Fermilab Library
0 1160 0041258 9

-UC-TP-2-95-

UNIVERSITY OF CRAIOVA

UC-TP-2/95
February 1995

FERMILAB
MAR
IBRA

The BRST Quantization of Second-Class Constrained Systems by Adding Extravariables. The Example of the Chiral Schwinger Model

C.Bizdadea and S.O.Saliu
University of Craiova, Department of Physics
13, A.I.Cuza Str., Craiova-1100, Romania

Abstract The problem of quantization of systems possessing only second-class constraints is considered in the framework of the BRST formalism based on path integral. In this paper it will be shown that extending the phase-space of the original system by adding some extravariabes, the quantization of the original second-class sytem is equivalent to the BRST quantization of a one-parameter family of first-class systems in a new phase-space. The general construction is exemplified on the gauge noninvariant chiral Schwinger model in the case $a > 1$. The family of first-class systems contains the gauge invariant chiral Schwinger model. The equality between the Lagrangian path integrals associated to the gauge noninvariant model and its family of first-class systems is not obtained for the gauge invariant formulation of the chiral Schwinger model.

Introduction

The problem of quantization of both first-class and second-class constrained systems in the framework of the path integral BRST formalism became of great interest during the past few years. The second-class constraints (SCC) can be eliminated in principle by passing to the Dirac bracket [1] as the theory based on the Dirac bracket and possesing only first-class constraints (FCC) is equivalent to the original theory. A different approach with no drop-out of the SCC is given in [2], the results derived there being phisically equivalent with the ones obtained in [3-5] . The path integral given in [2] is (for a certain class of gauge-fixing fermions) the same with the one obtained by canonical methods in [6,7] .

The theories with second-class constraints only (SCCO) need a special treatment. In this case the theory based on the Poisson bracket is no longer gauge invariant because the SCC do not generate gauge transformations. In order to apply the BRST formalism to this situation it is necessary to associate to the SCCO system an equivalent first-class system to whom one can apply further apply the BRST formalism making use of a certain gauge-fixing fermion.

The above mechanism was applied in [8] to a class of SCCO systems with the SCC $\chi_\alpha = 0$ such that the functions $\chi_\alpha = (G_a, C_a)$ satisfy the relations $\left[G_a, G_b\right] = C_{ab}{}^c G_c$ and the matrix $\Delta_{ab} = \left[C_a, G_b\right]$ do not depend on the canonical variables. In the above mentioned reference, both the SCCO system and the equivalent first-class system have the same phase-space described by the original local coordinates $\left(q^i, p_i\right)$, i.e. there were not introduced supplementary degrees of freedom. The path integral derived in [8] coincides with the one derived in [2,6,7] .

The method described in [8] can be extended as follows. We associate to the original SCCO system a one-parameter family of first-class systems (OPFFCS), but in a phase-space with the local coordinates $\left(q^i, p_i, z^a, \bar{p}_a\right)$ and apply the BRST formalism to the OPFFCS making use of a particular gauge-fixing fermion. The extravariables z^a and $\bar{p}_a$ are bosonic and canonically conjugated. The number of these canonical pairs is taken to be equal to half of the number of functions χ_α .

This method agrees with the idea of Fadeev and Shatashvili [9] to modify the canonical quantization and introduce new degrees of freedom through a Wess-Zumino action in the context of the chiral Schwinger model (CSM) . The quantization of the CSM is treated in [10] , but the quantization of the gauge noninvariant chiral Schwinger model (GNCSM) is not very clear.

In this work we shall expose in brief a new method of quantization for SCCO systems by adding extravariables in the context of the BRST formalism based on path integral. The method will be exemplified on the GNCSM [11] . Thus, we shall derive a OPFFCS associated to the GNCSM such that when the parameter takes the value $-e$ we reobtain the gauge invariant formulation of the CSM [9,12] . It will also be shown in the sequel that the Hamiltonian path integrals at the level of independent variables associated to the original SCCO system and to the OPFFCS (to these ones after the BRST quantization) coincide for the GNSCM ,as well as in general. In the same time, it will be shown that there exists a special value of the parameter such that the Lagrangian path integrals (LPI) of the GNSCM and of the associated OPFFCS coincide. Related to the BRST quantization, it will be applied the antibracket-antifield formalism following the same lines as in [13] .

