using a equation exactly like Eq.(6) and obtain the explicit expressions for universality $F^{NB}_q[F^{NB}_2(\delta\xi)]$. This universality in NB can be traced to the specific form of z^{NB}_p which leads a equation similar to Eq.(8), see the discussions in the text after Eq.(8).

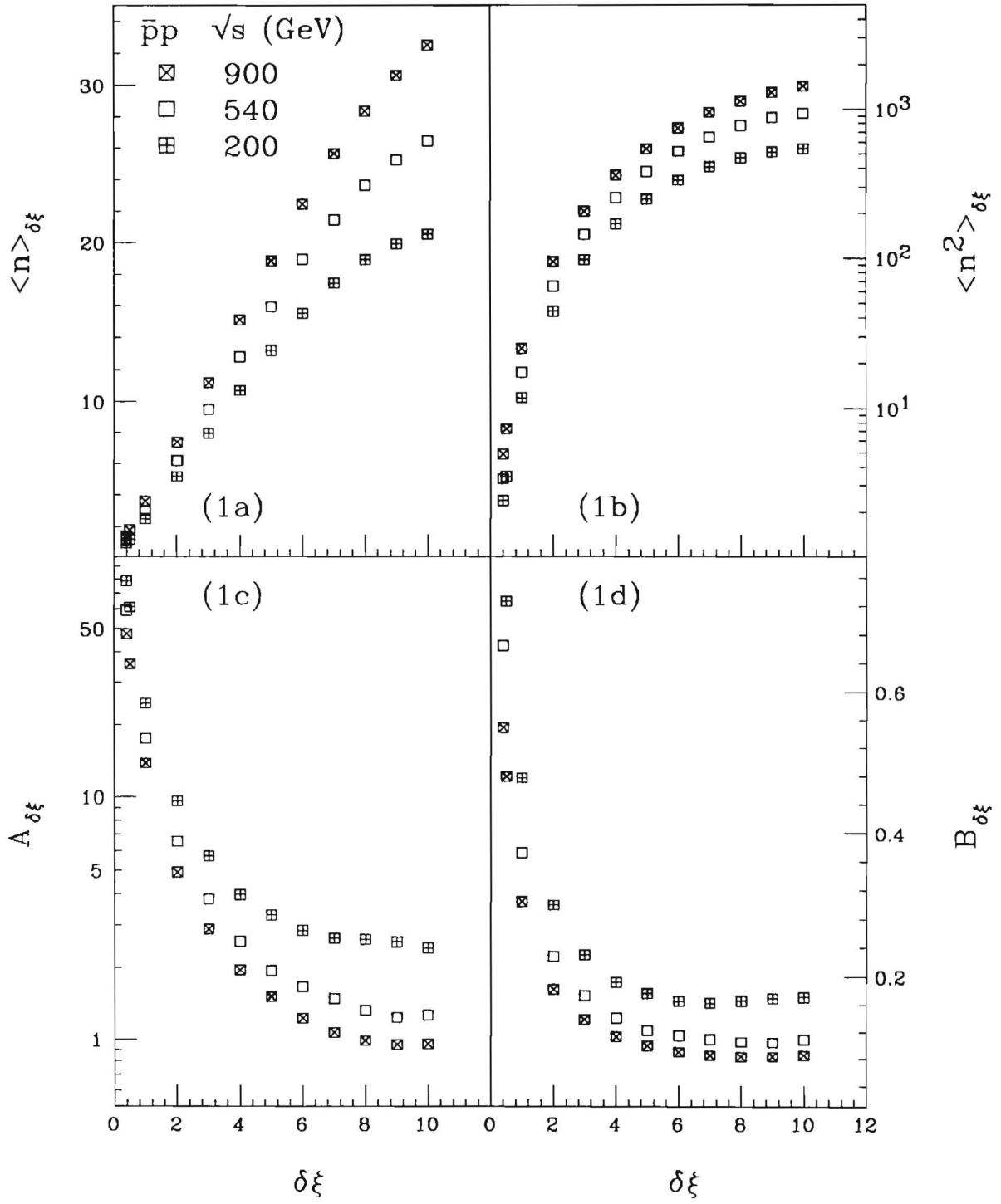

FIG. (1)

