Production of W-Bosons of the Left-Right–Symmetric Model in e^-e^- Collisions

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and

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Abstract

The new linear colliders (NLC, JLC, TESLA) can be constructed to produce e^-e^- collisions in the c.m. energy range of 1 – 2 TeV. In the present work, the pair production of the same-signed W bosons of a left-right-symmetric model has been investigated for this energy region. The possibility of these production channels arises on one hand from the Majorana neutrinos and from the double charged higgs boson of the model and on the other hand from the W_L and W_R mixing in the mass eigenstates.

The cross sections of the processes are proportional to the square of the neutrino mass if the mass is heavy enough (at least several TeV). In the case of a less heavy neutrino mass the cross section depends on it in a more complicated manner. The cross sections are proportional to $(K/MM')^2$, where M and M' refer to the W masses in the final state, and the parameter K is given by $K = \cos^2\zeta$ for the heavy W pair, $K = \sin\zeta\cos\zeta$ for the light and heavy pair, $K = \sin^2\zeta$ for the light pair, and ζ is the W mixing angle.

Our calculation shows that the total cross section of the heavy W pair production is at least of the order of 0.1 – 1 pbarn when the W mass is 0.5 TeV and the heavy neutrino mass 1 TeV. The cross sections can be greatly enhanced if the double charged higgs lies at the energy range of next generation colliders. Without this resonance production, the production channel of the same-signed light W, heavy W pair production is almost negligible, if ζ obeys its upper limit estimate (around 0.001) found in the literature and the particles have their masses in the region considered by us.

The dependence of the total cross sections on the masses of the neutrino, of the higgs, and of the heavy W boson have been illustrated by 12 figures.

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Chapter 1

Introduction

The next generation of linear colliders is just under the planning. The new colliders can be made, beside the e^+e^- mode, capable to produce high-energy e^-e^- collisions in the range of 1-2 TeV. This will offer new possibilities in testing of the standard model and of its extensions.

In our previous papers [7], we considered the pair production of the charged gauge bosons from e^+e^- collisions in the framework of the usual left-right symmetric model (LRM) based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)$, which has been regarded as the most natural extension of the standard electroweak theory. In the LRM there are most naturally two Majorana electron neutrinos, and our main motivation to that work was to try to find out if the measurement of the W-production cross sections could reveal the order of the magnitude of the second neutrino mass. The knowledge of this would cast light on the fundamental question of whether neutrinos are Dirac or Majorana particles.

Our results, however, were in this respect not quite satisfying, because the heavy neutrino had a negligible effect on the light W pair production, the dependence of the production rate of the heavy pair on the neutrino mass was complicated and the production rate of the light heavy pair was almost negligible. In addition, the production of different-signed W pairs is complicated due to the neutral higgs sector of the model.

In present work, we will continue our study on the role of Majorana neutrinos in the same model by calculating the production of the same-sign W pairs in electron-electron collisions[1]. These lepton number violating reactions are mediated by Majorana neutrinos and are forbidden in the case of Dirac neutrinos. Beside the neutrinos, which are exchanged in the t- and in the u-channels, a double charged higgs mediates the W pair productions in the s-channel. The different channels balance each other to achieve a good high-energy behaviour of the total cross section. The contribution from the neutrino exchange channels is proportional to the square of the heavy neutrino mass. For the balance, the higgs exchange should also produce terms which are proportional to the square of the neutrino mass. So, the production of the same-signed W pair could be expected to give a lot of information from the neutrino mass.

We begin our study by deriving the formulas for the contributions from various exchange channels in the lowest-order. Then we shall illustrate the dependence of the cross sections on our free parameters, including the heavy neutrino mass. We conclude that the production rate of the heavy W pair production is large enough for experiments in the new electron-electron colliders, if the heavy neutrino lies in the mass range of one hundred GeV or more, provided, of course, that the mass of the W pair is in the energy of the future colliders region.

[1] We thank J. Ellis, CERN, for drawing our attention to these processes.

Chapter 2

Basic Formulas

The left-right-symmetric model which we consider in this work has been discussed in detail in many articles (see f. ex. [8,6,9,3]). For this work it is essential that the particle spectrum of the model contains left- and right-handed lepton doublets l_L and l_R (consisting of the neutrino and of the electron components), left- and right-handed vector bosons V_L and V_R, and at least a right-handed triplet of the higgses Δ whose electric charges are 0, -1, and -2 in the units of the positron charge e.

The relevant parts of the Lagrangian for our work are: the Yukawa coupling $\mathcal{L}^{l-\Delta}$ of the triplet higgses to the leptons, the kinetic energy $\mathcal{L}_{\Delta}^{kin}$ of the triplet, and the charged current lagrangian $\mathcal{L}^{cc}$. The first one has the form [10]

$$\mathcal{L}^{l-\Delta} = f\,\overline{l}^{c}\,\vec{\Delta}\cdot(i\,\tau_2\,\vec{\tau})\,l + h.c., \tag{2.1}$$

where τ_k refers to the Pauli matrix and $\overline{l}^{c} = (\nu^T\,e^T)C$ is the adjoint of the charged conjugated spinor. We drop in these equations the subscripts L or R for a moment.

The kinetic energy of a higgs triplet is given by

$$\mathcal{L}_{\Delta}^{kin} = \{(\partial_\mu - g\vec{V}_\mu\times)\vec{\Delta}\}^{\dagger}\cdot\{(\partial^\mu - g\vec{V}^\mu\times)\vec{\Delta}\}, \tag{2.2}$$

and the charged current lagrangian reads in the mass eigenstate basis as follows [7]

$$\mathcal{L}^{cc} = \sum_{jl}\overline{\nu}_j\Gamma^{\mu}_{jl}e(W^{-}_{l\mu})^{\dagger} \quad + h.c. \tag{2.3}$$

In order to get to this equation one has to express the Cartesian components V_L and V_R in terms of the respective charged components $W_L^{\pm}$ and $W_R^{\pm}$ and then present these in terms of their mass eigenstates W_1 and W_2 :

$$\begin{pmatrix} W_L^{\pm} \\ W_R^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta \\ \sin\zeta & \cos\zeta \end{pmatrix}\begin{pmatrix} W_1^{\pm} \\ W_2^{\pm} \end{pmatrix}. \tag{2.4}$$

Furthermore, one has to form the mass eigenstates ν_1 and ν_2 of the neutrinos and write ν_L and ν_R in terms of them:

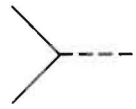
$$\begin{aligned} \nu_L &= \tfrac{1}{2}(1-\gamma^5)\,(\nu_1\cos\eta - \nu_2\sin\eta)\,, \\ \nu_R &= \tfrac{1}{2}(1+\gamma^5)\,(\nu_1\sin\eta + \nu_2\cos\eta)\,. \end{aligned} \tag{2.5}$$

We will make our calculations in unitary gauge and therefore we later have to re-express our quantities in the formulas (2.1) and (2.2) in the mass eigenstate basis, too.

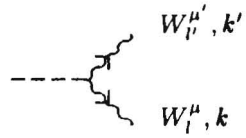
From the Lagrangians (2.1)–(2.3), we shall obtain the vertices for the lowest order diagrams associated with the W pair production in the electron- electron collisions. The vertices are presented

$\Delta_R^\dagger e\, e =$

$= -i\sqrt{2}\, f_R C(1+\gamma_5)$

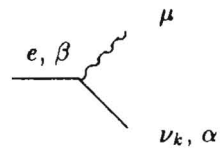


$\Delta_R\, W_l^\dagger W_{l'}^\dagger =$



$= -i\, 2 g_{\mu\mu'} g_R^2 < \Delta_R^0 > K_{ll'}(\zeta)$

$\bar{\nu}_k\ e\ W_l^\dagger =$



$= i\,(\Gamma_{kl}^{\mu})_{\alpha\beta}$

Figure 2.1: Relevant Feynman rules

in Figure 2.1. The detailed expressions, which lead to the rules of Figure 2.1, can be found in the sections 3 and 4. The Feynman diagrams for $e^-e^- \to W_l^- W_{l'}^-$ are presented in Figure 2.2 and they consist of a double charged Δ exchange in the s-channel and of neutrino exhanges in the t- and u-channels. We have checked our expressions for the Majorana neutrinos by comparing them with the ones given in the references [4] and [5].

The general form of the amplitude is given by

$$M_{ss'aa'} = u_s^T(p) T_{\mu\mu'} u_{s'}(p') \epsilon_a^{*\mu}(k) \epsilon_{a'}^{*\mu'}(k'), \tag{2.6}$$

where u_s and ϵ_a refer to the spinor of the electron and to the polarisation vector of the W boson, respectively, and T contains the contributions from the different channels:

$$T = T^{(s)} + T^{(t)} + T^{(u)}. \tag{2.7}$$

The squared matrix element summed over spins reads

$$\sum |M|^2 = Tr\left\{C T_{\mu\mu'}\ \not{p}'\gamma^0 (C T_{\nu\nu'})^\dagger \gamma^0\ \not{p}\right\} \left(g^{\mu\nu} - \frac{k^\mu k^\nu}{M^2}\right) \left(g^{\mu'\nu'} - \frac{k'^{\mu'} k'^{\nu'}}{M'^2}\right), \tag{2.8}$$

where C is the charge conjugation matrix. The total cross section of an unpolarized $W_l^- W_{l'}^-$ pair in a collision of unpolarized electrons is given by the formula

$$\sigma_{e^-e^- \to W_l^- W_{l'}^-} = \frac{1}{16\pi(1+\delta_{ll'})s^2} \int_{t_{min}}^{t_{max}} < \sum |M|^2 > dt, \tag{2.9}$$

with

$$< \sum |M|^2 > = \frac{1}{4} \sum |M|^2. \tag{2.10}$$

The Kronecker δ-symbol has been inserted into eq. (2.9) in order that one avoid double counting of identical final particles in the integration over the full 4π steradians.

