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ANYON SUPERFLUID WITH SUPPRESSED PARITY VIOLATION

24

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Abstract

We analyze an anyon superfluid of the standard kind. The model is composed of equal number density of two types of anyons carrying complementary statistics and interacting via a repulsive hard-core potential. This system shows a suppression of parity violation. The analysis is carried out in the limit of quasi-bosonic statistics, where we show that the interactions induced by the statistics tend to suppress the condensate.

1. Introduction

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In two spatial dimensions there exist the possibility of having elementary excitations whose statistics is neither of the Bose-Einstein nor of the Fermi-Dirac type [1,2]. The wave function of an N-particle system may acquire unremovable arbitrary phases under the interchange of identical particles. This is a consequence of the multiple connectivity of the configuration space in 2 dimensions [3], in contrast with its double connectivity in higher dimensions which restricts the statistics to be either bosonic or fermionic. On the other hand, the rotation group in 2 dimensions is abelian, thus allowing for arbitrary (non quantized) spin. The connection between spin and statistics for anyons has been discussed in ref. [4].

The study of excitations possessing fractional statistics is relevant in the description of the Fractional Quantum Hall effect [5,6], and it may also be of interest in connection with high T_c superconductivity [7].

The properties of a system of free anyons (no interaction beyond the statistics) are much harder to determine than in the case of free bosons or fermions. The reason for this is that the wave function of N free anyons cannot be obtained by suitable direct products of one-anyon wave functions, as it is the case for bosons and fermions. In particular, the determination of the thermodynamic properties of a free anyon gas is a difficult problem. At low densities, the system can be studied by means of the virial expansion, where the second virial coefficient is known exactly [8] and partial results exist for the third coefficient [9].

In ref. [10] it was found, within the random phase approximation, that in a system composed of a single type of free anyons at vanishing temperature the current-current correlators have a pole at zero momentum

for suitable values of the parameter determining the statistics [11]. The system is then a superfluid and shows the Meissner effect if the anyons carry electric charge, becoming a superconductor. Although the persistency of the zero momentum pole at finite temperature is controversial [12], this mechanism of superfluidity might be relevant in the description of layer high T_c superconductors [7]. A signal of this kind of anyonic superconductivity is parity violation. However, no experimental indication of parity violation in current layer superconductors has been found so far.

Motivated by this last observation, we consider here a system where parity violation is a subleading effect [13]. The system considered consists of two types of anyons with complementary statistics. When the chemical potentials for each type of anyon are equal, the Hamiltonian possesses a discrete symmetry, which we shall call charge conjugation, involving the exchange of both types of anyons. This symmetry leads to the suppression of parity violation. We study one such system with a hard core repulsive interaction between anyons in a standard superfluid phase. For quasi-bosonic statistics we can show that the long range interactions induced by the statistics tend to destabilize this phase, leading to a suppression of the condensate. As the repulsive interaction decreases, a phase transition should occur.

2. The model

We consider a system composed of two types of anyons of equal mass and complementary statistics in the sense made manifest below. The statistics is introduced in the usual way [14] by adding to a Lagrangian which describes bosons an auxiliary gauge field (which has no Maxwell term, and thus does not add degrees of freedom to the system) with a Chern-Simons term which accounts for the deviation of the statistics from the bosonic limit. In second quantization the Lagrangian density reads as follows:

$$\begin{aligned} L =& i\,\Psi_1^\dagger D_0 \Psi_1 - \frac{1}{2m}\,(D_i\Psi_1)^\dagger\, D_i\Psi_1 \\ &+ i\Psi_2^\dagger\, D_0\Psi_2 - \frac{1}{2m}\,(D_i\Psi_2)^\dagger\, D_i\Psi_2 \\ &+ \mu_1\,\Psi_1^\dagger\,\Psi_1 + \mu_2\,\Psi_2^\dagger\,\Psi_2 + \frac{1}{4\,\theta}\,\epsilon^{\mu\nu\rho}\,a_\mu\partial_\nu a_\rho - V(\Psi_1,\Psi_2) \end{aligned} \tag{2.1}$$

