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One-loop divergences in two-dimensional Maxwell-dilaton quantum gravity

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Abstract

Two-dimensional Maxwell-dilaton quantum gravity, which covers a large family of the actions for two-dimensional gravity (in particular, string-inspired models) is investigated. Charged black holes which appear in the theory are briefly discussed. The one-loop divergences in the linear covariant gauges are calculated. It is shown that for some choices of the dilaton potential and dilaton-Maxwell coupling, the theory is one-loop multiplicatively renormalizable (or even finite). A comparison with the divergences structure of four-dimensional Einstein-Maxwell gravity is given.

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1. *Introduction.* As is known, the string-inspired models of two-dimensional gravity contain black hole solutions [1] as well as the Hawking radiation [2], and can be exactly solvable classically [2]. These theories (scalar-tensor or dilaton gravity) give us the possibility to have a very useful toy model which can tell us a lot about the general properties of quantum gravity.

The study of the quantum properties of two-dimensional dilaton gravity [3-10] shows that for some choices of dilaton potential the theory can be renormalizable [5-10], or even finite [8,10]. This can be compared with four-dimensional Einstein gravity, which is one-loop finite (on shell) but, unfortunately, is not renormalizable. In the present letter we shall discuss quantum corrections (divergences) in Maxwell-dilaton two-dimensional quantum gravity with the action

$$S = \int d^2x \sqrt{g} e^{-2\phi} \left[R + \gamma g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{1}{4} e^{\epsilon(\phi)\phi} F^2_{\mu\nu} + V(\phi) \right], \qquad (1)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field-strength, ϕ the dilaton, and $V(\phi)$ the dilaton potential. Particular cases of action (1) describe a number of well-known models, like the Jackiw-Teitelboim model [3], the bosonic string effective action (for $\gamma = 4$, $F_{\mu\nu} = 0$, $V = \Lambda$) or the heterotic string effective action (for $\epsilon = 0$, $V = \Lambda$) [11].

The model with the action (1) is connected with four or higher-dimensional Einstein-Maxwell theories (or their generalizations), which admit charged black hole solutions [12]. The theory (1) can also be considered as a toy model for four-dimensional Einstein-Maxwell theory. Finally, as it has been shown in refs. [11,13], for $\epsilon =$ const. and dilaton potential of the type produced by string loops, the theory admits charged black hole solutions with multiple horizons.

Actually, the study carried out with the action of [11] can be generalized to the following one [14] (see also [13] for constant a and b)[2]

$$S = \int d^2x \sqrt{g} e^{-2\phi} \left[R + \gamma (\nabla\phi)^2 - \frac{1}{4} a(\phi) F^2_{\mu\nu} + b(\phi)(\nabla\psi)^2 + V(\phi,\psi) \right], \qquad (2)$$

where $a(\phi)$ and $b(\phi)$ are arbitrary functions of the dilaton field ϕ. The equations of motion for this generalized action are easily obtained:

$$2\nabla^2\phi - 4(\nabla\phi)^2 + \frac{1}{4} a(\phi) F^2_{\mu\nu} + V(\phi,\psi) = 0,$$

[2]In order to compare with the results in [13], we use here the same Minkowski notations of [13] when discussing the classical black hole solutions.

$$R_{\mu\nu} + 2\nabla_\mu\nabla_\nu\phi + (\gamma-4)\left[\nabla_\mu\phi\nabla_\nu\phi - g_{\mu\nu}(\nabla\phi)^2 + \frac{1}{2}g_{\mu\nu}\nabla^2\phi\right] - \frac{1}{2}a(\phi)F_\mu{}^\lambda F_{\nu\lambda} \tag{3}$$
$$+\frac{1}{16}a^{(1)}(\phi)g_{\mu\nu}F^2_{\alpha\beta} + b(\phi)\nabla_\mu\psi\nabla_\nu\psi - \frac{1}{4}b^{(1)}(\phi)g_{\mu\nu}(\nabla\psi)^2 - \frac{1}{4}g_{\mu\nu}\frac{\partial V(\phi,\psi)}{\partial\phi} = 0,$$
$$\nabla^\mu\left[F_{\mu\nu}a(\phi)e^{-2\phi}\right] = 0, \quad 2b(\phi)\nabla^2\psi + 2[b^{(1)}(\phi) - 2b(\phi)]\nabla\phi\cdot\nabla\psi - \frac{\partial V(\phi,\psi)}{\partial\psi} = 0,$$

