90-275

TUTP-89/29

DERIVATION OF LOW ENERGY EFFECTIVE ACTION FOR MESONS FROM QCD*

Qing Wang[1] and Yu-Ping Kuang[1,2,3]

FERMILAB
FEB 01 1990
LIBRARY

1. CCAST (World Laboratory), P.O.Box 8730, Beijing 100080, China
2. Department of Physics, Tsinghua University, Beijing 100084, China**
3. Institute of Theoretical Physics, Academia Sinica, Beijing, 100080, China

ABSTRACT

Low energy effective action for mesons is derived from the fundamental theory of QCD in different approaches. The obtained effective action is of the form of that in the chiral σ-model with vector and axial-vector mesons included but with fewer free parameters. Some of its consequences are discussed.

* Talk presented by Q.Wang at the Workshop on Weak Interactions and CP Violation, Aug.1989, Inst. High Energy Phys., Academia Sinica, Beijing, China. This work is supported by the National Natural Science Foundation of China

** Mailing Address

RETURN TO FERMI LIBRARY
LOCATION

TUTP-89-29

I. INTRODUCTION

Quantum chromodynamics (QCD) has been well studied at high energies where perturbation calculation is applicable and has proved to be successful. However, there are a lot of physically interesting low energy hadronic processes which are nonperturbative and remain unsolved in QCD. Usually low energy processes of mesons are studied by constructing effective model field theories from the consideration of symmetry requirements and phenomenological constraints, such as the linear and nonlinear σ-models. Although these models do reflect some essential properties of QCD and can lead to some successful results, the situation is still not satisfactory since it is not known how they are related to the first principle of QCD.

In this talk, we will give two different ways of deriving the low energy effective action for mesons from the first principle of QCD. Our starting point is the QCD generating functional

$$Z=\int \mathcal{D}B\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\omega\mathcal{D}\bar{\omega}\,\exp\{\int d^4x\{\mathcal{L}_g(B,\omega,\bar{\omega})+\bar{\psi}(i\not{D}-m_0-g_s\frac{\lambda_a}{2}\not{B}^a)\psi\}\} \qquad (1)$$

where ψ ($\bar{\psi}$), B^a_μ and ω ($\bar{\omega}$) are the quark field, gluon field and the ghost field, respectively, $\mathcal{L}_g(B,\omega,\bar{\omega})$ is the part of the QCD Lagrangian including the pure gluonic term, the gauge fixing term and the gauge compensation term, and m_0 is the current quark mass matrix. In Sec.II we shall give two different approximations at low energies in which after integrating out the gluon and ghost fields, the effective interactions between quarks are four-fermion interactions. As has been pointed out by several authors,[1][2] the four-fermion

interactions will lead to the σ-model type effective action for mesons through the introduction of auxiliary fields for the quark-anti-quark composite operators and the auxiliary fields are just the meson fields. In our effective action, there is one more free parameter than the set of free parameters in the fundamental theory of QCD due to the ignorance of the complicated loop contributions and nonperturbative effects in the gluon sector. The form of our effective action looks like that in the chiral σ-model with vector and axial-vector mesons included, but with fewer free parameters. Some of its qualitative consequences are discussed in Sec. III.

II. DERIVATION OF THE LOW ENERGY EFFECTIVE ACTION FOR MESONS

1. $1/g_s$ Expansion.

We are interested in low energy problems in which g_s is large. So we take $1/g_s$ as an expansion parameter to derive the low energy effective action. It is well-known that if we rescale the fields B^a_μ, ω and $\bar\omega$ by

$$B^a_\mu \to B^a_\mu/g_s\,, \qquad \omega \to \omega/g_s\,, \qquad \bar\omega \to \bar\omega/g_s\,,$$

the generating functional (1) can be written as

$$Z=\int \mathcal{D}B^a_\mu \mathcal{D}\psi \mathcal{D}\bar\psi \mathcal{D}\omega \mathcal{D}\bar\omega \exp i\int d^4x \left[\frac{1}{g_s^2}\mathcal{L}_g(B,\omega,\bar\omega)+\bar\psi\left(i\partial\!\!\!/-m_0-\frac{\lambda_a}{2}B^a\right)\right] \tag{2}$$

Let Λ be an energy scale below which g_s is large. In principle Λ is related

to the QCD scale parameter $\Lambda_{\overline{MS}}$ through the formula of the runnig of g_s. Unfortunately, the running of g_s at low energies is not known explicitly. So we simply take Λ as the free parameter instead of $\Lambda_{\overline{MS}}$ without considering the explicit relation between them (We expect that $\Lambda_{\overline{MS}}$ is not sensitive to the variation of Λ since at low energies g_s runs fast.). We know that physics at low energies should not be influenced much by the physics well above Λ. Thus, as the first approximation, we take into account only the contributions at energies below Λ. In this case we can implement $1/g_s$ expansion in (2) and to lowest order

$$Z \approx \int \mathcal{D}B\mathcal{D}\psi\mathcal{D}\bar\psi \exp i\int d^4x\, \bar\psi\left(i\partial\!\!\!/-m_0-\frac{\lambda_a}{2}B^a\right)\psi \tag{3}$$