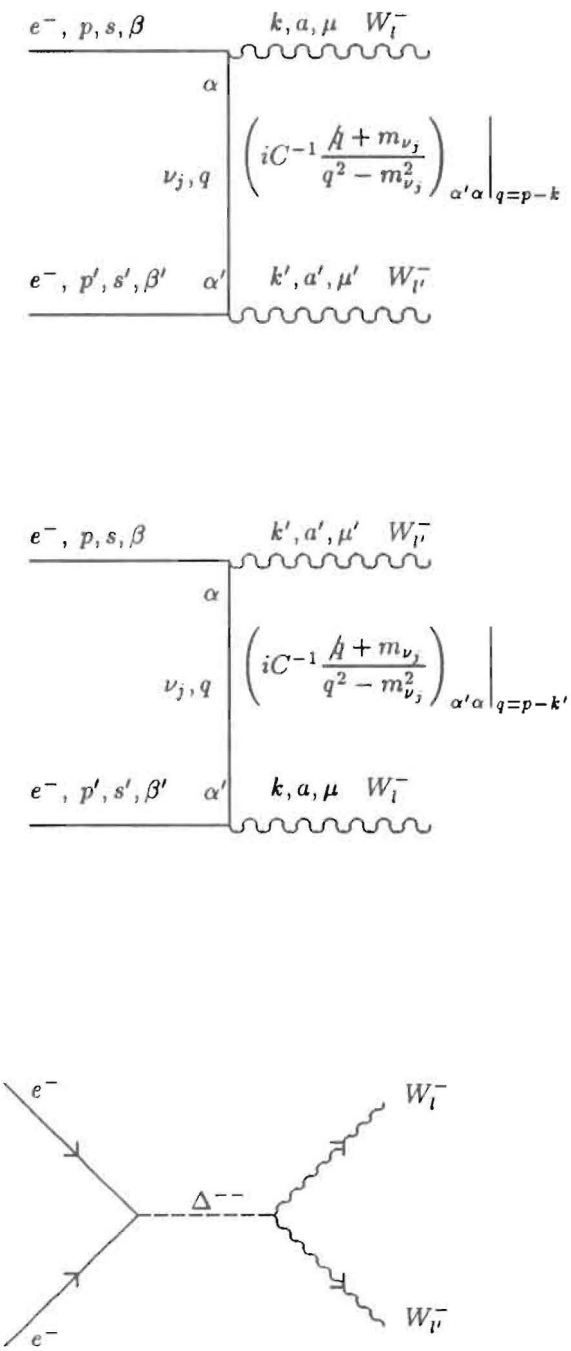


Figure 2.2: The t-, u- and s-channel Feynman graphs

The limits of integration are given by

$$t_{min/max} = \frac{a-s}{2} \pm \sqrt{\frac{(a-s)^2}{4} - b}, \tag{2.11}$$

where

$$\begin{aligned} a &= M^2 + M'^2, \\ b &= M^2 M'^2, \end{aligned} \tag{2.12}$$

and the W-masses are denoted by M and M'.

Chapter 3

Contributions from t- and u-channels

From the t-channel diagram 2.2 we obtain for the amplitude

$$T^{(t)}_{\mu\mu'}(jll') = i^3(\Gamma^{jl}_{\mu})^T \left(C^{-1}\frac{\not{q}+m_{\nu_j}}{q^2-m^2_{\nu_j}}\right)^T \Gamma^{jl'}_{\mu'}\Bigg|_{q=p-k}, \tag{3.1}$$

and from the u-channel for the amplitude

$$T^{(u)}_{\mu\mu'}(jll') = T^{(t)}_{\mu'\mu}(jl'l)\Big|_{q=p-k'}, \tag{3.2}$$

where

$$\Gamma^{\mu}_{jl} \equiv G^L_{jl}\gamma^{\mu}(1-\gamma^5) + G^R_{jl}\gamma^{\mu}(1+\gamma^5) = G^V_{jl}\gamma^{\mu} + G^A_{jl}\gamma^5\gamma^{\mu}, \tag{3.3}$$

and

$$G^V_{jl} = G^L_{jl} + G^R_{jl}, \qquad G^A_{jl} = G^L_{jl} - G^R_{jl}. \tag{3.4}$$

Introducing the notation

$$\overline{\Gamma}^{\mu}_{jl} \equiv -C(\Gamma^{\mu}_{jl})^T C^{-1} = G^V_{jl}\gamma^{\mu} - G^A_{jl}\gamma^5\gamma^{\mu} \tag{3.5}$$

and utilizing the symmetry relations

$$\begin{aligned} &\left[C^{-1}(\not{q}+m_{\nu_j})\right]^T = C^{-1}(\not{q}-m_{\nu_j}), \\ &\gamma^0(\Gamma^{jl}_{\mu})^{\dagger}\gamma^0 = \Gamma^{jl}_{\mu}, \end{aligned} \tag{3.6}$$

the quantities in the matrix element (2.8) can be cast to the forms

$$\begin{aligned} &CT^{(t)}_{\mu\mu'}(jll') = i\overline{\Gamma}^{jl}_{\mu}\frac{\not{q}-m_{\nu_j}}{q^2-m^2_{\nu_j}}\Gamma^{jl'}_{\mu'}\Bigg|_{q=p-k}, \\ &\gamma^0\left[CT^{(t)}_{\mu\mu'}(jll')\right]^{\dagger}\gamma^0 = -i\Gamma^{jl'}_{\mu'}\frac{\not{q}-m_{\nu_j}}{q^2-m^2_{\nu_j}}\overline{\Gamma}^{jl}_{\mu}\Bigg|_{q=p-k}. \end{aligned} \tag{3.7}$$

These can be rewritten as follows:

$$
\begin{aligned}
&\tfrac{1}{i}C(q^2-m_{\nu_j}^2)T^{(t)}_{\mu\mu'}(jll') \\
&\quad = \left[a_{jll'}\gamma_\mu \not{A}\gamma_{\mu'} + b_{jll'}\gamma_\mu \not{A}\gamma_{\mu'}\gamma^5\right] + m_{\nu_j}\left[c_{jll'}\gamma_\mu\gamma_{\mu'} - d_{jll'}\gamma_\mu\gamma_{\mu'}\gamma^5\right],
\end{aligned}
\tag{3.8}
$$

and

$$
\begin{aligned}
&i(q^2-m_{\nu_{j'}}^2)\gamma^0\left[CT^{(t)}_{\nu\nu'}(j'll')\right]^\dagger\gamma^0 \\
&\quad = \left[a_{j'll'}\gamma_{\nu'} \not{A}\gamma_\nu + b_{j'll'}\gamma_{\nu'} \not{A}\gamma_\nu\gamma^5\right] + m_{\nu_{j'}}\left[c_{j'll'}\gamma_{\nu'}\gamma_\nu + d_{j'll'}\gamma_{\nu'}\gamma_\nu\gamma^5\right]
\end{aligned}
\tag{3.9}
$$

by introducing the coefficients

$$
\begin{aligned}
a_{jll'} &= 2(G^L_{jl}G^R_{jl'} + G^R_{jl}G^L_{jl'}) \equiv 2(G_{LlRl'j} + G_{RlLl'j}), \\
b_{jll'} &= 2(G^L_{jl}G^R_{jl'} - G^R_{jl}G^L_{jl'}) \equiv 2(G_{LlRl'j} - G_{RlLl'j}), \\
c_{jll'} &= 2(G^L_{jl}G^L_{jl'} + G^R_{jl}G^R_{jl'}) \equiv 2(G_{LlLl'j} + G_{RlRl'j}), \\
d_{jll'} &= 2(G^L_{jl}G^L_{jl'} - G^R_{jl}G^R_{jl'}) \equiv 2(G_{LlLl'j} - G_{RlRl'j}).
\end{aligned}
\tag{3.10}
$$

The u-channel matrix element is obtained from the t-channel element just by making the replacements

$$
l \leftrightarrow l', \qquad k,a,\mu \leftrightarrow k',a',\mu', \qquad t=(p-k)^2 \to (p-k')^2 = u. \tag{3.11}
$$

In the next step we take into account that the trace (2.8) vanishes for odd number of γ-matrices and we use some symmetry properties of the traces. We obtain

$$
\begin{aligned}
&\sum |M^{(t)} + M^{(u)}|^2 \\
&= \sum_{jj'}\Bigg\{(a_{jll'}a_{j'll'} + b_{jll'}b_{j'll'})\left[\frac{F_1(t)}{(t-m^2_{\nu_j})(t-m^2_{\nu_{j'}})} + \frac{F_1(u)}{(u-m^2_{\nu_j})(u-m^2_{\nu_{j'}})}\right] \\
&\qquad\qquad +2(a_{jll'}a_{j'll'} - b_{jll'}b_{j'll'})\frac{H_1(t,u)}{(t-m^2_{\nu_j})(u-m^2_{\nu_{j'}})}\Bigg\} \\
&+\sum_{jj'} m_{\nu_j}m_{\nu_{j'}}\Bigg\{(c_{jll'}c_{j'll'} + d_{jll'}d_{j'll'})\Bigg[\frac{F_2(t)}{(t-m^2_{\nu_j})(t-m^2_{\nu_{j'}})} \\
&\qquad\qquad +\frac{F_2(u)}{(u-m^2_{\nu_j})(u-m^2_{\nu_{j'}})} + 2\frac{H_2(t,u)}{(t-m^2_{\nu_j})(u-m^2_{\nu_{j'}})}\Bigg]\Bigg\},
\end{aligned}
\tag{3.12}
$$