where $a^{(1)}$ means derivative with respect to ϕ. In the case of static spherical configurations (also considered in [13]): $F = f(r)dr\wedge dt$, $\phi(r)$, $\psi(r)$, with an asymptotically flat metric $ds^2 = -g(r)dt^2 + g(r)^{-1}dr^2$, and $g(r)\to 1$ as $r\to\infty$, we obtain:

$$(g\phi')' - 2g(\phi')^2 - \frac{a}{4}f^2 + \frac{1}{2}V(\phi,\psi) = 0, \qquad 2\phi'' + (\gamma-4)(\phi')^2 + b(\psi')^2 = 0,$$
$$\left(fae^{-2\phi}\right)' = 0, \qquad (g\psi')' + \left(\frac{b^{(1)}}{b} - 2\right)g\phi'\psi' - \frac{1}{2b}\frac{\partial V(\phi,\psi)}{\partial\psi} = 0, \tag{4}$$

where the prime means derivative with respect to r.

However, in the present paper, the emphasis will be put in the fact of allowing the Maxwell-dilaton coupling in (2) to be an arbitrary function of the field ϕ (through the exponent $\epsilon(\phi)$) and —for the sake of simplicity of the discussion— we shall not take into account the dependence on the scalar spectator field ψ. We shall now calculate the one-loop divergences in linear covariant gauges and find the form of V and $\epsilon(\phi)$ for which the theory is one-loop renormalizable. Then, we use these $\epsilon(\phi)$ and V for the discussion of the charged black hole solutions which generalize those of refs. [11,13].

2. *Calculation of the divergences.* To this end, we write the action (2) (without ψ field) in a slightly different form, which is more convenient for the investigation of quantum corrections:

$$S = \int d^2x\sqrt{g}e^{-2\phi}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \gamma R - \frac{1}{4}e^{\epsilon(\phi)\phi}F^2_{\mu\nu} + V(\phi)\right]. \tag{5}$$

This action will be now our starting point. With the change of field variable: $\tilde\phi = e^{-\phi}$, we can write it as follows

$$S = \int d^2x\sqrt{g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\tilde\phi\partial_\nu\tilde\phi + \gamma\tilde\phi^2 R - \frac{1}{4}\tilde\phi^{2-\epsilon(-\ln\tilde\phi)}F^2_{\mu\nu} + \tilde\phi^2 V(-\ln\tilde\phi)\right]. \tag{6}$$

Applying the transformation

$$c_1\varphi = \gamma\tilde\phi^2, \quad g_{\mu\nu} = e^{2\rho}\tilde g_{\mu\nu}, \quad \tilde\phi = \left(\frac{c_1\varphi}{\gamma}\right)^{1/2}, \quad \rho = \frac{\gamma\tilde\phi^2}{4c_1^2} - \frac{1}{8\gamma}\ln\tilde\phi, \tag{7}$$

expression (6) can be rewritten in the way

$$S = \int d^2x\sqrt{\tilde g}\left\{\frac{1}{2}\tilde g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + c_1\varphi\tilde R + e^{\varphi/2c_1}\left(\frac{c_1\varphi}{\gamma}\right)^{1-1/(8\gamma)}V\left(-\frac{1}{2}\ln\left(\frac{c_1\varphi}{\gamma}\right)\right)\right.$$
$$\left. - \frac{1}{4}e^{-\varphi/2c_1}\exp\left\{\left[1 - \frac{1}{2}\epsilon\left(-\frac{1}{2}\ln\left(\frac{c_1\varphi}{\gamma}\right)\right) + \frac{1}{8\gamma}\right]\ln\left(\frac{c_1\varphi}{\gamma}\right)\right\}\tilde F^2_{\mu\nu}\right\}, \tag{8}$$

(where $\tilde F^2_{\mu\nu} = \tilde g^{\mu\alpha}\tilde g^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$) or, dropping the tilde,

$$S = \int d^2x\sqrt{g}\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + c_1\varphi R - \frac{1}{4}f(\varphi)F^2_{\mu\nu} + V(\varphi)\right]. \tag{9}$$

Here the first two terms represent the action of dilaton gravity, the third one is the Maxwell term interacting with gravity, and the fourth one (the potential) has dimension $[V] = M^{-2}$. Note that the action (9), with arbitrary f and V, is the natural generalization of $d = 4$ Einstein-Maxwell or higher derivative gravity-Maxwell theory.

