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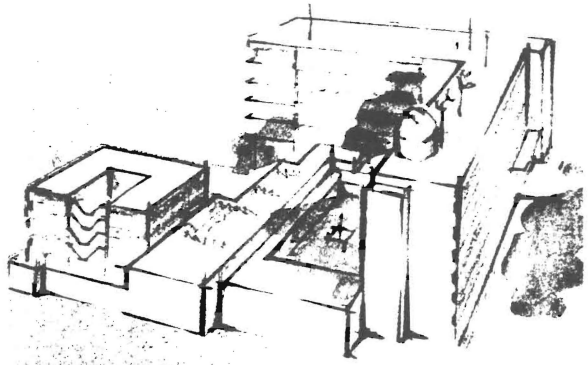

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February 1993

Renormalization-group improved effective potential for gauge theories in curved spacetime

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Abstract

The renormalization-group improved effective potential for an arbitrary renormalizable massless gauge theory in curved spacetime is found, thus generalizing Coleman-Weinberg's approach corresponding to flat space. Some explicit examples are considered, among them: $\lambda\varphi^4$ theory, scalar electrodynamics, the asymptotically-free SU(2) gauge model, and the SU(5) GUT theory. The possibility of curvature-induced phase transitions is analyzed. It is shown that such a phase transition may take place in a SU(5) inflationary universe. The inclusion of quantum gravity effects is briefly discussed.

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1. Introduction. It is well known in modern gauge theories that symmetry breaking and/or restoration can be caused by some external conditions, like temperature, external electric and magnetic fields, finite density, etc. Moreover, these changes in the vacuum structure (i.e., symmetry breaking and restoration) may be described as some phase transition (using the effective potential formalism). Such phase transitions are extremely important in early universe cosmology. Specifically, some models of inflationary universe (see [1,2] for a review and list of references) are based on the first-order phase transitions which take place during the reheating of the universe in the grand unification epoch.

However, it is clear that curved spacetime effects in the early universe (the GUT epoch) cannot be considered at all to be negligibly small. Therefore, GUTs corresponding to the very early universe ought to be treated as quantum field theories in *curved* spacetime (for a general review see [3]).

Unfortunately, at present we do not have a clear prescription how to combine quantum field theory at non-zero temperature and quantum field theory in curved spacetime (external temperature *and* external gravitational field). In such a situation, it seems reasonable to investigate in depth the two topics involved: field theory in curved space and field theory at non-zero temperature, always with the aim of combining them, later on. We will concentrate on the first topic here.

In the present paper, we shall study the effective potential of massless gauge theories in curved spacetime. We will work in the linear curvature approximation because, as it has been argued, at least linear curvature terms should be taken into account in the discussion of the effective potential corresponding to GUTs in the early universe. Also, quantum corrections with account to gravity effects should be even more important in a chaotic inflationary model[1].

By generalizing the Coleman-Weinberg approach [4] (for a review, see [5]) corresponding to the case of the effective potential in flat spacetime, at a first instance, we find the renormalization group (RG) improved effective potential in curved space. Hence, we extend the one-loop effective potential in curved spacetime, taking into account all logarithmic corrections. (The one-loop effective potential in curved spacetime has been calculated previously for the cases of scalar electrodynamics [6] and arbitrary massless gauge theory [7]. In these papers the corresponding curvature-induced phase transitions have been also discussed.)

We shall here present the explicit form of the RG improved effective potential for $\lambda\varphi^4$, scalar electrodynamics, the SU(2) gauge model, and SU(5) GUT. The possibility of corresponding curvature-induced phase transitions will be briefly discussed.

2. RG improved effective potential. Let us consider an arbitrary, renormalizable, massless gauge theory including scalars φ, spinors ψ, and vectors A_μ. Denote by $\tilde{g} \equiv (g, \lambda, h)$

the set of all coupling constants of the theory (g is the Yang-Mills, λ the scalar, and h the Yukawa coupling). The tree level potential reads

$$V^{(0)} = a\lambda\varphi^4 - b\xi R\varphi^2, \tag{1}$$

where a and b are some positive constants, ξ the conformal coupling [3], and R the scalar curvature.