where Ψ_1 and Ψ_2 are bosonic fields corresponding to the two types of anyons, a_μ is the auxiliary gauge field, and the indices μ, ν, ... refer to time $\mu = 0$ and to the two spatial dimensions $\mu = 1, 2$. The covariant derivatives are given by:

$$D_\mu\Psi_1 \equiv (\partial_\mu - ia_\mu)\Psi_1 \qquad D_\mu\Psi_2 \equiv (\partial_\mu + ia_\mu)\Psi_2 \tag{2.2}$$

where the difference in sign between the coupling of a_μ to Ψ_1 and Ψ_2 implements the complementary statistics we alluded to before, characterized by the real parameter θ. μ_1 and μ_2 are chemical potentials and V is a

repulsive interaction which is needed to stabilize the system in the approach followed in this work.

When a particle of type 1 revolves around another particle of type 1 the wave function acquires a phase θ. Exactly the same happens when a particle of type 2 revolves around another particle of type 2. When a particle of type 1 revolves around a particle of type 2 the wave function acquires a phase $-\theta$.

Besides the gauge invariance inherent to this formulation, when $\mu_1 = \mu_2$ the system possesses a discrete symmetry under the following operation, which we will call charge conjugation:

$$\begin{aligned} \Psi_1 &\longrightarrow \Psi_2 \\ \Psi_2 &\longrightarrow \Psi_1 \\ a_\mu &\longrightarrow -a_\mu \end{aligned} \tag{2.3}$$

It turns out that with this symmetry parity violation will be a non-leading effect for those systems composed by the same number of anyons of type 1 and 2. In what follows we set $\mu_1 = \mu_2 = \mu$. A model consisting of only one type of anyon, obtained for instance by setting $\Psi_2 = 0$ was studied in [15].

(2.1) is the Lagrangian written in first order formalism. The resulting Hamiltonian density and the only constraint are respectively given by:

$$\begin{aligned} H = &\frac{1}{2m}\Psi_1^\dagger(i\bar\partial + \bar a)^2\Psi_1 + \frac{1}{2m}\Psi_2^\dagger(i\bar\partial - \bar a)^2\Psi_2 \\ &+V(\Psi_1,\Psi_2) - \mu\,(\Psi_1^\dagger\,\Psi_1 + \Psi_2^\dagger\,\Psi_2) \end{aligned} \tag{2.4}$$

and

$$G = \Psi_1^\dagger\Psi_1 - \Psi_2^\dagger\Psi_2 + \frac{1}{2\theta}\,\epsilon^{ij}\,\partial_i a_j = 0 \tag{2.5}$$

The last equation is analogous to the Gauss law in electromagnetism. (2.5) implies that a_j is not a dynamical field since its transverse (gauge invariant) part is determined by Ψ_1 and Ψ_2. Choosing the transverse gauge ($\partial_i a_i = 0$), one obtains:

$$a_i(\bar x) = \frac{\theta}{\pi}\int d^2\bar y\;\epsilon^{ij}\,\frac{(\bar x - \bar y)_j}{(\bar x - \bar y)^2}\;(\Psi_1^\dagger\Psi_1 - \Psi_2^\dagger\Psi_2)(\bar y) \tag{2.6}$$

This equation contains the essential difference between the model (2.1) and the models containing a single type of anyon. Suppose we set $\Psi_2 = 0$. Then a finite density necessarily implies a finite fictitious magnetic field $b = \epsilon^{ij}\,\partial_i a_j$. Consequently, perturbation theory must be developed on Landau wave functions [11,12] in which parity violation is a leading effect. In (2.6) however, if we insist that the symmetry (2.3) be respected by the ground state, we must take $\langle 0 \mid \Psi_1^\dagger\Psi_1 \mid 0\rangle = \langle 0 \mid \Psi_2^\dagger\Psi_2 \mid 0\rangle$ and hence, the average magnetic field vanishes. Consequently perturbation theory must be developed on plane waves which leads to qualitatively different features. In particular, parity violation is not a leading effect and will appear only through the interactions. This is the type of configuration we will study in this letter.

