Fermilab Library
0 1160 0023316 7
UAB-FT-302

UAB-FT-302
Jan. 1993

Long-range forces between macroscopic unpolarised bodies induced by spin dependent interactions

J. A. Grifols
S. Tortosa

Grup de Física Teòrica
Universitat Autònoma de Barcelona
08193 Bellaterra, Barcelona, Catalonia, Spain

Abstract

We show that spin dependent couplings of fermions to massless (or almost massless) bosons generate long range, attractive and composition dependent forces between macroscopic neutral bodies even if unpolarised. The effect is genuinely quantum-mechanical and is due to the simultaneous exchange of two quanta. In the actual case of the Pauli magnetic moment interaction of nucleons and electrons the strength of the effect is very weak. We illustrate this fact by comparing it to the Casimir force in the case of two parallel plates.

It is a well-known fact that spin dependent interactions do not generally extend macroscopically over large distances. Indeed, one would need polarised samples for forces to show their influence coherently over macroscopic distances. We have in mind interactions such as pseudoscalar particles coupled to nucleon sources (or, in general, coupled to fermions) or Pauli magnetic moment couplings of fermions to the electromagnetic field. In both cases, even though the range of the interaction may be infinite (for a massless pseudoscalar and obviously for the photon), macroscopic bulk matter does not feel those forces unless the samples are polarised. Actually, it is the helicity-flip nature of such interactions that prevents the exchanged quantum to extend its influence over macroscopic bodies. This fact, however, corresponds to the situation where a single quantum is exchanged "at a time", *i.e.* it corresponds to the ladder aproximation in a Bethe-Salpeter approach of the bound state problem. Naturally then, the question arises as to what happens if two quanta are exchanged "at a time". That is, if we go beyond the ladder approximation. Intuitively, the helicity non-flip nature is restored in this case and, consequently, one should expect coherent effects to extend over macroscopic matter. In this paper we explore this possibility and find that indeed long-range macroscopic forces develop across extended bodies. Of course, as one could also foresee, these effects are extremely tiny.

We shall consider non-relativistic, non second quantised matter coupled to a pseudoscalar (vector) quantum field ϕ (A_μ). To be definite consider the interaction

$$\mathcal{L}_{int} = \frac{g}{2M}\,\chi^\dagger \vec{\sigma}\,\chi \cdot \nabla\phi(\vec{r}) \tag{1}$$

which is the static non-relativistic (NR) limit of $\overline{\psi}(x)\gamma_5\psi(x)\phi(x)$ (χ are the Pauli spinors, M is the fermion mass). Of course, if $\phi(x)$ were a scalar field then $\overline{\psi}(x)\psi(x)$ would be, in the NR-limit, just a number density of particles and hence bulk matter could act coherently as a source term, whereas for the spin-flip interaction in eq (1) this is not the case. For the electromagnetic case, we use the Pauli interaction $\frac{\mu}{2}\,\overline{\psi}\sigma_{\mu\nu}\psi\,F^{\mu\nu}$ where μ is the magnetic moment. It reduces to

$$\mathcal{L}_{int} = \mu\,\chi^\dagger \vec{\sigma}\,\chi \cdot \vec{B} \tag{2}$$

in the NR-limit. For the sake of simplicity of presentation we shall explicitly elaborate on the first case (pseudo-scalar interaction) and just give the results for the electromagnetic case.

Now, consider two point-like matter sources a distance R apart at positions $\vec{R}_1$ and $\vec{R}_2$, $\vec{R} = \vec{R}_1 - \vec{R}_2$. The interaction hamiltonian is

$$H_{int} = \int d^3\vec{r} \sum_{i=1,2} \frac{g}{2M}\,\delta(\vec{r} - \vec{R}_i)\,\vec{\sigma}^{(i)} \cdot \nabla\phi(\vec{r}) \tag{3}$$

where $\phi(\vec{r}) = \int \frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2\omega_k}[a_k e^{-i\vec{k}\cdot\vec{r}} + a_k^\dagger e^{i\vec{k}\cdot\vec{r}}]$ and $\omega_k = \sqrt{\vec{k}^2 + m^2}$ with m the mass of the pseudo-scalar quantum. Eventually we shall make $m \to 0$, since we are interested in long-range effects.