The RG equation for the effective potential in curved spacetime has the following form [3,7]:

$$\left(\mu\frac{\partial}{\partial\mu} + \beta_{\tilde{g}}\frac{\partial}{\partial\tilde{g}} + \delta\frac{\partial}{\partial\alpha} + \beta_\xi\frac{\partial}{\partial\xi} - \gamma\varphi\frac{\partial}{\partial\varphi}\right)V = 0, \tag{2}$$

where α is the gauge parameter. This RG equation is standard [4,5] but not for the term connected with ξ.

We now split V into $V \equiv V_1 + V_2 \equiv af_1(p,\varphi,\mu)\varphi^4 - bf_2(p,\varphi,\mu)R\varphi^2$, where f_1 and f_2 are some unknown functions and $p = \{\tilde{g},\alpha,\xi\}$. We also assume that both V_1 and V_2 satisfy the RG equation (2). (Notice that this is in fact a restriction, because in general only V should satisfy (2), and not V_1 and V_2 separately.) We choose the Landau gauge for the gauge fields, so that in the one-loop approximation $\delta = 0$ in (2). The other point is that $\varphi^2 >> |R|$, otherwise we cannot meaningfully expand the effective potential with an accuracy up to linear curvature terms.

With all these considerations in mind, we can now solve the RG equation (2) in the way

$$V = a\lambda(t)f^4(t)\varphi^4 - b\xi(t)f^2(t)R\varphi^2, \tag{3}$$

where $f(t) = \exp\left[-\int_0^t dt'\,\bar{\gamma}\left(\tilde{g}(t'),\alpha(t'),\xi(t')\right)\right]$, $t = \frac{1}{2}\ln(\varphi^2/\mu^2)$, $\dot{\tilde{g}}(t) = \bar{\beta}_{\tilde{g}}(t)$, $\dot{\alpha}(t) = \bar{\delta}(t)$, $\dot{\xi}(t) = \bar{\beta}_\xi(t)$, $\tilde{g}(0) = \tilde{g}$, $\alpha(0) = \alpha$, $\xi(0) = \xi$, and $(\bar{\beta}_{\tilde{g}},\bar{\delta},\bar{\beta}_\xi,\bar{\gamma}) = \frac{1}{1+\gamma}(\beta_g,\delta,\beta_\xi,\gamma)$. Notice that in the solution of Eq. (2) we have used the following initial conditions:

$$V_1(t=0) = a\lambda\varphi^4, \qquad V_2(t=0) = -b\xi R\varphi^2. \tag{4}$$

These initial conditions are only slightly different from the ones used in Refs. [4,7], while that for V_1 is the same as in Ref. [5]. The differences here will lead to some differences in the non-logarithmic φ^4 and $R\varphi^2$ terms.

In the one-loop approximation (in which we actually work in this paper), Eq. (3) remains formally the same, but now with

$$f(t) = \exp\left[-\int_0^t dt'\,\gamma\left(\tilde{g}(t'),\xi(t')\right)\right], \quad \dot{\tilde{g}}(t) = \beta_{\tilde{g}}(t), \quad \dot{\xi}(t) = \beta_\xi(t), \quad \tilde{g}(0) = \tilde{g}, \quad \xi(0) = \xi. \tag{5}$$

Expression (5) constitutes our main result for the one-loop RG improved potential, and can be applied to a variety of gauge theories.

Let us now consider some examples.