2. The chiral Schwinger model

The Lagrangian actions corresponding to the gauge noninvariant, respectively the gauge invariant formulations of the CSM are given by

$$S_0^{LN}[A,\varphi]=\int d^2x\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi+eA_\nu(\eta^{\mu\nu}-\varepsilon^{\mu\nu})\partial_\mu\varphi+\frac{1}{2}e^2aA_\mu A^\mu\right) \quad , (1)$$

$$S_0^{LI}[A,\varphi,\theta]=S_0^{LN}[A,\varphi]+S_0^{LWZ}[A,\theta] \quad , \qquad (2)$$

where

$$S_0^{LWZ}[A,\theta]=\int d^2x\left(\frac{1}{2}(a-1)\partial_\mu\theta\partial^\mu\theta-eA_\nu((a-1)\eta^{\mu\nu}+\varepsilon^{\mu\nu})\partial_\mu\theta\right) \quad . \qquad (3)$$

We use the conventions $\eta^{\mu\nu}=\mathrm{diag}(1,-1)$ and $\varepsilon^{01}=1$. In the action (2) appears an additional Wess-Zumino field , θ , such that (2) is invariant under the following gauge transformations: $\delta_\varepsilon A_\mu=-\partial_\mu\varepsilon,\delta_\varepsilon\varphi=e\varepsilon,\delta_\varepsilon\theta=-e\varepsilon$.

The theory is shown in [11] to be unitary and Lorentz invariant for $a>1$ with a massive $\left(m^2=e^2a^2/(a-1)\right)$. For $a=1$ one obtains a free field theory. The case $a=1$ is not interesting from our point of view and will be not treated here.

For $a>1$ the Hamiltonian analysis of (1) gives the primary constraint $G\equiv\pi^0=0$ and the canonical Hamiltonian

$$H_{SC}=\int dx^1\left(\frac{1}{2}\pi_1\pi_1+\frac{1}{2}\pi_\varphi^2-A_0\partial_1\pi^1-e\left(\pi_\varphi+\partial_1\varphi\right)\left(A_0-A_1\right)+\right.$$

$$\left.+\frac{1}{2}\partial_1\varphi\partial_1\varphi+\frac{e^2}{2}\left(A_0-A_1\right)^2-\frac{e^2a}{2}A_\mu A^\mu\right) \quad . \qquad (4)$$

Here, π^0,π^1 and π_φ are the canonical momenta respectively conjugated to A_0,A_1 and φ , and $\partial_1=\frac{\partial}{\partial x^1}$. The consistency condition on the primary constraint implies the secondary constraint $C\equiv\partial_1\pi^1+e\left(\pi_\varphi+\partial_1\varphi+e(a-1)A_0+eA_1\right)=0$. There are no further constraints, the two ones obtained above being SCC.

It is easy to see that $[C,G]=e^2(a-1),[C,C]=0,$ and $[G,G]=0$, where [,] denotes the Poisson bracket. We observe that C can be put in the form $C=C^{(0)}+C^{(1)}$, where $C^{(0)}\equiv\partial_1\pi^1+e\pi_\varphi$ and $C^{(1)}\equiv e\left(\partial_1\varphi+e(a-1)A_0+eA_1\right)$. It follows simply that $\left[C^{(0)},C^{(0)}\right]=\left[C^{(0)},C^{(1)}\right]=\left[C^{(0)},G\right]=0$ and $\left[C^{(1)},G\right]=e^2(a-1)$. It is then simple to check that $[G,[G,H]]=-e^2(a-1)\neq0$ and $[G,[G,\ldots,[G,H]\ldots]]=0$ for more than two G's. We also have $\left[C^{(0)},\left[C^{(0)},H\right]\right]\neq0$ and $\left[C^{(0)},\left[C^{(0)},\ldots,\left[C^{(0)},H\right]\ldots\right]\right]=0$ for more than two $C^{(0)}$'s. We mention that the above vanishing Poisson brackets hold strongly.