Here we have ignored the $\frac{1}{g_s^2}\mathcal{L}_g$ term. One may think that this is questionable since the term $-\frac{1}{4g_s^2}G^a_{\mu\nu}G^{a\mu\nu}$ includes important physics. Actually the physics of $-\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu}$ is not lost in (3) because after integrating over the quark fields in (3) we get

$$\begin{aligned} Z &= \exp i\,\mathrm{Tr}\ln\left(i\partial\!\!\!/-m_0-\frac{\lambda_a}{2}B^a\right) \\ &= \exp i\left\{-\frac{1}{4}I_0 G^a_{\mu\nu}G^{\mu\nu a}-\frac{1}{2}\delta\mu_1^2 B^a_\mu B^a_\mu - i\,\mathrm{Tr}'\ln\left(i\partial\!\!\!/-m_0-\frac{\lambda_a}{2}B^a\right)\right\} \end{aligned} \tag{4}$$

Where

$$I_0 = -2i\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\left[\frac{\left(\frac{p^2}{3}-m_0^2\right)p^2+m_0^4}{(p^2-m_0^2)^4}\right], \tag{5a}$$

$$\delta\mu_1^2 = -i\int\frac{d^4p}{(2\pi)^4}\,\mathrm{tr}\left[\frac{-\frac{p^2}{2}+m_0^2}{(p^2-m_0^2)^2}\right], \tag{5b}$$

$-i\mathrm{Tr}'\ln(i\not\partial - m_0 - \frac{\lambda_a}{2}\not B^a)$ means the part in $-i\mathrm{Tr}\ln(i\not\partial - m_0 - \frac{\lambda_a}{2}\not B^a)$ having lower powers of Λ ,and tr in (5) means the trace over the flavour degree of freedom.We see that an effective $-\frac{1}{4}l_0 G^a_{\mu\nu}G^{a\mu\nu}$ term emerges in (4), so that our approximation is just $l_0 + 1/g_s \approx l_0$. There is a gluon mass term $-\frac{1}{2}\delta\mu_1^2 B^a_\mu B^{a\mu}$ in (4) which is not gauge invariant.This is not surprising since the introduction of a cut-off parameter Λ does not preserve gauge invariance and we have not imposed the requirment of gauge invariance in our calculation. To make the final result consistent with gauge invariance, we should add a counter term $\frac{1}{2}\delta\mu^2 B^a_\mu B^{a\mu}$ in (3)[4].Here $\delta\mu^2$ is in general not equal to the $\delta\mu_1^2$ given in (5b) since $\frac{1}{2}\delta\mu^2 B^a_\mu B^{a\mu}$ is the counter term required by the complete theory including gluon loop contributions and nonperturbative effects.At present we cannot determine $\delta\mu^2$ theoretically so that we take it as an extra free parameter in our approach. Thus eq.(3) should be writtten as

$$Z \approx \int \mathcal{D}B\,\mathcal{D}\psi\,\mathcal{D}\bar\psi \exp i\int d^4x\{\bar\psi(i\not\partial - m_0 - \frac{\lambda_a}{2}\not B^a)\psi + \frac{1}{2}\delta\mu^2 B^a_\mu B^{a\mu}\} \tag{6}$$

The gluon field integration can be carried out and we obtain

$$Z \approx \int \mathcal{D}\psi\,\mathcal{D}\bar\psi \exp i\int d^4x\{\bar\psi(i\not\partial - m_0)\psi - \frac{1}{2\delta\mu^2}(\bar\psi\gamma^\mu\frac{\lambda_a}{2}\psi)(\bar\psi\gamma_\mu\frac{\lambda_a}{2}\psi)\} \tag{7}$$

We can apply Fierz reordering to the four-fermion interaction term to make the biliner operators colour singlet.Let t_l $(l=1,\cdots,N_f)$ be the generator of the flavour group normalized as

$$t_l(t_l t_{l'}) = \delta_{ll'} \tag{8}$$

After Fierz reordering, eq. (7) reads

$$Z \approx \int \mathcal{D}\psi\,\mathcal{D}\bar\psi \exp i\int d^4x\{\bar\psi(i\not\partial - m_0)\psi + \frac{1}{4\delta\mu^2}\{\frac{1}{N_f}(\bar\psi\psi)^2 + \frac{1}{N_f}(\bar\psi i\gamma_5\psi)^2 - (\frac{1}{2N_f} - \frac{1}{N_c})(\bar\psi\gamma^\mu\psi)^2 - \frac{1}{2N_f}(\bar\psi\gamma_5\gamma^\mu\psi)^2 + (\bar\psi t_l\psi)^2 + (\bar\psi i\gamma_5 t_l\psi)^2 - \frac{1}{2}(\bar\psi\gamma^\mu t_l\psi)^2 - \frac{1}{2}(\bar\psi\gamma_5\gamma^\mu t_l\psi)^2\}\} \tag{9}$$

The colour singlet quark-anti-quark bilinear operators $\bar\psi\psi$, $\bar\psi\gamma^\mu\psi$, etc have the quantum numbers of mesons.