(a) $\lambda\varphi^4$ theory. In this case $\gamma = 0$, and β_λ and β_ξ are well known (see for example [3,4]); we get from (4)[3]

$$V = \frac{\lambda\varphi^4}{4!\left(1 - \frac{3\lambda t}{(4\pi)^2}\right)} - \frac{1}{2}R\varphi^2\left[\frac{1}{6} + \left(\xi - \frac{1}{6}\right)\left(1 - \frac{3\lambda t}{(4\pi)^2}\right)^{-1/3}\right], \tag{6}$$

where $t = \frac{1}{2}\ln(\varphi^2/\mu^2)$. In the limit in which both λ and λt are small, our result agrees with the previously obtained one-loop potential [6,7]. However, as has been discussed in [4,5] for the case of flat space, Eq. (5) is actually valid for all t for which the potential does not diverge (in particular, for any negative value of t).

(b) Scalar electrodynamics. Using the well-known results [4,6,7]

$$\beta_\lambda = \frac{1}{4\pi^2}\left(\frac{5}{6}\lambda^2 - 3e^2\lambda + 9e^4\right), \quad \beta_\xi = \frac{\xi - \frac{1}{6}}{4\pi^2}\left(\frac{1}{3}\lambda - \frac{3}{2}e^2\right), \quad \gamma = -\frac{3e^2}{16\pi^2},$$

where γ is given in the Landau gauge, we obtain

$$\begin{aligned} V &= \frac{1}{4!}f^4(t)\varphi^4\left\{\frac{1}{10}e^2(t)\left[\sqrt{719}\tan\left(\frac{1}{2}\sqrt{719}\ln e^2(t) + \theta\right) + 19\right]\right\} \\ &- \frac{1}{2}R\varphi^2 f^2(t)\left[\frac{1}{6} + \left(\xi - \frac{1}{6}\right)\left(1 - \frac{e^2 t}{24\pi^2}\right)^{-26/5}\cos^{2/5}\left(\frac{1}{2}\sqrt{719}\ln e^2(t) + \theta\right)\right. \\ &\cdot \left.\cos^{-2/5}\left(\frac{1}{2}\sqrt{719}\ln e^2 + \theta\right)\right]. \end{aligned} \tag{7}$$

Here $e^2(t) = e^2\left(1 - \frac{e^2 t}{24\pi^2}\right)^{-1}$, θ is an integration constant, which should be chosen such that $\lambda(t) = \lambda$ when $e^2(t) = e^2$, and $f(t) = \left(1 - \frac{e^2 t}{24\pi^2}\right)^{-9/2}$. It is interesting to notice that for very small variations in $e^2(t)$ the argument of the tangent and cosinus can change by 2π, leading to a big difference in $\lambda(t)$ and $\xi(t)$.

A few remarks are in order. To compare with Coleman-Weinberg's result [4], we consider the one-loop non-improved effective potential with the standard proposal $\lambda \sim e^4$. Then one can get from (7)

$$V = \frac{\lambda}{4!}\varphi^4 + \frac{3e^4\varphi^4}{64\pi^2}\ln\frac{\varphi^2}{\mu^2} - \frac{1}{2}\xi R\varphi^2 - \frac{1}{(8\pi)^2}e^2 R\varphi^2\ln\frac{\varphi^2}{\mu^2}. \tag{8}$$

Choose now $\mu = <\varphi>$, where $<\varphi>$ is the vacuum (minimum) configuration. In flat space the equation $V'(<\varphi>) = 0$ gives the precise connection between λ and e^4. However, in curved space this is not the case, and

$$\frac{V'(<\varphi>)}{<\varphi>} = <\varphi>^2\left(\frac{\lambda}{6} + \frac{3e^4}{32\pi^2}\right) - R\left(\xi + \frac{e^2}{32\pi^2}\right) = 0. \tag{9}$$

[3]Note that the one-loop effective potential for the $\lambda\Phi^4$ theory has been calculated in Refs. [16] for different specific background spaces.

Hence, we obtain the connection between $<\varphi>$ and the curvature corresponding to the minimum.