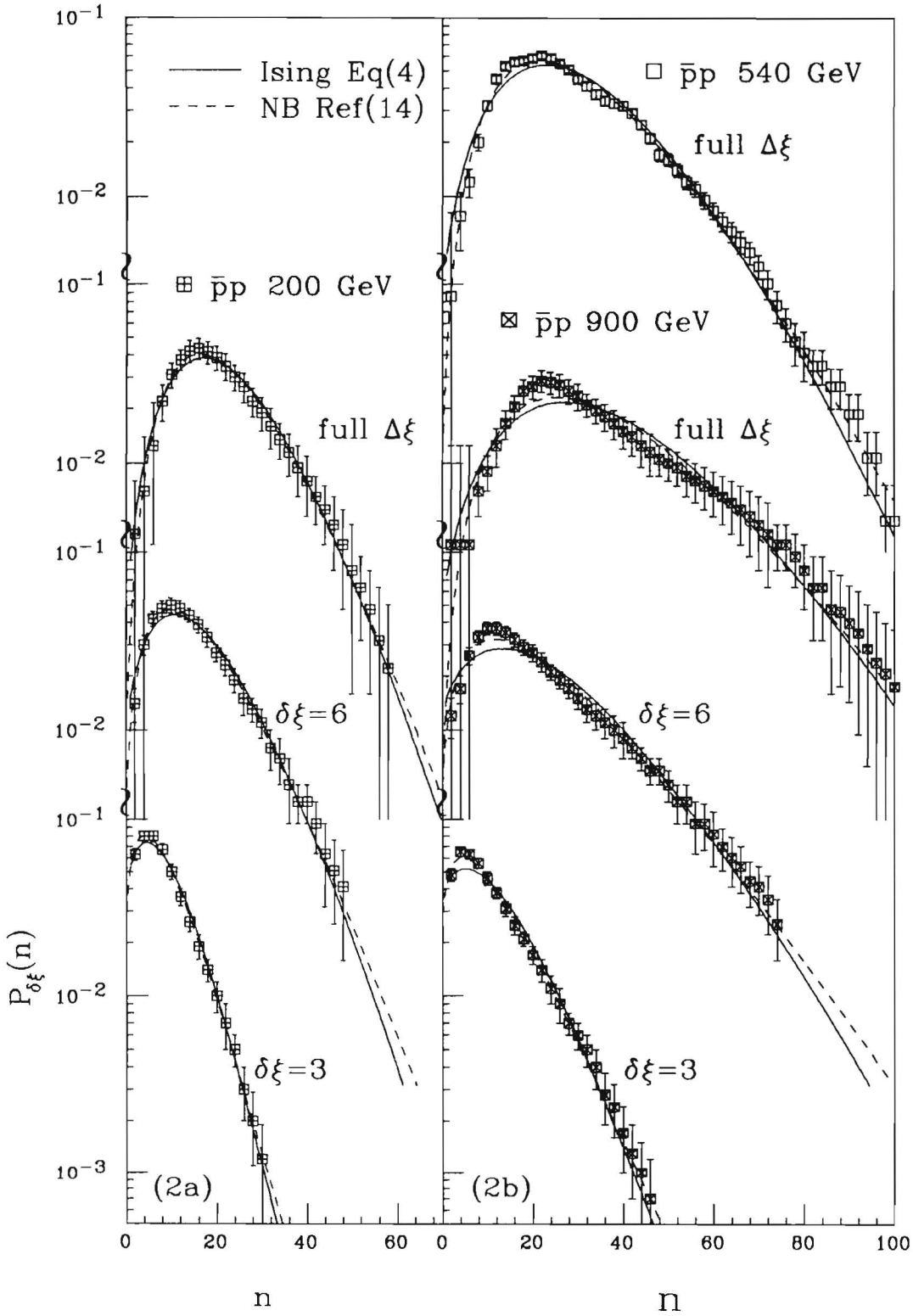
Ising Eq(4)
NB Ref(14)
p̄p 200 GeV
p̄p 540 GeV
p̄p 900 GeV
full Δξ
δξ=6
δξ=3
(2a)
(2b)
P_δξ(n)
n
n

