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Nonleptonic B, D, K Meson Decays and CP-Noninvariance

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ABSTRACT

An overview is given on the nonleptonic decays of the B, D, K mesons and their CP-noninvariance properties.

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I. Generic Ways to Study Nonleptonic Decays and CP-Noninvariance

In this section, I shall give some general remarks on generic ways of studying nonleptonic decays and CP-nonvariance via the Kobayashi-Maskawa (KM) scheme.

For a review see Ref. [1].

I.a. The Quark-Diagram Scheme

Nonleptonic weak decays, after taking out the first-order W-boson contribution, are essentially controlled by strong interactions. We do not have precise ways of calculating them. The explanation for $I = \frac{1}{2}$ dominance, which was observed almost 40 years ago in the suppression of $K^+ \to \pi^+\pi^o$, is still under debate. While we are trying to fully solve the strong interaction dynamics, it is useful to set up a framework, as model-independent as possible, to analyze these decays. This is the rationale for the quark diagram scheme. Since its introduction more than 10 years ago,[(2),(3)] it has proven to be very useful.

It has been shown that all nonleptonic meson decays can be expressed in terms of six quark-diagram amplitudes: $\mathcal{A}$, the external W-emission diagram; $\mathcal{B}$, the internal W-emission diagram; $\mathcal{C}$, the W-exchange diagram; $\mathcal{D}$, the W-annihilation diagram; $\mathcal{E}$, the horizontal W-loop diagram; and $\mathcal{F}$, the vertical W-loop diagram. These quark diagrams are specific and well-defined physical quantities. They are classified according to the topology of first-order weak interactions, but all QCD strong-interaction effects are included. (The reader can find figures for these amplitudes in Ref. [1] or Ref. [4])

It is important to emphasize that strong interactions do not alter the identity of these diagrams. Otherwise this scheme would be useless. The only approximations made are: restricting the weak interaction to first-order and assuming $SU(3)$ symmetry [which of course includes $SU(2)$]. These quark diagrams have a one-to-one correspondence to those amplitudes classified according to $SU(3)$ irreducible representations, which was shown in Ref. [2]. However, to describe a given decay, quark diagram amplitudes are much easier to write down than those $SU(3)$ amplitudes. Of course, for decay products involving only $SU(2)$ multiplets, one needs only to work with $SU(2)$ irreducible representations. Actually, these quark-diagram amplitudes are as well defined

as what we call the $\Delta I = 1/2, 3/2$ amplitudes for the K meson decays. But even in that simpler case the quark-diagram scheme works as well and can even bring in some new insight. For example, it was pointed out in Ref. [1] that the W-exchange diagram contributes the same way as the W-loop diagram (the Penguin) for the $\Delta I = \frac{1}{2}$ dominance in $K \to 2\pi$ and can be equally important. This has proven to be the case in lattice QCD calculations.

The quark-diagram scheme actually serves a purpose similar to the Fermi coupling constant G_F. When Fermi introduced G_F in 1932, what he did was to put down explicitly the known facts (*i.e.*, spin and Lorentz covariance) and parameterize the unknown by G_F. We hope that one day we will learn the precise content of the unknown in those quark diagrams, though they may not be as striking and as clear–cut as the intermediate boson propagator in G_F.

The quark-diagram scheme has so far served well in many ways:

(a) It has provided a framework with which to do model-independent data analysis and to make predictions from data.[(4)]

(b) It has provided a framework with which to make critical evaluations of model calculations.[(4),(5)] All existing theoretical model calculations are within the same general framework of approximations as the quark-diagram scheme (*i.e.*, first order in the weak interaction and using $SU(2)$ or $SU(3)$ symmetry); yet most model calculations pick only a very special set of Feynman diagrams to calculate. Without a general framework it is difficult to interpret or evaluate the comparisons of these model calculations with data. The quark-diagram scheme helps to pinpoint the difficulties of various model calculations and it should help future attempts to improve the calculations.

(c) The quark-diagram scheme also helps to provide a generic framework for analyzing CP-noninvariance. It is essential that these quark diagrams have a well-defined identity in order to discuss and calculate CP-noninvariance. A general survey using this method made was made in 1980.[(3)] It was pointed out then what the generic CP-noninvariance factor is in the KM scheme for decay-amplitude CP-noninvariance and the proper ways to see decay-

amplitude CP noninvariance: *i.e.*, via partial-decay-rate differences and tagged $K^0, \bar{K}^0$ decays. It was also pointed out that the partial-decay-rate differences can be large (as large as 10^{-1} in contrast to 10^{-2} to 10^{-3} in charm meson decays and in contrast to $|\epsilon'| \approx 10^{-3}|\epsilon| \approx 10^{-6}$ in K meson decays) in B decays, and that the search for the KM CP-noninvariance can be most fruitful in B decays. It has not generally been appreciated that the 10^{-2} suppression of the partial-decay-rate differences in D mesons comparing to those of B mesons is a generic result of the KM scheme, independent of model calculations. Most model calculations calculate specific sets of Feynmann diagrams and debate about which channels are important. We know that most of the state-of-the-art model calculations are unreliable. The experimental search for new decay-amplitude CP noninvariance should not rely upon these specific results of model calculations but should be based upon the generic observation that decay-amplitude CP noninvariance can be as large as fractional in B particle (mesons or baryons) decays and look for it in the channels easiest to search for experimentally. I will elaborate on these further in subsection I.b. and section V.

I.b. Invariants of the CKM Matrix and the Universal Decay-Amplitude CP-Nonvariance Factor X_{CP}

As was shown by Kobayashi and Maskawa (KM), the quark-mixing matrix for three generations of quarks, which we call the Cabibbo-Kobayashi-Maskawa (CKM) matrix, has four independent parameters: three angles and one phase. A very convenient way of parameterizing the CKM matrix was developed by L. Maiani, and by L.-L. Chau and W.-Y. Keung in Ref. [6] and has been adopted by the Particle Data Book:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}, \tag{I.1a}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_y & s_y \\ 0 & -s_y & c_y \end{bmatrix} \begin{bmatrix} c_z & 0 & s_z e^{-i\phi} \\ 0 & 0 & 0 \\ -s_z e^{i\phi} & 0 & c_z \end{bmatrix} \begin{bmatrix} c_x & s_x & 0 \\ -s_x & c_x & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$= \begin{bmatrix} c_x c_z & s_x c_z & s_z e^{-i\phi} \\ -s_x c_y - c_x s_y s_z e^{i\phi} & c_x c_y - s_x s_y s_z e^{i\phi} & s_y c_z \\ s_x s_y - c_x c_y s_z e^{i\phi} & -c_x s_y - s_x c_y s_z e^{i\phi} & c_y c_z \end{bmatrix},$$

$$\underset{\text{as } s_i \approx 0}{\approx} \begin{bmatrix} 1 & s_x & s_z e^{-i\phi} \\ -s_x - s_y s_z e^{i\phi} & 1 - s_x s_y s_z e^{i\phi} & s_y \\ s_x s_y - s_z e^{i\phi} & -s_y - s_x s_z e^{i\phi} & 1 \end{bmatrix}. \tag{I.1.b}$$

Of course physics does not depend upon how one makes the parameterization. However it is important to know what are the invariants which describe physical quantities and choose a parametrization that has the close relation to observables. This parameterization precisely has that important feature. It takes full advantage of the experimental information and that the phase factor appears in the smallest matrix element. Each sine is directly related to one type of experiments,

$$\begin{aligned} s_x &\approx |V_{us}| \approx |V_{cd}| \approx 0.22, \quad \text{determined from strange particle decays,} \\ s_y &\approx |V_{cb}| \approx 0.05, \quad \text{from the } b \text{ particle lifetime,} \\ s_z &\approx |V_{ub}| \approx 0.01, \quad \text{from the measurement of } (b \to u)/(b \to c), \end{aligned}$$

(the value of $|V_{ub}|$ is still under intensive study). These properties make the matrix easy to use.