In order to account for the mean effect of bulk matter we shall actually calculate $Tr(\rho H_{int})$ where ρ is the density matrix operator that describes the state of the system

in spin space of the two-fermion sources (we label the states that span the two-fermion space by $|i\rangle$, $i = 1, \ldots, 4$).

To do the calculation we shall resort to "old fashioned" perturbation theory. Since the hamiltonian in eq.(3) creates (annihilates) ϕ quanta from (into) the vacuum, the expectation value of H_{int} between vacuum pseudo-scalar states vanishes. There are no linear effects in g. But second order perturbation can, in principle, connect the vacuum with the vacuum, for H_{int} can create one pseudo-scalar and, on second application, annihilate it again into the vacuum.

A straightforward calculation then gives, to second order, for the interaction energy between the two sources,

$$\Delta E^{(2)}_{int} = \frac{g^2}{4M^2}\, \vec{\sigma}^{(1)}\cdot\nabla\, \vec{\sigma}^{(2)}\cdot\nabla\, \frac{e^{-mR}}{4\pi R} \tag{4}$$

where $\left(\vec{\sigma}^{(1)}\cdot\ \right)\left(\vec{\sigma}^{(2)}\cdot\ \right)$ has to be understood as an operator that acts on the 2×2 dimensional spin-space of the two-fermion system. We should point out that to reach eq.(4) we have substracted off the (infinite) self-interaction energies of each source.

Eq.(4) is a well-known result [1] and, clearly, when we perform the spin average in the tensor product space of the two Pauli spinors that corresponds to unpolarised sources, the interaction energy vanishes since

$$Tr(\vec{\sigma}\otimes\vec{\sigma}) = 0 \tag{5}$$

(the density matrix is just $\frac{1}{4}I$). Thus, we expect no macroscopic effect to this order as already advertised before.

We turn now to the next order in perturbation theory, *i.e.* $O(g^4)$ since only even powers of H_{int} are permitted. This corresponds to the emission and subsequent absorption of two quanta. Again we encounter in the calculation infinities associated to self-energies of the sources. Indeed, there are, to this order, the self-energies arising from the emission and absorption of one quantum by each source and emission and absorption of two quanta by either source. We simply substract off these infinities. But, there are also divergences corresponding to the case where one quantum is exchanged between both sources and the other one is emitted and reabsorbed by source 1 or source 2, respectively. These last terms, however, vanish when performing the statistical average over unpolarised samples.

The result, after statistical averaging is

$$\begin{aligned}\overline{\Delta E^{(4)}_{int}} &= -\frac{2}{(2\pi)^6}\left(\frac{g}{2M}\right)^4 Tr\left\{\left[(\vec{\sigma}\cdot\nabla\otimes\vec{\sigma}\cdot\nabla)\int\frac{d^3\vec{p}}{\omega_p^3}\cos(\vec{p}\cdot\vec{R})\right]\right.\\ &\left.\times\left[(\vec{\sigma}\cdot\nabla\otimes\vec{\sigma}\cdot\nabla)\int\frac{d^3\vec{p}'}{\omega_{p'}^2}\cos(\vec{p}'\cdot\vec{R})\right]\right\}\end{aligned} \tag{6}$$

which is certainly different from zero. Performing the integrals, eq.(6) can be put in the form

$$\overline{\Delta E^{(4)}_{int}} = -\frac{1}{4\pi^3}\left(\frac{g}{2M}\right)^4\sum_{i,j=1}^{3}\left(\frac{\partial}{\partial R_i}\frac{\partial}{\partial R_j}K_0[mR]\right)\left(\frac{\partial}{\partial R_i}\frac{\partial}{\partial R_j}\frac{e^{-mR}}{R}\right) \tag{7}$$

with K_0 the modified Bessel function. In the limit $m\to 0$, we obtain

$$\overline{\Delta E^{(4)}_{int}} = -\frac{g^4}{16\pi^3 M^4}\frac{1}{R^5} \tag{8}$$

for the interaction, which is attractive.