As it will be shown later on, the Hamiltonian (2.4) will present stability problems if one neglects the repulsive potential term and insists in analyzing the system within the Landau-Ginzburg approximation which we shall adopt here. In this case, stability is provided by such a repulsive interaction between anyons. One can, for instance, choose a hard core two body potential consistent with all the symmetries of the model of the following form:

$$V = \kappa\,(\Psi_1^\dagger\Psi_1 + \Psi_2^\dagger\Psi_2)^2 \tag{2.7}$$

where $\kappa > 0$ is a constant with dimension mass^{-1}.

In the following we consider θ to be a small parameter, i.e., the anyons are quasi-bosons, set T=0, and assume that charge conjugation symmetry is a symmetry of the ground state. We can write:

$$\Psi_i = \sqrt{\rho_0} + \Psi_i' \tag{2.8}$$

where ρ_0 is the number density of each type of anyon populating the Bose -Einstein condensate. The fluctuation fields Ψ_i' are supposed to be small such that a perturbation theory can be developed, as in the description of superfluids in 3 space dimensions [16]. At finite temperature the analysis must be modified due to the phase fluctuations which prevent long range diagonal order in two dimensions [17]. From (2.4), (2.6) and (2.8) we obtain the following effective Hamiltonian:

$$\begin{aligned}
H = \int d^2\bar{x}\Bigg(& -2\,\rho_0\,\mu - \mu\sqrt{8\,\rho_0}\,Re\chi(\bar{x}) \\
& -\chi^\dagger(\bar{x})(\frac{1}{2m}\nabla^2 + \mu)\chi(\bar{x}) - \Phi^\dagger(\bar{x})(\frac{1}{2m}\nabla^2 + \mu)\Phi(\bar{x}) \\
& -\frac{1}{2m}\frac{\theta}{\pi}\,i\,[\chi^\dagger\bar{\nabla}_i\Phi + \Phi^\dagger\bar{\nabla}_i\chi](\bar{x}) \\
& \times\int d^2\bar{y}\;\epsilon_{ij}\,\frac{(\bar{x}-\bar{y})^j}{(\bar{x}-\bar{y})^2}\,[\chi^\dagger\Phi + \Phi^\dagger\chi + \sqrt{8\,\rho_0}\,Re\Phi](\bar{y}) \\
& +\frac{1}{2m}\left(\frac{\theta}{\pi}\right)^2[2\,\rho_0 + \chi^\dagger\chi + \Phi^\dagger\Phi + \sqrt{8\,\rho_0}\,Re\chi](\bar{x}) \\
& \times\left[\int d^2\bar{y}\;\epsilon_{ij}\,\frac{(\bar{x}-\bar{y})^j}{(\bar{x}-\bar{y})^2}\,[\chi^\dagger\Phi + \Phi^\dagger\chi + \sqrt{8\,\rho_0}\,Re\Phi](\bar{y})\right]^2 + V\Bigg)
\end{aligned} \tag{2.9}$$

where we have defined

$$\begin{aligned}
\chi &\equiv \sqrt{\frac{1}{2}}\;(\Psi_1' + \Psi_2') \\
\Phi &\equiv \sqrt{\frac{1}{2}}\;(\Psi_1' - \Psi_2')
\end{aligned} \tag{2.10}$$

Under a charge conjugation transformation (2.3) we have:

$$\begin{aligned}
\chi &\longrightarrow \chi \\
\Phi &\longrightarrow -\Phi
\end{aligned} \tag{2.11}$$

which ensures that the modes χ and Φ will not be mixed by the interactions.

It is very important to notice that in (2.9) long range interactions induced by the statistics are present. In particular, the term bilinear in $\Phi^{(\dagger)}$ and proportional to $(\theta/\pi)^2$ gives rise to a self energy which diverges at vanishing wave number k, which is physically unacceptable. In fact, the presence of long range interactions points to the need to carry the analysis beyond tree level. The singularities at vanishing wave number are lifted if one performs a non-perturbative resumation, as described in next section, which leads to the screening of the interactions.