The renormalization of the action (9) without the Maxwell term has been investigated in linear covariant gauges in refs. [5-7] and in the conformal gauge in [10]. It has been shown that the theory is renormalizable[3] for some choices of the potential V (in particular, for $V = \Lambda$ or $V = \mu e^{\alpha\varphi}$). Note that, generally speaking, there exist gauges where the dilaton gravity is not one-loop renormalizable, and others where it is a one-loop finite theory [8].

Our main purpose in this paper will be to calculate the one-loop counterterms in the theory (Maxwell-dilaton gravity) given by the action (9) in the linear covariant gauge (harmonic type gauge).

We use the background field method. According to this procedure, we split the fields into their quantum and background parts:

$$g_{\mu\nu}\longrightarrow\bar g_{\mu\nu} = g_{\mu\nu} + h_{\mu\nu}, \quad \varphi\longrightarrow\bar\varphi = \phi + \varphi, \quad A_\mu\longrightarrow\bar A_\mu = B_\mu + A_\mu, \tag{10}$$

where the second terms $(h_{\mu\nu}, \varphi, A_\mu)$ are the quantum fields.

Let us choose the gauge fixing actions in the gravitational and electromagnetic field sectors, respectively, as the following (linear gauge):

$$S^g_{GF} = -\frac{c_1}{2}\int d^2x\sqrt{g}\left(\nabla_\nu h^\nu_\mu - \frac{1}{2}\nabla_\mu h - \frac{1}{\phi}\nabla_\mu\varphi\right)\phi\left(\nabla_\rho h^{\rho\mu} - \frac{1}{2}\nabla^\mu h - \frac{1}{\phi}\nabla^\mu\varphi\right), \tag{11}$$
$$S^{A_\mu}_{GF} = -\frac{1}{2}\int d^2x\sqrt{g}f(\phi)\left(\nabla_\mu A^\mu\right)^2. \tag{12}$$

[3]Power counting shows that the theory is renormalizable in the generalized sense (supposing the possible change of f and V under renormalization).

One should add the gauge-fixing actions (11) and (12) to the quadratic expansion ($S^{(2)}$) of (9) on the quantum fields.

Now let us recall a few simple expressions that are necessary for the one-loop counterterms calculation. The one-loop divergences of the euclidean effective action are given by

$$\Gamma_{div} = -\frac{1}{2}\mathrm{Tr}\,\ln\widehat{\mathcal{H}}\Big|_{div} + \mathrm{Tr}\,\ln\widehat{\mathcal{M}}_{gh}\Big|_{div}, \tag{13}$$

where $\widehat{\mathcal{H}}$ is defined through $S^{(2)} + S_{GF}$, and $\widehat{\mathcal{M}}_{gh}$ is the ghost operator corresponding to the gauge fixing action.

If $\widehat{\mathcal{H}}$ has the following form

$$\widehat{\mathcal{H}} = \hat{1}\Delta + 2\hat{E}^{\lambda}\nabla_{\lambda} + \hat{\Pi}, \tag{14}$$

where $\widehat{\mathcal{H}}$ acts in the space of all quantum fields (as well as $\hat{1}$, $\hat{E}^{\lambda}$ and $\hat{\Pi}$), and $\Delta = \nabla^{\mu}\nabla_{\mu}$; then, in dimensional regularization,

$$\mathrm{Tr}\,\ln\widehat{\mathcal{H}}\Big|_{div} = \frac{1}{\epsilon}\mathrm{Tr}\left(\hat{\Pi} - \hat{E}^{\lambda}\hat{E}_{\lambda}\right). \tag{15}$$

Here, the parameter of dimensional regularization is $\epsilon = 2\pi(n-2)$. Notice that we have dropped in (15) the surface terms like R. After the preceding remarks, we can now start the explicit calculation of the one-loop counterterms.

