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ASYMTOTICALLY FREE GRAND UNIFICATION THEORIES AND WORMHOLES

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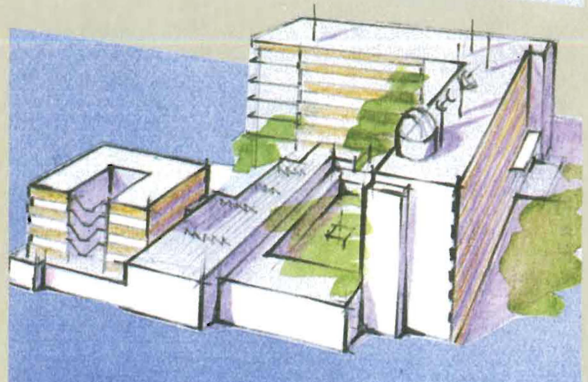

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Wormholes can provide the beautiful mechanizm [1-3] which can help to decide the long-standing cosmological constant problem (for a review, see ref [4]).It is not quite clear at the moment, is this the case (see papers [5] and references therein for the description of the different points of view and approaches) and can the wormholes really drive the cosmological constant to zero [1-3]? In any case the mechanizm of refs.[1-3] demands the further investigations.

In the present letter we consider asymptotically-free GUT's in curved space-time [6,7] (see [7],for a review).We argue that renormalisation group analysis of GUT's in curved space-time can support the wormholes mechanizm of the driving the cosmological constant to zero.

Let us start from the SU(2) gauge theory in curved space-time. The renormalized Lagrangian is [7]

$$
\begin{aligned}
L &= L_{ext} + L_m ,\\
L_{ext} &= aR^2 + bG + cC^2_{\mu\nu\alpha\beta} + d\Box R + \varkappa R + \Lambda ,\\
L_m &= L_{YM} + \frac{1}{2}(D_\mu^{ab}\varphi^b)^2 + \frac{1}{2}\xi R\varphi^a\varphi^a - \frac{1}{4!}f(\varphi^a\varphi^a)^2 +\\
&+ i\sum_{k=1}^{n}\bar{\Psi}^a_{(k)}(\gamma^\mu D_\mu^{ab} - h\varepsilon^{acb}\varphi^c)\psi^b_{(k)} + \frac{1}{2}m^2\varphi^a\varphi^a
\end{aligned}
\qquad (1)
$$

where $a = 1,2,3$, $n = 1$ or $n = 2$ (two different models),for simplicity, we take only scalar masses to be non-zero. The renormalization group equations for the coupling constants [7] lead to the following asymptotically free regime in sector of L_m:

$$
\begin{aligned}
&g^2(t) = g^2\left(1 + \frac{B^2 g^2 t}{(4\pi)^2}\right)^{-1}, \quad h^2(t) = \kappa_1 g^2(t),\\
&f(t) = \kappa_2 g^2(t), \quad \xi(t) = \frac{1}{6} + \left(\xi - \frac{1}{6}\right)\left(1 + \frac{B^2 g^2 t}{(4\pi)^2}\right)^{b},\\
&m^2(t) = m^2 e^{-2t}\left(1 + \frac{B^2 g^2 t}{(4\pi)^2}\right)^{b_6}
\end{aligned}
\qquad (2)
$$

where the constants, B^2, K_1, K_2, b are given in ref.[7],(we do not need the expicit values of these constants), b<0 when n=1 and b>0 when n=2. Here, as usually scale transformation of the metric [7]

$$\tilde{g}_{\mu\nu} \to e^{-2t} g_{\mu\nu} \tag{3}$$

is used to formulate the renormalization group in curved space. Under this transformation $\tilde{R}^2 \to e^{4t} R^2$, i.e. $t \to \infty$ corresponds to the large scalar curvature (small distances or high energies).

In ref.[2] Coleman made the simplest assumption that the Euclidean functional integral is dominated by the saddle points of the effective action. In his work the effective action Γ has been obtained by integrating out the fluctuations of all the fields except $g_{\mu\nu}$. Now as in ref.[8] we suppose that this effective action is produced by the renormalized theory of quantum gravity.(Asymtotically-free GUT's on fixed curved background can be considered as the one-loop approximation to such a theory). Using this proposal we can write (at least, quilitatively) the effective action as

$$\Gamma = \int d^4x\sqrt{g}\,\left(\Lambda(t) + \mathcal{X}(t)R + \ldots\right) \tag{4}$$

where if $\Lambda(t)$ is small, then the stationary points of Γ are very smooth manifolds and higher derivative terms in (4) are negligible.