FIG. (2)

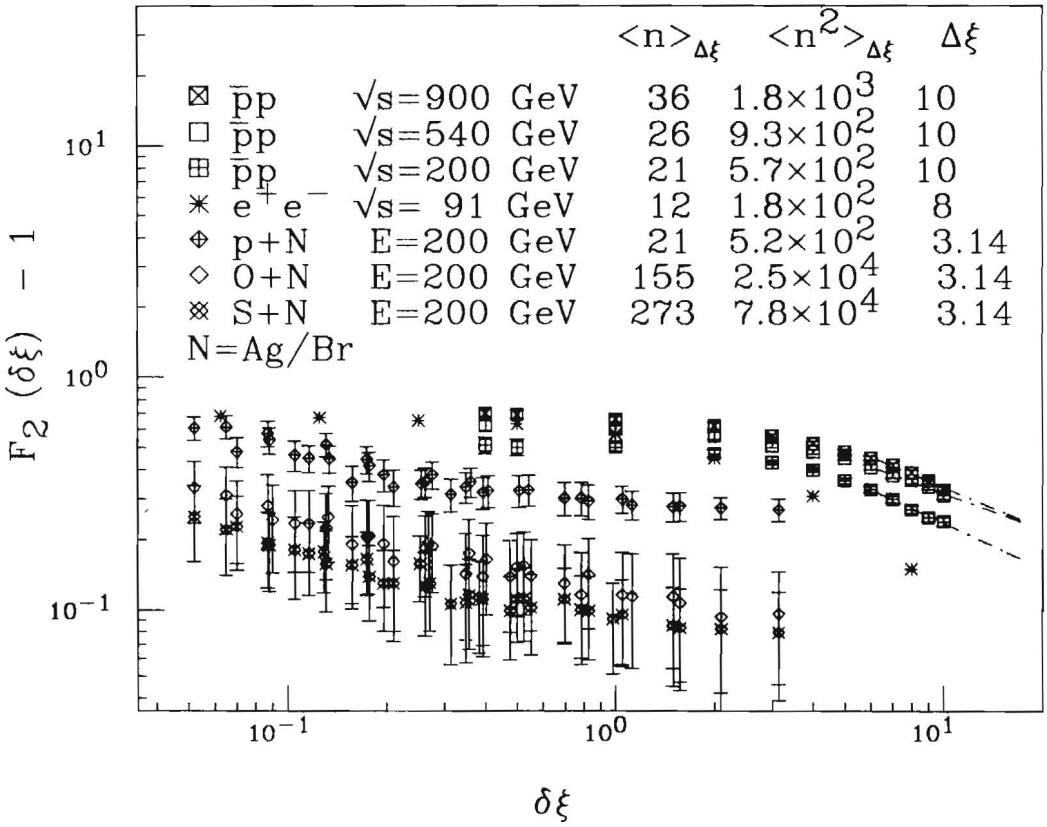
$\langle n \rangle_{\Delta\xi}$ $\langle n^2 \rangle_{\Delta\xi}$ $\Delta\xi$
$\bar{p}p$ $\sqrt{s}$=900 GeV 36 1.8×10^3 10
$\bar{p}p$ $\sqrt{s}$=540 GeV 26 9.3×10^2 10
$\bar{p}p$ $\sqrt{s}$=200 GeV 21 5.7×10^2 10
e^+e^- $\sqrt{s}$= 91 GeV 12 1.8×10^2 8
p+N E=200 GeV 21 5.2×10^2 3.14
O+N E=200 GeV 155 2.5×10^4 3.14
S+N E=200 GeV 273 7.8×10^4 3.14
N=Ag/Br
$F_2(\delta\xi) - 1$
10^1
10^0
10^{-1}
10^{-1}
10^0
10^1
$\delta\xi$

FIG. (3)