2a. *The dilaton-gravitational background (Maxwell sector contributions).* In order to simplify the calculus we will do it in two steps. In the first step we will be interested in the contribution of the Maxwell sector to the action of dilaton gravity (the first two terms and V of (1)). In this case we can put the background vector field $B_{\mu} = 0$. Moreover, one can immediately see that, due to the presence of only quadratic terms in A_{μ}, in $S^{(2)}$, the Maxwell sector decouples from the gravitational sector. The corresponding term in the path integral has the form (with account of the gauge fixing term (12))

$$\int \mathcal{D}A^{\mu}\exp\left(-\frac{1}{2}\int d^2x\,\sqrt{g}f(\phi)A_{\mu}\widehat{H}^{\mu\nu}A_{\nu}\right)\Big|_{div} = -\frac{1}{2}\mathrm{Tr}\,\ln\widehat{H}^{\mu\nu}\Big|_{div}, \tag{16}$$

where

$$\widehat{H}^{\mu\nu} = g^{\mu\nu}\Delta - R^{\mu\nu} - \frac{1}{f(\phi)}\left[\delta^{\mu}_{\alpha}\left(\nabla^{\nu}f(\phi)\right) - \delta^{\nu}_{\alpha}\left(\nabla^{\mu}f(\phi)\right) - g^{\mu\nu}\left(\nabla_{\alpha}f(\phi)\right)\right]\nabla^{\alpha}, \tag{17}$$

the local factor $f(\phi)$ not giving any contribution to divergences. One can see that the operator $\widehat{H}^{\mu\nu}$ has exactly the form (14), with

$$\hat{\Pi} = -R^{\mu\nu}, \quad \hat{E}_{\alpha} = -\frac{1}{2f(\phi)}\left[\delta^{\mu}_{\alpha}\left(\nabla^{\nu}f(\phi)\right) - \delta^{\nu}_{\alpha}\left(\nabla^{\mu}f(\phi)\right) - g^{\mu\nu}\left(\nabla_{\alpha}f(\phi)\right)\right]. \tag{18}$$

The simple calculation of the one-loop counterterms with this $\widehat{H}^{\mu\nu}$ shows that there is no non-trivial contribution from the Maxwell sector to the dilaton gravity sector (except for the trivial surface terms which we consequently drop). The ghost operator corresponding to the gauge fixing (12) is Δ, which again gives only contribution to the surface terms. Thus, the complete Maxwell sector does not provide any contribution to the quantum gravitational sector.

2b. *Arbitrary vector and constant dilaton background.* We proceed now with the calculation of the one-loop counterterm for the Maxwell action. For the sake of simplicity, one can put $\phi = \text{const.}$ and $g_{\mu\nu} = \delta_{\mu\nu}$ in the background field splitting (10). Note that the calculation of the one-loop counterterms in the gravitational sector has been already done for the gauge under discussion in [5,7]. It is not influenced by the addition of $F_{\mu\nu}$, as we showed before; and that is why one can choose $g_{\mu\nu} = \delta_{\mu\nu}$.

First of all, we should write the quadratic expansion of the action (9) with account to the corresponding gauge fixing terms. A straightforward (although lengthy) calculation gives, for the background under consideration:

$$S^{(2)} + S^{g}_{GF} + S^{A_{\mu}}_{GF} = \frac{1}{2}\int d^2x\,\sqrt{g}\,\left(A_{\rho}\;\;\bar{h}_{\mu\nu}\;\;h\;\;\varphi\right)\widehat{\mathcal{H}}\begin{pmatrix} A_{\tau} \\ \bar{h}_{\alpha\beta} \\ h \\ \varphi \end{pmatrix}, \tag{19}$$