3. The family of first-class systems

Our starting point is a system with the canonical Hamiltonian H , described by N bosonic canonical pairs $\left(q^i,p_i\right)$ and subject to the SCC $\chi_\alpha=0$. We assume that the constraint functions $\chi_\alpha=(G_a,C_a)$ are such that the G_a's are first-class among themselves. Because for the GNCSM we have $[G,G]=0$ we will consider only the case $\left[G_a,G_b\right]=0$ strongly. In addition we shall make the assumption that $G_a=0$ are primary constraints, while $C_a=0$ are secondary ones. We shall also presume that $C_a=C_a^{(0)}+C_a^{(1)}$ such that $\left[C_a^{(0)},C_b^{(0)}\right]=\left[C_a^{(0)},G_b\right]=0$ strongly and $\left[C_a^{(1)},G_b\right]=\Delta_{ab}$ don't depend on the canonical variables. As $\chi_\alpha=0$ are SCC , it follows that $\det\left(\left[\chi_\alpha,\chi_\rho\right]\right)\equiv\det\left(C_{\alpha\rho}\right)=\left(\det\left(\Delta_{ab}\right)\right)^2\neq0$. The path integral correspondent to the above SCCO system takes the form [13]

$$Z=\int DqDpD\mu\left(\det\Delta_{ab}\right)\exp\left(i\tilde{S}_0\right) \quad , \qquad (5)$$

where

$$\tilde{S}_0[q,p,\mu]=\int dt\left(\dot{q}^ip_i-H-\mu^\alpha\chi_\alpha\right) \quad . \qquad (6)$$

The functions μ_α represent the Lagrange multipliers corresponding to the SCC .

In the case of the GNSCM the correspondences are

$$q^i\leftrightarrow\left(A_\mu,\varphi\right)\equiv\left(A_0,A_1,\varphi\right) \quad , \qquad (7)$$

$$p_i\leftrightarrow\left(\pi^\mu,\pi_\varphi\right)\equiv\left(\pi^0,\pi^1,\pi_\varphi\right) \quad , \qquad (8)$$

$$\chi_\alpha=(G_a,C_a)\leftrightarrow\left(\pi^0,\partial_1\pi^1+e\left(\pi_\varphi+\partial_1\varphi+e(a-1)A_0+eA_1\right)\right) \quad , \qquad (9)$$

$$H\leftrightarrow H_{SC} \quad . \qquad (10)$$

Next, we shall associate to the SCCO system a OPFFCS in a phase-space with the local coordinates $\left(q^i,p_i,z^a,\bar{p}_a\right)$. This construction will be accomplished in two steps. The first step consists in associating to the original SCCO system a first-class one in the original phase-space, having $G_a=0$ as primary and the only ones constraints and the first-class Hamiltonian $\bar{H}$ such that $\left[\bar{H},G_a\right]=f^b{}_aG_b$, with $f^b{}_a$'s some functions of q's and p's. The general form of $\bar{H}$ is given in [8] . In the case of the GNCSM it is enough to find $\bar{H}$ such that $\left[\bar{H},G_a\right]=0$ strongly. Because H_{SC} is quadratic in $C^{(1)}$ the first-class Hamiltonian for the GNSCM reads

$$\bar{H}_{SC}=H_{SC}+\frac{1}{e^2(a-1)}\int dx^1\left(\frac{1}{2}\left(C^{(1)}\right)^2+C^{(0)}C^{(1)}\right) \quad . \qquad (11)$$

Thus, the first step is implemented.

With the aid of the first-class system constructed above we shall build in the second step a OPFFCS with the constraints $G_a = 0$, $\gamma_a = 0$ and the first-class Hamiltonian H^* such that $[G_a, H^*] = V_a{}^b \gamma_b$ and $[\gamma_a, H^*] = 0$, where $V_a{}^b$ are some functions of q's and p's. In order to build the Hamiltonian H^* we introduce a number of additional canonical pairs $(z^a, \bar{p}_a)$ equal with the number of functions G_a . The new variables are bosonic and fulfill $\left[z^a, \bar{p}_b\right] = \delta^a{}_b$. Then, H^* takes the form

$$H^* = \bar{H} + \frac{\lambda^2}{2}\Delta^{ab}\bar{p}_a\bar{p}_b - \left(\lambda\bar{p}_a + C_a^{(0)}\right)\Delta^{ab}C_b^{(1)} + g(q,p,z) \quad , \tag{12}$$

with $g(q,p,z)$ such that $\left[\lambda\bar{p}_a + C_a^{(0)}, \bar{H} + g\right] = 0$. In (12) Δ^{ab} is the inverse of the matrix Δ_{ab} and λ is the parameter of the OPFFCS. It is easy to see from (12) that $\gamma_a \equiv \lambda\bar{p}_a + C_a^{(0)}$. It is clear that $\left[G_a, \gamma_b\right] = 0$ strongly, so that the above constraints are first-class. The consistency conditions on $\gamma_a = 0$ don't imply further constraints due to the construction of $g(q,p,z)$. Thus, the second step is also implemented.