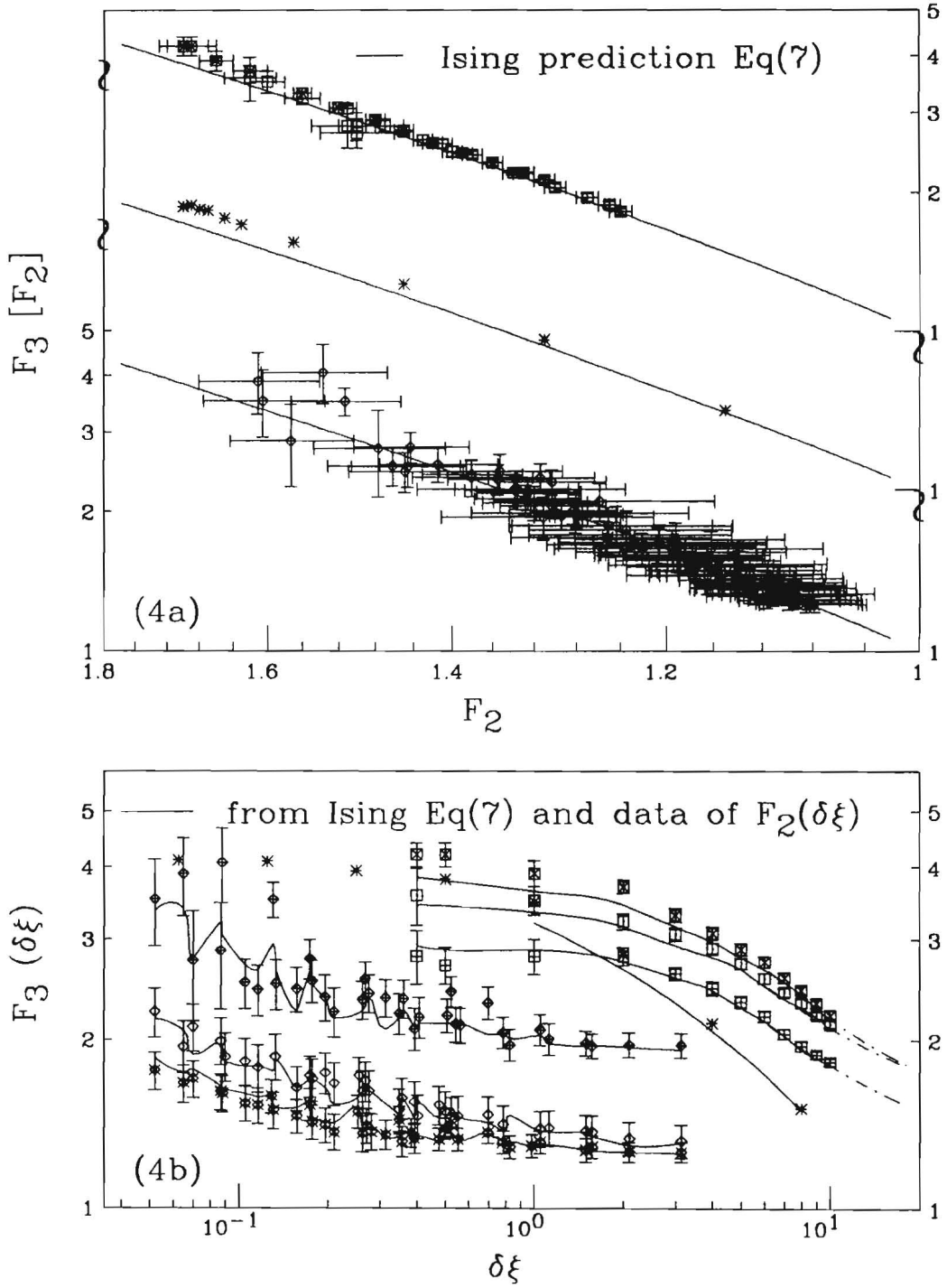
Ising prediction Eq(7)
(4a)
F_3 [F_2]
F_2
from Ising Eq(7) and data of $F_2(\delta\xi)$
(4b)
F_3 ($\delta\xi$)
$\delta\xi$

FIG. (4)