the 16 components of $\widehat{\mathcal{H}}$ being given by

$$\begin{aligned}
H_{11} &= f(\phi)g^{\sigma\tau}\Delta, H_{12} = -f(\phi)\left[B^{\alpha\rho}\nabla^{\beta} - \delta^{\rho\beta}B^{\alpha\sigma}\nabla_{\sigma}\right] - \frac{1}{2}f(\phi)\left[(\nabla^{\beta}B^{\alpha\rho}) - \delta^{\rho\beta}(\nabla_{\sigma}B^{\alpha\sigma})\right], \\
H_{13} &= -\frac{1}{2}f(\phi)B^{\sigma\rho}\nabla_{\sigma} - \frac{1}{4}f(\phi)(\nabla_{\sigma}B^{\sigma\rho}), \quad H_{14} = f^{(1)}(\phi)B^{\sigma\rho}\nabla_{\sigma} + \frac{1}{2}f^{(1)}(\phi)(\nabla_{\sigma}B^{\sigma\rho}), \\
H_{21} &= f(\phi)\left[B^{\nu\tau}\nabla^{\mu} - \delta^{\tau\mu}B^{\nu\sigma}\nabla_{\sigma}\right] - \frac{1}{2}f(\phi)\left[(\nabla^{\mu}B^{\nu\tau}) - \delta^{\tau\mu}(\nabla_{\sigma}B^{\nu\sigma})\right], \\
H_{22} &= \frac{1}{2}\left(c_1\phi\Delta - \tilde{V}\right)\hat{P}^{\mu\nu,\alpha\beta} - f(\phi)B^{\mu\sigma}B^{\rho}_{\sigma}\delta^{\nu\tau}\hat{P}^{\alpha\beta}_{\rho\tau}, \quad H_{23} = -\frac{f(\phi)}{4}B^{\mu\sigma}B^{\nu}_{\sigma} = H_{32},
\end{aligned}$$

$$
\begin{aligned}
H_{24} &= \frac{f^{(1)}(\phi)}{2} B^{\mu\sigma} B^{\nu}{}_{\sigma} = H_{42}, \quad H_{31} = \frac{1}{2} f(\phi) B^{\sigma\tau} \nabla_{\sigma} - \frac{1}{4} f(\phi) (\nabla_{\sigma} B^{\sigma\tau}), \quad H_{33} = 0, \\
H_{34} &= \frac{1}{2} \left(-c_1 \Delta + \tilde{V}^{(1)} + \frac{f^{(1)}(\phi)}{2} B^2_{\mu\nu} \right) = H_{43}, \quad H_{41} = -f^{(1)}(\phi) B^{\sigma\tau} \nabla_{\sigma} + \frac{1}{2} f^{(1)}(\phi) (\nabla_{\sigma} B^{\sigma\tau}), \\
H_{44} &= \left(\frac{c_1}{\phi} - 1 \right) \Delta + 2\tilde{V}^{(2)},
\end{aligned} \tag{20}
$$

where $\widehat{P} \equiv \delta^{\mu\nu,\alpha\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta}$, $h_{\mu\nu} \equiv \bar{h}_{\mu\nu} + \frac{1}{2} \delta_{\mu\nu} h$, $B_{\mu\nu} \equiv \nabla_{\mu} B_{\nu} - \nabla_{\nu} B_{\mu}$, $\tilde{V} \equiv V(\phi) - \frac{1}{4} f(\phi) B^2_{\mu\nu}$, and $\tilde{V}^{(1)}$ and $\tilde{V}^{(2)}$ are the first and second derivatives of the potential with respect to ϕ, ∇_{ν} being a flat derivative. The contraction of the projector $\widehat{P}$ of terms with $\alpha\beta$ and $\mu\nu$ indices should also be done (because $\bar{h}_{\alpha\beta} = \widehat{P}^{\rho\tau}_{\alpha\beta} \bar{h}_{\rho\tau}$, $\bar{h}_{\mu\nu} = \widehat{P}^{\rho\tau}_{\mu\nu} \bar{h}_{\rho\tau}$).

One easily sees that the operator $\widehat{\mathcal{H}}$ (20) does not have the canonical structure (14) and, therefore, we cannot use the algorithm (15). In order to render it possible the application of this algorithm, we use a trick introduced in refs. [5,7]. We express

$$
\widehat{H} = \widehat{K} \widehat{\mathcal{H}}, \tag{21}
$$

where $\widehat{K}$ is the constant matrix:

$$
\widehat{K} = \begin{pmatrix} \frac{\delta^{\mu\nu}}{f(\phi)} & 0 & 0 & 0 \\ 0 & \frac{2}{c_1 \phi} \widehat{P} & 0 & 0 \\ 0 & 0 & \frac{4}{c_1} \left(\frac{1}{c_1} - \frac{1}{\phi} \right) & -\frac{2}{c_1} \\ 0 & 0 & -\frac{2}{c_1} & 0 \end{pmatrix}, \tag{22}
$$

and the operator $\widehat{H}$ —which can be easily evaluated from (21)— has now the canonical structure (14). It is evident that, due to the fact that $\widehat{K}$ is a constant matrix, we have

$$
\mathrm{Tr} \ln \widehat{H} \Big|_{div} = \mathrm{Tr} \ln \widehat{\mathcal{H}} \Big|_{div}, \tag{23}
$$

so that we can now concentrate on the operator $\widehat{H}$ only.