In the case of the GNCSM, in agreement with the previous general discussion, we introduce a single canonical pair $(z, \bar{p})$, so that

$$H^*_{SC} = \int dx^1 \left(\frac{1}{2}\pi_1\pi_1 + \frac{1}{2}\pi_\varphi^2 + \frac{\lambda^2}{2e^2(a-1)}\bar{p}^2 - \frac{\lambda}{e(a-1)}\bar{p}\left(\partial_1\varphi + e(a-1)A_0 + eA_1\right) - \right.$$

$$-A_0\partial_1\pi^1 - e\pi\varphi\left(A_0 - A_1 - \frac{1}{\lambda}\partial_1 z\right) + \frac{e^2a^2}{2(a-1)}A_1^2 + \frac{a}{2(a-1)}\partial_1\varphi\partial_1\varphi +$$

$$\left. +eaA_1\left(\frac{e}{\lambda}\partial_1 z + \frac{1}{(a-1)}\partial_1\varphi\right) + \frac{e^2a}{2\lambda^2}\partial_1 z\partial_1 z\right) \quad , \tag{13}$$

and also

$$\gamma \equiv \lambda\bar{p} + \partial_1\pi^1 + e\pi\varphi = 0 \quad . \tag{14}$$

From (13) and (14) one checks explicitly that $\left[\pi^0, H^*_{SC}\right] = \gamma$ and $\left[\gamma, H^*_{SC}\right] = 0$.

In this way we associate to the original SCCO system described by the action (6) a OPFFCS in a phase-space with the local coordinates $\left(q^i, p_i, z^a, \bar{p}_a\right)$, the last system being described by the extended action

$$S_0^E[q,p,z,\bar{p},v,u] = \int dt\left(\dot{q}^i p_i + \dot{z}^a\bar{p}_a - H^* - v^a G_a - u^a\gamma_a\right) \quad , \tag{15}$$

with v^a and u^a the Lagrange multipliers associated to the FCC. The action (15) is invariant under the following gauge transformations

$$\delta_\varepsilon q^i = \left[q^i, G_a\right]\varepsilon_1^a + \left[q^i, C_a^{(0)}\right]\varepsilon_2^a \quad ,(16a) \quad \delta_\varepsilon z^a = \lambda\varepsilon_2^a, \quad (16c) \quad \delta_\varepsilon v^a = \dot{\varepsilon}_1^a \quad , \tag{16e}$$

$$\delta_\varepsilon p_i = \left[p_i, G_a\right]\varepsilon_1^a + \left[p_i, C_a^{(0)}\right]\varepsilon_2^a \quad ,(16b) \quad \delta_\varepsilon\bar{p}_a = 0, (16d) \quad \delta_\varepsilon u^a = \dot{\varepsilon}_2^a + \bar{V}^a{}_b\varepsilon_2^b + \varepsilon_1^a \quad , (16f)$$

where $\bar{V}^a{}_b = -\Delta^{ac}\left[C_b^{(0)}, C_c^{(1)}\right]$.

4. The BRST quantization of the first-class family

Now, we pass to the quantization of the OPFFCS described by the action (15) in the framework of the BRST antibracket-antifield formalism. We assume the constraints $G_a = 0$ and $\gamma_a = 0$ to be irreducible (as they actually are in the case of the investigated model). In agreement with the general formalism [13] we associate to every variable in the theory its antifield, denoted by $q_i^*, p^{*i}, z_a^*, \bar{p}^{*a}, v_a^*$ and u_a^* . As the variables appearing in (15) are all bosonic, it follows that the Grassmann parities and ghost numbers of every antifield are respectively equal to one and minus one. The constraints being irreducible and abelian, the minimal ghost spectrum contains only the ghosts η_1^a and η_2^a associated to the gauge parameters ε_1^a and ε_2^a . The Grassmann parities and ghost numbers of the two ghosts are equal to one, respectively one. With the non-minimal sector of the form $\left(B_1^a, B_{1a}^*, B_2^a, B_{2a}^*, \bar{\eta}_1^a, \bar{\eta}_{1a}^*, \bar{\eta}_2^a, \bar{\eta}_{2a}^*\right)$, the non-minimal solution of the master equation reads