Since we are working in the linear curvature approximation (supposing that the curvature correction is small), we may just impose (without much error) the flat space condition $\lambda/6 = -3e^4/(32\pi^2)$ by hand. Then, from (9) we get

$$\xi = -\frac{e^2}{32\pi^2}, \tag{10}$$

and

$$V = \frac{3e^4\varphi^4}{64\pi^2}\left(\ln\frac{\varphi^2}{<\varphi>^2} - \frac{1}{2}\right) - \frac{e^2R\varphi^2}{64\pi^2}\left(\ln\frac{\varphi^2}{<\varphi>^2} - 1\right). \tag{11}$$

Eq. (11) constitutes the generalization to curved space of the famous Coleman-Weinberg result (Eq. (4.9) in [4]) given in universal form. Notice that, from (11), we immediately obtain the scalar mass which takes into account curvature effects:

$$m^2(s) = V''(<\varphi>) = \frac{3e^4<\varphi>^2}{8\pi^2} - \frac{e^2R}{32\pi^2}. \tag{12}$$

(c) The SU(2) gauge model. Let us now consider the SU(2) gauge model of Ref. [9] with one multiplet of scalars (φ^a, $a = 1,2,3$) taken in the adjoint representation of SU(2) and one or two multiplets of spinors also taken in the adjoint representation. The Yukawa coupling acts through only one of the spinor multiplets (see [9] for the precise Lagrangian). This theory is asymptotically free for all coupling constants [9] and in the case of only one spinor multiplet it is also asymptotically conformal invariant.

The RG improved effective potential of this theory can be calculated to be

$$V = \frac{1}{4!}\varphi^4 f^4(t) k_1 g^2(t) - \frac{1}{2}R\varphi^2 f^2(t)\left[\frac{1}{6} + \left(\xi - \frac{1}{6}\right)\left(1 + \frac{a^2g^2t}{(4\pi)^2}\right)^{-(12-5k_1/3-8k_2)/a^2}\right], \tag{13}$$

where $\varphi^2 = \varphi^a\varphi^a$, $\lambda(t) = k_1 g^2(t)$, $h^2(t) = k_2 g^2(t)$, $g^2(t) = g^2\left(1 + \frac{a^2g^2t}{(4\pi)^2}\right)^{-1}$, being the values of the numerical constants k_1, k_2 and a^2 given in Ref. [9], and $f(t) = \left(1 + \frac{a^2g^2t}{(4\pi)^2}\right)^{(6-4k_2)/a^2}$.

In the same way one can find the RG improved effective potential in asymptotically free GUTs (for a review and a list of references, see [3]).

(d) The SU(5) GUT. Let us now study the RG improved potential for the SU(5) GUT [8]. In flat space this theory has been used for the discussion of inflationary cosmology [1,2]. The one-loop potential in the linear curvature approximation has been given in Ref. [7].

The tree-level potential has the form

$$V_{\text{tree}} = \frac{1}{4}\lambda_1(\mathrm{Tr}\,\phi^2)^2 + \frac{1}{2}\lambda_2\mathrm{Tr}\,\phi^4 - \frac{1}{2}\xi R\mathrm{Tr}\,\phi^2, \tag{14}$$

where λ_1 and λ_2 are scalar couplings, and for simplicity we suppose that there are no fermions in the theory. Even in this case, the system of RG equations for the coupling constants is quite complicated and can be solved only numerically. This is why we will just consider the vector loop contributions to the β-functions. Presumably [5] such approach gives qualitatively the same results that would be obtained with the inclusion of scalar couplings.

We assume that the breaking SU(5) $\to$ SU(3) x SU(2) x U(1) has taken place. Then $\phi = \varphi$ diag $(1,1,1,-\frac{3}{2},-\frac{3}{2})$ and

$$V_{\text{tree}} = \frac{15}{16}(15\lambda_1 + 7\lambda_2)\varphi^4 - \frac{15}{4}\xi R\varphi^2. \tag{15}$$