Introduce auxiliary fields $S^0, S^l, P^0, P^l, V^0_\mu, V^l_\mu, A^0_\mu, A^l_\mu$ which are scalars,pseudoscalars,vectors and axial-vectors,respectively.The following Gaussian integral

$$\int \mathcal{D}S^0\mathcal{D}S^l\mathcal{D}P^0\mathcal{D}P^l\mathcal{D}V^0_\mu\mathcal{D}V^l_\mu\mathcal{D}A^0_\mu\mathcal{D}A^l_\mu \exp i\int d^4x\{-\delta\mu^2(S^0 - \frac{\bar\psi\psi}{2\sqrt{N_f}\,\delta\mu^2})^2 - \delta\mu^2(S^l - \frac{\bar\psi t_l\psi}{2\delta\mu^2})^2 - \delta\mu^2(P^0 - \frac{\bar\psi i\gamma_5\psi}{2\sqrt{N_f}\,\delta\mu^2})^2 - \delta\mu^2(P^l - \frac{\bar\psi i\gamma_5 t_l\psi}{2\delta\mu^2})^2 + \frac{\delta\mu^2}{\frac{1}{2N_f} - \frac{1}{N_c}}(\frac{1}{\sqrt{N_f}}V^0_\mu + \frac{(\frac{1}{2N_f} - \frac{1}{N_c})}{2\delta\mu^2}\bar\psi\gamma_\mu\psi)^2 + 2\delta\mu^2(V^l_\mu + \frac{\bar\psi\gamma_\mu t_l\psi}{4\delta\mu^2})^2 + 2\delta\mu^2(A^0_\mu + \frac{\bar\psi\gamma_5\gamma_\mu\psi}{4\sqrt{N_f}\,\delta\mu^2})^2 + 2\delta\mu^2(A^l_\mu + \frac{\bar\psi\gamma_5\gamma_\mu t_l\psi}{4\delta\mu^2})^2\} \tag{10}$$

can be inserted into (9) to eliminate the four-fermion interactions and so the integration over ψ and $\bar\psi$ can be carried out.We then obtain

$$Z = \mathcal{D}S^0\mathcal{D}S^l\mathcal{D}P^0\mathcal{D}P^l\mathcal{D}V^0_\mu\mathcal{D}V^l_\mu\mathcal{D}A^0_\mu\mathcal{D}A^l_\mu \exp i l_{eff} \tag{11}$$

Where

$$l_{eff}\{S^0, S^l, P^0, P^l, V^0_\mu, V^l_\mu, A^0_\mu, A^l_\mu\} \equiv -i\mathrm{Tr}\ln\{i\not\partial - m_0 + \frac{1}{\sqrt{N_f}}(S^0 + i\gamma_5 P^0 + \gamma^\mu V^0_\mu + \gamma_5\gamma^\mu A^0_\mu) + t_l(S^l + i\gamma_5 P^l + \gamma^\mu V^l_\mu + \gamma_5\gamma^\mu A^l_\mu)\} - \delta\mu^2\int d^4x\{(S^0)^2 + (S^l)^2 + (P^0)^2 + (P^l)^2 - \frac{1}{\frac{1}{2} - \frac{N_f}{N_c}}V^0_\mu V^{0\mu} - 2A^0_\mu A^{0\mu} - 2V^l_\mu V^{l\mu} - 2A^l_\mu A^{l\mu}\} \tag{12}$$

The auxiliary fields $S^0, S^\ell, P^0, P^\ell, V^0_\mu, V^\ell_\mu, A^0_\mu, A^\ell_\mu$ are just the effective local fields for scalar, pseudoscalar, vector and axial-vector mesons and l_{eff} is the low energy effective action for the meson fields.

The functional integration over V^0_μ in (10) needs special consideration. The integral is Gaussian (convergent) in Euclidean space-time provided $\frac{\delta\mu^2}{\frac{1}{2N_f}-\frac{1}{N_c}}>0$, i.e. $N_c>2N_f$ which is not realistic. Thus for the isospin singlet vector meson sector, the present lowest order approximation works only in an unrealistic case ($N_c>2N_f$) and one should go beyond the lowest order to study the V^0_μ sector in the real world. This problem does not happen to other mesons.