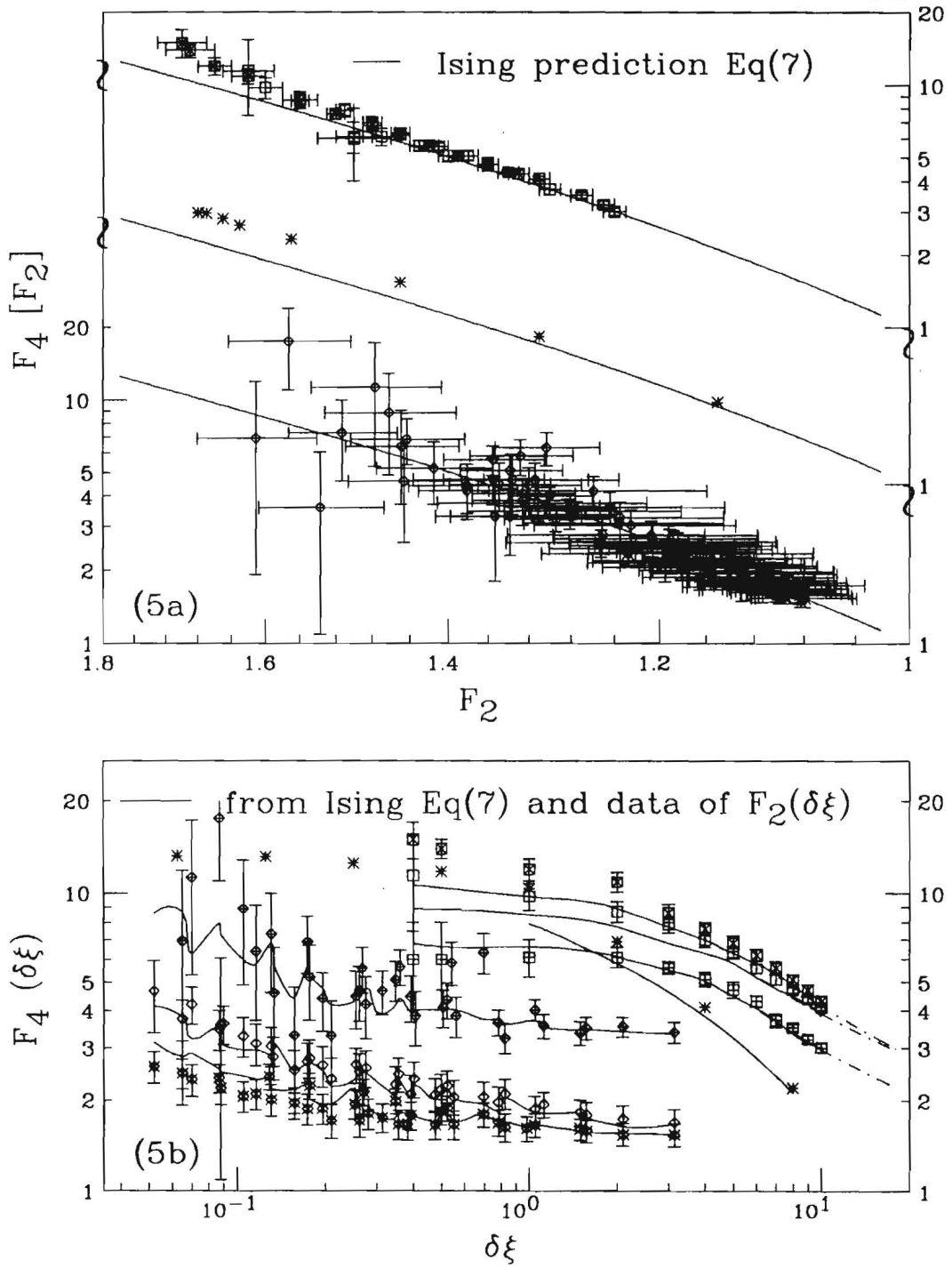
Ising prediction Eq(7)
F_4 [F_2]
F_2
(5a)
from Ising Eq(7) and data of $F_2(\delta\xi)$
F_4 ($\delta\xi$)
$\delta\xi$
(5b)

FIG. (5)