Within our approach

$$\begin{aligned}\frac{dg(t)}{dt} &= -\frac{5g^3(t)}{6\pi^2}, \quad \frac{d}{dt}\left[\frac{15}{4}(15\lambda_1 + 7\lambda_2)\right] \equiv \frac{d\Lambda(t)}{dt} = \frac{5625}{128\pi^2}g^4(t),\\ \frac{d\xi(t)}{dt} &= -\frac{30}{16\pi^2}\left(\xi(t) - \frac{1}{6}\right)g^2(t), \quad \gamma = -\frac{15g^2}{16\pi^2}.\end{aligned} \tag{16}$$

Solving Eq. (16) and substituting the result into (5) we get the RG improved effective potential:

$$V = \frac{3375}{512}\left(g^2 - \frac{g^2}{1 + \frac{5g^2t}{3\pi^2}}\right)\varphi^4 f^4(t) - \frac{15}{4}\left[\frac{1}{6} + \left(\xi - \frac{1}{6}\right)\left(1 + \frac{5g^2t}{3\pi^2}\right)^{-9/8}\right]R\varphi^2 f^2(t), \tag{17}$$

where $f(t) = \left(1 + \frac{5g^2t}{3\pi^2}\right)^{9/16}$.

This finishes our calculation of several RG improved effective potentials corresponding to different massless gauge theories in curved spacetime.

3. Phase transitions. As stated previously, it is very common nowadays to think that the very early universe experienced several phase transitions before it could reach its present state. It is certainly possible that a phase transition could be induced by the resulting (very strong) external gravitational field existing at this epoch [6,7]. We will now discuss such possibility by using our simple (but, on the other hand, quite general) RG improved effective potential.

We shall be concerned with first-order phase transitions where the order parameter φ experiences a quick change for some critical value, R_c, of the curvature. Let us first consider the $\lambda\varphi^4$ theory, in which case we can write

$$\frac{V}{\mu^4} = \frac{\lambda x^2}{4!\left(1 - \frac{3\lambda\ln x}{32\pi^2}\right)} - \frac{1}{2}\epsilon y x\left[\frac{1}{6} + \left(\xi - \frac{1}{6}\right)\left(1 - \frac{3\lambda\ln x}{32\pi^2}\right)^{-1/3}\right], \tag{18}$$

where $x = \varphi^2/\mu^2$, $y = |R|/\mu^2$, and $\epsilon = \text{sgn } R$. The critical parameters, x_c, y_c, corresponding to the first-order phase transition are found from the conditions

$$V(x_c, y_c) = 0, \qquad \left.\frac{\partial V}{\partial x}\right|_{x_c,y_c} = 0, \qquad \left.\frac{\partial^2 V}{\partial x^2}\right|_{x_c,y_c} > 0. \tag{19}$$

For the one-loop effective potential the two equations (19) can be solved analytically [7]. However, for the RG improved potential they lead to some transcendental equations which cannot be solved analytically. For instance, in the case of the $\lambda\varphi^4$-theory effective potential (18), they are

$$T = -\frac{\lambda}{16\pi^2}\frac{\frac{1}{4} + \left(\xi - \frac{1}{6}\right)T^{-1/3}}{\frac{1}{6} + \left(\xi - \frac{1}{6}\right)T^{-1/3}}, \qquad \left(16\pi^2 + \frac{\lambda}{T}\right)x + \epsilon y = 0, \tag{20}$$

where $T = 1 - (32\pi^2)^{-1}3\lambda \ln x$. The simplest case, with an analytical solution, is $\xi = 1/6$. Here we have found no phase transition (since $y_c \sim \pi^2 x_c$, which lies outside our approach $x_c >> y_c$). In the same way we can see that there is no phase transition for $\xi = 1/6$ in the asymptotically free SU(2) model.

Consider now the RG improved potential (17). For the sake of simplicity, let us put $f(t) = 1$. (To take into account a non-zero anomalous dimension presumably only rescales φ [5].) After choosing $\xi = 1/6$ and solving (19), we obtain

$$x_c \simeq 10^2, \qquad \epsilon y_c \simeq g^4 x_c. \tag{21}$$

Thus, it turns out that a curvature induced phase transition is possible in this model even in the simplest situation where $\xi = 1/6$ (there are no radiative corrections to the $R\varphi^2$-term.)