In the chiral limit $m_0\to 0$, the QCD Lagrangian respects a chiral symmetry $SU(N_f)_L * SU(N_f)_R$ (An extra $U_A(1)$ symmetry is broken by the axial-vector current anomaly[3] which is not considered here.). A direct consequence of the effective action (12) is that $SU(N_f)_L * SU(N_f)_R$ is spontaneously broken into a diagonal $SU(N_f)$. This can be seen as follows. Since vector and axial-vector meson fields cannot develop a nonvanishing vacuum expectation value (VEV) due to Lorentz invariance, the effective potential can be seen as a function of S^0, S^ℓ, P^0 and P^ℓ. Take $N_f=2$ as an example. The effective potential obtained from (12) is

$$V_{eff}(S,P)=\delta\mu^2(S^2+P^2)-\frac{N_c}{32\pi^2}\int_0^{\Lambda^2}dq^2q^2\ln\{[(q^2+m_0^2-\sqrt{2}m_0S^0+\tfrac{1}{2}(S^2+P^2))^2-(\varepsilon_{\ell jk}S^jP^k+S^0S^\ell+P^0P^\ell-\sqrt{2}m_0S^\ell)^2]*[(q^2+m_0^2-\sqrt{2}m_0S^0+\tfrac{1}{2}(S^2+P^2))^2-(-\varepsilon_{\ell jk}S^jP^k+S^0S^\ell+P^0P^\ell-\sqrt{2}m_0S^\ell)^2]\} \tag{13}$$

Where $\ell=1,2,3$ and S^2+P^2 stands for $(S^0)^2+(S^\ell)^2+(P^0)^2+(P^\ell)^2$. The stability of

V_{eff} leads to

$$\delta\mu^2\langle S^0\rangle+\frac{N_c}{16\pi^2}(\sqrt{2}m_0-\langle S^0\rangle)\int_0^{\Lambda^2}dq^2\frac{q^2}{q^2+\frac{1}{2}(\sqrt{2}m_0-\langle S^0\rangle)^2}=0 \tag{14a}$$

$$\langle P^0\rangle=\langle P^\ell\rangle=\langle S^\ell\rangle=0 \qquad \ell=1,2,3. \tag{14b}$$

When $m_0\to 0$, (14a) reads

$$\langle S^0\rangle\{\delta\mu^2-\frac{N_c}{16\pi^2}\int dq^2\frac{q^2}{q^2+\frac{1}{2}\langle S^0\rangle^2}\}=0 \tag{15}$$

Which has two solutions

$$\langle S^0\rangle=0 \tag{16a}$$

$$\delta\mu^2-\frac{N_c}{16\pi^2}\int_0^{\Lambda^2}dq^2\frac{q^2}{q^2+\frac{1}{2}\langle S^0\rangle^2}=0 \tag{16b}$$

It is easy to check that the $\langle S^0\rangle=0$ solution (16b) gives the true minimum of V_{eff} and hence $SU(2)_L*SU(2)_R$ is spontaneously broken into the diagonal SU(2) and the pseudoscalar mesons are Goldstone bosons.

Shifting the S^0 field by its VEV $\langle S^0\rangle$ in eq.(12), we obtain the final form of l_{eff}

$$l_{eff}=-i\mathrm{Tr}\ln\{i\not\partial-m+\frac{1}{\sqrt{N_f}}(S^0+i\gamma_5P^0+\gamma^\mu V^0_\mu+\gamma_5\gamma^\mu A^0_\mu)+t_\ell(S^\ell+i\gamma_5P^\ell+\gamma^\mu V^\ell_\mu+\gamma_5\gamma^\mu A^\ell_\mu)\}-$$
$$-\delta\mu^2\int d^4x\{(S^0)^2+(S^\ell)^2+(P^0)^2+(P^\ell)^2-\frac{1}{\frac{1}{2}-\frac{N_f}{N_c}}V^0_\mu V^\mu_0-2A^0_\mu A^\mu_0-2V^\ell_\mu V^\mu_\ell-2A^\ell_\mu A^\mu_\ell\}, \tag{17}$$

Where

$$m=m_0-\frac{\langle S^0\rangle}{\sqrt{N_f}} \tag{18}$$

is the constituent quark mass. Eq.(17) is similar to the effective action in

Ref.[2] but with fewer free parameters.

To see the relation between our effective action (17) and the conventional σ-model, we expand the logarithm in (17) in powers of the meson fields and their derivatives[1][2]. For simplicity, we keep, in the coefficients, only the terms containing $\ln\Lambda^2$ and positive powers of Λ. Using the method of Ref.[4], we obtain, up to dimension-four operators,