where

$$
\begin{aligned}
F_1(t) &= Tr\left\{\gamma^\mu(\not{p}-\not{k})\gamma^{\mu'}\not{p}'\gamma^{\nu'}(\not{p}-\not{k})\gamma^\nu\not{p}\right\}(\mu\nu,k)(\mu'\nu',k'), \\
F_2(t) &= Tr\left\{\gamma^\mu\gamma^{\mu'}\not{p}'\gamma^{\nu'}\gamma^\nu\not{p}\right\}(\mu\nu,k)(\mu'\nu',k'), \\
H_1(t,u) &= Tr\left\{\gamma^\mu(\not{p}-\not{k})\gamma^{\mu'}\not{p}'\gamma^\nu(\not{p}-\not{k}')\gamma^{\nu'}\not{p}\right\}(\mu\nu,k)(\mu'\nu',k'), \\
H_2(t,u) &= Tr\left\{\gamma^\mu\gamma^{\mu'}\not{p}'\gamma^\nu\gamma^{\nu'}\not{p}\right\}(\mu\nu,k)(\mu'\nu',k'), \\
F_1(u) &= Tr\left\{\gamma^{\mu'}(\not{p}-\not{k}')\gamma^\mu\not{p}'\gamma^\nu(\not{p}-\not{k}')\gamma^{\nu'}\not{p}\right\}(\mu\nu,k)(\mu'\nu',k'), \\
F_2(u) &= Tr\left\{\gamma^{\mu'}\gamma^\mu\not{p}'\gamma^\nu\gamma^{\nu'}\not{p}\right\}(\mu\nu,k)(\mu'\nu',k'),
\end{aligned}
\tag{3.13}
$$

and we have used the abbreviation

$$(\mu\nu, k) \equiv \left(g_{\mu\nu} - \frac{k_\mu k_\nu}{M^2} \right). \tag{3.14}$$

The last two traces of (3.13) originate in the u-channel amplitude and are given by the same functions F_1 and F_2 as the first two t-channel traces but with the variable t changed to u.

The traces (3.13) have been calculated using the symbolic manipulation program *Reduce* and the following expressions for the functions $F_1, \ldots, H_2$ have been derived:

$$\begin{aligned}
F_1(t) &= -\tfrac{2}{b}[t^4 + (s-a)t^3 + (5b - 2as)t^2 + 4(s-a)bt + 4b^2], \\
F_2(t) &= \tfrac{2}{b}[(s-2a)t^2 - 2a(s-a)t + 2b(2s-a)], \\
H_1(t,u) &= -\tfrac{2}{b}[t^2u^2 + (2as - b)tu - 4abs], \\
H_2(t,u) &= \tfrac{2}{b}[(s-2a)tu + 2ab].
\end{aligned} \tag{3.15}$$

Here s refers to the square of the c.m. energy. In principle, the equations (3.13) can be directly used for numerical computations. We will, however, develope the formulas further in order to get a better insight into the results . Therefore we break the sums $aa + bb$ and the sums $cc + dd$ in the following way:

$$\sum_{jj'} \frac{a_{jll'}a_{j'll'} + b_{jll'}b_{j'll'}}{(t - m^2_{\nu_j})(u - m^2_{\nu_{j'}})} = 8 \sum_j \frac{G_{LlRl'j}}{t - m^2_{\nu_j}} \sum_j \frac{G_{LlRl'j}}{u - m^2_{\nu_j}} \quad + \quad (l \leftrightarrow l'), \tag{3.16}$$

and

$$\begin{aligned}
&\sum_{jj'} \frac{c_{jll'}c_{j'll'} + d_{jll'}d_{j'll'}}{(t - m^2_{\nu_j})(u - m^2_{\nu_{j'}})} m_{\nu_j} m_{\nu_{j'}} \\
&\qquad = 8 \sum_j \frac{G_{LlLl'j} m_{\nu_j}}{t - m^2_{\nu_j}} \quad \sum_j \frac{G_{LlLl'j} m_{\nu_j}}{u - m^2_{\nu_j}} \quad + (L \to R).
\end{aligned} \tag{3.17}$$

If the + sign on the left-hand side of eq. (3.16) is replaced by the - sign, then the indeces l and l' in the second sums of the products on the right-hand side must be exchanged.

In our previous paper [7], we derived expressions for the G-coefficients in terms of the W- and ν-mixing angles ζ and η, respectively. We give them in the Tables 3.1–3.2. From Table 3.1 we find that

$$\begin{aligned}
&\sum_j \frac{G_{LlRl'j}}{t - m^2_{\nu_j}} \\
&= G_L G_R \sin\eta \cos\eta \left(\frac{1}{t - m^2_{\nu_2}} - \frac{1}{t - m^2_{\nu_1}} \right) \begin{cases} \sin\zeta\cos\zeta, & l = l' = 1 \\ -\sin\zeta\cos\zeta, & l = l' = 2 \\ \cos^2\zeta, & l = 1, l' = 2 \\ -\sin^2\zeta, & l = 2, l' = 1 \end{cases}
\end{aligned} \tag{3.18}$$

and so we obtain for the sums $aa \pm bb$ of eq. (3.12):

$$\begin{aligned}
&\sum_{jj'} \frac{a_{jll'}a_{j'll'} \pm b_{jll'}b_{j'll'}}{(t - m^2_{\nu_j})(u - m^2_{\nu_{j'}})} \\
&= 8 G_L^2 G_R^2 \sin^2\eta \cos^2\eta \left(\frac{1}{t - m^2_{\nu_2}} - \frac{1}{t - m^2_{\nu_1}} \right) \left(\frac{1}{u - m^2_{\nu_2}} - \frac{1}{u - m^2_{\nu_1}} \right) \\
&\qquad \begin{cases} 2\sin^2\zeta\cos^2\zeta, & l = l' \\ \sin^4\zeta + \cos^4\zeta, & l \neq l', t = u \\ -2\sin^2\zeta\cos^2\zeta, & l \neq l', t \neq u. \end{cases}
\end{aligned} \tag{3.19}$$

Here $G_L = g_L/2\sqrt{2}$ and $G_R = g_R/2\sqrt{2}$. We shall use the subscript 1 for the contributions arising from these terms in the formula (3.12). Into the sum of the squared matrix element they produce the part

$$\sum |M_1^{(t)} + M_1^{(u)}|^2$$

$$= \begin{cases} 16G_L^2 G_R^2 (\sin\eta\cos\eta\sin\zeta\cos\zeta)^2 (m_{\nu_1}^2 - m_{\nu_2}^2)^2 \\ \left[\dfrac{F_1(t)}{(t-m_{\nu_1}^2)^2(t-m_{\nu_2}^2)^2} + \dfrac{F_1(u)}{(u-m_{\nu_1}^2)^2(u-m_{\nu_2}^2)^2}\right. \\ \qquad \left. +2\dfrac{H_1(t,u)}{(t-m_{\nu_1}^2)(t-m_{\nu_2}^2)(u-m_{\nu_1}^2)(u-m_{\nu_2}^2)}\right], & l = l' \\ \\ 8G_L^2 G_R^2 \sin^2\eta\cos^2\eta (m_{\nu_1}^2 - m_{\nu_2}^2)^2 \\ \left[(\sin^4\zeta + \cos^4\zeta)\left(\dfrac{F_1(t)}{(t-m_{\nu_1}^2)^2(t-m_{\nu_2}^2)^2} + \dfrac{F_1(u)}{(u-m_{\nu_1}^2)^2(u-m_{\nu_2}^2)^2}\right)\right. \\ \qquad \left. -4\sin^2\zeta\cos^2\zeta\dfrac{H_1(t,u)}{(t-m_{\nu_1}^2)(t-m_{\nu_2}^2)(u-m_{\nu_1}^2)(u-m_{\nu_2}^2)}\right], & l \neq l'. \end{cases} \tag{3.20}$$

To obtain their contribution to the total cross section we have to integrate the expressions (3.20). In the integration the t-channel and the u-channel yields the same contribution as can be checked by integrating the pure u-channel part over u instead of t. This was our motivation for writing the limits of the integration in the form (2.11) where $a - s = t + u$ and thus the limits do not change when the integration variable t is replaced by u.

The essential integrals are given by the analytical formulas

$$\begin{aligned} &\int_{t_{min}}^{t_{max}} F_1(t)\left(\frac{1}{t-x} - \frac{1}{t-y}\right)^2 dt \\ &= \Big/ \left\{-\frac{2}{b}(x-y)^2 t - \frac{F_1(x)}{t-x} - \frac{F_1(y)}{t-y} + F_1'(x)\ln|t-x| + F_1'(y)\ln|t-y| \right. \\ &\qquad \left. -2\frac{F_1(x)\ln|t-x| - F_1(y)\ln|t-y|}{x-y}\right\}, \end{aligned} \tag{3.21}$$

and

$$\begin{aligned} &\int_{t_{min}}^{t_{max}} H_1(t,u)\left(\frac{1}{t-x} - \frac{1}{t-y}\right)\left(\frac{1}{u-x} - \frac{1}{u-y}\right) dt \\ &= \Big/ \left\{-\frac{2}{b}(x-y)^2 t + 2\left[-\frac{2}{b}x^2(x-y) - \frac{H_1(x,x)}{s-a+2x} + \frac{H_1(x,y)}{s-a+x+y}\right]\ln|t-x| \right. \\ &\qquad \left. +2\left[-\frac{2}{b}y^2(y-x) - \frac{H_1(y,y)}{s-a+2y} + \frac{H_1(x,y)}{s-a+x+y}\right]\ln|t-y|\right\}. \end{aligned} \tag{3.22}$$