A reasonable estimation [6,7] shows that in the GUT epoch

$$10^{-7} \leq |y| \leq 10^{-5}. \tag{22}$$

And for a standard choice $g^2 \sim 1/3$ and $\epsilon y_c \simeq 10$, what seems to be too large and non-realistic. However, we could argue that at the begining of inflation g^2 corresponds to the *running* $g^2(t)$. In this case, a natural choice is [7] $g^2 \simeq 10^{-3}$ and this gives $\epsilon y_c \simeq 10^{-4}$. This value is already very close to the upper border of the estimation (22), what is quite remarkable. Of course, the inclusion of scalar loops and/or the estimation of phase transitions for other choices of ξ can also lower the value of y_c. This is an interesing analysis which demands for numerical calculations.

4. Conclusions. To summarize, we have developed an explicit formalism for the determination of the RG improved effective potential for massless gauge theories in curved spacetime. We have shown the plausible possibility of a curvature-induced phase transition taking place

for the SU(5) RG improved inflationary potential. It would be interesting to understand the further influence of quantum-gravitational effects in the above described picture.

In principle, this can be done, at least for multiplicatively-renormalizable R^2-gravity (see [3] for a review). Starting from the following Lagrangian (in Euclidean notation)

$$L = \tilde{\alpha}R^2 + \tilde{\beta}R_{\mu\nu}R^{\mu\nu} + \xi R\varphi^2 + \frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + \frac{\lambda}{4!}\varphi^4, \tag{23}$$

and forgetting for a moment about the unitarity problem (see [3] for a list of references concerning this point), we can employ the background field method in order to show that the theory given by (23) can be asymptotically free for all coupling constants [3] (see also [10]).

The formalism developed at point 2 can be extended to the theory (23) (see [11]) —by just adding $\tilde{\alpha}$ and $\tilde{\beta}$ to the set $\tilde{g}$— working in the background-field method and in the one-loop approximation (then the anomalous dimension for the background gravitational field is zero). We cannot pretend to find the $f(t)$ of Eq. (4) explicitly either, because to this end we would need to work out γ in a class of gauges dependent on the parameter. However, as before, we can perform this calculation for the simplest gauge. Hence, we drop the γ-dependence and put $f(t) = 1$ in (4). With all this in mind, the RG improved effective potential in the linear curvature approximation is again given by (4). In particular, for one of the regimes of asymptotic freedom of the theory given by (23) [10], we get

$$V = \frac{4.72}{4!}\varphi^4\tilde{\beta}^{-1}(t) - 0.03\,R\varphi^2, \tag{24}$$

where

$$\tilde{\beta}(t) = \tilde{\beta}(0) + \frac{799}{60(4\pi)^2}t.$$

We thus see that we actually may take into account the quantum gravitational effects in a rather simple way.

The other interesting topic is the derivation of RG improved effective potentials in massive theories [5,12][4]. There has been some recent activity in this direction [13], readdressing the question in flat space. It is our impression that quantum field theory in curved spacetime can really help to answer some questions about the RG improved effective potential for massive theories even in flat space. Work along this line is in progress. Notice, finally, that the generalization to the multiscale RG can be carried out as in Ref. [15] in flat space.

[4]It would be meaningful also to improve the effective potential in the models which have composite Higgs scalars in curved space [14].

Acknowledgments.

S.D.O. wishes to thank JSPS (Japan) for financial support and the Particle Group at Hiroshima University for kind hospitality, and E.E. the DGICYT (Spain), for financial help through research project PB90-0022. We are very grateful to T. Muta for his interest in this work and for helpful discussions.