Using the explicit form of $\widehat{H}$, we get

$$
\widehat{H} = \widehat{1} \Delta + 2 \widehat{E}^{\lambda} \nabla_{\lambda} + \widehat{\Pi}, \tag{24}
$$

where the only non-zero components of the matrix E_{λ} are:

$$
\begin{aligned}
(E_{\lambda})_{12} &= -\frac{1}{2} \left[B^{\alpha\rho} \delta^{\beta}_{\lambda} - B^{\alpha}{}_{\lambda} \delta^{\rho\beta} - \delta^{\alpha\beta} B_{\lambda}{}^{\rho} \right], \quad (E_{\lambda})_{13} = -\frac{1}{4} B_{\lambda}{}^{\rho}, \\
(E_{\lambda})_{14} &= \frac{f^{(1)}(\phi)}{2 f(\phi)} B_{\lambda}{}^{\rho}, \quad (E_{\lambda})_{21} = \frac{f(\phi)}{c_1 \phi} \left(B^{\nu\tau} \delta^{\mu}_{\lambda} - B^{\nu}{}_{\lambda} \delta^{\tau\mu} - g^{\mu\nu} B_{\lambda}{}^{\tau} \right), \\
(E_{\lambda})_{31} &= \frac{1}{c_1} \left[\left(\frac{1}{c_1} - \frac{1}{\phi} \right) f(\phi) + f^{(1)}(\phi) \right] B_{\lambda}{}^{\tau}, \quad (E_{\lambda})_{41} = -\frac{f(\phi)}{2 c_1} B_{\lambda}{}^{\tau}.
\end{aligned} \tag{25}
$$

Concerning the operator $\widehat{\Pi}$, we need only its diagonal components for the calculation of divergences. They are

$$
\begin{aligned}
\Pi_{11} = 0, \quad \Pi_{22} &= -\frac{\tilde{V}}{c_1 \phi} \widehat{P}^{\alpha\beta,\mu\nu} - \frac{2 f(\phi)}{c_1 \phi} \widehat{P}^{\alpha\beta}_{\mu'\nu'} B^{\mu'\sigma} B^{\rho}{}_{\sigma} \delta^{\nu'\tau} \widehat{P}^{\mu\nu}_{\rho\tau}, \\
\Pi_{33} = \Pi_{44} &= -\frac{\tilde{V}^{(1)}}{c_1} - \frac{f^{(1)}(\phi)}{2 c_1} B^2_{\mu\nu}.
\end{aligned} \tag{26}
$$

Now, it is straightforward to apply the algorithm (15) and to calculate the one-loop divergences:

$$
\Gamma_{div} = -\frac{1}{2} \mathrm{Tr} \ln \widehat{H} = \frac{1}{\varepsilon} \int d^2x \sqrt{g} \left\{ \frac{V(\phi)}{c_1 \phi} + \frac{V^{(1)}(\phi)}{c_1} - \frac{1}{2} B^2_{\mu\nu} \left[\frac{f(\phi)}{2 c_1^2} + \frac{f^{(1)}(\phi)}{2 c_1} \right] \right\}. \tag{27}
$$

Notice, as is easy to see, that the ghosts corresponding to the gauge fixing actions (11) and (12) give contributions to the surface terms only. Here, the expression (27) is the final one for the one-loop divergences of the effective action. We can also see that the one-loop counterterms for the potential V (which come from the gravitational sector) coincide with the corresponding expressions found independently in refs. [5,7] (using the same gauge condition).