Next we concentrate ourselves to sum (3.17). We will refer to the contributions arising from these terms by the subscript 2. From Table 3.2 we obtain

$$\sum_j \frac{G_{LlLl'j} m_{\nu_j}}{t - m_{\nu_j}^2} = G_L^2\left(\frac{m_{\nu_1}\cos^2\eta}{t - m_{\nu_1}^2} + \frac{m_{\nu_2}\sin^2\eta}{t - m_{\nu_2}^2}\right)\begin{cases} \cos^2\zeta, & l = l' = 1 \\ \sin^2\zeta, & l = l' = 2 \\ -\sin\zeta\cos\zeta, & l \neq l', \end{cases} \tag{3.23}$$

and

$$\sum_j \frac{G_{RlRl'j} m_{\nu_j}}{t - m_{\nu_j}^2} = G_R^2 \left(\frac{m_{\nu_1} \sin^2 \eta}{t - m_{\nu_1}^2} + \frac{m_{\nu_2} \cos^2 \eta}{t - m_{\nu_2}^2} \right) \begin{cases} \sin^2 \zeta, & l = l' = 1 \\ \cos^2 \zeta, & l = l' = 2 \\ \sin \zeta \cos \zeta, & l \neq l'. \end{cases} \tag{3.24}$$

Our typical sum is now of the form

$$\begin{aligned} &\sum_{jj'} \frac{c_{jll'} c_{j'll'} + d_{jll'} d_{j'll'}}{(t - m_{\nu_j}^2)(u - m_{\nu_{j'}}^2)} m_{\nu_j} m_{\nu_{j'}} \\ &= 8G_L^4 \left[\frac{m_{\nu_1}^2 \cos^4 \eta}{(t - m_{\nu_1}^2)(u - m_{\nu_1}^2)} + \frac{m_{\nu_1} m_{\nu_2} \sin^2 \eta \cos^2 \eta}{(t - m_{\nu_1}^2)(u - m_{\nu_2}^2)} + \frac{m_{\nu_1} m_{\nu_2} \sin^2 \eta \cos^2 \eta}{(t - m_{\nu_2}^2)(u - m_{\nu_1}^2)} \right. \\ &\qquad \left. + \frac{m_{\nu_2}^2 \sin^4 \eta}{(t - m_{\nu_2}^2)(u - m_{\nu_2}^2)} \right] \begin{cases} \cos^4 \zeta, & l = l' = 1 \\ \sin^4 \zeta, & l = l' = 2 \\ \sin^2 \zeta \cos^2 \zeta, & l \neq l' \end{cases} \\ &+ 8G_R^4 \left[\sin \eta \to \cos \eta, \quad \sin \zeta \to \cos \zeta, \quad \cos \zeta \to -\sin \zeta \right]. \end{aligned} \tag{3.25}$$

Introducing the notation

$$\begin{aligned} &T_2(j, j') \\ &= m_{\nu_j} m_{\nu_{j'}} \left[\frac{F_2(t)}{(t - m_{\nu_j}^2)(t - m_{\nu_{j'}}^2)} + \frac{F_2(u)}{(u - m_{\nu_j}^2)(u - m_{\nu_{j'}}^2)} + \frac{2\, H_2(t, u)}{(t - m_{\nu_j}^2)(u - m_{\nu_{j'}}^2)} \right], \end{aligned} \tag{3.26}$$

and

$$\begin{aligned} T_{2L} &= \cos^4 \eta\, T_2(1,1) + \sin^4 \eta\, T_2(2,2) + \sin^2 \eta \cos^2 \eta\, (T_2(1,2) + T_2(2,1)), \\ T_{2R} &= \sin^4 \eta\, T_2(1,1) + \cos^4 \eta\, T_2(2,2) + \sin^2 \eta \cos^2 \eta\, (T_2(1,2) + T_2(2,1)) \end{aligned} \tag{3.27}$$

the contribution to the sum of the squared matrix element can be given by

$$\frac{1}{8} \sum |M_2^{(t)} + M_2^{(u)}|^2 = \begin{cases} G_L^4 T_{2L} \cos^4 \zeta + G_R^4 T_{2R} \sin^4 \zeta, & l = l' = 1 \\ G_L^4 T_{2L} \sin^4 \zeta + G_R^4 T_{2R} \cos^4 \zeta, & l = l' = 2 \\ (G_L^4 T_{2L} + G_R^4 T_{2R}) \sin^2 \zeta \cos^2 \zeta, & l \neq l'. \end{cases} \tag{3.28}$$

The contribution to the total cross section is again derived by the similar integration as previously using now the formulas

$$\begin{aligned} &\int_{t_{min}}^{t_{max}} \frac{F_2(t)}{(t - x)(t - y)} \\ &= \begin{cases} \Big/ \left\{ \frac{2}{b}(s - 2a)t + \frac{F_2(x)}{x - y} \ln|t - x| - \frac{F_2(y)}{x - y} \ln|t - y| \right\} & \text{if } x \neq y \\ \Big/ \left\{ \frac{2}{b}(s - 2a)t - \frac{F_2(x)}{t - x} + F_2'(y) \ln|t - x| \right\} & \text{if } x = y \end{cases} \end{aligned} \tag{3.29}$$

and

$$\begin{aligned} &\int_{t_{min}}^{t_{max}} \frac{H_2(t, u)}{(t - x)(u - y)} \\ &= \Big/ \left\{ \frac{2}{b}(s - 2a) \left[t + x \ln|t - x| + y \ln|t - y| \right] \right. \\ &\qquad \left. - \frac{H_2(x, y)}{s - a + x + y} \left[\ln|t - x| + \ln|t - y| \right] \right\}. \end{aligned} \tag{3.30}$$

	$G_{LlRl'j}/G_L G_R = 8\, G^L_{jl}\, G^R_{jl'}/g_L g_R$	
	$j=1$	$j=2$
$l=l'=1$	$-\sin\eta\cos\eta\sin\zeta\cos\zeta$	$sin\eta\cos\eta\sin\zeta\cos\zeta$
$l=l'=2$	$\sin\eta\cos\eta\sin\zeta\cos\zeta$	$-\sin\eta\cos\eta\sin\zeta\cos\zeta$
$l=1,\, l'=2$	$-\sin\eta\cos\eta\cos^2\zeta$	$\sin\eta\cos\eta\cos^2\zeta$
$l=2,\, l'=1$	$\sin\eta\cos\eta\sin^2\zeta$	$-\sin\eta\cos\eta\sin^2\zeta$

Table 3.1: The coefficients $G_{LlRl'j}$ from [7].

	$G_{LlLl'j}/G_L^2 = 8\, G^L_{jl}\, G^L_{jl'}/g_L^2$		$G_{RlRl'j}/G_R^2 = 8\, G^R_{jl}\, G^R_{jl'}/g_R^2$	
	$j=1$	$j=2$	$j=1$	$j=2$
$l=l'=1$	$\cos^2\eta\cos^2\zeta$	$\sin^2\eta\cos^2\zeta$	$\sin^2\eta\sin^2\zeta$	$\cos^2\eta\sin^2\zeta$
$l=l'=2$	$\cos^2\eta\sin^2\zeta$	$\sin^2\eta\sin^2\zeta$	$\sin^2\eta\cos^2\zeta$	$\cos^2\eta\cos^2\zeta$
$l\neq l'$	$-\cos^2\eta\sin\zeta\cos\zeta$	$-\sin^2\eta\sin\zeta\cos\zeta$	$\sin^2\eta\sin\zeta\cos\zeta$	$\cos^2\eta\sin\zeta\cos\zeta$

Table 3.2: The coefficients $G_{LlLl'j}$ and $G_{RlRl'j}$ from [7]. Here, $G_{L/R} = g_{L/R}/2\sqrt{2}$, $g_{L/R}$ is the coupling constant of the group $SU(2)_{L/R}$, ζ and η refer to the mixing angles of the W-bosons and of the neutrinos, respectively.

The equations (3.20)–(3.22) and (3.28)–(3.30) form the basis both for the numerical computations and for the analytical considerations as comes to the t- and u-channels.

Chapter 4

Contribution from s-channel and final formulas

The Yukawa coupling of a higgs triplet to the lepton doublets is given by eq. (2.1). The connection between the charged states and the cartesian states is given by

$$\begin{cases} \Delta^{--} &= \frac{1}{\sqrt{2}}(\Delta^1 - i\Delta^2) \\ \Delta^0 &= \frac{1}{\sqrt{2}}(\Delta^1 + i\Delta^2) \\ \Delta^- &= \Delta^3. \end{cases} \tag{4.1}$$

The relevant part of the lagrangian (2.1) for the annihilation vertex of the s-channel diagram is

$$-\sqrt{2}\, f_R\, e_R^T\, C\, e_R (\Delta_R^{--})^\dagger. \tag{4.2}$$

The ΔWW vertex comes from the kinetic energy (2.2) of the triplet higgs, whose relevant part is

$$-\, g_R^2 < \Delta_R^0 > g^{\mu\nu} (W_{R\mu}^-)^\dagger (W_{R\nu}^-)^\dagger \Delta_R^{--}. \tag{4.3}$$

The weak boson state W_R^- is a mixture of the mass eigenstates $W_{1,2}^-$. The expressions for these vertices have been given in Figure 2.1.

From these rules we derive the amplitude

$$T_{\mu\mu'}^R(ll') = \sqrt{2} f_R < \Delta_R^0 > g_R^2\, C \frac{i\, g_{\mu\mu'}}{s - M_\Delta^2} \begin{cases} \sin^2\zeta, & l = l' = 1 \\ \cos^2\zeta, & l = l' = 2 \\ \sin\zeta\cos\zeta, & l \neq l', \end{cases} \tag{4.4}$$

and for a left-handed triplet we would analogously have

$$T_{\mu\mu'}^L(ll') = \sqrt{2} f_L < \Delta_L^0 > g_L^2\, C \frac{i\, g_{\mu\mu'}}{s - M_\Delta^2} \begin{cases} \cos^2\zeta, & l = l' = 1 \\ \sin^2\zeta, & l = l' = 2 \\ -\sin\zeta\cos\zeta, & l \neq l'. \end{cases} \tag{4.5}$$

In general, one may have both Δ_L and Δ_R triplets the physical states being their mixings in the theory. To avoid extra parameters we choose the phenomenologically acceptable situation where we have only the right-handed triplet, that is, just the amplitude (4.4).