Appendix. We shall here study the behaviour of the RG improved effective potential (18) in the $\lambda\varphi^4$ model. We shall also compare it with that of the one-loop effective potential for the same theory, which is

$$\frac{V_{\text{one-loop}}}{\mu^4} = \frac{\lambda x^2}{4!}\left(1+\frac{3\lambda\ln x}{32\pi^2}\right) - \frac{1}{2}\epsilon y x\left[\xi + \left(\xi - \frac{1}{6}\right)\frac{\lambda\ln x}{32\pi^2}\right]. \tag{25}$$

(This is just an expansion of (18), taking $\lambda << 1$ and $|\lambda \ln x| << 1$.) A very important difference between (18) and (25) lies in the fact that while the RG improved effective potential (18) has a pole for $x = x_p \equiv \exp[32\pi^2/(3\lambda)]$, the one-loop effective potential (25) is a smooth function for the whole range of values of x and y. Notice however, in particular, that expression (18) is finite for all negative values of $\ln x$. The fact that the $\lambda\varphi^4$ potential (25) exists for *all* values of φ makes this expression obviously wrong.

We perform the usual analysis of extrema of (18) and (25). Calling $V(x,y)$ the potential in each case, we shall look for critical points (x_c, y_c) defined by the simultaneous conditions (19). The first two equations (19) yield, for the RG improved action (18),

$$y_c = \frac{\epsilon\lambda x_c}{2u\left[1+(6\xi-1)u^{-1/3}\right]}, \quad \frac{32\pi^2}{3\lambda}u - \frac{1}{3\left[1+(6\xi-1)u^{1/3}\right]} + 1 = 0, \quad u \equiv 1 - \frac{3\lambda\ln x_c}{32\pi^2}, \tag{26}$$

and, for the one-loop action (25),

$$y_c = \frac{\epsilon\lambda(1+3v)x_c}{12\left[\xi+\left(\xi-\frac{1}{6}\right)v\right]}, \quad \frac{32\pi^2}{\lambda}\left(v+\frac{1}{3}\right) - \frac{\lambda}{96\pi^2}\left(\xi-\frac{1}{6}\right)\frac{1+3v}{\xi+\left(\xi-\frac{1}{6}\right)v} = 0, \quad v \equiv \frac{\lambda\ln x_c}{32\pi^2}. \tag{27}$$

The following models, which are particularly interesting for different reasons, will be considered in further detail.

(a) Chaotic inflationary model. We consider first the potentials (18) and (25) for $\lambda = 10^{-13}$ and $\xi = 0$, both for positive $\epsilon = 1$ and negative $\epsilon = -1$ curvature. The results can be summarized as follows. For the RG improved effective action, a critical value appears, which lies close to the pole

$$x_c = \exp\left(-\frac{2}{3}10^{-15}\right)x_p, \quad y_c = -\epsilon\, 10^{-3}\, x_c. \tag{28}$$

Moreover, this point is a minimum of (18) (as are all the similar points obtained below). That is, all three equations (19) are indeed satisfied. On the contrary, the one-loop effective action (25) does not yield any phase transition, the solutions of Eqs. (27) being $x_c = 0$, $y_c = 0$, and $x_c = 1$, $y_c = \epsilon\, 10^{17}\, x_c$.

(b) Variable Planck-mass model. It has been considered in [17]. For λ we take a typical value corresponding to particle physics models, e.g. $\lambda = 0.05$. For ξ we choose two different values: (i) $\xi = -10^4$ (which actually corresponds to ref.[17]) and (ii) $\xi = 1/6$, respectively.

In case (i), the critical point corresponding to (18) is obtained for

$$x_c = e^{2/3}\, x_p, \quad y_c = -5\cdot 10^{-[illegible]}\, x_c, \tag{29}$$

both for positive and for negative curvature. For the one-loop action, the only solution is again the trivial one $x_c = y_c = 0$.

In case (ii), the critical point for the RG improved action (18) is at

$$x_c = e\, x_p, \quad y_c = -\epsilon\, 50\, x_c, \tag{30}$$

which is not consistent with our approximation $x_c >> y_c$. For the one-loop effective action (25), $x_c = y_c = 0$ is again the only solution of (27).

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