Then, the stationary points approximately satisfy

$$R_{\mu\nu} = -\frac{\Lambda(t)}{2\mathcal{X}(t)} g_{\mu\nu}$$

According to Coleman, the dominant stationary point is the one with the lowest action. For a positive cosmological constant, this is the large 4-sphere

(De Sitter space where $r^2 = -\frac{6\mathcal{X}(t)}{\Lambda(t)}$). However,in this situation we can also assume[8] that not only $g_{\mu\nu}$, but also $\tilde{g}_{\mu\nu}$ corresponds to 4-sphere. Here $t = \frac{1}{2}\ln(\tilde{R}/R) = \ln(r/\tilde{r})$. Thus, we obtain some self-consistent equation for r^2 :

$$r^2 = -6\,\mathcal{X}\left(\ln\frac{\tilde{r}}{r}\right)\Big/\Lambda\left(\ln\frac{\tilde{r}}{r}\right) \tag{6}$$

The action turns out to be

$$\Gamma = -\frac{3\left(16\pi\mathcal{X}(t)\right)^2}{\Lambda(t)} \tag{7}$$

As it has been shown in [2] the Euclidean path integral in a large universe has a sharp peak at $\mathcal{X}^{-2}(t)\,\Lambda(t) = 0^+$. Now we can check the Coleman's proposal for some GUTs.If $\mathcal{X}^{-2}(t)\Lambda(t) \xrightarrow[t\to\infty]{} 0^+$ (strong curvature),then we can say that such a model supports (at least, quialitativly) the Coleman proposal (as it has been checked in [8] this is the case for supersymmetric GUTs). Now one can write the renormalisation group equations for $\Lambda(t)$, $\mathcal{X}(t)$ in the model under investigation :

$$\frac{d\Lambda(t)}{dt} = \left\{\frac{3m^4(t)}{2(4\pi)^2} - 4\Lambda(t)\right\},$$

$$\frac{d\mathcal{X}(t)}{dt} = \left\{\frac{3m^2(t)(\xi(t)-\frac{1}{6})}{(4\pi)^2} - 2\mathcal{X}(t)\right\} \tag{8}$$

The solutions of these equations (with functions (2)) are:

$$\Lambda(t) = e^{-4t}\left(\Lambda + \frac{3m^4}{2B^2g^2(2b+1)}\left(\left(1+\frac{B^2g^2t}{(4\pi)^2}\right)^{2b+1}-1\right)\right),$$

$$\varkappa(t) = e^{-2t}\left(\varkappa + \frac{3m^2(\xi-\frac{1}{6})}{B^2g^2(2b+1)}\left(\left(1+\frac{B^2g^2t}{(4\pi)^2}\right)^{2b+1}-1\right)\right) \qquad (9)$$

In the strong gravitational field limit $t \to \infty$ we get (in the case b>0 which corresponds to the first model of (1)):

$$\Gamma \sim -\frac{18(16\pi)^2}{B^2g^2(2b+1)}\left(\xi-\frac{1}{6}\right)^2\left(1+\frac{B^2g^2t}{(4\pi)^2}\right)^{2b+1} \to$$

$$\to -\infty . \qquad (10)$$

Now, one can say that there exists the class of asymptotically free GUT's which supports the Coleman proposal. Of course, for real Universe $t \to \infty$ means that t is big (but finite). Here, one say that the effective cosmological constant is tending to zero when the curvature is growing (at least quilitatively).

Now, as in ref.[8] we can look to the self-consistent equation for radii (6). One get $(t \to \infty,\ r^2 >> \tilde{r}^2)$

$$\tilde{r}^2 \simeq -\frac{12(\xi-\frac{1}{6})}{m^2} \qquad (11)$$

To be more realistic, let us look to the asymptotically-free realistic E_6 GUT of ref. [10]. In curved space this model has been investigated in ref.[9], where the Lagrangian is written. The free part of the Lagrangian connected with the scalars is (again, only the scalars have the masses)

$$L^{(scalars)} = \frac{1}{2}\left[D_\mu^{\alpha\gamma}\varphi^\gamma\right]^2 + \frac{1}{2}\xi_\varphi R\varphi^\gamma\varphi_\gamma + \frac{1}{2}m_\varphi^2\varphi^\gamma\varphi_\gamma$$
$$+ \left[D_\mu^{ad}M_d\right]^2 + \xi_M RM^+M + m_N^2M^+M +$$
$$+ \left[D_\mu^{ad}N_d\right]^2 + \xi_N RN^+N + m_N^2N^+N + \ldots \qquad (12)$$

where φ^γ, M_d, N_d are the scalars, α=1,..,78, a=1,...,27, for more detail, see refs.[9,10].