3. Non perturbative resumation and screening

In order to proceed with the analysis, we first define the propagators in the usual manner [16]:

$$\begin{aligned} i\,G_\alpha(\bar{x},t) &\equiv \langle 0|\,T\,\alpha^\dagger(\bar{x},t)\,\alpha(0,0)\,|0\rangle \\ i\,F_\alpha(\bar{x},t) &\equiv \langle 0|\,T\,\alpha(\bar{x},t)\,\alpha(0,0)\,|0\rangle \\ i\,F^\dagger_\alpha(\bar{x},t) &\equiv \langle 0|\,T\,\alpha^\dagger(\bar{x},t)\,\alpha^\dagger(0,0)\,|0\rangle \qquad \alpha=\Phi,\chi \end{aligned} \tag{3.1}$$

where F_α refers to anomalous propagators, which in an homogeneous system are self conjugated [16]. Choosing ρ_0 such that:

$$\langle 0|\,\Psi'_i\,|0\rangle = 0 \tag{3.2}$$

the density of excitations above the Bose-Einstein condensate is given by:

$$\rho_\alpha = \int \frac{d^2\bar{k}\,d\omega}{(2\pi)^3}\, i\,G_\alpha(\bar{k},\omega)\,\exp(i\omega\epsilon) \qquad \epsilon\to 0^+ \tag{3.3}$$

$(\alpha=\Phi,\chi)$ and the total density turns out to be:

$$\rho = 2\,\rho_0 + \rho_\Phi + \rho_\chi \tag{3.4}$$

Following standard procedures, the propagators in momentum representation can be written in terms of the self energies Σ^α_{11} , Σ^α_{02} as follows:

$$\begin{aligned} G_\alpha(\bar{k},\omega) &= \frac{1}{D_\alpha}\,[\omega + \frac{\bar{k}^2}{2m} - \mu + \Sigma^\alpha_{11}(\bar{k},-\omega)] \\ F_\alpha(\bar{k},\omega) &= -\frac{1}{D_\alpha}\,\Sigma^\alpha_{02}(\bar{k},\omega) \\ D_\alpha &= {\Sigma^\alpha_{02}}^2(\bar{k},\omega) \\ &\quad - [\Sigma^\alpha_{11}(\bar{k},\omega) - \omega + \frac{\bar{k}^2}{2m} - \mu - i\,\epsilon]\,[\Sigma^\alpha_{11}(\bar{k},-\omega) + \omega + \frac{\bar{k}^2}{2m} - \mu - i\,\epsilon] \\ \epsilon &\to 0^+ \qquad \alpha=\chi,\Phi \end{aligned} \tag{3.5}$$

where Σ^α_{11} is defined as the self energy with one ingoing and one outgoing excitation whereas Σ^α_{02} is the anomalous self energy with two ingoing or two outgoing excitations. Since $F^\alpha(\bar{k},\omega) = F^\alpha(\bar{k},-\omega)$, we have $\Sigma^\alpha_{02}(\bar{k},\omega) = \Sigma^\alpha_{02}(\bar{k},-\omega)$. Such a property is not necessarily shared by Σ^α_{11}. As observed before, the symmetry transformation (2.11) implies that the self energies will be diagonal in the $(\chi;\Phi)$ space.

In the present model we have explicitly checked in a diagrammatic expansion that the Hugenholtz-Pines relation [18]:

$$\Sigma^{\chi}_{11}(0,0) - \mu = \Sigma^{\chi}_{02}(0,0) \tag{3.6}$$

is satisfied. No such relation is satisfied by the mode Φ. Thus, for θ small enough the system is a superfluid and the mode χ describes the phonon excitations. There is however an important remark: superfluidity in this model is not a result of the interactions induced by the statistics. It is rather a consequence of the repulsive interaction one needs to add in order to stabilize the system. It is however remarkable that (3.6), which holds for $\theta = 0$ (i.e. for bosons), continues to hold when the statistics becomes exotic. Thus, at small enough θ the system should retain its superfluidity.

Qualitatively, the spectrum of the model at small wave number consists of phonons associated with the excitations χ which propagate with velocity $c = \sqrt{\Sigma^{\chi}_{02}(0,0)/m}$ and the excitations Φ will in general show a gap Δ and an effective mass m^*. For θ sufficiently small all the quantities characterizing the spectrum will be determined by the repulsive interactions, and can be calculated in the standard manner. For $\theta = 0$ the hard core repulsion gives $\Sigma^{\chi}_{02} = \mu = 4\,\rho_0\,\kappa$. The gap and Σ^{Φ}_{02} vanish at order κ. The first contribution to Δ is positive and is of order κ^2: $\Delta = 2\,\rho_0 m\kappa^2/\pi$, while Σ^{Φ}_{02} still vanishes at this order, implying that $m^* = m$.