In order to write the final answer we should take into account the counterterms of the gravitational sector (arbitrary dilaton-graviton background). These counterterms have been calculated in refs. [5,7] (with the gauge fixing action (11), in particular). As it has been shown above (point 2a), the Maxwell sector does not give here any contribution. So, finally, the one-loop divergences of the effective action for the theory (9) are

$$
\Gamma_{div} = \frac{1}{\varepsilon} \int d^2x \sqrt{g} \left[\frac{3}{\phi^2} (\nabla_{\lambda} \phi) (\nabla^{\lambda} \phi) + \frac{V(\phi)}{c_1 \phi} + \frac{V^{(1)}(\phi)}{c_1} - \frac{1}{4} B^2_{\mu\nu} \left(\frac{f(\phi)}{c_1^2} + \frac{f^{(1)}(\phi)}{c_1} \right) \right]. \tag{28}
$$

This expression constitutes the main result of our work.

2c. *Renormalization.* Let us now discuss the renormalization of the theory under consideration. It follows from (28) that the one-loop renormalized action is

$$
S_R = \int d^2x \sqrt{g} \left[\frac{1}{2} \left(1 - \frac{6}{\varepsilon \varphi^2} \right) g^{\mu\nu} \partial_{\mu} \varphi \partial_{\nu} \varphi + c_1 \varphi R + V(\varphi) \left(1 - \frac{1}{\varepsilon c_1 \varphi} \right) \right.
$$

$$- \frac{V^{(1)}(\varphi)}{\varepsilon c_1} - \frac{f(\varphi)}{4} F^2_{\mu\nu} \left(1 - \frac{1}{\varepsilon c_1^2} - \frac{f^{(1)}(\varphi)}{\varepsilon c_1 f(\varphi)}\right)\Bigg]. \tag{29}$$

Choosing the one-loop renormalization of fields and constant in the dilaton-gravity sector as the following (see [7] for a discussion of this point)

$$\varphi = \varphi_R, \quad c_1 = c_{1R}, \quad g_{\mu\nu} = e^{2\sigma(\varphi, c_1)} \tilde{g}_{\mu\nu}, \tag{30}$$

where, in the one-loop approximation, $\sigma(\varphi, c_1) = 3/(2c_1\varepsilon\varphi)$, we obtain

$$\begin{aligned} S_R &= \int d^2x \sqrt{\tilde{g}} \Bigg[\frac{1}{2}\tilde{g}^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi + c_1\varphi\tilde{R} + V(\varphi)\left(1 - \frac{1}{\varepsilon c_1\varphi} + \frac{3}{\varepsilon c_1\varphi}\right) \\ &- \frac{V^{(1)}(\varphi)}{\varepsilon c_1} - \frac{f(\varphi)}{4}\tilde{F}^2_{\mu\nu}\left(1 - \frac{1}{\varepsilon c_1^2} - \frac{3}{\varepsilon c_1\varphi} - \frac{f^{(1)}(\varphi)}{\varepsilon c_1 f(\varphi)}\right)\Bigg], \end{aligned} \tag{31}$$

where $\tilde{F}^2_{\mu\nu} = \tilde{g}^{\mu\alpha}\tilde{g}^{\nu\beta}F_{\mu\nu}F_{\alpha\beta}$, being $\tilde{g}^{\mu\alpha}$ the renormalized metric tensor.

It follows from expression (31) that, in the gauge under consideration, Maxwell-dilaton gravity is one-loop renormalizable for the following choices of the functions V and f:

$$\begin{aligned} V(\varphi) &= \mu\varphi^2 e^{-a\varphi}, \qquad \mu = \left(1 - \frac{a}{\varepsilon c_1}\right)\mu_R, \\ f(\varphi) &= \frac{f_0}{\varphi^3}\exp\left[-\left(\frac{1}{c_1} + b\right)\varphi\right], \qquad f_0 = \left(1 - \frac{b}{\varepsilon c_1}\right) f_0^R. \end{aligned} \tag{32}$$

Here a and b are arbitrary functions of c_1 (note that μ and f_0 can be choosen to be zero). Thus, we have shown that the theory under consideration is one-loop multiplicatively renormalizable in the gauge (11), (12), for special choices of V and f (32).