$$
\begin{aligned}
I_{eff} = \int d^4x \{ & I_0 \,\mathrm{tr}\Big(\big[\partial_\mu - i\big(\tfrac{1}{\sqrt{N_f}} V^0_\mu + t_\ell V^\ell_\mu\big), \tfrac{1}{\sqrt{N_f}} S^0 + t_\ell S^\ell + m_0 - m\big]_- + i\tfrac{1}{\sqrt{N_f}} A^0_\mu + t_\ell A^\ell_\mu, \tfrac{1}{\sqrt{N_f}} P^0 + t_\ell P^\ell \Big]_+ \Big)^2 \\
& + I_0 \,\mathrm{tr}\Big(\big[\partial_\mu - i\big(\tfrac{1}{\sqrt{N_f}} V^0_\mu + t_\ell V^\ell_\mu\big), \tfrac{1}{\sqrt{N_f}} P^0 + t_\ell P^\ell \big]_- - i\tfrac{1}{\sqrt{N_f}} A^0_\mu + t_\ell A^\ell_\mu, \tfrac{1}{\sqrt{N_f}} S^0 + t_\ell S^\ell + m_0 - m \Big]_+ \Big)^2 - \\
& - \tfrac{1}{3} I_0 \big(F^0_{\mu\nu} F^{0\mu\nu} + F^\ell_{\mu\nu} F^{\ell\mu\nu} + G^0_{\mu\nu} G^{0\mu\nu} + G^\ell_{\mu\nu} G^{\ell\mu\nu} \big) - \big(\delta\mu^2 - 2I_2 \big) \big((S^0 + \sqrt{N_f}(m_0 - m))^2 + (S^\ell)^2 \big) - \\
& - \big(\delta\mu^2 - 2I_2 + 4m^2 I_0 \big) \big((P^0)^2 + (P^\ell)^2 \big) + \Big(\frac{1}{\frac{1}{2} - \frac{N_f}{N_c}} - I_2 - m^2 I_0 \Big) V^0_\mu V^{0\mu} + \big(2\delta\mu^2 - I_2 - m^2 I_0 \big) V^\ell_\mu V^{\ell\mu} + \\
& + \big(2\delta\mu^2 - I_2 + 3m^2 I_0 \big) \big(A^0_\mu A^{0\mu} + A^\ell_\mu A^{\ell\mu} \big) + I \Big(\big[\tfrac{1}{\sqrt{N_f}} S^0 + t_\ell S^\ell + m_0 - m, \tfrac{1}{\sqrt{N_f}} P^0 + t_\ell P^\ell \big]_+^2 - \\
& - \Big(\big(\tfrac{1}{\sqrt{N_f}} S^0 + t_\ell S^\ell + m_0 - m \big)^2 - m^2 + \tfrac{1}{\sqrt{N_f}} P^0 + t_\ell P^\ell \Big)^2 - 4 \Big(\big(\tfrac{1}{\sqrt{N_f}} S^0 + t_\ell S^\ell + m_0 - m \big)^2 - m^2 \Big) \big(\tfrac{1}{\sqrt{N_f}} P^0 + t_\ell P^\ell \big)^2 \Big) \} + \\
& + \cdots\cdots
\end{aligned}
\tag{19}
$$

Where I_0 is given in (5a),

$$
I_2 \equiv \frac{N_c}{16\pi^2} \int_0^{\Lambda^2} dq^2 \frac{q^2}{q^2 + m^2} \tag{20}
$$

$$
\begin{aligned}
& F^0_{\mu\nu} \equiv \partial_\mu V^0_\nu - \partial_\nu V^0_\mu \qquad G^0_{\mu\nu} \equiv \partial_\mu A^0_\nu - \partial_\nu A^0_\mu \\
& F^\ell_{\mu\nu} \equiv \partial_\mu V^\ell_\nu - \partial_\nu V^\ell_\mu + c_{\ell jk} (V^j_\mu V^k_\nu - A^j_\mu A^k_\nu), \qquad G^\ell_{\mu\nu} \equiv \partial_\mu A^\ell_\nu - \partial_\nu A^\ell_\mu + c_{\ell jk} (A^j_\mu V^k_\nu - V^j_\mu A^k_\nu)
\end{aligned}
\tag{21}
$$

in which $c_{\ell jk}$ is the structure constant of the flavour group.

After rescaling the meson fields by

$$
\begin{aligned}
& S^0 \to \sqrt{\frac{1}{I_0}}\, S^0, \quad S^\ell \to \sqrt{\frac{1}{I_0}}\, S^\ell, \quad P^0 \to \sqrt{\frac{1}{I_0}}\, P^0, \quad P^\ell \to \sqrt{\frac{1}{I_0}}\, P^\ell, \\
& V^0 \to \sqrt{\frac{3}{4I_0}}\, V^0, \quad V^\ell_\mu \to \sqrt{\frac{3}{4I_0}}\, V^\ell_\mu, \quad A^0_\mu \to \sqrt{\frac{3}{4I_0}}\, A^0_\mu, \quad A^\ell_\mu \to \sqrt{\frac{3}{4I_0}}\, A^\ell_\mu,
\end{aligned}
\tag{22}
$$

the scalar and pseudoscalar mesons in (19) behave like the fields in the linear σ-model and the vector and axial-vector mesons in (19) like massive gauge fields.

Since Λ should not be very large, the terms omitted in the coefficients in (19) (terms independent of Λ and terms containing negative powers of Λ) may not be completely negligible. Therefore (19) can show the qualitative feature of I_{eff} but may not be sufficient for making quantitative predictions.