$$S^E = S_0^E + \int dt\left(q_i^*\left(\frac{\partial G_a}{\partial p_i}\eta_1^a + \frac{\partial C_a^{(0)}}{\partial p_i}\eta_2^a\right) - p^{*i}\left(\frac{\partial G_a}{\partial q^i}\eta_1^a + \frac{\partial C_a^{(0)}}{\partial q^i}\eta_2^a\right) + \right.$$

$$\left. +\lambda z_a^*\eta_2^a + v_a^*\dot{\eta}_1^a + u_a^*\left(\dot{\eta}_2^a + \bar{V}^a{}_b\eta_2^b + \eta_1^a\right) + \bar{\eta}_{1a}^*B_1^a + \bar{\eta}_{2a}^*B_2^a\right) \quad . \tag{17}$$

In order to compare the path integral correspondent to the gauge-fixed action deriving from (17) with the path integral (5) we choose the gauge-fixing fermion implementing the canonical gauge conditions $C_a = 0$ and $z^a = 0$ of the form

$$\Psi = -\int dt\left(\bar{\eta}_1^a C_a + \lambda\bar{\eta}_2^a\Delta_{ab}z^b\right) \quad . \tag{18}$$

Eliminating the antifields from (17) with the relations $\Phi_A^* = \frac{\overleftarrow{\partial}\Psi}{\partial\Phi^A}$, where $\Phi^A = \left(q^i, p_i, z^a, \bar{p}_a, B_1^a, B_2^a, \bar{\eta}_1^a, \bar{\eta}_2^a\right)$ and Φ_A^* their antifields, we obtain the gauge-fixed action $S_\Psi^E = S^E\left(\Phi^A, \Phi_A^* = \frac{\overleftarrow{\partial}\Psi}{\partial\Phi^A}\right)$ of the form

$$S_\Psi^E = S_0^E - \int dt\left(\bar{\eta}_1^a\Delta_{ab}\eta_1^b + \lambda^2\bar{\eta}_2^a\Delta_{ab}\eta_2^b + B_1^aC_a + \lambda B_2^a\Delta_{ab}z^b\right) \quad . \tag{19}$$

The action (19) is invariant under the following gauge-fixed BRST transformations

$sq^i = \left[q^i, G_a\right]\eta_1^a + \left[q^i, C_a^{(0)}\right]\eta_2^a$,(20a) $\quad s\eta_1^a = 0$,(20e) $\quad sB_1^a = 0$,(20i)

$sp_i = \left[p_i, G_a\right]\eta_1^a + \left[p_i, C_a^{(0)}\right]\eta_2^a$,(20b) $\quad s\eta_2^a = 0$,(20f) $\quad sB_2^a = 0$,(20j)

$sz^a = \lambda\eta_2^a$, (20c) $\quad sv^a = \dot{\eta}_1^a$,(20g) $\quad s\bar{\eta}_1^a = -B_1^a$,(20k)

$s\bar{p}_a = 0$,(20d) $\quad su^a = \dot{\eta}_2^a + \bar{V}^a{}_b\eta_2^b + \eta_1^a$,(20h) $\quad s\bar{\eta}_2^a = -B_2^a$.(20l)

The path integral correspondent to the OPFFCS is then given by

$$Z_\Psi^H = \int DqDpDzD\bar{p}DvDuD\bar{\eta}_1D\eta_1D\bar{\eta}_2D\eta_2DB_1DB_2\exp\left(iS_\Psi^E\right) \quad . \tag{21}$$

If one applies the previous results to the GNCSM one gets the path integral corresponding to its OPFFCS as

$$Z^H_{SC} = \int DA_\mu D\pi^\mu D\varphi D\pi_\varphi DzD\bar{p}DvDuD\bar{\eta}_1 D\eta_1 D\bar{\eta}_2 D\eta_2 DB_1 DB_2 \exp\left(iS^E_{\Psi_{SC}}\right) \quad , \quad (22)$$

where

$$S^E_{\Psi_{SC}} = \int d^2x\Big(\dot{A}_\mu\pi^\mu + \dot{\varphi}\pi_\varphi + \dot{z}\bar{p} - H^*_{SC} - v\pi^0 - u\gamma - \\ -e^2(a-1)\left(\bar{\eta}_1\eta_1 + \lambda^2\bar{\eta}_2\eta_2\right) - B_1C - \lambda e^2(a-1)B_2 z\Big) \quad . \quad (23)$$

The BRST transformations leaving the action (23) invariant are:

$sA_0 = \eta_1, sA_1 = -\partial_1\eta_2, s\pi^0 = s\pi^1 = s\bar{p} = s\pi_\varphi = s\eta_1 = s\eta_2 = sB_1 = sB_2 = 0, s\varphi = e\eta_2,$
$sz = \lambda\eta_2, sv = \dot{\eta}_1, su = \dot{\eta}_2 + \eta_1, s\bar{\eta}_1 = -B_1, s\bar{\eta}_2 = -B_2$.