Besides the absolute values of each matrix element $|V_{ij}|$, which are related to the rates of decays, there is one single universal invariant that captures the essence of the KM scheme for CP-noninvariance. In a general survey of decay-amplitude CP-noninvariance reported in the 1980 VPI conference[3] and later crystalized in Ref. [6], we showed that for three generations of quarks all decay-amplitude CP-noninvariance is proportional to one single universal invariant

parameter:

$$X_{CP} = I_m\left[V_{il}V_{jl}^*(V_{im}V_{jm}^*)^*\right], \tag{I.2a}$$
$$= I_m\left[V_{il}V_{im}^*(V_{jl}V_{jm}^*)^*\right], \quad i \neq j = u,c,t; \quad l \neq m = d,s,b,$$
$$= s_x s_y s_z s_\phi \; c_x c_y c_z^2 \approx s_x s_y s_z s_\phi. \tag{I.2b}$$

It can be shown, using this universality of X_{CP}, as discussed in Ref. [3] and in section V, that B decays are the fruitful reactions to search for decay-amplitude CP-noninvariance. This has also been substantiated by many explicit calculations.[8] I will elaborate more in section V.

The universal decay-amplitude CP-noninvariance factor X_{CP} was later noted by C. Jarlskog[9] to be also related to an invariant of the quark mass matrix.

The parameterization of Ref. [6] was inspired by Wolfenstein's parameterization, but satisfies exactly the unitarity constraints. One has to be cautious in using Wolfenstein's original parameterization; it can give wrong results in CP-noninvariance. For example, using the original Wolfenstein parameterization, ϵ' would not have a contribution from charm quark in the W-loop (Penguin) diagram; thus Gilman and Wise[7] would not have made the original observation that $\epsilon' \neq 0$ in the KM scheme! (I am glad to see that after my remark at the beginning of this conference, people began to pay attention to this fact and correction terms were put in for the Wolfenstein parameterization.)

The geometrical meaning of the sine of the phase ϕ, s_ϕ, was also given in Ref. [6]. It is the angle between the two complex numbers $V_{ud}V_{ub}{}^*$, and $(V_{cd}V_{cb}{}^*)^*$ of the triangle formed by the unitarity condition of the KM matrix:

$$V_{ud}V_{ub}{}^* + V_{cd}V_{cb}{}^* + V_{td}V_{tb}^* = 0, \tag{I.3}$$
$$s_\phi \approx I_m\left[V_{ud}V_{ub}{}^*(V_{cd}V_{cb}{}^*)^*\right]/(|V_{ud}|\,|V_{ub}|\,|V_{cd}|\,|V_{cb}|),$$
$$\approx X_{CP}/s_x s_y s_z \tag{I.4}$$

Due to the experimental accident that $|V_{cd}| \approx |V_{us}|$, it is also the angle between the two complex numbers $V_{us}V_{ub}{}^*$ and $(V_{cs}V_{cb}{}^*)^*$ of the unitary

triangle in the Gauss-Argonne plot formed by the three complex numbers in the constraint,

$$V_{us}V_{ub}{}^* + V_{cs}V_{cb}{}^* + V_{ts}V_{tb}^* = 0, \tag{I.5}$$

$$s_\phi \approx I_m\left[V_{us}V_{ub}{}^*(V_{cs}V_{cb}{}^*)^*\right]/(|V_{us}|\,|V_{ub}|\,|V_{cs}|\,|V_{cb}|),$$
$$\approx X_{CP}/s_x s_y s_z. \tag{I.6}$$

Actually there are altogether six such unitary conditions:

$$\sum_{i=u,c,t} V_{il}V_{im}^* = 0, \qquad l \neq m = s,d,b; \tag{I.7a}$$
$$\sum_{l=d,s,b} V_{il}V_{jl}^* = 0, \qquad i \neq j = u,c,t. \tag{I.7b}$$

Each equation gives one triangle with the area given by half of the products of the absolute values of two neighboring complex numbers and the sine of the angle between the two complex numbers. However, it takes a proof to show that all six of these triangles have the same area, and this was done in Ref. [6]. The universal area is precisely the expression of Eqs.(I.2) for X_{CP}, *i.e.*,

$$\text{Area} = \frac{1}{2}X_{CP}. \tag{I.8}$$

Each unitary triangle has three angles. So, altogether, there are 18 angles (some are accidentally equal according to experiments. See the two different expressions for s_ϕ given in Eqs. (I.4) and (I.6). These angles do not have fundamental importance. The important fact is that all six of these triangles have the same area ($= \frac{1}{2}X_{CP}$) and their edges are given by products of the absolute values of the KM matrix elements. Therefore all angles can be written as X_{CP} divided by products of four absolute values of the KM matrix element.

$$\sin\phi_i = X_{CP}/|V_{il}|\ |V_{jl}|\ |V_{im}|\ |V_{jm}|, \tag{I.9a}$$

or

$$= X_{CP}/|V_{il}|\ |V_{im}|\ |V_{jl}|\ |V_{jm}|, \tag{I.9b}$$

where $l \neq m = d, s, b;\ i \neq j = u, c, t$.

To conclude this subsection, we see that it is natural to pick the four independent invariants $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and X_{CP} for the four independent CKM parameters. They are the ones precisely picked in Ref. (6): s_x, s_y, s_z and $s_\phi \approx X_{CP}/s_x s_y s_z$.

I.b.2. Implications of Measuring Partial-Decay-Rate Asymmetries in $B^\pm$ Decays and in Neutral B Decays Such as $B^0, \bar{B}^0 \to K_S J/\psi$

Recently it has become very popular to discuss the partial-decay-rate differences of $B^o\bar{B}^o$ and even to design B factories for measuring them; for example, $B_d^o\bar{B}_d^o \to K_S J/\psi$ is the most popular. Indeed, assuming that the so called "Penguin polution" (what a down-turn of the fate of Penguin diagrams) is small (which is yet to be proven experimentally), the decay-rate asymmetry in the $Im(\Delta m \cdot t)$ term is:

$$\begin{aligned} a(K_S J/\psi) &= -I_m \left\{ \frac{q_B}{p_B} \frac{\bar{A}(\bar{B}^o \to K_S J/\psi)}{A(B^o \to K_S J/\psi)} \frac{q_K}{p_K} \right\} \\ &= I_m \left\{ \frac{V_{td}V_{tb}^*}{V_{td}^*V_{tb}} \frac{V_{cs}{}^*V_{cb}}{V_{cs}V_{cb}{}^*} \frac{V_{cs}V_{cd}{}^*}{V_{cs}{}^*V_{cd}} \right\} \\ &= \sin 2\phi_1, \end{aligned} \tag{I.10}$$

(see Ref. [10] and Lim's talk at this workshop). Therefore this decay becomes a measurement of the angle ϕ_1.

However, I must stress that without first establishing that the KM scheme is the valid one for CP-noninvariance, an assertion claiming that $a(K_S J/\psi)$ measures ϕ_1 is meaningless. Furthermore $a(K_S J/\psi) \neq 0$ does not establish the KM scheme for CP-noninvariance at all. It may come entirely from the mass-matrix part, *i.e.*, the (q/p) factors in Eq.(I.10).

To establish the KM scheme for CP-noninvariance unambiguously, we must observe pure decay-amplitude CP-noninvariance, with no matrix-matrix contribution mixed-in. It is essential first to unambiguously observe effects such as ϵ' of the K meson decays and the partial-decay-rate differences of $B^\pm$ meson decays. Therefore, for B factory builders, I must stress the importance of observing partial-decay-rate differences in $B^\pm$ decays. Furthermore, their measurements should be the first-generation experiments because of their experimental simplicity. They are minimum-bias experiments: no tagging and no time-evolution measurements are needed, in contrast to measuring the asymmetries in neutral B^o meson decays such as $B^o, \bar{B}^o \to K_S J/\psi$.

II. Nonleptonic Decays of D Mesons: from CKM Non- and Singly-Suppressed Decays to the Predictions of Doubly-Suppressed Decays

Here I shall briefly summarize our results from analyzing nonleptonic decays of D mesons using the quark-diagram scheme. I refer the reader to the original papers[4,11] for details.

II.a. Charm Meson Decays into Vector and Pseudoscalar Bosons, $D \to VP$

II.a.1. The CKM Non- and Singly- Suppressed $D \to VP$ Decays

Our analysis of recent data of charm mesons decaying into a vector meson and a pseudoscalar meson $D \to VP$ have established the following:[4]

- The final-state interactions are definitely important in $D \to \bar{K}^*\pi$, (as in the $D \to PP$ pseudo-pseudo meson decays).
- The $D^0 \to \phi\bar{K}^0$ decay is solely from the W-exchange diagram, with final-state interactions taken into consideration. Its observation and size shows that the W-exchange diagram $\underline{\mathcal{C}'}$ is significant.
- The $D^0 \to \bar{K}^{0*}\pi^0$ decays, $K^{*-}\pi^+$, associated with the measurements of $D^+ \to \phi\pi^+$ and $D_s^+ \to \phi\pi^+$, give the value of the W-exchange diagram $\mathcal{C}'$. But we found that $\mathcal{C}' \neq \underline{\mathcal{C}'}$, which implies important SU(3) symmetry breaking.
- From the $D_s^+ \to \bar{K}^{0*}K^+$, $D_s^+ \to \phi\pi^+$, $\bar{K}^{*0}\pi^+$, and $D^+ \to \phi\pi^+$ decays, we obtained the information that the W-annihilation diagram $\mathcal{D}$ is important.