We should note that the statistical averaging in eq.(6) involves the introduction of a finite renormalization of the two-fermion wavefunction. In fact, to the order we are working, the statistical weights ω_i in the density matrix ρ are related to the lowest order density matrix elements that enter in eq.(6) through a rescaling $\omega_i = \omega^{(0)} Z_i^{-1}$, where Z_i is the (finite) wavefunction renormalization of the perturbed states, *i.e.* $Z_i^{-1} = \langle i|i\rangle$.

Starting from the dipole interaction eq.(2) we can reach a similar conclusion to eq.(8):

$$\overline{\Delta E^{(4)}_{int}} = -\frac{\mu^4}{\pi^3}\frac{1}{R^5} \tag{9}$$

valid for $R\neq 0$.

Whereas the effect in eq.(8) depends on the existence of an hypothetical massless pseudo-scalar particle the effect described by eq.(9) is a real effect that must be present in bulk matter. The resulting force is attractive, composition dependent and very small. Indeed the interaction energy associated to the magnetic moment of protons in two samples of, say, copper a distance R apart (R much larger than the size of the samples), as compared to the gravitational attraction is

$$\left(\frac{E_{m.d.m.\,prot}}{E_{gravity}}\right)_{Cu} = 7.2\times 10^{-29}(R_{[m]})^{-4} \tag{10}$$

and the interaction due to the magnetic moment of the electrons in the same matter samples,

$$\left(\frac{E_{m.d.m.\,elect}}{E_{gravity}}\right)_{Cu} = 1.4\times 10^{-17}(R_{[m]})^{-4} \tag{11}$$

This is extremely weak, indeed. However, if we compare it to the long-range force associated to two neutrino exchange [2] (recently, there has been a revived interest in this effect [3]) it is 10^9 (10^{20}) times larger for protons (electrons)!

To quantify further the effect, consider the force between two large parallel plates of thickness d separated by a distance a. Ignoring edge effects (*i.e.* a,d much smaller than the size of the plates) one obtains for copper,

$$\left(\frac{F_{m.d.m.}}{S}\right)_{Cu} = -7.6\times 10^{-8}\left(\frac{1}{(a+2d)^2}+\frac{1}{a^2}-\frac{2}{(a+d)^2}\right)\ N/m^2 \tag{12}$$

for the attractive force per unit surface area where a,d are in μm. Of course, we took copper just to be definite but it could equally well be any other material, conducting or dielectric. Let us compare this to the Casimir force between the plates [4] (in this case the plates must be necessarily conducting)

$$\frac{F_{Cas}}{S} = -1.3\times 10^{-3}\frac{1}{a^4}\ N/m^2 \tag{13}$$

Notice that for very thin plates ($d \ll a$) the a behaviour of both forces is identical. As to their mutual strength it can be inferred from fig.1. We also plot in fig.1 the (constant) force of gravity.

To conclude, we have established the existence of spin dependent macroscopic attractive forces between unpolarised neutral matter. The effect is of quantum mechanical origin and is associated to the exchange of two quanta "at a time". These quanta may correspond both to a putative new (fifth) force [5] or to photons. In this latter case, the effect must be really present in ordinary bulk matter and we have estimated its strength in simple experimental instances. The force is really small but worth to keep in mind, specially in view of recent progress in testing cavity QED effects in the laboratory [6].

References

[1] J. J. Sakurai, *Advanced Quantum Mechanics*, Addison-Wesley (1967).

[2] G. Feinberg and J. Sucher, Phys. Rev. 166 (1968) 1638.

[3] J. Bernabeu, Nucl. Phys. B (Proc. Suppl.) 28 A (1992) 500.
S. D. H. Hsu and.P. Sikivie, preprint HUTP-92-A041/UFIFT-HEP-92-28.