In this case the interaction (2.1) of the right-handed triplet is responsible to the Majorana mass terms of the neutrinos. The relevant part of (2.1) is now

$$\sqrt{2}\, f_R\, \nu_R^T\, C\, \nu_R\, \Delta_R^0, \tag{4.6}$$

which leads to the identification (see f. ex. [8])

$$m_{\nu_2} \simeq \sqrt{2}\, f_R < \Delta_R^0 > \tag{4.7}$$

and to the s-channel amplitude

$$T^{(s)}_{\mu\mu'}(ll') = -g_R^2\, m_{\nu_2}\, C(1+\gamma^5)\frac{i\, g_{\mu\mu'}}{s - M_\Delta^2} \begin{cases} \sin^2\zeta, & l = l' = 1 \\ \cos^2\zeta, & l = l' = 2 \\ \sin\zeta\cos\zeta, & l \neq l'. \end{cases} \tag{4.8}$$

From this we can derive for the factors appearing in the trace (2.8) the following expression:

$$\begin{aligned} CT^{(s)}_{\mu\mu'} &= i m_{\nu_2} g_R^2 K(\zeta)\frac{g_{\mu\mu'}}{s - M_\Delta^2}(1+\gamma^5), \\ \gamma^0 (CT^{(s)}_{\nu\nu'})^\dagger \gamma^0 &= -i m_{\nu_2} g_R^2 K(\zeta)\frac{g_{\nu\nu'}}{s - M_\Delta^2}(1-\gamma^5), \end{aligned} \tag{4.9}$$

with

$$K_{ll'}(\zeta) = \begin{cases} \sin^2\zeta, & l = l' = 1 \\ \cos^2\zeta, & l = l' = 2 \\ \sin\zeta\cos\zeta, & l \neq l'. \end{cases} \tag{4.10}$$

A straightforward trace calculation gives for the sum of the squared matrix element

$$\sum |M^{(s)}|^2 = m_{\nu_2}^2\, g_R^4\, \frac{s\,[8 + \frac{(s-a)^2}{b}]}{(s - M_\Delta^2)^2} \begin{cases} \sin^4\zeta, & l = l' = 1 \\ \cos^4\zeta, & l = l' = 2 \\ \sin^2\zeta\cos^2\zeta, & l \neq l'. \end{cases} \tag{4.11}$$

We are left with the st and su cross terms. From relations (3.7) and (4.9) we have

$$\begin{aligned} \sum M^{(s)} M^{(t+u)*} &= \frac{K(\zeta) g_R^2 m_{\nu_2}^2 (c_{2ll'} - d_{2ll'})}{s - M_\Delta^2} g^{\mu\mu'}(\mu\nu, k)(\mu'\nu', k') \\ &\quad \times \left[\frac{Tr\{\not p' \gamma^{\nu'}\gamma^\nu \not p\}}{t - m_{\nu_2}^2} + \frac{Tr\{\not p' \gamma^\nu \gamma^{\nu'} \not p\}}{u - m_{\nu_2}^2}\right], \end{aligned} \tag{4.12}$$

and the trace calculation results in

$$\begin{aligned} &\sum (M^{(s)} M^{(t+u)*} + M^{(s)*} M^{(t+u)}) = 2\, Re \sum M^{(s)*} M^{(t+u)} \\ &= \tfrac{1}{4} m_{\nu_2}^2 g_R^4 \cos^2\eta \left[\frac{s[4 - (s-a)t/b]}{(s - M_\Delta^2)(t - m_{\nu_2}^2)}\right. \\ &\quad \left. + \frac{s[4 - (s-a)u/b]}{(s - M_\Delta^2)(u - m_{\nu_2}^2)}\right] \begin{cases} \sin^4\zeta, & l = l' = 1 \\ \cos^4\zeta, & l = l' = 2 \\ \sin^2\zeta\cos^2\zeta, & l \neq l'. \end{cases} \end{aligned} \tag{4.13}$$

We note that in the s-channel amplitude we have approximated the mixing angle η of the neutrino mass eigenstates to vanish which allows us to use the mass m_{ν_2} as a parameter in our expressions. This is a good approximation as long as the Dirac neutrino mass, which equals the electron mass, is small compared with the Majorana mass given in (4.7). For the t- and u-channel results (3.20)–(3.22) and (3.28)–(3.30) we will use the same approximation. The integrated equations obtained from them and from eqs (4.11) and (4.13) for all channels are given by

$$\int_{t_{min}}^{t_{max}} \sum |M^{(s)}|^2 = g_R^4\, (m_{\nu_2}^2/b)\, K_{ll'}^2(\zeta)\, \frac{(s-a)^2 + 8b}{(s - M_\Delta{}^2)^2}\, s\, (t_{max} - t_{min}), \tag{4.14}$$

$$
\begin{aligned}
\int_{t_{min}}^{t_{max}} \sum |M^{(t)}|^2 &= \int_{t_{min}}^{t_{max}} \sum |M^{(u)}|^2 \\
&= g_R^4 \, (m_{\nu_2}^2/2b) \, K_{ll'}^2(\zeta) \, [(s-2a)(t_{max}-t_{min}) \\
&- \frac{b}{2} \, F_2(m_{\nu_2}^2) \, (\frac{1}{t_{max}-m_{\nu_2}^2} - \frac{1}{t_{mix}-m_{\nu_2}^2}) \\
&+ \frac{b}{2} \, F_2'(m_{\nu_2}^2) \, \log \left| \frac{t_{max}-m_{\nu_2}^2}{t_{min}-m_{\nu_2}^2} \right|, \qquad (4.15)
\end{aligned}
$$

$$
\begin{aligned}
2 \int_{t_{min}}^{t_{max}} Re \sum M^{(s)*} M^{(t)} &= 2 \int_{t_{min}}^{t_{max}} Re \sum M^{(s)*} M^{(t)} \\
&= -g_R^4 \, (m_{\nu_2}^2/b) \, K_{ll'}^2 \, (\zeta) \frac{(s-a)}{s-{M_\Delta}^2} \, s \, (t_{max}-t_{min}) \\
&+ (m_{\nu_2}^2 - \frac{4b}{s-a}) \, \log \left| \frac{t_{max}-m_{\nu_2}^2}{t_{min}-m_{\nu_2}^2} \right|, \qquad (4.16)
\end{aligned}
$$

$$
\begin{aligned}
2 \int_{t_{min}}^{t_{max}} Re \sum M^{(t)*} M^{(u)} &= g_R^4 \, (m_{\nu_2}^2/b) \, K_{ll'}^2(\zeta) \, [(s-2a)(t_{max}-t_{min}) \\
&- [2(s-2a)m_{\nu_2}^2 - \frac{b \, H_2(m_{\nu_2}^2, m_{\nu_2}^2)}{s-a+2m_{\nu_2}^2}] \\
&\times \log \left| \frac{t_{max}-m_{\nu_2}^2}{t_{min}-m_{\nu_2}^2} \right|. \qquad (4.17)
\end{aligned}
$$

Our computations are based on these formulas although our computer program can process the case where m_{ν_1} and η do not vanish.

Chapter 5

Results and discussion

First of all we note the cancellation of the terms which would lead to a constant or a linear or worse behaviour of the cross section as a function of the c.m. energy squared. Such terms cancel each other in the integral of $\sum |M_1^{(t)} + M_1^{(u)}|^2$. In the integral of $\sum |M_1^{(t)} + M_1^{(u)}|^2$ the highest terms are quadratic in s and the sum of the pure t-channel and the pure u-channel terms, which always are equal, is just the sum of their cross terms. Further, their total sum is just equal to the highest term arising from the s-channel. The cancellation happens due to the negative (and equal) cross terms of the s-channel with the t- and the u-channels. Thanks to this cancellation the high-energy behaviour of the total cross section is of the usual type of $\log s/s$. This is, of course, what one should expect according to the gereral theorems of the asymptotic smoothness of gauge theories.

The second remarkable point is that the expressions always have an overall factor, which is proportional to the neutrino masses. This causes the cross sections to vanish with the vanishing neutrino masses. The third noteworthy feature is the dominance of the Δ^{--} resonance in the s-channel. As there is substantial uncertainty associated with the mass of Δ^{--} we are not able to make any unique predictions of the cross sections which quite crucially depend on how close we are to the Δ^{--} pole. Fourth, we note that although we do not consider the angular dependence of the differential cross section in this work, it is obvious that the s-channel can be separated from the t- and from the u- channels in experiments because the s-channel results do not depend on the scattering angle.

We have made our calculation under the simplifying assumptions of the vanishing light neutrino mass m_{ν_1} and respectively of the vanishing neutrino mixing angle η. In this case the contributions from $\sum |M_1^{(t)} + M_1^{(u)}|^2$ vanish and the contribution of $\sum |M_2^{(t)} + M_2^{(u)}|^2$ arises merely from $T_2(2,2)$ in T_{2R} (see eq. (3.27)). This gives the overall factor

$$\frac{g_R^4 m_{\nu_2}^2}{M^2 M'^2} K_{l,l'}^2(\zeta) \tag{5.1}$$

to the cross section. From this we see the dependence of the cross section on the gauge coupling constant g_R and on the W mixing angle ζ (see eq. (4.10)). Because ζ is at most of the order of 0.001 [2] and because we deal with the case of $M_{W_2} \sim (6-10) \times M_{W_1}$ the production rate of the pair $W_1 W_2$ is only about one ten thousendth of the production rate of the heavy pair $W_2 W_2$ (outside the possible Δ^{--} resonance region). For the same reason, the production rate of the light pair $W_1 W_1$ is really negligible (about $10^{-8}\times$ the production rate of $W_2 W_2$).