- From the $D_s^+ \to \rho^+\pi^0$ decay it follows that $\mathcal{D} \approx \mathcal{D}'$.
- From the Fermilab E691 result that D_s^+ does not decay into $\pi^+\omega$, $D_s^+ \overset{\times}{\longrightarrow} \pi^+\omega$, it follows that the decay amplitudes have the following relations: $|\underline{\mathcal{D}}| >> |\mathcal{D}| \approx |\mathcal{D}'|$.
- Perturbative QCD calculations always give $|\mathcal{A}'| > |\mathcal{B}'|$ due to color suppression in the internal W-emission diagram $\mathcal{B}'$.[11],[12] The experimental fact that $|\mathcal{A}'| < |\mathcal{B}'|$ clearly indicates the importance of nonperturbative effects (e.g., soft-gluon exchange). The decay rates of some color-mismatched channels, e.g., $D^0 \to \omega\bar{K}^0$, $\phi\bar{K}^0$, $\pi^0\bar{K}^0$, and $D_s \to \bar{K}^{*0}K^+$, are not suppressed relative to that of color-matched modes. From the quark-diagram scheme we can predict that

$$Br(D^0 \to \phi\pi^0) = \frac{1}{2}(0.05)\,\frac{\Gamma(D^0)}{\Gamma(D^+)}\,Br(D^+ \to \phi\pi^+).$$

For other comments on QCD calculations,[11] see Ref. [5].

The values of the amplitudes and phase shifts are given next:

From the measured decay rates of $D_s^+ \to \phi\pi^+, D^+ \to \phi\pi^+, \bar{K}^{*0}\pi^+$ and $D^0 \to \phi\bar{K}^0$ we obtained the values for various quark-diagram amplitudes

$$\begin{aligned} &\mathcal{A}' = (2.01 \pm 0.33) \times 10^{-6}, \qquad \mathcal{B}' = -(3.12 \pm 0.44) \times 10^{-6}, \\ &|\underline{\mathcal{C}}'| = (1.77 \pm 0.21) \times 10^{-6}, \end{aligned} \tag{II.1}$$

where the primed amplitudes denote the case that the vector meson arises from the decay of the charm quark, and the under-lined amplitude refers to the amplitudes of graphs involving the creation of $\bar{s}s$. From the measurements of $D_s^+ \to \bar{K}^{*0}K^+$ and $D^0 \to K^{*-}\pi^+$, $\bar{K}^{*0}\pi^0$ we found

$$\begin{aligned} &\mathcal{C}' = -0.43 \times 10^{-6}, \qquad \underline{\mathcal{D}} = (1.43 \pm 0.46) \times 10^{-6}\ , \\ &\Delta_{K^*\pi} \equiv (\delta_{1/2} - \delta_{3/2})_{K^*\pi} = (97 \pm 13)^\circ\ . \end{aligned} \tag{II.2}$$

For unprimed amplitudes, three sets of solutions[4] can be derived from the observed rates of $D^+ \to \rho^+\bar{K}^0$, $D^0 \to \omega\bar{K}^0$, $\rho^0\bar{K}^0$ and ρ^+K^-. However,

the recent ARGUS result $Br(D^o \to K^o\bar{K}^{*0}) < 3 \times 10^{-4}$ implies $\mathcal{C} \sim \mathcal{C}'$. This favors the following solution:

$$\begin{aligned} &\mathcal{A} = (3.50 \pm 0.05) \times 10^{-6}, \qquad \mathcal{B} = -(1.97 \pm 0.12) \times 10^{-6}\ , \\ &\mathcal{C} = -(0.39 \pm 0.05) \times 10^{-6}, \qquad \Delta_{\rho K} \equiv (\delta_{1/2} - \delta_{3/2})_{\rho K} = (0 \pm 26)^\circ\ . \end{aligned} \tag{II.3}$$

The other two solutions give values for the amplitude $\mathcal{C}$ which are too large. The small rate for $D_s^+ \to \rho^+\pi^o$ indicates[4] $\mathcal{D} \approx \mathcal{D}'$. The recent E691 result for negligible $D_s^+ \to \pi^+\omega$ implies that $|\underline{\mathcal{D}}| >> |\mathcal{D}| \approx |\mathcal{D}'|$. Motivated by $\mathcal{C} \sim \frac{1}{4}\underline{\mathcal{C}}'$, we will assume that $\mathcal{D} \sim \mathcal{D}' \sim \frac{1}{4}\underline{\mathcal{D}}$ for the purpose of giving specific $D \to VP$ rates.

II.a.2. The CKM Doubly-Suppressed $D \to VP$ Decays

With the quark-diagram amplitudes and the isospin phase-shift differences given by Eqs.(II.1) - (II.3), we are able to compute the branching fractions for all doubly suppressed $D \to VP$ decays[13] except $D_s \to K^*K$.[14] The readers are referred to Ref. [13] for explicit results. Note that our predictions do take into account the effects of final-state interactions obtained from data analyses in the quark diagram scheme, as indicated by the phase shifts in Eqs. (II.2) and (II.3). Without final-state interactions, the predicted branching ratio for $D \to K^{*0}\pi^0$ will be smaller by a factor of three. Such final-state interaction effects are still beyond the capability of most theoretical model calculations.

It is also worth emphasizing that our analysis demonstrates the non-negligible effects from the W-annihilation diagram $\underline{\mathcal{D}}$ and the W-exchange diagrams, $\mathcal{C}$ and $\mathcal{C}'$, the so-called nonspectator diagrams, as evidenced by Eqs. (II.2) and (II.3), in contrast to most model calculations.[12],[15] For example, in our scheme $D^+ \to \phi K^+$ arises solely from the W-annihilation diagram $\underline{\mathcal{D}}$.

II.b. Charm Meson Decays into Pseudoscalar-Pseudoscalar Mesons, $D \to PP$

II.b.1. The CKM Non- and Singly- Suppressed $D \to PP$ Decays

From analyzing the recent data of charm mesons decaying into two pseudoscalar mesons, we have established the following:[4]

- The final-state interactions are important[10] in $D \to \bar{K}\pi$.
- The W-annihilation diagram $\mathcal{D}$ is important in $D_s \to PP$.
- The $D^0 \to K^0\bar{K}^0$ decay can only come from effects of final-state interactions and/or with SU(3) breaking. Its observation by Fermi lab experiments E400, E691 establish such effects.
- The loop-diagram contributions in D decays only occur in the SU(3)-breaking forms $(\mathcal{E}-\mathcal{E})$ and $(\mathcal{F}-\mathcal{F})$ because $V_{us}V_{cs}^* + V_{ud}V_{cd}^* \approx 0$. They are shown to be small by the $D^0 \to \bar{K}^0K^0$ measurement.

With the new data of $D^0 \to \bar{K}^0\eta'$, $D_s^+ \to \pi^+\eta$ and $D_s^+ \to \pi^+\eta'$, a unique set of solutions for the $D \to PP$ quark-diagram amplitudes $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{C}$ and $\mathcal{D}$ can now be obtained. More precisely, the six amplitudes determined from the measured decay rates of $D^+ \to \bar{K}^0\pi^+$, $\bar{K}^0K^+$, $D^0 \to \bar{K}^0\eta, \bar{K}^0\eta'$, $K^-\pi^+$, $\bar{K}^0\pi^0$, and $D_s^+ \to \bar{K}^0K^+$, $\pi^+\eta$ are (in units of GeV):

$$\begin{aligned}
&[\mathcal{A}]_{PP} \sim 3.4\times10^{-6}, \quad [\mathcal{B}]_{PP} \sim 2.1\times10^{-6}, \quad [\mathcal{C}]_{PP} \sim -1.8\times10^{-6}\ ,\\
&[\mathcal{D}]_{PP} \sim -0.5\times10^{-6}, \quad [\mathcal{C}]_{PP} \sim -0.8\times10^{-6}, \quad [\mathcal{D}]_{PP} \sim -0.4\times10^{-6}\ ,\\
&\Delta_{K\pi} \equiv (\delta_{1/2}-\delta_{3/2})_{K\pi} = (79\pm11)^\circ\ .
\end{aligned} \tag{II.4}$$

Evidently, the nonspectator contributions, especially the W-exchange diagrams, play an essential role in $D \to PP$ decays.