Determining the corrections to the self energies due to the interactions induced by the statistics demands certain care. Due to their long range, a non-perturbative resumation is required. This is similar to the RPA for the Coulomb gas [19], and it consists in adding a chain of loops in the channel which shows the most singular behaviour as the corresponding momentum tends to zero. Fig. 1 shows an example of the construction of

a three- and a four-leg effective vertex following this procedure. We have constructed all three- and four-leg effective vertices which are relevant in the present analysis. For reasons of length we postpone the details to a forthcoming publication [20]. The resumation leads to a screening of the interactions induced by the statistics. In momentum representation, the factors $1/k^2$ characteristic of the long range interactions are replaced in a first approximation by:

$$\frac{1}{k^2 + \lambda^{-2}} \tag{3.7}$$

The screening length λ can be expressed as follows:

$$\lambda = \sqrt{\frac{m}{8\,\rho_0\,\theta^2\,I(0,0)}} \sim \frac{d}{\theta}\sqrt{\frac{m}{8\,I(0,0)}} \tag{3.8}$$

where d is the average distance between anyons of the same type and $I(0,0)$ is the following loop integral:

$$I(k,\omega) \equiv -i \int \frac{d^2\bar{k}'\,d\omega'}{(2\,\pi)^3}\,\Delta(\bar{k},\omega;\bar{k}',\omega') \tag{3.9}$$

where:

$$\Delta(\bar{k},\omega;\bar{k}',\omega') = -\,[\,G_{\chi}(\bar{k},\omega)\,G_{\Phi}(\bar{k}',\omega') + F_{\chi}(\bar{k},\omega)\,F_{\Phi}(\bar{k}',\omega') + \begin{matrix}\omega \to -\omega \\ \omega' \to -\omega'\end{matrix}\,] \tag{3.10}$$

$I(0,0)$ is estimated by taking $\Sigma_{02}^{\Phi} = 0$, i.e., $F_{\Phi} = 0$, and neglecting any dependence of the self-energies on the wave number. This is the appropriate approximation as far as the corrections to the self-energies induced by the statistics are small. We arrive at the following expression:

$$I(0,0) = \frac{m}{2\pi}\int_1^{\infty} dx\,(\frac{x}{\sqrt{x^2-1}} - 1)/(x - 1 + \sqrt{x^2-1} + \frac{\Delta}{\Sigma_{02}^{X}}) \qquad (3.11)$$

This integral, which can be done explicitly, diverges logarithmically as the gap Δ tends to zero. Physically, two anyon probes added to the system will behave with respect to ea ch other as bosons if their separation is much greater than λ. It should be emphasized that in the determination of λ the repulsive interactions due to the potential only played the role of fixing the ratio $\Delta/\Sigma_{02}^{X} \simeq \frac{m\kappa}{2}$.

An analysis of $I(k,\omega)$ shows that it diverges logarithmically as $\omega \to \omega_{\Phi}(k)$, the energy of an excitation Φ of wave number k. In general (3.7) should read as follows:

$$f(k,\omega) \equiv \frac{1}{k^2 + \frac{8\rho_0\theta^2}{m} I(k,\omega)} \qquad (3.12)$$

it turns out that this implies that phonons, as expected, in the limit of vanishing wave number will decouple from the massive excitations described by Φ. This decoupling only occurs logarithmically, in contrast with the linear decoupling observed in three spatial dimensions.

4. Corrections to the self-energies by the statistics

In this section we discuss the corrections to the self-energies induced by the statistics. Since the details are lengthy, we will present them in [20].

Our purpose is to show that in our model the interactions induced by the statistics tend to destabilize the superfluid phase. In particular, this implies that a phase transition must occur as the coupling constant κ decreases at fixed $\theta \neq 0$.

It is sufficient to consider those corrections to the self-energies which only involve the effective interaction vertices induced by the statistics. As we show later, corrections which involve one of these vertices and a vertex induced by the hard core repulsion turn out to be negligible. The corrections are calculated in the usual manner [16] in terms of effective vertices and full propagators. As illustration, Fig. II shows some of the diagrams involved.

