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MATTER FIELDS IN THE LOOP REPRESENTATION OF THE PARTITION FUNCTION

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Abstract

We present the extension of the Lagrangian *loop* representation in such a way to introduce matter fields. The partition function of lattice compact U(1) Gauge-Higgs model is expressed as a sum over closed as much as open surfaces. These surfaces correspond to world sheets of loop-like pure electric flux excitations and open electric flux tubes carrying matter fields at their ends. There is a duality transformation between this description in terms of loop world sheets and the *topological* representation in terms of world sheets of Nielsen-Olesen strings both closed and open joining pairs of monopoles.

In a previous paper [1] we showed a natural procedure in order to set up the **Lagrangian** *loop* representation correlative to the original Hamiltonian *loop* formulation [2]. In reference [1], starting with the Villain form of the action, was obtained straightforwardly the partition function of 4D lattice pure QED as a sum of closed world sheets of electric loops. In the present paper we shall extend this Lagrangian loop approach in such a way to include matter fields. We consider the lattice compact U(1) Gauge-Higgs model which describes the interaction of a compact gauge field θ_μ with the scalar field $\phi = |\phi|e^{i\varphi}$. The self-interaction of the scalar field is given by the potential $\lambda(|\phi|^2 - |\phi_0^2|)^2$. For simplicity we shall consider the limit $\lambda \to \infty$ which freezes the radial degree of freedom of the Higgs field (it is known that the numerical results obtained already at $\lambda = 1$ are indistinguishable from the frozen case). Thus the dynamical variable is compact, i.e. $\varphi \in (-\pi, \pi)$. This model is known to possess three phases, namely confining, Higgs and Coulomb [3]. The Higgs phase splits into a region where magnetic flux can penetrate in form of vortices (Nielsen-Olesen strings) and a region where the magnetic flux is completely expelled [4], the relativistic version of Meissner effect in superconductivity. Relying on this, we call this two subregions: Higgs I and II in analogy with superconducting materials.

The Villain action of this lattice model is given by

$$Z = \int (d\theta) \sum_n \int (d\varphi) \sum_l \exp(-\frac{\beta}{2} \parallel \nabla\theta - 2\pi n \parallel^2 - \frac{\kappa}{2} \parallel \nabla\varphi - 2\pi l - \theta \parallel^2) \quad (1)$$

where we use the notations of the calculus of differential forms on the lattice of [5]. In the above expression: $\beta = \frac{1}{e^2}$, θ is a real compact 1-form defined in each link of the lattice and φ is a real compact 0-form defined on the sites of the lattice, ∇ is the co-boundary operator, n are integer 2-forms defined at the lattice plaquettes, and $\parallel . \parallel^2 = < .,. >$.

If we use the Poisson sumation formula $\sum_n f(n) = \sum_s \int_{-\infty}^{\infty} d\phi f(\phi) e^{2\pi i \phi s}$ for each of the compact variables, the partition function (1) transforms into

$$Z = \sum_s \sum_t \int (d\theta) \int (d\varphi) \int_{-\infty}^{+\infty} (d\psi) \int_{-\infty}^{+\infty} (d\chi) \exp(-\frac{\beta}{2} \parallel \nabla\theta - 2\pi\psi \parallel^2 - \frac{\kappa}{2} \parallel \nabla\varphi - 2\pi\chi - \theta \parallel^2) e^{i<s,\psi>} e^{i<l,\chi>}. \quad (2)$$

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Integrating in the ψ and χ variables

$$Z \propto \sum_s \sum_t \int (d\theta) \int (d\varphi) \exp(-\frac{1}{2\beta} \mid\mid s \mid\mid^2 - \frac{1}{2\kappa} \mid\mid t \mid\mid^2) \times e^{i<s,\nabla\theta>} e^{i<t,\nabla\varphi>} \tag{3}$$

using the partial integration rule $< \psi, \nabla\phi >=< \partial\psi, \phi >$ ($\partial = *\nabla*$ is the boundary operator which maps k-forms into (k-1)-forms) and integrating over the compact φ and θ we get the constrains $\delta(\partial t = 0)$ and $\delta(\partial s = t)$ and thus, we finally arrive to

$$Z \propto \sum_s \exp[-\frac{1}{2\beta} \mid\mid s \mid\mid^2 - \frac{1}{2\kappa} \mid\mid \partial s \mid\mid^2] \tag{4}$$

or

$$Z \propto \sum_s \exp[-\frac{1}{2\beta} < s, \frac{\nabla\partial + M^2}{M^2} s >] \tag{5}$$

where $M^2 = \frac{\kappa}{\beta}$ is the mass acquired by the gauge field due to the Higgs mechanism. If we consider the intersection of one of the surfaces defined by the integer 2-forms s (open and closed surfaces) with a t=constant plane we get pure electric loop as much as 'electromesons' configurations. Thus, we have arrived to an expression of the partition function in terms of the world sheets of string-like configurations: the *loop* (Lagrangian) representation. In this representation the matter fields are naturally introduced by means of open surfaces which correspond to the world sheets of open paths, i.e. 'meson-like' configurations. The corresponding Hamiltonian description in terms of gauge invariant path-dependent operators is the so-called P-representation [6],[7]. The creation operators of loop-states[1] are the $\Phi(C)$ for pure gauge excitations and $\Phi(P_x^y)$ for 'meson-like' configurations, that is

$$\Phi(C)|0>= \prod_{l\in C)} U(l)|0>= |C> \tag{6}$$

[1] We still use the term 'loop' for the configurations in presence of matter fields albeit in a relaxed sense which covers both closed as much as open paths.