There has been an outstanding prediction since 1983:[2]

$$\Gamma(D^+ \to \pi^+\pi^0)/\Gamma(D^+ \to \bar{K}^0\pi^+) = \frac{1}{2}|V_{cd}/V_{cs}|^2 \approx 0.01.$$

II.b.2. The CKM Doubly–Suppressed $D \to PP$ Decays

The quark-diagram amplitudes given by Eq. (II.4), lead us straight toward computing the branching fractions for quark-mixing doubly–suppressed $D \to PP$ decays.[13] Again, it should be noted that nonspectator diagrams and final-state interactions are included in our calculations. In addition to the predictions based upon detailed information of Eq. (II.4), we have also the

following general relations between the doubly–suppressed decays and non-suppressed D^0 decays:[13]

$$\begin{aligned}
\Gamma(D^0 \to K^+\pi^-) &= \left(\frac{s_1}{c_1}\right)^4 \Gamma(D^0 \to K^-\pi^+);\quad \Gamma(D^0 \to K^0\pi^0)\\
&= \left(\frac{s_1}{c_1}\right)^4 \Gamma(D^0 \to \bar{K}^0\pi^0);\\
\Gamma(D^0 \to K^0\eta) &= \left(\frac{s_1}{c_1}\right)^4 \Gamma(D^0 \to \bar{K}^0\eta);\ \backslash Gamma(D^0 \to K^0\eta') =\\
&\left(\frac{s_1}{c_1}\right)^4 \Gamma(D^0 \to \bar{K}^0\eta'),
\end{aligned} \tag{II.5}$$

where

$$\begin{aligned}
&s_1 \approx |V_{cd}^*| \simeq |V_{us}|;\quad c_1 \approx |V_{cs}^*| \approx |V_{ud}|;\\
&\eta = \eta_8\cos\theta + \eta_0\sin\theta,\quad \eta' = -\eta_8\sin\theta + \eta_0\cos\theta;\quad \theta \approx 20^\circ\ .
\end{aligned} \tag{II.6}$$

II.c. Charm Meson Decays into Vector-Vector Mesons, $D \to VV$

The quark-diagram scheme description for D decaying into vector-vector mesons is much neater than that for D decaying into vector-pseudoscalar mesons and that for D decaying into pseudoscalar-pseudoscalar mesons. For doubly–suppressed $D \to VV$ decays,[13] we have the following predictions:

The observation of $D^+ \to \phi K^{*+}$ will establish and measure the W-annihilation diagram $\mathcal{D}$.

The observation of $D^0 \to \phi K^{*0}$ will establish and measure the W-exchange diagram $\mathcal{C}$, and we predict

$$\begin{aligned}
\Gamma(D^0 \to \phi K^{*0}) &= \left(s_1/c_1\right)^2 \Gamma(D^0 \to \phi\overline{K}^{*0});\quad \Gamma(D^0 \to \omega K^{*0})\\
&= \left(s_1/c_1\right)^2 \Gamma(D^0 \to \omega\overline{K}^{*0});
\end{aligned} \tag{II.7}$$

$$\Gamma(D^0 \to \rho^- K^{*+}) = \left(s_1/c_1\right)^2 \Gamma(D^0 \to K^{*-}\rho^+); \quad \Gamma(D^0 \to \rho^0 K^{*0})$$
$$= \left(s_1/c_1\right)^2 \Gamma(D^0 \to \overline{K}^{*0}\rho^0); \tag{II.8}$$

We also predict

$$\Gamma(D_s^+ \to K^{*+}K^{*0}) = \left(s_1/c_1\right)^2 \Gamma(D^+ \to \rho^+\overline{K}^{*0}). \tag{II.9}$$

Using the recent ARGUS measurement $Br(D^0 \to \bar{K}^{*0}\omega) = (2.0 \pm 0.5 \pm 0.3)\%$, we predict $Br(D^0 \to K^{*0}\omega) \sim 6 \times 10^{-5}$.

III. Nonleptonic Decays of B Mesons

III.a. The CKM Non-Suppressed Decays

Using the quark-diagram scheme, studies similar to those with the D mesons can be made for the B meson decays. However, at this time, we need more complete data for such an analysis.

III.b. Interesting Features in the Rare B Meson Decays

III.b.1. Unambiguous Ways to Observe $|V_{ub}| \neq 0$ in Nonleptonic Decays

Theoretically the simplest way to measure $|V_{ub}|$ is to observe $B_u \to \tau\nu_\tau$. Unfortunately, the branching ratio for such a decay is very small, $[Br(B_u \to \tau\nu_\tau) \lesssim 10^{-5}]$, using the current estimate of $|V_{ub}/V_{cb}|^2 \approx (\frac{1}{5})^2$. It is therefore important to find other decay channels which are non–zero only when $|V_{ub}|$ is non–zero. Certainly the observation of a charmless semi-leptonic decay $b \to \ell^-\overline{\nu}_\ell X_{c=0}$ inclusively or exclusively will establish $|V_{ub}| \neq 0$. Here, we shall list some pure nonleptic decays which are non-zero only if $|V_{ub}| \neq 0$. Such a selection is possible based upon the general model-independent quark-

diagram scheme:[16]

$$\begin{aligned}
B_u^+ \to\ & F^+X_{c=0,s=0}: \\
& F^+\pi^0 \times \sqrt{2},\ F^+\eta_8 \times \sqrt{6},\ F^+\eta_0 \times \sqrt{3},\ F^+\rho^0 \text{ etc.}: && V_{cs}V_{ub}^* (\mathcal{A}); && \text{(III.1)}\\
& F^+\phi, \text{ etc.}: && V_{cs}V_{ub}^* (\mathcal{D}); && \text{(III.2)}\\
\to\ & DX_{c=0,s=+1}: \\
& D^0K^+,\ D^0K^{*+},\ D^{*0}K^+,\ D^{*0}K^{*+}, \text{ etc.}: && V_{cs}V_{ub}^* (\mathcal{B}+\mathcal{D}); && \text{(III.3)}\\
B_d^0 \to\ & F^+X_{c=0,s=0}: \\
& F^+\pi^-,\ F^+\rho^-, \text{ etc.}: && V_{cs}V_{ub}^* (\mathcal{A}); && \text{(III.4)}\\
\to\ & DX_{c=0,s=+1}: \\
& D^0K^0,\ D^0K^{0*},\ D^{*0}K^0,\ D^{*0}K^{0*}, \text{ etc.}: && V_{cs}V_{ub}^* (\mathcal{B}); && \text{(III.5)}\\
B_s^0 \to\ & F^+X_{c=0,s=-1}: \\
& F^+K^-,\ F^+K^{*-}, \text{ etc.}: && V_{cs}V_{ub}^* (\mathcal{A}+\mathcal{C}); && \text{(III.6)}\\
\to\ & DX_{c=0,s=o}: \\
& D^0\pi^0 \times \sqrt{2},\ D^0\eta_8 \times \sqrt{6},\ D^0\eta_0 \times \sqrt{3},\ D^+\pi^-, \\
& D^+\rho^-,\ D^0J/\psi, \text{ etc.}: && V_{cs}V_{ub}^* (\mathcal{C}). && \text{(III.7)}
\end{aligned}$$

For simplicity, we use here the same amplitude symbols for final states from different multiplets. But keep in mind that they are, in general, different. Notice that all these decays have a net one unit of charm and strangeness quantum numbers in the final state. Essentially they are all from graphs with $\overline{b} \to \overline{u}W^+$ and $W^+ \to c\overline{s}$; therefore, the combination $V_{cs}V_{ub}{}^*$ always appears in these decays. There are also decays involving $V_{cd}V_{ub}{}^*$, e.g. $B \to D^+\pi^-$, $D^0\pi^0$, which are non–zero only if $|V_{ub}| \neq 0$ but their branching ratios are suppressed by a factor $|V_{cd}/V_{cs}|^2 \approx 0.05$ compared to the ones in Eqs. (III.1) to (III.7).