In the asymptotically free regime [10]

$$g^2(t) = g^2\left[1 + 32\frac{g^2t}{(4\pi)^2}\right]^{-1} \qquad (13)$$

and all other coupling constants are proportional to (13). The solution of the effective equations for $\xi_\varphi(t), \ldots, m_N^2(t)$ is [9]:

$$\xi_\varphi(t) = \frac{1}{6} + \left[0.0341\left(\xi_\varphi-\frac{1}{6}\right) + 0.1067\left(\xi_M+\xi_N-\frac{1}{3}\right)\right]\times$$
$$\times\left(\frac{g^2(t)}{g^2}\right)^{-2.1246} + \left[0.9659\left(\xi_\varphi-\frac{1}{6}\right) - 0.1067\left(\xi_M +\right.\right.$$
$$\left.\left.+\ \xi_N-\frac{1}{3}\right)\right]\left[\frac{g^2(t)}{g^2}\right]^{0.5034}$$

$$m_\varphi^2(t) = e^{-2t}\Big\{\Big[0.0341\, m_\varphi^2 + 0.1067\,(m_M^2 + m_N^2)\Big]\Big(\frac{g^2(t)}{g^2}\Big)^{2.1246} + \Big[0.9659\, m_\varphi^2 - 0.1067\,(m_M^2 + m_N^2)\Big]\Big(\frac{g^2(t)}{g^2}\Big)^{0.5034}\Big\},$$

$$\xi_{M,N}(t) = \frac{1}{6} \pm (\xi_M - \xi_N)\Big(\frac{g^2(t)}{g^2}\Big)^{2.0584} + \Big[0.1541\Big(\xi_\varphi - \frac{1}{6}\Big) + 0.4830\Big(\xi_M + \xi_N - \frac{1}{3}\Big)\Big]\Big(\frac{g^2(t)}{g^2}\Big)^{2.1246} + \Big[-0.1541\Big(\xi_\varphi - \frac{1}{6}\Big) + 0.0170\Big(\xi_M + \xi_N - \frac{1}{3}\Big)\Big]\Big(\frac{g^2(t)}{g^2}\Big)^{0.5034},$$

$$m_{M,N}^2(t) = e^{-2t}\Big\{\pm(m_M^2 - m_N^2)\Big(\frac{g^2(t)}{g^2}\Big)^{2.0584} + \Big[0.1541\, m_\varphi^2 + 0.4830\,(m_M^2 + m_N^2)\Big]\Big(\frac{g^2(t)}{g^2}\Big)^{2.1246} + \Big[-0.1541\, m_\varphi^2 + 0.0170\,(m_M^2 + m_N^2)\Big]\Big(\frac{g^2(t)}{g^2}\Big)^{0.5034}\Big\} \qquad (14)$$

The renormalization group equations for $\Lambda(t)$, $\varkappa(t)$ are:

$$\frac{d\Lambda(t)}{dt} = \Big\{\frac{1}{2(4\pi)^2}\big(78\, m_\varphi^4(t) + 54\, m_M^4(t) + 54\, m_N^4(t)\big) - 4\Lambda(t)\Big\},$$

$$\frac{d\varkappa(t)}{dt} = \Big\{\frac{1}{(4\pi)^2}\Big(78\, m_\varphi^2(t)\Big(\xi_\varphi(t) - \frac{1}{6}\Big) + 54\, m_M^2(t) \times \Big(\xi_M(t) - \frac{1}{6}\Big) + 54\, m_N^2(t)\Big(\xi_N(t) - \frac{1}{6}\Big)\Big)\Big\} \qquad (15)$$

It is evident that

$$\Lambda(t) = e^{-4t}\big(\Lambda + a t^{5.2492} + \dots\big)$$
$$\varkappa(t) = e^{-2t}\big(\varkappa + b t^{5.2492} + \dots\big) \qquad (16)$$

where a,b are the constant parameters depending on the initial values of masses and conformal couplings, the terms with the degrees of t which are less than 5.2492 are not written explicitly in (16). Again, when $t \to \infty$, $\Gamma \to -\mathrm{const}\cdot t^{5.2492}$, and Coleman's proposal can be realized in the E_6 GUT. It is not difficult also to check that the number of SU(N) GUTs listed in reviews [7] supports the Coleman's wormholes mechanizm. Note that one can analyse more complicated case where the fermionic masses in GUTs under consideration are not zero. The conclusion will be the same.

The most interesting topic for further investigation seems to be connected with the inclusion into the analysis the quantum gravitational effects.

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