A first interesting result is that the singularity at vanishing wave number present in the self-energies of Φ at tree level (see terms quadratic in Φ in eq. (2.9)) is lifted by the one loop corrections. This is a screening effect similar to that observed in the effective vertices, and agrees with the fact that at long distances two anyon probes added to the system behave as bosons with respect to each other.

As Fig. II shows, diagrams can be classified according to whether they involve one or two propagators. The latter give contributions to the self-energies which are of the general form (we set the momenta at which we evaluate the corrections to the self-energies to zero):

$$\delta\Sigma \sim i\,\rho_0\left(\frac{8\,\rho_0\,\theta^2}{m}\right)^2 \int \frac{d^2\bar{k}\,d\omega}{(2\,\pi)^3}\,[f(k,\omega)]^n\,\bar{k}^{2(n-2)} \times \left((G_\alpha(\bar{k},\omega);F_\alpha(\bar{k},\omega)) \times (G_\beta(\bar{k},\pm\omega);F_\beta(\bar{k},\pm\omega))\right) \tag{4.1}$$

where for $\delta\Sigma^\chi$ we have $\alpha = \beta = \Phi$ and $n = 2,3,4$, while for $\delta\Sigma^\Phi$ we have $\alpha = \chi$, $\beta = \Phi$ and $n = 2$. The form (4.1) for $\delta\Sigma^\Phi$ assumes that one neglects corrections which vanish when evaluated at $(k = 0, \omega = \omega_\Phi(0))$.

We find that the contributions to the self-energies of the diagrams with two propagators are all negative. The explicit evaluation of the integrals can be done by rotating the integration path to imaginary ω, approximating the self-energies by the leading contributions given by the repulsive potential (which imply $\Sigma^\Phi_{02} = 0$), and using $f(k,\omega)$ as in (3.6). In this manner we obtain the following estimate for the corrections to $\delta\Sigma^\chi$:

$$\delta\Sigma^\chi \sim -\frac{\theta^2\,\rho_0^2}{\pi\,m\,\Delta\,I(0,0)} \tag{4.2}$$

Within this approximation, the choice of the positive sign in the argument ω of the propagator in (4.1) leads to a vanishing result. These corrections $\delta\Sigma^\Phi$ can also be explicitly calculated leading to complicated expressions involving various scale s Δ, Σ^χ_{02}, etc.. They are not very useful for our purpose and therefore we shall only use the corrections to Σ^χ_{02} in our further considerations.

The fact that the corrections are negative is of general character: it follows from the stability requirements that the gap and Σ^χ_{02} must be positive, and the property that the effective three-leg vertices have the same sign.

The diagrams containing only one propagator involve the four-leg effective vertices. We have checked that these vertices correspond to repulsive interactions, which, as observed before, vanish when the wave number of the phonon modes tends to zero. This is because the effective vertices contain factors of $f(k,\omega)$, which in that limit are evaluated at $\omega = \omega_\Phi(k)$, where f vanishes. We focus on the corrections to Σ^χ at vanishing wave number. The integrals involved are of the following form:

$$\delta\Sigma^\chi \sim i\,\frac{8\,\rho_0\,\theta^2}{m}\int \frac{d^2\bar{k}\,d\omega}{(2\,\pi)^3}\,\bar{k}^{2(n-1)}\,[\,f(k,\omega)]^n\,G_\Phi(\bar{k},\omega) \tag{4.3}$$

where $n = 1,2$. The prescription to calculate these integrals is the same as for the Bose liquid [16], and consists in multiplying the integrand by the factor $e^{i\omega\epsilon}$, where $\epsilon = 0^+$, and the integral becomes well defined by closing the integration contour over the upper ω plane. Since $G_\Phi(k,\omega)$ only has a pole at $\omega = \omega_\Phi(k)$, and $f(k,\omega)$ has discontinuities on the real axis for $|\,\omega\,| \geq \omega_\Phi(k)$, the integral reduces to an integral of the discontinuity of the integrand on the negative real axis. From the discontinuity of f for $\omega < 0$ given by:

$$d_f(k,\omega) \equiv f(k,\omega - i\epsilon) - f(k,\omega + i\epsilon) = 2\,i\,\mathrm{Im}(f(k,\omega - i\epsilon)) \tag{4.4}$$

and the property that $\mathrm{Im}(f(k,\omega - i\epsilon)) < 0$, as it can be deduced from (3.9) and (3.12), one concludes that $\delta\Sigma^\chi$ must be negative. It is possible to give bounds on $\delta\Sigma^\chi$ by using the following inequality valid for $\omega < -\omega_\Phi(k)$:

$$\frac{1}{2\,\omega} \geq \frac{1}{\omega - \omega_\Phi(k)} = G_\Phi(\bar{k},\omega) \geq \frac{1}{\omega} \tag{4.5}$$

which implies that:

$$2\,B < \delta\Sigma^{\chi} < B$$

$$B = i\,\frac{8\,\rho_0\,\theta^2}{m}\int\frac{d^2\bar{k}}{(2\,\pi)^3}\,\bar{k}^{2(n-1)}\int_{-\infty}^{-\omega_\Phi(k)}\frac{d\omega}{\omega}\,d_{f^n}(k,\omega)\;<0 \tag{4.6}$$

While $I(k,\omega)$ admits a representation in terms of an unsubtracted dispersion integral, $[f(k,\omega)]^n$ requires a subtraction. This is done at $\omega=\infty$, where it has the value $1/k^2$. Using the property that $d_{f^n}(k,\omega)=d_{f^n}(k,-\omega)$ (here n refers to the nth power of f), the integral over $d\omega$ in (4.6) is readily expressed in terms of $[f(k,0)]^n-[f(k,\infty)]^n$:

$$B=\frac{8\,\rho_0\,\theta^2}{m}\int\frac{d^2k}{(2\,\pi)^3}\,k^{2(n-1)}\left([\,f(k,0)]^n-[\,f(k,\infty)]^n\right) \tag{4.7}$$

The integration over k of the term proportional to $f^n(k,\infty)$ diverges logarithmically at the origin. This problem is entirely related to the two dimensionality of space. A proper treatment, where the fields are separated into fast and slow variables [21], should resolve this problem. The cut-off required here should then be related to the scale separating the two types of variables.

The leading corrections due to the diagrams containing one propagator are then approximately given by:

$$\delta\Sigma^{\chi}\sim-\frac{\rho_0\,\theta^2}{\pi\,m}\log\left(\frac{\Lambda^2\lambda^2+1}{\Lambda^2\lambda^2}\right) \tag{4.8}$$

where Λ is the cut-off. Its precise choice is not very relevant for the conclusions we will draw bellow.

Finally, diagrams containing a three-leg vertex induced by the hard core potential (2.7), like the one depicted in Fig. II (b), give corrections of the form:

$$\delta\Sigma^{\chi}\sim i\,\frac{\theta^2\,\rho_0^2\,\kappa}{m}\int\frac{d^2\bar{k}\,d\omega}{(2\,\pi)^3}\,k^{2(n-1)}\,[f(k,\omega)]^n\,[G_\Phi(k,\omega)]^2\quad,\qquad n=1,2 \tag{4.9}$$

where in writing this expression we use the property that the hard core three-leg vertices have one ingoing and one outgoing Φ excitation, and that $F_\Phi\sim 0$. Within the same approximations which lead to (4.2), and by the comment given thereafter, it turns out that (4.9) vanishes.

Comparing (4.2) with (4.8), using that $\Sigma^{\chi}\sim\rho_0\,\kappa$, $\Delta=2\,\rho_0\,m\,\kappa^2/\pi$ and $I(0,0)\sim m\log(\frac{1}{m\,\kappa})$, we see that as κ decreases the contributions depicted in (4.2) become dominant over those in (4.8). This is a consequence of the factor $1/\Delta$ in (4.2) whose origin is traced back to the long wave length contributions in the loop integrals with two propagators. These type of contributions are absent in the case of (4.3). From (4.2) we can draw an estimate of the order of magnitude of the critical κ below which the superfluid phase becomes unstable:

$$\kappa_{crit}^3\log\left(\frac{1}{m\,\kappa_{crit}}\right)\approx\frac{\theta^2}{\pi\,m^3} \tag{4.10}$$

Below κ_{crit} the positivity conditions that Σ^{χ} should satisfy are not fulfilled and the approach we used breaks down. A new phase is then expected to emerge. Since in the determination of κ_{crit} we only used Σ^{χ}, the possibility is open for the inestability to occur at a larger value of κ, for instance, if Δ becomes negative.