Performing in (22) the integration over $A_0, \pi^0, z, \bar{p}, v, u, \bar{\eta}_1, \eta_1, \bar{\eta}_2, \eta_2, B_1$ and B_2 we get

$$Z^H_{\Psi_{SC}} = \int DA_1 D\pi^1 D\varphi D\pi_\varphi \exp\left(i\bar{S}_{SC}\right) \quad , \quad (24)$$

where

$$\bar{S}_{SC} = \int d^2x\Big(\dot{A}_1\pi^1 + \dot{\varphi}\pi_\varphi - \frac{1}{2}\pi_1\pi_1 - \frac{a}{2(a-1)}\pi^2_\varphi - \frac{1}{2e^2(a-1)}\left(\partial_1\pi^1\right)^2 - \\ -\frac{1}{e(a-1)}\pi_\varphi\left(\partial_1\pi^1 + e\partial_1\varphi + e^2aA_1\right) - \frac{ea}{a-1}A_1\partial_1\varphi - \\ -\frac{1}{e(a-1)}\partial_1\pi^1\left(eA_1 + \partial_1\varphi\right) - \frac{a}{2(a-1)}\partial_1\varphi\partial_1\varphi - \frac{e^2a^2}{2(a-1)}A_1^2\Big) \quad . \quad (25)$$

Relation (24) expresses the path integral of the OPFFCS associated to the GNCSM in terms of the independent variables. Due to (6) applied to our model it results

$$Z_{SC} = \int DA^\mu D\pi^\mu D\varphi D\pi_\varphi \delta\left(\pi^0\right)\delta\left(A_0 + \frac{1}{e^2(a-1)}\left(\partial_1\pi^1 + e\pi_\varphi + e\partial_1\varphi + e^2A_1\right)\right)\times \\ \times\exp\left(i\int d^2x\left(\dot{A}_\mu\pi^\mu + \dot{\varphi}\pi_\varphi - h_{SC}\right)\right) \quad , \quad (26)$$

where h_{SC} represents the Hamiltonian density of H_{SC} (given by (4)) . Integrating in (26) over A_0, π^0 we obtain

$$Z^H_{SC} = Z_{SC} \quad . \quad (27)$$

The last equality shows clearly that the GNCSM and its OPFFCS are equivalent at the level of the Hamiltonian path integrals over independent variables. The result given by (25) is identical with the one derived in [10] .

Equation (27) also takes place in general. Within the hypotheses mentioned earlier, if one takes $\bar{H} = H + \frac{1}{2}\Delta^{ab}C^{(1)}_a C^{(1)}_b + \Delta^{ab}C^{(0)}_a C^{(1)}_b$, introduces the last relation in (21) and integrates over $z^a, \bar{p}_a, v^a, u^a, \bar{\eta}^a_1, \eta^a_1, \bar{\eta}^a_2, \eta^a_2, B^a_1$ and B^a_2 , one gets $Z^H_\Psi = Z$ (if one also integrates in (6) over μ^α).The exponents of Z^H_Ψ and Z differ by the term $\int d^2x\frac{1}{2}\Delta^{ab}C_aC_b$, but this fact doesn't affect the prevoius equality because of the factor $\prod_a \delta(C_a)$, which is present in the measure of Z^H_Ψ after integration over B^a_1 . This supplementary term helps us to make a simple switch from the Z^H_Ψ to its correspondent LPI. We shall be concerned with this subject within the next section, which will be devoted to obtaining the LPI forms of (22) and of the one deriving from the Lagrangian action of the OPFFCS associated to the GNCSM.