2. 1/N Expansion.

The effective action (19) can also be derived from a different approximabtions——1/N expansion. To do this, we first state here a theorem about functional integrations which is proved in Ref.[5].

Let $\phi(x)$ be an arbitrary field, where x stands for a set of indices including the space-time coordinates and internal symmetry indices, and $\sigma(x)$ be an operator which can be $\phi(x)$ or a composite operator composed of $\phi(x)$. Consider now a vertex of the type $V(x)\sigma^2(x)$, where $V(x)$ is an arbitrary function of x indepedent of ϕ. Let us cut the vertex in the way that the two $\sigma(x)$'s are separated as is shown in Fig.1.

A vertex $V\sigma^2$ in a vacuum diagram is said to be reducible (R) if, by cutting this vertex, the vacuum diagram is divided into two disconnected parts. If a vertex $V\sigma^2$ in a vacuum diagram is not reducible, it is said to be irreducible (I). $V\sigma^2$ vertices in

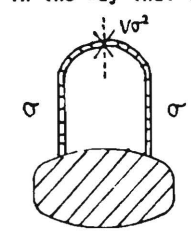

Fig.1 Cutting the vertex V by the dashed line.

a vacuum diagram can be either reducible or irreducible. As an example, we take $\phi(x)$ to be a fermion field ψ_a with an internal index a, $\sigma(x)$ be the composite operator $\bar{\psi}_a\psi_a$,and $V(x)\sigma^2(x) = g^2(\bar{\psi}_a\psi_a)(\bar{\psi}_b\psi_b)$. The vertex is depicted in Fig.2 with a bar indicating the pairing of the indices.The cut of the vertex should be made along the bar . An example of a vacuum diagram having reducible and irreducible vertices is shown in Fig.3 and the reducible and irreducible vertices are labled by R and I,respectively.

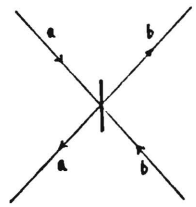

Fig.2 The vertex $V\sigma^2=g^2(\bar{\psi}_a\psi_a)(\bar{\psi}_b\psi_b)$

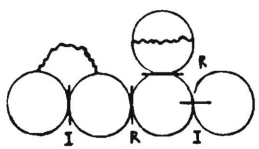

Fig.3. A vacuum diagram in which reducible and irreducible vertices are labled by R and I,respectively.

The theorem proved in Ref.[5] states that

$$\exp i\{ I(\phi)+\int dx\, V(x)\cdot(\Delta^2(x)-2\Delta(x)\sigma(x)) \}$$

$$= \exp i\{ I(\phi)+\int dx\, V(x)\cdot \sigma^2(x)\big|_R \} \tag{23a}$$

$$= \exp i\{ I(\phi)+\int dx\, (V(x)\sigma^2(x)-V(x)\sigma^2(x)\big|_I) \} \tag{23b}$$

Where $I(\phi)$ is an arbitrary action of the ϕ field, $\Delta(x)$ is the VEV of the operator $\sigma(x)$,$V(x)\sigma^2(x)\big|_R$ and $V(x)\sigma^2(x)\big|_I$ are reducible and irreducible vertices,respectively.We shall express the equivalence of the vertex contributions in (23) by the shortened notation

$$\int dx\, V(x)\, (\Delta^2(x) -2\Delta(x)\sigma(x)) \approx \int dx\, V(x)\, \sigma^2(x)\big|_R \tag{24a}$$

$$\approx \int dx\, (V(x)\sigma^2(x) - V(x)\sigma^2(x)\big|_I) \tag{24b}$$

Now we apply (24) to QCD.Let us consider a fixed gluon field configuration $B^a_\mu(x)$ and take $\sigma_B(x) = -g_s B^a_\mu \bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi$. Taking $V(x)=\frac{1}{2\Delta_B(x)}$ in (24a), we have

$$-g_s\, \bar{\psi}\gamma^\mu B^a_\mu\frac{\lambda^a}{2}\psi \approx -\frac{1}{2\Delta_B}(\bar{\psi}g_s\gamma^\mu B^a_\mu\frac{\lambda_a}{2}\psi)^2\big|_R+\frac{1}{2}\Delta_B \equiv -V^{ab}_{\mu\nu}(B)(\bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi)(\bar{\psi}\gamma^\nu\frac{\lambda_b}{2}\psi)+\frac{1}{2}\Delta_B \tag{25}$$

The effective gluon-field dependent vertex $V^{ab}_{\mu\nu}(B)$ defined in (25) can always be written in the form [5]

$$V^{ab}_{\mu\nu}(B)= V(B)\,\delta_{ab}\, g_{\mu\nu}+ V'^{ab}_{\mu\nu}(B) \tag{26}$$