5. Final remarks

We have investigated a model consisting of two types of anyons carrying complementary statistics. For equal number density of each type of anyon, the model possesses a charge conjugation symmetry (2.3) which leads to a suppression of parity violation. We considered one such a system in the quasi-bosonic limit, and found that the standard superfluid phase tends to be destabilized by the interactions induced by the statistics. This phase can only exist if a hard core repulsion between anyons is present. If this interaction is too weak (see eq. (4.10)), the system should choose a different phase. The nature of this phase is not clear to us. One could conjecture that it corresponds to pairing correlations between anyons of type 1 and type 2, as in the mechanism suggested for semions [22]. Another possibility is the spontaneous breaking of the charge conjugation symmetry by the ground state. It is clearly not excluded that the new phase can still be a superfluid. The problem of establishing the nature of this phase seems very interesting to us. Finally, we conjecture that the results of this paper are general, applying beyond the quasi-bosonic limit.

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Figure captions

Fig. I: Example of self-consistency equations for (a) three-leg, and (b) four-leg effective vertices. The solid line corresponds to the mode Φ and the wavy line to χ. The dashed line indicates the presence of a factor $\frac{1}{k^2}$ in the bare vertices.

Fig. II: Some diagrams contributing to the corrections to Σ^{χ} induced by the statistics. The second diagram in (b) contains a vertex, hc, induced by the hard core repulsive potential (2.7).

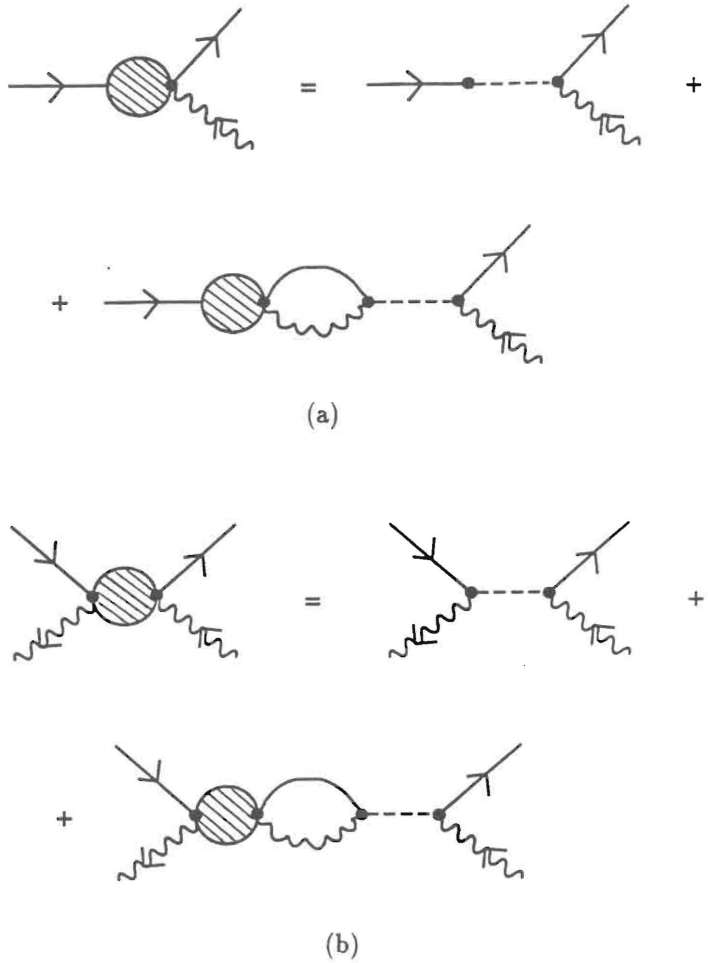

Fig. I

(a)

(b)

Fig. II