In the decay

$$B_u^+ \to \pi^+\pi^\circ: \quad V_{ud}V_{ub}\frac{1}{\sqrt{2}}(\mathcal{A}+\mathcal{B}), \tag{III.8}$$

there is cancellation among the loop-graph ϵ contributions with coefficient $V_{cd}V_{cb}^*$. Thus the observation of $B_u^+ \to \pi^+\pi^\circ$ is clear indication that $|V_{ub}| \neq 0$. $B_u^+ \to \rho^+\rho^\circ$ decays also occur only when $|V_{ub}| \neq 0$; but experimentally it is not as easy as $B_u^+ \to \pi^+\pi^0$ to observe.

Other decays with $V_{ud}V_{ub}{}^*$ factors in general can have contributions from other diagrams involving $V_{cd}V_{cb}^*$ which has absolute value much larger than that of $V_{ud}V_{ub}^*$. For example, the charmless decay $B_d^0 \to \pi^+\pi^-$ is given by the following quark amplitudes and quark mixing factors:

$$B_d^0 \to \pi^+\pi^- : \; V_{ud}V_{ub}^*(\mathcal{A}+\mathcal{C}+\mathcal{E}_{u-t}) + V_{cd}V_{cb}^*(\mathcal{E}_{c-t}). \qquad \text{(III.9)}$$

We see that even when $|V_{ub}| = 0$, $B_d^0 \to \pi^+\pi^-$ may not be zero due to the loop diagram (here $\mathcal{E}_{u-t} \equiv \mathcal{E}_u - \mathcal{E}_t$).

Hence the search for $|V_{ub}|$ is, in general, not likely to be most successful in the charmless decays of beauty particles, with the exception of case $B_u^+ \to \pi^+\pi^\circ$. However, if we can establish experimentally that the W-loop diagram contributions, even with their large mixing matrix coefficient, are negligible compared to the tree-diagram contributions (in many cases with very small mixing matrix coefficients), then the charmless decays such as $B_d^0 \to \pi^+\pi^-$ can give estimates on $|V_{ub}|$.

III.b.2. Decays of B Mesons Solely from the W-loop diagrams

Here again the Quark Diagram Scheme is useful. The following decays are from W-loop diagrams only:[16]

$$\begin{array}{lll}
B_d^0 \to K^0\phi: & V_{us}V_{ub}^* & (\mathcal{E}'_{u-t}) + V_{cs}V_{cb}^*(\mathcal{E}'_{c-t}), \\
\to K^{*0}\phi: & V_{us}V_{ub}^* & (\mathcal{E}_{u-t}) + V_{cs}V_{cb}^*(\mathcal{E}_{u-t}), \\
\to K^{*0}\overline{K}^0: & V_{ud}V_{ub}^* & (\mathcal{E}_{u-t}+\mathcal{F}_{u-t}) \\
 & +V_{cd}V_{cb}^* & (\mathcal{E}_{c-t}+\mathcal{F}_{c-t}), \\
\to K^0\overline{K}^{*0}: & V_{ud}V_{ub}^* & (\mathcal{E}'_{u-t}+\mathcal{F}'_{u-t}) \\
 & +V_{cd}V_{cb}^* & (\mathcal{E}'_{c-t}+\mathcal{F}'_{c-t}), \\
\to K^0\overline{K}^0: & V_{ud}V_{ub}^* & (\mathcal{E}_{u-t}+\mathcal{F}_{u-t}) \\
 & +V_{cd}V_{cb}^* & (\mathcal{E}_{c-t}+\mathcal{F}_{c-t}), \\
\to \phi\phi: & V_{ud}V_{ub}^* & (\mathcal{F}_{u-t}) + V_{cd}V_{cb}^*(\mathcal{F}_{c-t}); \\
B_s^0 \to \overline{K}^0\phi: & V_{ud}V_{ub}^* & (\mathcal{E}_{u-t}) + V_{cd}V_{cb}^*(\mathcal{E}_{c-t}), \\
\to \overline{K}^{*0}\phi: & V_{ud}V_{ub}^* & (\mathcal{E}'_{u-t}) + V_{cd}V_{cb}^*(\mathcal{E}'_{c-t}), \\
\to K^0\overline{K}^{0*}: & V_{us}V_{ub}^* & (\mathcal{E}_{u-t}+\mathcal{F}_{u-t}) \\
 & +V_{cs}V_{cb}^* & (\mathcal{E}_{c-t}+\mathcal{F}_{c-t}),
\end{array}$$

$$\begin{array}{lll}
\to \phi\phi: & V_{us}V_{ub}^* & (\mathcal{E}_{u-t}+\mathcal{F}_{u-t}) \\
 & +V_{cs}V_{cb}^* & (\mathcal{E}_{c-t}+\mathcal{F}_{c-t}), \\
\to K^{0*}\overline{K}^0: & V_{us}V_{ub}^* & (\mathcal{E}'_{u-t}+\mathcal{F}'_{u-t}) \\
 & +V_{cs}V_{cb}^* & (\mathcal{E}'_{c-t}+\mathcal{F}'_{c-t}), \\
\to K^0\overline{K}^0: & V_{us}V_{ub}^* & (\mathcal{E}_{u-t}+\mathcal{F}_{u-t}) \\
 & +V_{cs}V_{cb}^* & (\mathcal{E}_{c-t}+\mathcal{F}_{c-t}).
\end{array} \qquad \text{(III.10)}$$

Note that $B_d \to \phi\phi$ come only from the vertical W-loop diagram $\mathcal{F}$; and $B_d \to K^0\phi$, $K^{*0}\phi$, $B_s^0 \to \overline{K}^0\phi$, $\overline{K}^{*0}\phi$ come only from the horizontal loop diagram $\mathcal{E}$. See Ref. [9] for a model calculation of these decays.

III.b.3. Sensitive Dependence of $B_d^o\bar{B}_d^o$ mixing on the KM Phase.

The ARGUS' observation of $B_d^o \leftrightarrow \bar{B}_d^o$ mixing initially came as a surprise. Here I would like to tell an interesting story – especially to those of you in the audience who are young physicists:

As we all know, the formula for B^0-mixing is

$$r_B \equiv \frac{P(B^0 \to \ell^-)}{P(B^0 \to \ell^+)} = \frac{P(e^+e^- \to B^0B^0 \to \ell^-\ell^-)}{P(e^+e^- \to B^0\overline{B}^0 \to \ell^+\ell^-)} = \frac{(\Delta m/\tilde{\Gamma})^2 + (\Delta\Gamma/2\tilde{\Gamma})^2}{(\Delta m/\tilde{\Gamma})^2 - (\Delta\Gamma/2\tilde{\Gamma})^2 + 2}. \qquad \text{(III.11)}$$

The large mixing occurs either through $\Delta\Gamma/2\tilde{\Gamma} \approx 1$, which was in fact true in the K-meson decays, *i.e.*, $\Delta\Gamma_K \approx \Gamma_{K_S} \approx 2\tilde{\Gamma}$; or through $\Delta m/\tilde{\Gamma} >> 2$, which is the case for the B- meson decays. In both cases, the box graph which gives Δm and $\Delta\Gamma$ must compete with the decay graphs which give $\tilde{\Gamma}$. Thus the particle–antiparticle mixing is more favorable in the decays of K and B mesons where the tree graphs are suppressed due to the quark-mixing suppression, and, in contrast, the mixing of the decays of charm mesons and top mesons is less favorable due to supression of the box graph relative to the tree graphs, which is quark-mixing nonsuppressed.