The factor (5.1) does not tell us all about the behaviour of the cross section as a function of the neutrino mass m_{ν_2}, because there are complicated mass depending terms of the t- and the u-channel amplitudes.

Let us now turn to our numerical results. We have concentrated upon the planned energy region of the future colliders and in this work we use two values, 0.5 TeV and 0.8 TeV for the

heavy W-boson mass M_{W_2}. The first one is just above the present experimental minimum value [2]. We present the cross section as a function of the energy and of the neutrino mass by fixing the mass of the higgs to some illustrating values. In these pictures, the charged W-mixing angle ζ has taken to have the value 0.001, which is experimentally allowed [2]. To keep the consistency of the parameters we use the value 2.073 for the right-handed coupling constant g_R, which has been computed in our previous work [7]. For the electric coupling constant α_{em} we take the Z-boson threshold value 1/128. Actually this should be appropriately corrected for the TeV region, but it would not make notable change of our results.

Only the process $e^- e^- \rightarrow W_2^- W_2^-$ has surely the large enough cross section for experiments provided that the particles had their parameters in the region assumed by us. Let us now consider our results for this process. The figures of this chapter can be found in the end of this paper.

In Figures 1-2, the total cross section is presented as a function of the center of mass energy for $M_{W_2} = 0.5$ TeV and $m_{\nu_2} = 1.0$ TeV. In Fig 1 $M_\Delta = 0.5$ TeV and in Fig 2 0.8 TeV. The form of the curves is typical for the cases when the Δ^{--} resonance lies below the threshold of the process. We note that the maximum value of the cross section has been removed from 0.55 pbarn in the first picture to 2.35 pbarn in the second picture. This is explained by the enhancement in the s-channel when the Δ^{--} resonance is approached.

In Figures 3–4, we see the resonance region when $m_{\nu_2} = 1.0$ TeV. In Fig 3 $M_\Delta = 1.5$ TeV and the rise at the threshold is really steep. In Fig 4 $M_\Delta = 5.0$ TeV and the rise is now slower because the s-channel contribution has dropped slightly at the threshold region.

In Figures 5-6 the cross section is presented as a function of the energy for various values of M_Δ when $m_{\nu_2} = 1.0$ TeV. In Fig 5 $M_{W_2} = 0.5$ TeV and in Fig 6 $M_{W_2} = 0.8$ TeV. The order of the magnitude of the cross sections has decreased approximately according to the front factor (5.1). Once again the strong enhancement of the cross section in the cases where M_Δ approaches to the center-of-mass energy is a striking feature of the curves. The s-channel begins to dominate already quite far from the resonance point as can be seen in the curve where $M_\Delta = 4$ TeV. In Fig 5, it can also be seen that a cross section of about 0.3 pbarn is always reachable.

In Fig 7, the cross section has further been shown as a function of the energy for $M_\Delta = 0.5$ TeV but now m_{ν_2} varies: $m_{\nu_2} = 0.5$ TeV, 1.0 TeV and 1.5 TeV. This picture reveals that the cross section increases when m_{ν_2} increases but the increase is not proportional to the square of m_{ν_2}. This gives ground to the following pictures.

In Figures 8–9 we see the cross section as a function of m_{ν_2} for various values of M_Δ when the energy is 1.5 TeV and 1.2 TeV. These again are typical pictures where the steepest curves are obtained when the energy is near to the Δ^{--} resonance. In these Figures we also see the complicated behaviour of the cross section as a function of m_{ν_2} if we are far from the resonance region. This structure has further been studied by dividing the cross section by $m_{\nu_2}^2$. The s-channel and its cross terms with the t- and the u-channels contribute a constant which varies from one curve to another. The result is presented in Figures 10–11. We see that the expected behaviour of the cross section as a function of the neutrino mass is achieved when the mass exceeds some limit which is around 5 TeV in Fig 11.

Finally we show an example where the light heavy pair $W_1 W_2$ has been produced in electron-electron collisions. The light W mass M_{W_1} has the value 80.14 GeV [1] and the other parameters have the same values as in Fig 1. The maximum value of the cross section is about 0.095 fbarn as it for the heavy pair is about 0.55 pbarn. The order of the cross section of light heavy pair is what can be expected from the factor (5.1).

Chapter 6

Conclusions

The computated cross sections for the heavy W pair production $e^- e^- \mapsto W_2^- W_2^-$ are of the order of 0.1 - 1 pbarn at the energy range of the next generation colliders, provided that the particles have the parameters in the region assumed and that the Δ^{--} resonance does not ly in the vicinity. The production rates for the other same-signed W pair channels are negligible. The cross section vanishes in the limit of the vanishing neutrino masses but its behaviour as a function of the 'heavy' neutrino mass is complicated until the mass increases to a several TeV scale, after which the cross section is proportional to the square of the mass. The Δ^{--} exchange channel should be possible to separate from the neutrino exchange channels by using the scattering angle distributions but without a more detailed study we can not say if the order of the heavy neutrino mass could be determined from these scattering experiments under some conditions.