[4] H. B. G. Casimir, Proc. Kon. Ned. Akad. Wet. 51 (1948) 793

[5] For a review, see E. Fishbach and C. Talmadge, Nature 356 (1992) 207.

[6] See *e.g.* E. A. Hinds in *Progress in Atomic Physics, Neutrinos and Gravitation*, Proc.XXVIIth Rencontre de Moriond, Editions Frontières (1992) 321.

FIGURE CAPTION

Fig.1 Attractive force described in the text in eq.(12) *(full lines)* between two copper plates of widths $A = 1\mu m$, $B = 10\mu m$, $C = 100\mu m$, $D = 1000\mu m$ as compared to their gravitational attraction *(dotted lines)* and the Casimir force *(dashed line)*, as a function of their distance a apart.

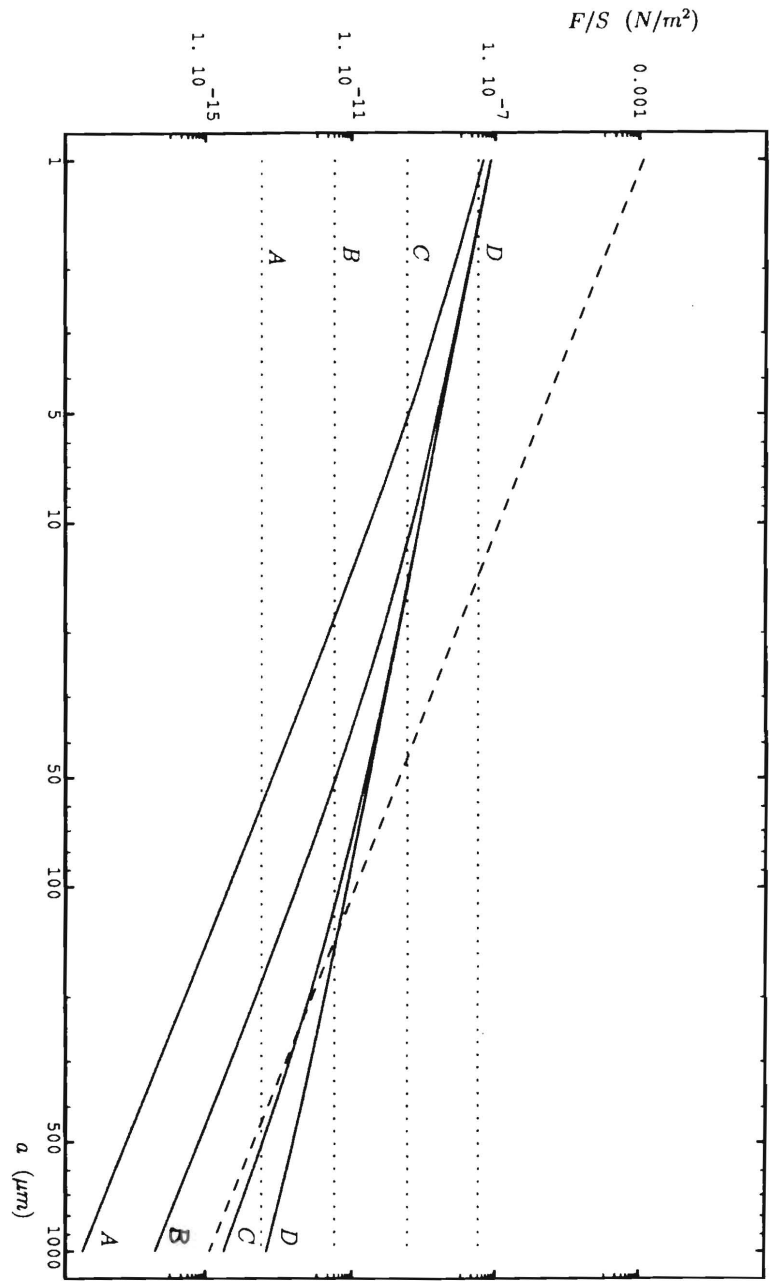

fig.1