Where V(B) is gauge invariant.Then using (24b) we have

$$V^{ab}_{\mu\nu}(B)(\bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi)(\bar{\psi}\gamma^\nu\frac{\lambda_b}{2}\psi)\big|_R \approx V(B)(\bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi) -V(B)(\bar{\psi}\gamma^\nu\frac{\lambda_b}{2}\psi)^2\big|_I +V'^{ab}_{\mu\nu}(B)(\bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi)(\bar{\psi}\gamma^\nu\frac{\lambda_b}{2}\psi)\big|_R \tag{27}$$

and our generating functional (1) can be written as

$$Z=\int \mathcal{D}B\,\mathcal{D}\psi\mathcal{D}\bar{\psi}\mathcal{D}\omega\mathcal{D}\bar{\omega}\exp i\int d^4x\{\mathcal{L}_g(B,\omega,\bar{\omega})+\bar{\psi}(i\partial\!\!\!/-m_0)\psi -V(B)(\bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi)^2+$$
$$+V(B)(\bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi)^2\big|_I -V'^{ab}_{\mu\nu}(B)(\bar{\psi}\gamma^\mu\frac{\lambda_a}{2}\psi)(\bar{\psi}\gamma^\nu\frac{\lambda_b}{2}\psi)\big|_R +\frac{1}{2\Delta_B}\} \tag{28}$$

The integrations over the gluon and ghost fields are difficult to carry

out, so that we have to make approximations. As has been argued by Witten[6] that when we calculate the VEV of a gauge invariant operator composed of gluon fields in the $N_c \to \infty$ limit, the gluonic functional integration can be represented by the contribution of an effective classical configuration of the gluon field $B^a_{\mu cl}$. We apply this idea to (28). Of course, when N_c is finite, this is a mean field approximation. The term $\mathcal{L}_g + \frac{1}{2\Delta_B}$ in (28) is independet of the quark field and is thus irrelevant in the derivation of the effective action for mesons. In the terms $-V(B)(\bar\psi\gamma_\mu\frac{\lambda_a}{2}\psi)^2$ and $V(B)(\bar\psi\gamma_\mu\frac{\lambda_a}{2}\psi)^2\big|_I$, the effective vertex $V(B)$ is gauge invariant and thus the above idea can be applied, i.e. $V(B)$ is taken to be $V(B_{cl})$ without doing the gluonic integration. The term $-V'^{ab}_{\mu\nu}(B)(\bar\psi\gamma^\mu\frac{\lambda_a}{2}\psi)(\bar\psi\gamma^\nu\frac{\lambda_b}{2}\psi)\big|_R$ is complicated since $V'^{ab}_{\mu\nu}(B)$ is not gauge invariant. However the steps (25)-(26) for treating the gauge dependent vertex $g_s B^a_\mu$ can now be applied to this term. After the steps (25)-(26), we obtain three terms containing operators with eight fermion fields (dimension-12 operators) two of which are gauge invariant and the other one is gauge dependent. This process can be carried on further to the remaining gauge dependent term and we get operators of even higher dimensions. Assuming colour confinement, we conclude that eventually only the gauge invariant terms with colour singlet operators will contribute to the effective action. For these terms, we can apply the above idea and take only the contribution of $B^a_{\mu cl}$ instead of doing the gluonic integration. We can also introduce auxiliary fields for the higher dimensional operators and they correspond to higher-quark meson states. Since these higher-quark mesons are

not Goldstone bosons in the chiral limit $m_0 \to 0$, the typical mass M of these mesons must be characterized mainly by the cut-off parameter Λ which is larger than the masses of the low lying mesons. The gauge invariant effective vertex of a dimension-6n operator must be proportional to $(M)^{4-6n}$. From dimensional analysis, for a low energy meson process characterized by an energy scale $\sqrt{s} < M$, the heavy mesons contribute only through internal lines and thus their contributions must be suppressed by positive powers of $\sqrt{s}/M$. As an approximation, we neglect these contributions, i.e. we drop the $-V'^{ab}_{\mu\nu}(B)(\bar\psi\gamma^\mu\frac{\lambda_a}{2}\psi)(\bar\psi\gamma^\nu\frac{\lambda_b}{2}\psi)\big|_R$ terms in (28). Then (28) is approximately

$$Z=\int\mathcal{D}\psi\mathcal{D}\bar\psi\,\exp i\int d^4x\{\bar\psi(i\not\partial-m_0)\psi-V(B_{cl})(\bar\psi\gamma_\mu\frac{\lambda_a}{2}\psi)^2+V(B_{cl})(\bar\psi\gamma_\mu\frac{\lambda_a}{2}\psi)^2\big|_I \tag{29}$$