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Appendix A

Programs

A.1 Program for Computation of Cross Section

```
    {PROGRAM e_e_W_W_.pas}
{-----------------------------}
    {CROSS SECTION FOR W PAIR PRODUCTION IN ELECTRON-ELECTRON COLLISION}

PROGRAM eeWW(input,output,kirjdata);

uses Crt;
 A:  VARIABLES AND SUBROUTINES}
{-----------------------------}

LABEL ensidata,toinendata,lopputila,energiavali,evaihto,loppu;


TYPE VEKTORI = ARRAY[1..2] OF REAL;
     KERTOIMET = ARRAY[0..4] OF REAL;
     INDEKSIT=1..2;
     MATRIISI=ARRAY[INDEKSIT,INDEKSIT] OF REAL;


VAR
     Alkutila,STKoodi,LKoodi:integer;
     KoordY: byte;
     val,kirjval,pit,val1 : char;
     kirj : text;
     kirjdata : string[12]; teksti:string;
     tyhja10 : string[10]; tyhja12 : string[12]; yhta54 : string[54];
     logoteksti : array[1..10] of string[34];
     II : integer;
     M,N:VEKTORI;            {M mass of W, N mass of neutrino}
     KERROIN:REAL;           {factor of cross section}
     MH:REAL;                {mass of higgs}
     SETA,SETA2,SETA4,CETA2,CETA4,SKSI,SKSI2,SKSI4,CKSI,CKSI2,CKSI4:REAL;
     G,SUHDE:real;       {left coupl constant and ratio of R and L couplings}
     L1,L2:INDEKSIT;                          {Indices of final state}
     SMM,TMM:REAL;                     {sum and product of squares of W masses}
```

```
     ENERGIA,EMIN,EMAX,EVALI,S:REAL; {CM-ENERGY,limits,step,square of energy}
     Z:VEKTORI;              {Limits ofintegration}
     TX_1,LNTX,NN:VEKTORI;
     C:KERTOIMET;            {Coefficients of trace polynomial}
     X,Y,TERO,APU,APU1S,APU2S,SUHDE4,KULMASTU:REAL;

     TT1OSA,TU1OSA,T1OSA:REAL;
     TT2OSA,TU2OSA,T2OSA:REAL;
     TT2L,TT2R,TU2L,TU2R,TT_OSA,TU_OSA,UU_OSA:REAL;
     ST_OSA,SU_OSA,S_OSA,T_OSA,SIGMA:REAL;

     ITT2,ITU2,IT2:MATRIISI;

     I,J1,J2:INDEKSIT;

FUNCTION POT(X:REAL;N:INTEGER):REAL;
VAR I:INTEGER; APU:REAL;
BEGIN
APU:=1;
     FOR I:=1 TO N DO
     BEGIN
  APU:=APU*X;
     END;
POT:=APU;
END;


FUNCTION ITT1(X,Y,S,SMM,TMM,TERO:REAL;TX_1,LNTX:VEKTORI):REAL;
VAR FOX,FOY,F1X,F1Y,APUX,APUY:REAL;
    N:INTEGER;
    C:KERTOIMET;

BEGIN

{The reduced coefficients of the power series ( in Mandelstams t) of trace}
C[4]:=1; C[3]:=S-SMM; C[2]:=-2*S*SMM+5*TMM; C[1]:=4*(S-SMM)*TMM;
C[0]:=4*SQR(TMM);

FOX:=C[0]; FOY:=C[0]; F1X:=0; F1Y:=0; APUX:=1; APUY:=1;

  FOR N:=1 TO 4 DO
  BEGIN
       F1X:=F1X+N*C[N]*APUX;
       F1Y:=F1Y+N*C[N]*APUY;
       APUX:=APUX*X;
       APUY:=APUY*Y;
       FOX:=FOX+C[N]*APUX;
       FOY:=FOY+C[N]*APUY;
  END;

APUX:=C[4]*(X-Y)*(X-Y)*TERO-FOX*TX_1[1]-FOY*TX_1[2];
APUY:= (F1X - 2*FOX/(X-Y))*LNTX[1] + (F1Y + 2*FOY/(X-Y))*LNTX[2];
```

```
ITT1:=-2*(APUX + APUY)/TMM;

END;

FUNCTION ITU1(X,Y,S,SMM,TMM,TERO:REAL;LNTX:VEKTORI):REAL;
VAR FXX,FYY,FXY,XX,YY,XY,XYERO,NIM,HAKAX,HAKAY,CXY,CO:REAL;
BEGIN
XX:=X*X; XY:=X*Y; YY:=Y*Y; XYERO:=X-Y;NIM:=S-SMM+X+Y;
CXY:=2*SMM*S-TMM; CO:=4*SMM*TMM*S;
FXX:=XX*(XX+CXY)-CO;
FYY:=YY*(YY+CXY)-CO;
FXY:=XY*(XY+CXY)-CO;
HAKAX:= XX*XYERO - FXX/(NIM+XYERO) + FXY/NIM;
HAKAY:=-YY*XYERO - FYY/(NIM-XYERO) + FXY/NIM;

ITU1:=-2*(XYERO*XYERO*TERO + 2*HAKAX*LNTX[1] + 2*HAKAY*LNTX[2])/TMM;
END;

FUNCTION TU2(X,Y,S,SMM,TMM,TERO,LNTXERO,LNTYERO:REAL):REAL;
VAR FXY,NIM,CXY:REAL;

BEGIN
CXY:=S-2*SMM; FXY:=CXY*X*Y + 2*SMM*TMM; NIM:=S-SMM+X+Y;
TU2:= -2*(CXY*TERO+(CXY*X-FXY/NIM)*LNTXERO+(CXY*Y-FXY/NIM)*LNTYERO)/TMM;
END;

PROCEDURE LaskeTT2(VAR JTT2:MATRIISI;S,SMM,TMM,TERO:REAL;XX,TJ_1,LNTJ:VEKTORI);
VAR F1X,CXY:REAL;
    FX:VEKTORI;
    J1:INDEKSIT;
    BEGIN
CXY:=S-2*SMM;

FOR J1:=1 TO 2 DO
    BEGIN
    FX[J1]:=XX[J1]*(CXY*XX[J1] -2*SMM*(S-SMM))+2*TMM*(2*S-SMM);
    F1X:=2*(CXY*XX[J1]-SMM*(S-SMM));
    JTT2[J1,J1]:=2*(CXY*TERO-FX[J1]*TJ_1[J1]+F1X*LNTJ[J1])/TMM;
    END;
JTT2[1,2]:=2*(CXY*TERO+(FX[1]*LNTJ[1] - FX[2]*LNTJ[2])/(X-Y))/TMM;
JTT2[2,1]:=JTT2[1,2];

END;

PROCEDURE LueRLuku(VAR A:real;teksti:string);
VAR STKoodi:integer;
BEGIN
repeat
WRITE(teksti);
{$I-}
READLN(A);
  {$I+}
```

```
  STKoodi := IOResult;
  if STKoodi <> 0 then Writeln('INPUT ERROR: Give again')
until STKoodi = 0;

END;

PROCEDURE Lue2RLukua(VAR A,B:real;teksti:string);
VAR STKoodi:integer;
BEGIN
repeat
WRITE(teksti);
{$I-}
READLN(A,B);
  {$I+}
  STKoodi := IOResult;
  if STKoodi <> 0 then Writeln('INPUT ERROR: Give again')
until STKoodi = 0;

END;

PROCEDURE Lue3RLukua(VAR A,B,C:real;teksti:string);
VAR STKoodi:integer;
BEGIN
repeat
WRITE(teksti);
{$I-}
READLN(A,B,C);
  {$I+}
  STKoodi := IOResult;
  if STKoodi <> 0 then Writeln('INPUT ERROR: Give again')
until STKoodi = 0;

END;

{MAIN PROGRAM BEGINS}

BEGIN
Alkutila := LastMode;
TextAttr := LightGray;
ClrScr;

{logo-part}
begin

  tyhja10 := '          ';
  tyhja12 := '            ';
  FillChar(yhta54, SizeOf(yhta54),Chr(205));
  yhta54[0] := #54;

  writeln(tyhja10,Chr(213),yhta54,Chr(184));
  logoteksti[1] := '=======    e_e_ -  W_W_   ======= ';
  logoteksti[2] := '                                    ';
```

```
  logoteksti[3] := 'Program  for  production of vector';
  logoteksti[4] := '   bosons in electron-electron    ';
  logoteksti[5] := '  collisions in left-right model  ';
  logoteksti[6] := logoteksti[2];
  logoteksti[7] := '    Programmed by A. Pietilä      ';
  logoteksti[8] := 'Depart. of Physics, Univ. of Turku';
  logoteksti[9] := 'Vesilinnantie 5,   SF-20500 Turku ';
  logoteksti[10] := 'Date of release:     23. 4. 1992  ';

  writeln(tyhja10, Chr(179), tyhja10, logoteksti[2], tyhja10, Chr(179));
  for II := 1 to 10 do
  writeln(tyhja10, Chr(179), tyhja10, logoteksti[II], tyhja10, Chr(179));
  writeln(tyhja10,Chr(212),yhta54,Chr(189));
  writeln;
end; {logo-osalle}

{writing on the screen or by printer or into file}
HighVideo;
GotoXY(1,25);
write('output on screen n or by printer p or into file t. Select: ');
readln(kirjval);
case kirjval of
  'p','P': begin; kirjval:='p'; Assign(kirj,'lpt1');
   writeln('output by printer');rewrite(kirj); end;
  't','T': begin; kirjval:='t';
   writeln('output into file, name later');end;
      else begin; kirjval := 'n'; AssignCrt(kirj);
   writeln('output on screen');rewrite(kirj);end;
   {use slow output Assign(kirj,'') if Crt does not work}
      end;

{selection of legnth of output}
{short output especially for into file output}

writeln('Normal output contains data and main results');
writeln('Short output contains energy and cross section in exponent form');
writeln('Long output also contains intermediate results');
write('Select normal n or short l or long p: ');
readln(pit);
case pit of
    'n','N': pit:='n';
    'p','P': pit:='p';
else pit:='l'; end;

{B: Input and other preparation}
{-----------------------------}

{Initial values for basic data}

M[1]:=0.08014; M[2]:=M[1]; N[1]:=0; G := 2.072; SUHDE:= 1;
SETA := 0.000; SKSI := 0.001;
```

```
ensidata:
writeln('NOTE: Separate the figures by space if you give more than one');
writeln('Basic data is below:');
writeln('M(W1)     M(n1)     g(L)     g(R)/g(L)  sin(eta)   sin(ksi)  ovat:');
writeln(M[1]:7:5,N[1]:10:5,G:10:5,SUHDE:10:5,SETA:10:5,SKSI:10:5);
write('Accept by letter k, change by e: ');readln(val);

if (val = 'e') or (val = 'E') then
   begin;
  teksti:='Give light boson mass and light neutrino mass:';
  Lue2RLukua(M[1],N[1],teksti);
  teksti:='Give coupl cons g(L) and ratio g(R)/g(L): ';
  Lue2RLukua(G,SUHDE,teksti);
  teksti:='Give sin(eta) ja sin(ksi): ';
  Lue2RLukua(SETA,SKSI,teksti);
    end; {for if}

{Factors and angle parameters}

KERROIN:=SQR(137/128)*0.130282*POT(G,4)/8;
{This includes 1/4 from electron spin average}
SUHDE4 := POT(SUHDE,4);

SETA2:=SQR(SETA);CETA2:=1-SETA2;SETA4:=SQR(SETA2);CETA4:=SQR(CETA2);
SKSI2:=SQR(SKSI);SKSI4:=SQR(SKSI2);CKSI2:=1-SKSI2;CKSI4:=SQR(CKSI2);
CKSI:=SQRT(CKSI2);

lopputila:
repeat