$$\Phi(P_x^y)|0>= \phi^\dagger(x)U(P_x^y)\phi(y)|0>= |P_x^y> \tag{7}$$

where $|0>$ is the zero loop state (strong coupling vacuum of the system), $U(l)$ are the lattice gauge group operators, $\phi(x)$ are the matter field operators and $U(P_x^y)$ corresponds to the product of the $U(l)$ along the path P with ends x and y.

Another equivalent description of the Villain form is the *topological* or BKT (for Berezinskii-Kosterlitz-Thouless) representation in terms of the topological objects. As our model has two compact variables we have two topological excitations: monopoles and Nielsen-Olesen strings [8]. The BKT expression for the partition function of compact scalar QED is obtained via the '$Banks - Kogut - Myerson$' transformation [9] (see Appendix) and is given by

$$Z \propto \sum_{n(m)} \exp[-2\pi^2\beta < *n(m), \frac{M^2}{\partial\nabla + M^2} * n(m) >] \tag{8}$$

where the $*$ denotes forms on the dual lattice, $m = \partial * n$ are closed integer 1-forms attached to links which represent monopole-loops and $*n(m) = *n - \partial * q$ are integer 2-forms attached to plaquettes corresponding to the world sheets of both Dirac and Nielsen-Olesen strings (with monopole loops as borders). Thus, comparing (5) and (8) we can observe a complete parallelism: in both representations we have a sum over surfaces, and intersecting with a plane t=constant we get closed as much as open strings with point charges at their ends. In the first case this string-like excitations are 'electric' whilst in the second they are 'magnetic'. *Loop* and *topological* representations are connected by a duality transformation. While loops provide the most natural description of the strong coupling confining phase, Nielsen-Olesen vortices are the relevant excitations in the weak Higgs II sector of the phase diagram. We want to remark that there is a slight difference between both equivalent descriptions. In the BKT representation monopoles occur at the ends of both the Nielsen-Olesen strings (physical excitations) and the Dirac strings (non physical gauge variant objects) so we have the corresponding two types of world sheets mixed in the 2-form $*n(m)$ of equation (8). On the other hand the gauge invariant *loop* description is simpler and completely transparent from the geometrical point of view.

In summary, we have shown that the partition function of U(1) Gauge-Higgs theory can be represented as the sum over world sheets of loops (open and closed). A correspondence between the *loop* and the BKT descriptions is patent and suggests some kind of dual connection between the confining and the Higgs II regions of the phase diagram.

The next step will be the implementation of the Pauli exclusion principle in the Lagrangian loop approach in order to include fermionic fields. This task has been accomplished in the context of the Hamiltonian *loop* formalism in reference [6] where a transparent geometrical description of 'full' QED was given.

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Appendix

To obtain the monopole representation (8) we start with the Villain form (1) and fix the gauge $\nabla\varphi = 0$. Then, we parameterize the $n = \nabla q + \bar{n}(v)$, where q run over arbitrary 1-forms and v over all co-closed 3-forms ($\nabla v = 0$). $\bar{n}(v)$ is a solution of $\nabla n = v$. If we perform a translation $l \longrightarrow l + q$ we get the expression

$$Z = \int_{\infty}^{+\infty} (dA) \sum_{q} \sum_{\substack{v \\ (\nabla v = 0)}} \sum_{l} \exp(-\frac{\beta}{2} \| \nabla A + 2\pi\bar{n} \|^2 - \frac{\kappa}{2} \| A + 2\pi l \|^2) \qquad (9)$$

where $A = \theta + 2\pi q$ is a non-compact variable. By shifting A by $2\pi l$ we find dependence on the combination $\bar{n} + \nabla l$, which turns to be a solution of $\nabla n = v$ so we can eliminate the l variable and perform the gaussian integration yielding

$$Z = \sum_{\substack{v \\ (\nabla v = 0)}} \exp[-2\pi^2\beta < \bar{n}(v), \frac{M^2}{\nabla\partial + M^2}\bar{n}(v) >] \qquad (10)$$

Performing a duality transformation we get (8) where $m = *v$ are now integer closed 1-forms ($\partial m = 0$) and $*n(m) = *\bar{n}(v)$ so $\partial * n = m$. It is possible to express (10) in terms of the l instead of the n variables which reflects the presence of Dirac and Nielsen-Olesen sheets.

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