For the B^0 mesons it is reasonable to assume that Δm comes mainly from the box graph – the so called short-distance contributions – which is dominated by the t-quark term:[18]

$$\Delta m \approx \frac{G_F^2 B_B f_B^2 m_b}{6\pi^2} m_t^2 F(m_t^2/m_w^2)|V|^2, \qquad \text{(III.12)}$$

where $|V|^2 = |V_{td}V_{tb}^*|^2$, for B_d-mixing; and $|V|^2 = |V_{ts}V_{tb}^*|^2$, for B_s-mixing; and

$$F(x) = 1 - (3/4)(x + x^2)/(1-x)^2 - (3/2)x^2 \ln x/(1-x)^3,$$

which is a smooth and slowly decreasing function. $\Delta\Gamma$ comes from the absorptive part of the box diagram and therefore the t quark intermediate state cannot contribute. Thus $\Delta\Gamma << \Delta m$ is expected, and Eq. (III.11) simplifies to

$$r_B \approx (\Delta m/\tilde{\Gamma})^2/[(\Delta m/\tilde{\Gamma})^2 + 2]. \qquad \text{(III.13)}$$

It is obvious that as $B_B f_B^2$ and m_t^2 increase, the mixing of r_B increases. The reason that B_s-mixing in general is bigger than B_d-mixing is that $|V_{ts}|^2 > |V_{td}|^2$. What was noted in Refs. [1] and [17] is that $|V_{td}|$ is very sensitive to the CP-invariance violating phase. This is most easily seen by using the parameterization given in Ref. [6] and in Eq. (I.1.b)

$$V = \begin{pmatrix} c_x & s_x & s_z e^{-i\phi} \\ -(s_x + s_y s_z e^{i\phi}) & c_y & s_y \\ s_y[s_x - (s_z/s_y)e^{i\phi}] & -s_y[1 + s_x(s_z/s_y)e^{i\phi}] & 1 \end{pmatrix}. \qquad \text{(III.14)}$$

From the expression (III.14) we see that $|V_{td}| = s_y[|s_x - (s_z/s_y)\cos\phi|^2 + |(s_z/s_y)\sin\phi|^2]^{\frac{1}{2}}$ is very sensitive to the sign of $\cos\phi$, and it increases as ϕ increases from $0°$ to $180°$. As the value of $|V_{ub}/V_{cb}|$ becomes closer to the value of $|V_{us}|$, the dependence of $|V_{td}|$ on ϕ becomes more dramatic. The dependence of $|V_{ts}|$ on ϕ is the opposite, *i.e.*, it decreases as ϕ increases from $0°$ to $180°$. However, since the quantity $s_x(s_z/s_y) < 1$, the dependence of $|V_{ts}|$ on ϕ is less than that of $|V_{td}|$. Therefore it was noted in Refs. [1] and [17] that γ_{Bd} can change from 10^{-5} to 10^{-1} as ϕ changes from $\phi \approx 0$ to $\phi \approx 180°$. The value of top quark mass used was $m_t \approx 60$ GeV. We thought that we were brave in using such a large top quark mass. Our results of 1983 were forgotten, however, because there was a time in the mid-1980s when UA1 announced the observation of a top particle with a mass of about 35 GeV, and the value of $|V_{ub}/V_{cb}|$ had diminished. Both results made $B_d^o \leftrightarrow \bar{B}_d^o$ mixing negligible. It is amusing to reflect that we are today talking about a top quark mass larger than 100 GeV!

In any case, $B_d^o \leftrightarrow \bar{B}_d^o$ mixing is sensitive to the value of the KM phase and indications are that the phase ϕ is large.

IV. CP-Noninvariance in K Meson Decays

IV.a. Implications of the Measurement of Re(ϵ'/ϵ)

Now, after almost a quarter of a century has passed since the first observation of CP-noninvariance ϵ in the K_L^0 decays, there is finally a hint of a new type of CP-noninvariance ϵ' that has been observed, again in the K_L^0 meson. The significance of NA31's positive results of a non–zero ϵ':

$$R_e\left(\frac{\epsilon'}{\epsilon}\right) = \frac{1}{6}\left[|\eta_{+-}/\eta_{00}|^2 - 1\right] = 0.0035 \pm 0.0007 \pm 0.0004 \pm 0.0010 \qquad \text{(IV.1)}$$

where

$$\eta_{+-} \equiv A(K_L \to \pi^+\pi^-)/A(K_S \to \pi^+\pi^-),$$
$$\eta_{00} \equiv A(K_L \to \pi^0\pi^0)/A(K_S \to \pi^0\pi^0).$$

It was originally observed by Gilman and Wise[7] that the KM scheme gives $\epsilon' \neq 0$. One can easily show[19] that $|\eta_{+-}/\eta_{00}| \neq 1$ necessarily indicates decay-amplitude CP-noninvariance. From the definitions of η_{+-}, η_{00},

$$\eta_{+-} = \frac{A_{+-}/\overline{A}_{+-} - p/q}{A_{+-}/\overline{A}_{+-} + p/q} = \epsilon + \epsilon',$$
$$\eta_{00} = \frac{A_{00}/\overline{A}_{00} - p/q}{A_{00}/\overline{A}_{00} + p/q} = \epsilon - 2\epsilon', \qquad \text{(IV.2)}$$

where $\overset{(-)}{A}_i = A(\overset{(-)}{K} \to f_i)$, and $p = 1 - \bar{\epsilon}$, $q = 1 + \bar{\epsilon}$; ϵ is the CP-violating parameter of the amplitude of isospin zero of the two pions; and ϵ', of the isospin two,[15] we see that both the mass-matrix and the decay-amplitude CP violations can give nonvanishing η_{+-}, η_{00}. In the ratios $A_{+-}/\overline{A}_{+-}$, $A_{00}/\overline{A}_{00}$, the strong-interaction phase cancels. Without decay-amplitude CP violation in weak interactions, any phase in $A_{+-}/\overline{A}_{+-}$ and $A_{00}/\overline{A}_{00}$ can be simultaneously removed by choosing a proper phase convention so that

$A_{+-}/\overline{A}_{+-} = 1 = A_{00}/\overline{A}_{00}$; thus $\eta_{+-}/\eta_{00} = 1$. The reverse is also true, *i.e.*, if $\eta_{+-}/\eta_{00} = 1$, then $A_{+-}/\overline{A}_{+-} = A_{00}/\overline{A}_{00}$, and $|A_{+-}/\overline{A}_{+-}| = |A_{00}/\overline{A}_{00}| = 1$, (that the second relation must also be true is due to the fact that the K^0, $\overline{K}^0 \to 2\pi$ decays are closed systems and the CPT theorem applies and it implies $\Gamma(K^0 \to \pi^+\pi^-) + \Gamma(K^0 \to \pi^0\pi^0) = \Gamma(\overline{K}^0 \to \pi^+\pi^-) + \Gamma(\overline{K}^0 \to \pi^0\pi^0)$. Therefore, if $|A_{+-}/\overline{A}_{+-}| < 1$, then $|A_{00}/\overline{A}_{00}| = |\overline{A}_{+-}/A_{+-}| > 1$. We cannot have a situation where $|A_{+-}/\overline{A}_{+-}| = |A_{00}/\overline{A}_{00}| \neq 1$. There are decay-amplitude CP violations if $|A/\overline{A}| \neq 1$, or even when both $|A_{+-}/\overline{A}_{+-}| = |A_{00}/\overline{A}_{00}| = 1$, but no phase convention can be found such that $A_{+-}/\overline{A}_{+-}$ and $A_{00}/\overline{A}_{00}$ are both real. Therefore $|\eta_{+-}/\eta_{00}| \neq 1$, *i.e.*, $Re(\epsilon'/\epsilon) \neq 0$, is a necessary and sufficient indication of decay-amplitude CP violation. This is what NA31 and Fermilab experiments measure.

Such decay-amplitude CP violating effects can be measured,[(1),(3),(20)] in different ways: directly in tagged K^0, $\overline{K}^0 \to \pi^+\pi^-$, $\pi^0\pi^0$, $\gamma\gamma$, and $K^\pm \to 3\pi$, as well in $\Lambda, \bar{\Lambda}$ and $\Sigma, \bar{\Sigma}$ decays, which I shall discuss in the next section.

IV.b. Other Important Searches for Decay-Amplitude CP Noninvariance in the Strange Particles

IV.b.1. Tagged $K^0, \bar{K}^0 \to 2\pi$ Experiments

As indicated in the proof of $\epsilon' \neq 0$ being solely in the decay amplitude, $\epsilon' \neq 0$ is a direct result of the partial-decay-rate difference in $K^0, \bar{K}^0$, *i.e.*,

$$\begin{aligned} &A(K^0 \to \pi^+\pi^-)/\bar{A}(\bar{K}^0 \to \pi^+\pi^-) \neq 1, \\ &\text{and} \\ &A(K^0 \to \pi^0\pi^0)/\bar{A}(\bar{K}^0 \to \pi^0\pi^0) \neq 1. \end{aligned} \tag{IV.3}$$

Thus, a natural way to measure decay-amplitude CP-noninvariance is to measure the partial decay-rate difference of $K^0, \bar{K}^0$,[(3)] *i.e.*,

$$\begin{aligned} R_{+-} &\equiv \frac{\Gamma(K^0 \to \pi^+\pi^-)}{\Gamma(\bar{K}^0 \to \pi^+\pi^-)} = 1 + 2Re(\epsilon'); \\ R_{00} &\equiv \frac{\Gamma(K^0 \to \pi^0\pi^0)}{\Gamma(\bar{K}^0 \to \pi^0\pi^0)} = 1 - 4Re(\epsilon'). \end{aligned} \tag{IV.4}$$

Here the partial decay-rate-difference effects exist only when $K^0, \bar{K}^0$ are tagged, since $K^0, \bar{K}^0$ mix maximally.