5. The Lagrangian form of the path integral

If one performs in (23) the transformation $C \to C - \frac{e^2(a-1)}{2}B$, which doesn't affect the BRST invariance of $S^E_{\Psi_{SC}}$, and introduces this new action in (22) , one gets after integration over $\pi^\mu, \pi_\varphi, \bar{p}, z, v, u, \bar{\eta}_1, \eta_1, \bar{\eta}_2, \eta_2, B_1$ and B_2 the LPI for the GNCSM of the form

$$Z^L_{SC} = \int DA_\mu D\varphi\left(e\sqrt{a-1}\right)\exp\left(iS^{LN}_0\right) \quad . \quad (28)$$

On the other hand we can evidence the LPI of the OPFFCS built earlier. In order to infer its Lagrangian action we impose the gauge conditions $u^a = 0$ [13] . The action derived from (15) with the aid of the previous conditions is called the total action and reads

$$S^T_0[q, p, z, \bar{p}, v] = \int dt\left(\dot{q}^i p_i + \dot{z}^a\bar{p}_a - H^* - v^aG_a\right) \quad . \quad (29)$$

The gauge invariances of the action (29) result from (16 a-f) via the relations $\delta_\varepsilon u^a = 0$.

In the case of our model the correspondent total action is

$$S^T_{0_{SC}}\left[A, \varphi, \pi, \pi_\varphi, z, \bar{p}, v\right] = \int d^2x\left(\dot{A}_\mu\pi^\mu + \dot{\varphi}\pi_\varphi + \dot{z}\bar{p} - H^*_{SC} - v\pi^0\right) \quad , \quad (30)$$

and is invariant under the following gauge transformations(modulo the identification $\varepsilon_2 \equiv \varepsilon$):

$\delta_\varepsilon A_\mu = -\partial_\mu\varepsilon, \delta_\varepsilon\varphi = e\varepsilon, \delta_\varepsilon z = \lambda\varepsilon, \delta_\varepsilon v = -\ddot{\varepsilon}, \delta_\varepsilon\pi^\mu = \delta_\varepsilon\pi_\varphi = \delta_\varepsilon\bar{p} = 0$. If one eliminates $\pi^\mu, \pi_\varphi, \bar{p}$ and v on their equations of motion because they are auxiliary [14] , we obtain the Lagrangian action of the OPFFCS correspondent to the GNCSM

$$S^L_{0_{OPFFCS}}[A, \varphi, z] = S^{LN}_0[A, \varphi] + \int d^2x\Big(\frac{e^2(a-1)}{2\lambda^2}\partial_\mu z\partial^\mu z + \\ +\frac{e^2}{\lambda}A_\nu((a-1)\eta^{\mu\nu} + \varepsilon^{\mu\nu})\partial_\mu z\Big) \quad . \quad (31)$$

Strictly speaking, (31) should also contain the term $\frac{e}{\lambda}\int d^2x\varepsilon^{\mu\nu}\partial_\mu z\partial_\nu\varphi$, but this term doesn't contribute to the equations of motion. It is, in fact, an anomalous term and was neglected vanishing the integral of the divergence. The last relation exhibits that at the Lagrangian level there is a OPFFCS corresponding to the original GNCSM . For $\lambda = -e$ the second term in the right side of (31) represents the Wess-Zumino action (3) modulo the identification $\theta \equiv z$.

At this point we are able to clarify the problem of the equivalence at the level of path integrals between the GNCSM and the OPFFCS . The action (31) is invariant under the gauge transformations: $\delta_\varepsilon A_\mu = -\partial_\mu\varepsilon, \delta_\varepsilon\varphi = e\varepsilon, \delta_\varepsilon z = \lambda\varepsilon$. Applying the antibracket-antifield BRST formalism to the action (31) we find the non-minimal solution of the master equation

$$S^L_{OPFFCS} = S^L_{0_{OPFFCS}} + \int d^2x\left(-A^{*\mu}\partial_\mu\eta + e\varphi^*\eta + \lambda z^*\eta + \bar{\eta}^*B\right) \quad , \quad (32)$$

where $A^{*\mu}, \varphi^*, z^*$ are the antifields of the fields A_μ, φ, z and η represents the Lagrangian ghost field, while $(B, B^*, \bar{\eta}, \bar{\eta}^*)$ give the non-minimal sector.