It is easy to see that the vacuum diagrams containing only irreducible vertices are at least smaller than the diagrams containing only reducible vertices by a factor of $1/N_f$. Thus from (24b), to lowest order of $1/N_f$ expansion, the term $V(B_{cl})(\bar\psi\gamma_\mu\frac{\lambda_a}{2}\psi)^2\big|_I$ in (29) does not contribute and so (29) reduces to

$$Z=\int\mathcal{D}\psi\mathcal{D}\bar\psi\,\exp i\int d^4x\{\bar\psi(i\not\partial-m_0)\psi-\frac{1}{2\delta\mu^2}(\bar\psi\gamma_\mu\frac{\lambda_a}{2}\psi)^2\} \tag{30}$$

where

$$\delta\mu^2 \equiv \frac{1}{2V(B_{cl})}. \tag{31}$$

Eq. (30) is exactly of the form of (7) except that the meaning of $\delta\mu^2$ (cf. (31)) is now different. Lacking of the knowledge about $B^a_{\mu cl}$, we also take $\delta\mu^2$ as a free parameter here. The form (30) (or (7)) leads directly to the

effective action (17) through the steps (9)-(17). Thus, to lowest order, the two different approximations lead to the same effective action (17), but their higher order corrections are different.

DISCUSSIONS

Finally we discuss some of the consequences of the effective action (17).

1. As has been pointed out by Volkov and Ebert[2] that after gauging the effective action like (17) to incorporate the mesons with photon, both quark-loop effect and vector-dominance are contained in the effective action which are both successful in describing low energy hadronic processes. Since they are derived from a single fundamental theory, there is no double counting in this kind of effective action[2].

2. A specially interesting quark-loop effect is the axial-vector current anomaly. Volkov and Ebert calculated the amplitude of $\pi^0 \longrightarrow 2\gamma$ from their effective action and the result is the same as the one obtained from the triangle anomaly calculation[2]. The situation of our effective action (19) is exactly the same. On the other hand, in the nonlinear σ-model, the effect of the triangle anomaly resides in the Wess-Zumino term[7]. The Wess-Zumino term has also other effects. For instance, it gives rise to the reaction $K^+K^- \to \pi^0\pi^+\pi^-$[7]. To see the relation between the nonlinear σ-model and our linear σ-model type effective action (17), we can search for the interaction which gives rise to $K^+K^- \longrightarrow \pi^0\pi^+\pi^-$ in our effective action. We keep calculating the unwritten dimension-5 operators in (19), and find that there

is indeed a term causes $K^+K^- \longrightarrow \pi^0\pi^+\pi^-$[5]. Therefore the effects of the Wess-Zumino term in the nonlinear σ-model are equivalently contained in the logarithm term in our effective action (17).

3. As a rough estimate, we derive the relation between the meson masses from the explicitly written terms in (19). From (19) and (22) we see that, except for V^0_μ, there is a simple mass relation for the mesons $S^0, S^\ell, P^0, P^\ell, V^\ell_\mu, A^0_\mu$ and A^ℓ_μ, namely

$$m_A^2 - m_V^2 = \frac{3}{4}(m_S^2 - m_P^2) \tag{32}$$

To compare this relation with experiments, we identify the isospin triplet pseudoscalar meson P^ℓ with the pion, the isospin triplet vector meson V^ℓ_μ with the ρ-meson, the isospin singlet scalar meson S^0 with $f_0(975)$, the isospin triplet scalar meson S^ℓ with $a_0(980)$, the isospin singlet axial-vector meson A^0_μ with $h_1(1170)$, and the isospin triplet axial-vector meson A^ℓ_μ with $b_1(1235)$. In (19) and (22), the masses of the isospin singlet and isospin triplet states are degenerate in the scalar and axial-vector meson sectors. Thus m_S^2 and m_A^2 in (32) should be taken as the averaged values

$$m_S^2 = \frac{1}{4}(m_{f_0}^2 + 3m_{a_0}^2) = 0.96 \text{ Gev}$$
$$m_A^2 = \frac{1}{4}(m_{h_1}^2 + 3m_{b_1}^2) = 1.48 \text{Gev}$$

Comparing the experimental data with the relation (32), we find that the derivation is only 20 %. The result is not bad. A systematic study of the quantitative predictions of the effective action (17) will be given elsewhere.

ACKNOWLEGEMENT

We would like to thank many of our colleagues of Peking University, Institute of Theoretical Physics and Institute of High Energy Physics of Academia Sinica for interesting discussions.

REFERENCES

[1] T.Eguchi, Phy. Rev.D14 (1976) 2755

[2] M.K.Volkov and D.Ebert, Sov.J.Nucl.Phys.36(5)(1982)736; M.K.Volkov, Sov.J.Part.Nucl.17(3)(1986)186

[3] G.'tHooft, Phys.Rev.Lett.37(1976)8

[4] C.M.Fraser, Z.Phys.C28(1985)101

[5] Q.Wang, Ph.D.Thesis (Tsinghua University)

[6] E.Witten, Lecture at Harvard University (1979)

[7] E.Witten, Nucl.Phys. B223(1983)422

[8] Particle Data Group, Phys.Lett.B204(1988)1