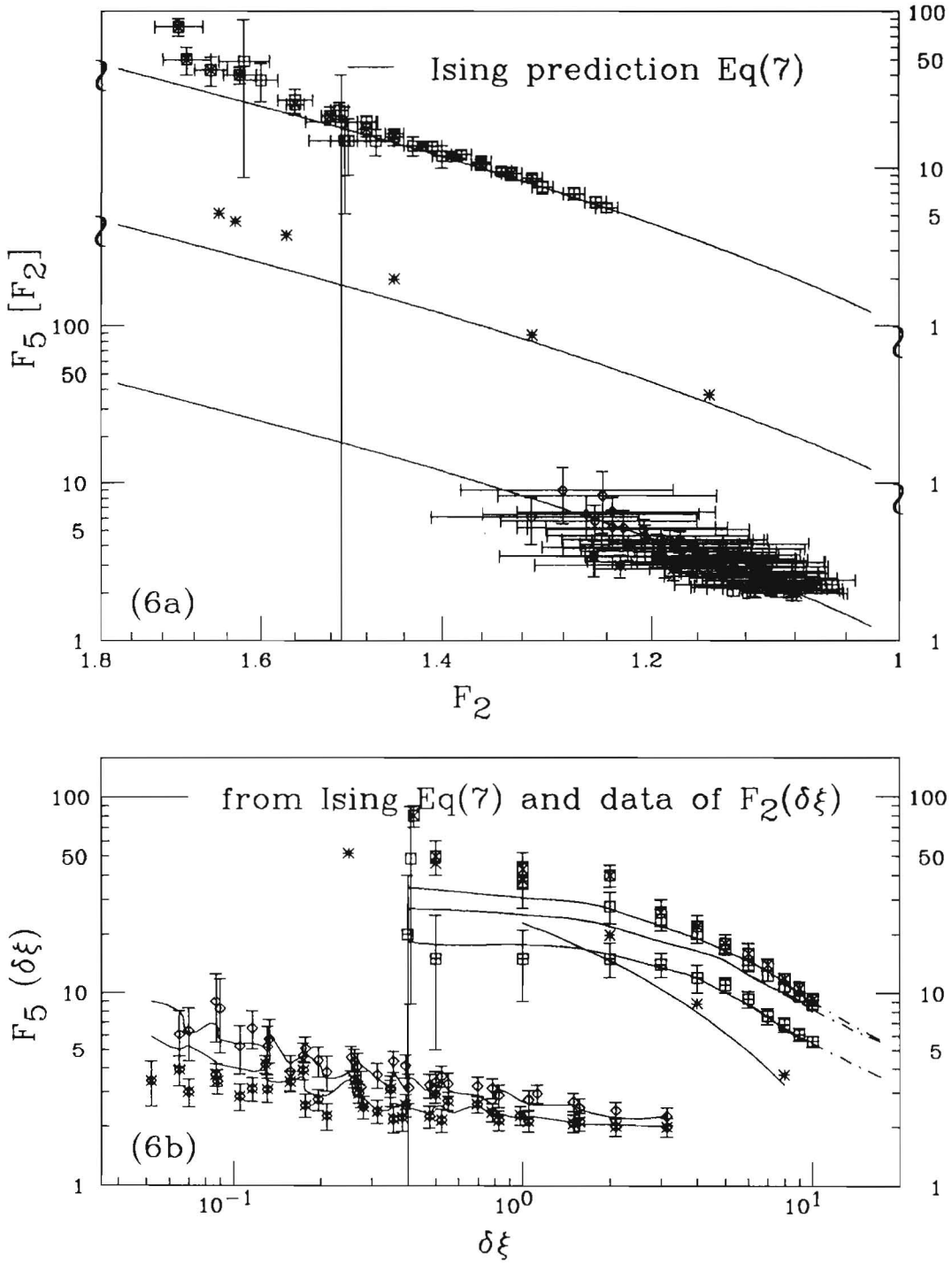
Ising prediction Eq(7)
(6a)
F_5 [F_2]
F_2
from Ising Eq(7) and data of $F_2(\delta\xi)$
(6b)
F_5 ($\delta\xi$)
$\delta\xi$

FIG. (6)