In order to compare the path integral deriving from (32) with (28) it is natural to take a gauge-fixing fermion implementing the canonical gauge condition $z = 0$. Thus, it has the form $\Psi_L = \int d^2x(\bar{\eta}z)$. Eliminating as usually the antifields from (32) one infers the gauge-fixed Lagrangian action associated to the OPFFCS of the form

$$S^L_{\Psi_L\,\mathrm{OPFFCS}} = S^L_{0\,\mathrm{OPFFCS}} + \int d^2x(\lambda\bar{\eta}\eta + Bz) \quad . \tag{33}$$

The action (33) is invariant under the following gauge-fixed BRST transformations: $sA_\mu = -\partial_\mu\eta, s\varphi = e\eta, sz = \lambda\eta, s\bar{\eta} = B, s\eta = sB = 0$. After performing the integration over $\bar{\eta}, \eta, B$ and z the path integral correspondent to the action (33) takes the form

$$Z^L_{\Psi_L\,\mathrm{OPFFCS}} = \int DA_\mu D\varphi\,\lambda\,\exp\left(iS_0^{LN}\right) \quad . \tag{34}$$

Because (28) and (34) must coincide it follows that

$$\lambda = e\sqrt{a-1} \quad . \tag{35}$$

It is then clear that from the entire OPFFCS only the one with the above value of the parameter is equivalent at the level of the path integral with the GNCSM, but it does not reveal the Wess-Zumino action. We can always modify the gauge-fixing fermion such that $\Psi_L = \beta\int d^2x(\bar{\eta}z)$ without affecting (34) as β is present in every of the last two terms in (33). It follows that for $\lambda = -e$ it is impossible to recover the factor $e\sqrt{a-1}$ from the measure of (28), so that the path integral correspondent to the action (1) and (2) (after the BRST quantization for this one) don't coincide in general.

In [10] Srivastava compares the gauge-fixed action resulting after the quantization of the two formulations of the CSM at the level of independent variables. The corresponding path integrals coincide because the factor in the measure is well-known to be equal to one at this level. The equivalence between the path integrals mentioned before must be checked at the Lagrangian level. As we notified earlier, the equivalence takes place only for λ given by (35).

6.Conclusion

If one extends the original phase-space by adding some extravariables, the BRST quantization of the original SCCO system means, in fact, the BRST quantization of a OPFFCS. The path integral of the OPFFCS (after the BRST quantization) coincides with the one of the original SCCO system at the level of indepedent variables. At this level the path integral of the OPFFCS does not depend on the parameter λ. The LPI correspondent to the original SCCO system and to the OPFFCS coincide for a unique value of the parameter λ. For the CSM this value is given by (35). For the previous value of λ the contribution of the extravariables to the Lagrangian action doesn't coincide with the Wess-Zumino action. The Wess-Zumino action is recovered for $\lambda = -e$. The method exposed in this paper can be extended to the case of the canonical Hamiltonian of the original SCCO system more than quadratic in the functions C_a. This case will reported elsewhere.

References

CJMAA

[1] P.A.M.Dirac, Can. J. Math. **2** (1950) 129; Lectures on Quantum Mechanics (Yeshiva University, 1964).

[2] J. M. L. Fisch, Mod. Phys. Lett. A **5** (1990) 195. MPLAE

[3] J. M. L. Fisch and M. Henneaux, Phys. Lett. B **226** (1989) 80.

[4] C. Battle, J. Gomis, J. Paris and J. Roca, Phys. Lett. B **224** (1989) 288.

[5] I. A. Batalin and E. S. Fradkin, Riv. Nuovo Cimento **9** (1986) 1.

[6] L. D. Fadeev, Theor. Math. Phys. **1** (1970) 1. TMPHA

[7] P. Senjanovic, Ann. Phys. **100** (1976) 227.

[8] C. Bizdadea and S.O.Saliu, The BRST Quantization of Second-Class Constrained Systems, University of Craiova preprint UC-TP-1/95 (1995).

[9] L. D. Fadeev and S. L. Shatashvili, Phys. Lett. B **176** (1986) 225.

[10] P. P. Srivastava, Phys. Lett. B **235** (1990) 287.

[11] R. Jackiw and R. Rajaraman, Phys. Rev. Lett17. **54** (1985) 1219, and references therein

[12] N. K. Falck and G. Kramer, Ann. Phys. **6** (1987) 330; APNYA
H. O. Girotti and H. J. Rothe, Int. J. Mod. Phys. A **4** (1989) 3041, and references therein IMPAE

[13] M. Henneaux and C. Teitelboim, Quantization of Gauge Systems, Princeton Univ. Press, Princeton 1992.

[14] M. Henneaux, Phys. Lett. B **238** (1990) 299.