Thus, to see such effects directly, tagging and time-evolution measurements are necessary. The CERN LEAR experiment PS195 is designed to make such measurements:

$$R_{+-} = \frac{\Gamma(P\overline{P} \to \pi^-K^+\overline{K}^0 \to \pi^-K^+\pi^+\pi^-)}{\Gamma(P\overline{P} \to \pi^+K^-K^0 \to \pi^+K^-\pi^+\pi^-)} = 1 + 2R_e(\epsilon'), \tag{IV.5a}$$

$$R_{00} = \frac{\Gamma(P\overline{P} \to \pi^-K^+\overline{K}^0 \to \pi^-K^+\pi^0\pi^0)}{\Gamma(P\overline{P} \to \pi^+K^-K^0 \to \pi^+K^-\pi^0\pi^0)} = 1 - 4R_e(\epsilon'). \tag{IV.5b}$$

Such tagged $K^0, \bar{K}^0$ partial-decay-rate-difference measurements are more direct and general ways to measure decay-amplitude CP-noninvariance (see later discussion in Section V on CP-noninvariance in B and charm mesons). Another important reason for doing such tagged $K^0, \bar{K}^0$ measurements are that it provides a different measurement of ϵ'. Note that NA31 measures $Re(\epsilon'/\epsilon)$, *i.e.*, the projection of ϵ' on ϵ, while tagged $K^0, \bar{K}^0$ experiments measure $Re(\epsilon')$. Under CPT invariance, one can show that [phase (ϵ)] $\approx 90° - (\delta_2 - \delta_1) = 90° + (-45.3° \pm 4.6°) \approx 45°$, which is very close to [phase (η_{+-})] $= \phi_{+-} = 44.6° \pm 1.20°$. So ϵ' is expected to be aligned with ϵ. Therefore, from the CPT theorem we expect:

$$|Re(\epsilon')| \approx |Re(\epsilon'/\epsilon) \cdot \epsilon \cdot \cos 45°| = |0.0035 \cdot \epsilon \cdot \cos 45°|. \tag{IV.6}$$

A measurement of $Re(\epsilon')$ different from this predicted value is a clear indication of CPT invariance violation.[(21)]

IV.b.2. High Precision Measurement of ϕ_{00}, ϕ_{+-}

Currently the experimental values of

$$\phi_{00} = 54.5° \pm 5.3°; \quad \phi_{+-} = 44.6° \pm 1.2° \tag{IV.7}$$

have a difference larger than allowed by the observed value of $Re(\epsilon'/\epsilon)$. Therefore more accurate measurements of ϕ_{00}, ϕ_{+-} are very important. If the difference of ϕ_{00}, ϕ_{+-} remains larger than allowed by the measured value of $Re(\epsilon'/\epsilon)$, again CPT invariance is violated. For a detailed discussion see Ref. [22].

IV.b.3. Decay-Amplitude CP-Noninvariance in $K^0, \bar{K}^0 \to 2\gamma$

The measurement of ϵ' is a definite indication of decay amplitude CP-noninvariance. Unfortunately ϵ' is a CP-noninvariance effect in the $\Delta I = \frac{3}{2}$ amplitude, which is known from experiments to be suppressed in the amplitude by a factor of 20 compared with the $\Delta I = \frac{1}{2}$ part. Historically, this suppression was important for the observation by Dalitz of the $\theta(K^+ \to \pi^+\pi^0)$, $\tau(K^+ \to \pi^+\pi^+\pi^-)$ puzzle, which eventually led to the discovery of parity-invariance violation. However, ironically, the same factor now is important in making $|\epsilon'/\epsilon|$ small. Therefore Cheng and I[23] began a search for other K-decay channels where decay-amplitude CP-noninvariance has no suppression which compares to the mass-matrix CP-noninvariance. We found that such a possibility existed in K^0, $\overline{K}^0 \to 2\gamma$ from the interference between the 2π intermediate-state contribution in $K_S \to 2\gamma$ and the π, 3π, η, η' intermediate-state contributions in $K_L \to 2\gamma$. For such an interference effect to be appreciable, the $K_S \to 2\gamma$ rate needs to be comparable with $K_L \to 2\gamma$. Our theoretical calculation predicted[23] $\Gamma(K_S \to 2\gamma)/\Gamma(K_L \to 2\gamma) \approx 2.4$ and the NA3 measurement indeed gave[24]

$$\Gamma(K_S \to 2\gamma)/\Gamma(K_L \to 2\gamma) \equiv R = 2.4 \pm 1.1. \qquad \text{(IV.8)}$$

As indicated in Ref. [7], for such a ratio of comparable $K_S \to 2\gamma$ vs $K_L \to 2\gamma$, the decay–amplitude CP-noninvariance effect can be as large as the mass–matrix CP-noninvariance ϵ. It is interesting and important to note that the appropriate ways for observing such a CP-noninvariance effect are measuring the tagged K^0, $\overline{K}^0 \to 2\gamma$ time-dependent decay–rate difference measurements. It is impossible to see this effect in separate K_L, $K_S \to 2\gamma$ decays. For details, see Ref. [23].

IV.b.4. Decay-Amplitude CP-Noninvariance in $K \to 3\pi$ decays[1]

We should also expect decay–amplitude CP-noninvariance in $K^\pm \to 3\pi$ decays; *i.e.*, $\Gamma(K^+ \to \pi^+\pi^+\pi^-)/\Gamma(K^- \to \pi^-\pi^-\pi^+) = 1 + \Delta$, where $\Delta \neq 0$. However, since $K \to 3\pi$ decays do not mix with $K \to 2\pi$, the total rate of $K^+ \to 3\pi$ must be the same as $K^- \to 3\pi$ from the CPT theorem. Thus the difference in the charged 3π decays must be balanced by the $\pi^\pm\pi^0\pi^0$ channels.

A good place to study decay–amplitude CP-noninvariance in $K \to 3\pi$ is the measurement of the ratio of the ratios of the following decays:

$$\frac{\Gamma(K^+ \to \pi^+\pi^-\pi^-)/\Gamma(K^+ \to \pi^+\pi^0\pi^0)}{\Gamma(K^- \to \pi^-\pi^+\pi^+)/\Gamma(K^- \to \pi^-\pi^0\pi^0)} = 1 + 2\Delta_{3\pi}. \qquad \text{(IV.9)}$$

The advantage of such measurements of the ratio are two–fold: firstly, it gains a factor of two in the difference, secondly many experimental errors are cancelled. Unfortunately though, $\Delta_{3\pi}$ is expected to be only a fraction of the value of ϵ'.

One cautious note should be given to the implication of measuring $\eta_{+-0} \equiv A(K_S^0 \to \pi^+\pi^-\pi^0)/A(K_L^0 \to \pi^+\pi^-\pi^0)$ for CP noninvariance. Unlike that $\eta_{+-} \neq 0$ always indicates CP-noninvariance $\eta_{+-0} \neq 0$ does not necessarily imply CP-noninvariance, since $\pi^+\pi^-\pi^0$ can be in both CP odd as well as in CP even states. One has to look at the particular region of the Dalitz plot where $\pi^+\pi^-\pi^0$ forms a CP odd state to look for CP-noninvariance effects in η_{+-0}.

IV.b.5. Decay-Amplitude CP-Noninvariance in Hyperon Decays

Through the general analysis one can show[3],[24] that there can be partial-decay-rate differences $\Gamma(\Lambda \to \pi^- p)/\Gamma(\bar{\Lambda} \to \pi^+\bar{p}) = 1 + \Delta$, and $\Gamma(\Lambda \to \pi^0 n)/\Gamma(\bar{\Lambda} \to \pi^0\bar{n}) = 1 - \Delta$. Currently the estimate is $\Delta \sim 10^{-4}$ to 10^{-5}. There has been a proposal made at LEAR to measure

$$\frac{\Gamma(P\overline{P} \to \Lambda\overline{\Lambda} \to \pi^- p\pi^0\bar{n})}{\Gamma(P\overline{P} \to \Lambda\overline{\Lambda} \to \pi^0 n\pi^+\bar{p})} = 1 + 2\Delta. \qquad \text{(IV.10)}$$

It is well known that $\sum$ is produced highly polarized and the π distribution with respect to the polarization of $\sum$ is asymmetric. CP-noninvariance can lead to a difference in the asymmetry between $\sum^+ \to \pi^+ n$, and $\sum^- \to \pi^-\bar{n}$. The estimate given on such an asymmetry was comparable to the size of ϵ. This may be a very interesting effect to observe. Table 1 lists some other interesting CP-noninvariance effects in hyperon decays.