By expanding in $e^{-\phi}$, this V is seen to correspond to a special case of the type of dilaton potential one expects from closed string loop corrections [11]. In fact, in terms of the original dilaton field ϕ (see (5)), we get

$$V(\phi) = \mu' e^{\alpha\phi} e^{A'e^{-2\phi}}, \qquad a(\phi) = a' \quad e^{-(\alpha-6)\phi} e^{B'e^{-2\phi}}. \tag{33}$$

These are indeed very nice functions of the variable ϕ. In particular, for positive values of all the coefficients involved, V develops a minimum for a rather small value of ϕ (depending, of course, on the precise values of the constants), while a has a monotonical, exponentially decreasing shape. When A' and B' are negative, V is an exponentially increasing function of ϕ while a has a maximum previous to an exponential decrease. V also develops a minimum for negative values of μ', α and A'. Notice that if one uses the

expansion on ϕ of $\exp(\exp(-2\phi))$ (as in the perturbative case of refs. [11,13]), then the minimum is never seen.

Let us now compare the one-loop structure of the theory (9) with the structure of four-dimensional Maxwell-Einstein theory, which is known to be one-loop non-renormalizable [15], even on shell. First of all, one can see that there is the contribution to the gravitational sector (R^2-terms) from the electromagnetic sector in four dimensions [15]. As we have seen, in the two-dimensional case there is no contribution to the dilaton-gravity sector. Second, while there is no contribution from the gravitational sector to $F^2_{\mu\nu}$ in four dimensions, here, on the contrary, we see that for Maxwell-dilaton gravity there is such contribution from the dilaton-gravity sector, even if the function $f(\varphi)$ is choosen to be constant. Third, new counterterms appear [15] which differ from the original action and which, in the end, lead to the non-renormalizability of four-dimensional Maxwell-Einstein theory even on shell. Such terms have not appeared in the theory (9), which is one-loop multiplicatively renormalizable off-shell for V and f given in (32). (Note that if, moreover, we take $a = 0$ and $b = 0$ in (33), then two-dimensional Maxwell-dilaton gravity is actually one-loop finite,[4] compare with [8,10]). Finally, we must remark that for the two-dimensional theory under consideration, as well as for the four-dimensional Maxwell-Einstein theory, one-loop counterterms (off-shell) depend on the gauge. Here, presumably, the use of a gauge independent effective action (see [16,17] for a discussion of gauge independent effective action in two dimensional gravity) can add some information on the renormalization structure of the theory.

2d. *Charged black holes.* The solutions of the two last eqs. (4) (when ψ is absent from the begining) are:

$$f(r) = f_0 a^{-1}(\phi(r)) e^{2\phi(r)}, \qquad \phi(r) = \begin{cases} \phi_0 + \frac{\alpha}{2}\ln r, & \text{for } \gamma \neq 4, \\ \phi_0 - \frac{Q}{2} r, & \text{for } \gamma = 4, \end{cases} \tag{34}$$

where $\alpha \equiv 4/(\gamma - 4)$, and f_0, ϕ_0 and Q are arbitrary constants (see also [13]). The solution for $g(r)$ (coming from the first of eqs. (4)) is

$$g(r) = \begin{cases} r^{\alpha+1}\left[-2m - \frac{1}{\alpha}\int^r ds\, s^{-\alpha} W(\phi(s))\right], & \text{for } \gamma \neq 4, \\ e^{-Qr}\left[-2m + \frac{1}{Q}\int^r ds\, s^{-\alpha} W(\phi(s))\right], & \text{for } \gamma = 4, \end{cases} \tag{35}$$

[4] Perturbative properties (in particular, two-loop finiteness) of the supersymmetric extension of matter-dilaton gravity ($d = 2$ supergravity) have been recently investigated in ref. [19].

where $W(\phi) \equiv V(\phi) - f_0^2 e^{4\phi}/(2a(\phi))$ and m is a new constant (it turns out to be the black hole's mass).

This solution for the potential (33) describes a Reissner-Nordstrøm black hole. We do not discuss the properties of (34)-(35) here due to lack of space.

In summary, we have studied the renormalization structure of two-dimensional Maxwell-dilaton quantum gravity and showed that this theory can be one-loop multiplicatively renormalized. This theory admits also charged black holes. However, it seems [13,18] that these solutions do not satisfy scalar no-hair theorems. In order to have these theorems we ought to introduce scalar spectator fields [13] (as in eqs. (2)), which arise in the superstring context from the Ramond-Ramond field. Then it would be interesting to investigate the one-loop structure of such theory. Work along this line is in progress.

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