Table 1. CP violation effects in hyperon decays, from Ref. [24] of Chau and Cheng.

	Δ/X_{CP}	$\mathcal{A}/X_{CP}$	$[\mathcal{B}/\alpha - \tan(\delta_p - \delta_s)]/X_{CP}$
Λ^0_-	8×10^{-2}	-2×10^{-2}	1.2×10^{-1}
Λ^0_0	-1×10^{-1}	4×10^{-2}	-4×10^{-1}
Σ^+_0	-8×10^{-3}	-4×10^{-2}	---
Σ^+_+	2×10^{-2}	80	---
Σ^-_-	0	-4×10^{-1}	4

V. Some Generic Properties of Decay-Amplitude CP-Noninvariance

As we have already seen in some specific decay modes in strange meson and hyperon decays, the general and direct way to analyze and measure decay-amplitude CP-noninvariance is to measure the partial-decay-rate differences in both neutral and charged particles. Such a general view has led to a different way to measure ϵ', *i.e.*, measuring the tagged K^0, $\overline{K}^0 \to 2\pi$ decay-rate differences, and to the predictions of CP-noninvariance effects in partial-decay-rate differences in many other systems. A general survey of such effects in the KM scheme was accomplished (see Ref. [3]) in 1980. That such a general survey of CP-noninvariance was possible was based upon the observation that meson weak decays are given unambiguously in terms of the quark-mixing matrix and six decay amplitudes $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$, $\mathcal{E}$, $\mathcal{F}$. To explain how particle-antiparticle decay-amplitude differences arise, I use an explicit example: The process $B_u^+ \to K^+\rho^0$ is given in terms of the quark diagrams as follows,

$$A = V_{ub}V_{us}^*(\mathcal{A}+\mathcal{B}+\mathcal{D}+\mathcal{E}_{u-t}) + V_{cb}V_{cs}^*(\mathcal{E}_{c-t}) \equiv V_{ub}V_{us}^*(A_1) + V_{cb}V_{cs}^*(A_2),$$

and $B_u^- \to K^-\rho^0$ is given by

$$\overline{A} = V_{ub}^*V_{us}(\mathcal{A}+\mathcal{B}+\mathcal{D}+\mathcal{E}_{u-t}) + V_{cb}^*V_{cs}(\mathcal{E}_{c-t}) \equiv V_{ub}^*V_{us}(A_1) + V_{cb}^*V_{cs}(A_2). \tag{V.1a, V.1b}$$

Note that the difference between particle and antiparticle decays is in the

exchange of $V_{ij} \rightleftarrows V_{ij}^*$. The amplitudes A_1 and A_2 are not changed because of CP invariance in strong reactions. The partial-decay-rate difference is then given by

$$\Delta \equiv \frac{|A|^2 - |\overline{A}|^2}{|A|^2 + |\overline{A}|^2} = \frac{-4Im[(V_{ub}V_{us}^*(V_{cb}V_{cs}^*)^*]Im(A_1A_2^*)}{|V_{ub}V_{us}^*A_1 + V_{cb}V_{cs}^*A_2|^2 + |V_{ub}^*V_{us}A_1 + V_{cb}^*V_{cs}A_2|^2}. \tag{V.2}$$

Here we see that the important ingredients for a non–zero partial-decay rate difference are: first, that $X_{CP} \equiv Im[V_{ub}V_{us}^*(V_{cb}V_{cs}^*)^*] \neq 0$, which is provided by the KM mechanism from the interference of at least two distinct weak decay amplitudes; hence such decay-amplitude CP noninvariances will only happen in mixing-matrix suppressed decays; and second, that $Im(A_1A_2^*) \neq 0$, which means that there must be two independent types of nonleptonic decay amplitudes, e.g., the interference of the I = 0 and I = 2 amplitudes in $K \to 2\pi$ and the interference of the s and p wave amplitudes in $\Lambda \to \pi N$, $\bar{\Lambda} \to \pi\bar{N}$ decays.

The same formulation applies to other B meson decays as well as decays of charm, strange and top-quark mesons. The interesting point here is that for the three–generation quark-mixing matrix, shown in Ref. [6], the same X_{CP} appears in the numerator for all partial-decay-rate-differences, *i.e.*, the partial-decay rate differences have the following generic form

$$\Delta = \frac{X_{CP}\ Im(A_1A_2^*)}{|A|^2 + |\overline{A}|^2}, \tag{V.3}$$

where X_{CP} is the universal KM CP-noninvariance factor as given in Eq. (I.2).

Phenomenologically the uniqueness of X_{CP} for all CP-noninvariant decays in the three generations of quarks carries an important message. Even without knowing the detailed dynamics, we can conclude that we will have better luck in getting large partial decay rate differences in those channels where the decay rates are suppressed, *i.e.*, where the denominators in Eq. (V.3) are suppressed. This fact has been born out by many model calculations,[19] and the basic reasons were given in detail in Ref. [3]. The partial decay rates in many rare B meson decays can be as large as a few $\times$ 10% in constrast to $|\epsilon'| \approx 10^{-3}|\epsilon| \approx 10^{-6}$. The reason can be simply seen as follows. If there is

no reason for dynamical suppression,

$$\Delta_B \sim \sin\phi, \tag{V.4}$$

which can be of order one when ϕ is near 90°. However, in charm decays they are typically less than 1%:

$$\Delta_C \approx 10^{-2} \sin\phi. \tag{V.5}$$

This is merely a reflection of the larger decay rates of quark-matrix suppressed decays of charm mesons compared with those of B meson decays. Actually this is the same reason that the K and B meson systems are the best systems for neutral particle-antiparticle mixing. Thus we anticipate that the B meson will be a very useful source for interesting physics just as the K mesons have been. Recently, we have made detailed state-of-the-art calculations of rates and CP-noninvariance effects in Rare B decays.[8]

V.a. Decay-Amplitude CP-Noninvariance in B Meson Decays

From the general arguments just given, we see that the rare B decay channels are the most likely reactions in which to find fractional partial-decay-rate differences. Among all the channels considered, $B^\pm \to K^\pm\rho^0$, $\pi^\pm\rho^0$ have the final decay products that are most accessible for experimental detections. Channels involving charm mesons are more difficult to observe due to the need for charm-particle reconstructions. To observe partial decay rate differences in neutral B^0, $\overline{B}^0$ mesons, one must tag the B^0, $\overline{B}^0$ mesons. In contrast to the "neat" ways to tag K^0, $\overline{K}^0$ in $\overline{P}P \to \pi^+K^-K^0$, $\pi^-K^+\overline{K}^0$ and to tag D^0, $\overline{D}^0$ in $e^+e^- \to \psi(3370, 4030) \to \pi^+D^-D^0$, $\pi^-D^+\overline{D}^0$, clever arrangements have not been found for tagging the B^0, $\overline{B}^0$ mesons. Theoretically, the sources of partial-decay-rate differences in neutral mesons are also complicated (see discussions in subsection I.b.2.). They can involve all three effects of mixing: B^0, $\overline{B}^0$ mixing, mass matrix CP-noninvariance, and decay-amplitude CP-noninvariance (see Lim's talk at this conference and Eq. (I.10)).

V.b. Decay-Amplitude CP-Noninvariance in Charm Meson Decays

As discussed before, in the case of three generations of quarks, the partial-decay rate difference in charm mesons are, in general, a factor of 10-100 smaller than in the B mesons, and they thus require 10^2–10^4 more events. The partial-decay-rate differences in charm particle decays can be large if there are more than three generations of quarks;[25] this however is not indicated by experiments.

V.c. Decay-Amplitude CP-Noninvariance in Top-Quark Particle Decays

The decay-amplitude CP-noninvariance in top-quark particle decays can be as large as in B meson decays. But the two-body and three-body decay channels of top-particle decays have branching ratios that are too small ($\approx 10^{-8}$ for a top quark mass >50 GeV). Therefore, it will be difficult to see decay-amplitude CP-noninvariance in top quark decays.

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