WRITE('Give indices of final state ( 1 1 or 1 2 or 2 2) : ');
{$I-}
READLN(L1,L2);
  {$I+}
  STKoodi := IOResult;
  LKoodi := ABS((L1-1)*(L1-2)) + ABS((L2-1)*(L2-2)) + ABS(STKoodi);
  if LKoodi <> 0 then Writeln('Input error: Give again')
until LKoodi = 0;

IF (L1=2) OR (L2=2) THEN
   BEGIN
   teksti:='Give mass of W2: ';
   LueRLuku(M[2],teksti);
   END;

SMM:=SQR(M[L1])+SQR(M[L2]);TMM:=SQR(M[L1]*M[L2]);

toinendata:
{Masses of higgs and heavy neutrino}


teksti:='Give Higgs mass and heavy neutrino mass: ';
Lue2RLukua(MH,N[2],teksti);
```

```
X:=SQR(N[1]); NN[1]:=X;
Y:=SQR(N[2]); NN[2]:=Y;

{Input of C.M. energy and parameters depending on it, REPEAT-loop}

energiavali:

{Chose of file and opening}
if kirjval = 't' then
  begin
  {$I-}
  repeat
  write('Give output file name: ');
  readln(kirjdata);
  Assign(kirj,kirjdata);
  write('do I open new file or add to old one  new=u old=v : ');
  readln(val);
   if (val='v') or (val='V') then

   begin val := 'v';Append(kirj);STKoodi := IOResult;end
   else begin Rewrite(kirj);STKoodi := IOResult;end;

   if STKoodi <> 0 then Writeln('Error in opening: Give again')
   until STKoodi = 0;
   {$I+}              {Check of error}
   if val = 'v' then writeln('adding to file ',kirjdata)
   else writeln('opening new file ',kirjdata);

   end;


teksti:='Give lower and upper limits and step of energy: ';
Lue3RLukua(EMIN,EMAX,EVALI,teksti);

ENERGIA := EMIN;

      {OUTPUT}
if kirjval = 'n' then begin NormVideo;ClrScr;end;
if (pit='n') or (pit='p') then
   begin
   writeln(kirj);
   writeln(kirj,'                          USED PARAMETERS AND RESULTS');
   writeln(kirj,'     final state indices and W masses ',
      L1:2,L2:2,M[L1]:10:5,M[L2]:8:5);
   writeln(kirj,'     g g(R)/g(L) sin(eta) sin(ksi):  ',
      G:8:5,SUHDE:8:5,SETA:8:5,SKSI:8:5);
   writeln(kirj,'     higgs and neutriino masses    : ',
      MH:8:5,N[1]:8:5,N[2]:8:5);

Writeln(kirj,'-----------------------------------------------------
      ----------------------------');
```

```
end;
if pit='n' then begin
WRITELN(kirj,'  E        ','SIGMA       ','S-CHANNEL ',
       'ST-CROSS  ','SU-CROSS  ','T-CHANNEL ','U-CHANNEL ','TU-CROSS');
Writeln(kirj,'-----------------------------------------------------
      ----------------------------');
end;

REPEAT   {Energy loop begins}

{Treatment of error in energy value}
IF (ENERGIA < M[L1]+M[L2]) OR (ABS(ENERGIA-MH)<0.01) THEN
    begin
    KoordY := WhereY - 1;
    GotoXY(1,25); HighVideo;
    Write('Energy',ENERGIA:5:2,' ERRONEOUS, do I jump over yes=k no=e: ');
    readln(val);
    if (val='k') or (val='K') then
    begin GoToXY(1,24);DelLine;GotoXY(1,KoordY);NormVideo;GOTO evaihto;end
    else begin close(kirj);GOTO toinendata;end;
    end;

S:=SQR(ENERGIA);

APU1S:=(SMM-S)/2; APU2S:=SQRT(SQR(APU1S)-TMM);

Z[1]:=APU1S - APU2S;   {Low limit of integration}
Z[2]:=APU1S + APU2S;   {Upper limit of integration}
TERO:=2*APU2S;
TX_1[1]:= -TERO/(TMM+(S-SMM)*X+SQR(X));
TX_1[2]:= -TERO/(TMM+(S-SMM)*Y+SQR(Y));
LNTX[1]:=LN(ABS((APU1S-X+APU2S)/(APU1S-X-APU2S)));
LNTX[2]:=LN(ABS((APU1S-Y+APU2S)/(APU1S-Y-APU2S)));

{Real calculation begins}

 {C:  CHANNELS T AND U}
    {-----------------------------}

{All traces calculated by REDUCE which give them only one quarter}
{which together with the common factor in t powers has been taken into acc.}
{sign/ 2 TMM has been taken into account in TT1OSA-UU2OSA }

    {OSA I or CONTRIBUTIONS FROM QQ TERMS OF TRACES}

{TT1-OSA = UU1-OSA = pure T- and U- channel contributions}

TT1OSA := ITT1(X,Y,S,SMM,TMM,TERO,TX_1,LNTX)/2;


{TU1-OSA =  cross term of T and U channels}
```

```
TU1OSA:= ITU1(X,Y,S,SMM,TMM,TERO,LNTX)/2;


{OSA2 or CONTRIBUTIONS FROM NON-QQ TERMS OF TRACES}

{TT2- = UU2-OSA = pure T- and U- channel contributions}

LaskeTT2(ITT2,S,SMM,TMM,TERO,NN,TX_1,LNTX);

APU := 2*N[1]*N[2]*SETA2*CETA2*ITT2[1,2];
TT2L := X*CETA4*ITT2[1,1] + Y*SETA4*ITT2[2,2] + APU;
TT2R := X*SETA4*ITT2[1,1] + Y*CETA4*ITT2[2,2] + APU;


{TU2-OSA = cross term of T and U channels}

FOR J1:=1 TO 2 DO
    FOR J2:=1 TO 2 DO
    ITU2[J1,J2]:=TU2(NN[J1],NN[J2],S,SMM,TMM,TERO,LNTX[J1],LNTX[J2]);

APU := N[1]*N[2]*SETA2*CETA2*(ITU2[1,2] + ITU2[2,1]);
TU2L := - 2 * (X*CETA4*ITU2[1,1] + Y*SETA4*ITU2[2,2] + APU);
TU2R := - 2 * (X*SETA4*ITU2[1,1] + Y*CETA4*ITU2[2,2] + APU);


{LONG OUTPUT}

If pit = 'p' then begin
{for alternative caculation}
FOR J1:=1 TO 2 DO
    FOR J2:= 1 TO 2 DO
IT2[J1,J2]:=2*ITT2[J1,J2]-2*ITU2[J1,J2];

WRITELN(kirj,'  ENERGIA',ENERGIA:7:2);
WRITELN(kirj,'     TT1OSA  ','     TU1OSA  ','   IT2[1,1]  ',
     '   IT2[2,2]  ','   IT2[1,2]');
WRITELN(kirj,TT1OSA:12,' ',TU1OSA:12,' ',IT2[1,1]:12,' ',
     IT2[2,2]:12,' ',IT2[1,2]:12);
WRITELN(kirj,'  ITT2[1,1]','    ITT2[1,2]','    ITT2[2,2]',
     '       TT2L','         TT2R');
WRITELN(kirj,ITT2[1,1]:12,' ',ITT2[1,2]:12,' ',ITT2[2,2]:12,' ',
     TT2L:12,' ',TT2R:12);
WRITELN(kirj,'  ITU2[1,1]','    ITU2[1,2]','    ITU2[2,2]',
     '       TU2L','         TU2R');
WRITELN(kirj,ITU2[1,1]:12,' ',ITU2[1,2]:12,' ',ITU2[2,2]:12,' ',
     TU2L:12,' ',TU2R:12);
end;
   {TOTAL CONTRIBUTION FROM T AND U CHANNELS}

{For non-identical particles in final state}
IF L1 <> L2
   THEN
```

```
      BEGIN   {begin-command 1}

      KULMASTU := SKSI2*CKSI2; {angular factor for ss-, st- ja su-terms}

      APU := SETA2*CETA2*(SKSI4+CKSI4)*SQR(SUHDE);
      TT1OSA := APU * TT1OSA;
      APU := SETA2*CETA2*SKSI2*CKSI2;
      TU1OSA := - 4*APU*TU1OSA; {TU1OSA has own angular dependence}
      APU := SKSI2*CKSI2;
      TT2OSA := APU * (TT2L+SUHDE4*TT2R)/2;
      TU2OSA := APU * (TU2L+SUHDE4*TU2R)/2;
      END  {begin-command 1}
   ELSE
      {For identical particles in final state}
      BEGIN     {begin-command 2}
      APU := SETA2*CETA2*2*SKSI2*CKSI2*SQR(SUHDE);
      TT1OSA := APU * TT1OSA;
      TU1OSA := APU *2*TU1OSA;

      IF L1=1
  THEN
    BEGIN    {begin-command 3}
    KULMASTU := SKSI4;    {Angular factor for ss-, st- ja su-terms}
    TT2OSA :=  (TT2L*CKSI4 + SUHDE4*TT2R*SKSI4)/2;
    TU2OSA :=  (TU2L*CKSI4 + SUHDE4*TU2R*SKSI4)/2;
    END      {begin-command 3}
  ELSE
    BEGIN    {begin-command 4}
    KULMASTU := CKSI4;     {Angular factor for ss-, st- ja su-terms}
    TT2OSA := (TT2L*SKSI4 + SUHDE4*TT2R*CKSI4)/2;
    TU2OSA := (TU2L*SKSI4 + SUHDE4*TU2R*CKSI4)/2;
  END;      {begin-command 4}
      END;          {begin-command 2}
    {if-command}


{Total QQ contribution from T and U channels or contribution 1}
T1OSA := 2*TT1OSA + TU1OSA;

{Total non-QQ cintribution from T and U channels}
T2OSA := 2*TT2OSA + TU2OSA;



   {D:  CONTRIBUTIONS FROM S AND FROM CROSS TERMS OF S  WITH T AND U }
   {-----------------------------------------------------------------}

{CHANNEL S}
S_OSA:=SUHDE4*Y*S*(SQR(S-SMM)/TMM+8)*(Z[2]-Z[1])/SQR(S-SQR(MH))/4;


{CROSS TERMS}
```

```
ST_OSA := - S*(Y*(S-SMM)/TMM - 4)*LN(ABS((Z[2]-Y)/(Z[1]-Y)));
ST_OSA := ST_OSA - S*(S-SMM)*(Z[2]-Z[1])/TMM;

SU_OSA := S*(Y*(S-SMM)/TMM - 4)*LN(ABS((Z[2]+S-SMM+Y)/(Z[1]+S-SMM+Y)));
SU_OSA := SU_OSA - S*(S-SMM)*(Z[2]-Z[1])/TMM;

ST_OSA:=Y*ST_OSA*SUHDE4/(S-SQR(MH));
SU_OSA:=Y*SU_OSA*SUHDE4/(S-SQR(MH));

       {E:  CROSS SECTION AND OUTPUT}
      {-----------------------------}

{ADDING OF FACTORS}

APU := KERROIN / SQR(S);
IF L1=L2 THEN APU := APU/2;

S_OSA  := APU*KULMASTU*S_OSA * 4;
ST_OSA := APU*KULMASTU*ST_OSA;     {Here factor 2 from S-graph added}
SU_OSA := APU*KULMASTU*SU_OSA;     {and factor from G g ratio}
T_OSA  := APU*(T1OSA+T2OSA)/4;
TT_OSA := APU*(TT1OSA+TT2OSA)/4;
TU_OSA := APU*(TU1OSA+TU2OSA)/4;
UU_OSA := TT_OSA;
SIGMA  := S_OSA+ST_OSA+SU_OSA+T_OSA;

{OUTPUT}
case pit of
 'n':
WRITELN(kirj,ENERGIA:6:2,' ',SIGMA:12,' ',S_OSA:9,' ',ST_OSA:9,' ',
     SU_OSA:9,' ',TT_OSA:9,' ',UU_OSA:9,' ',TU_OSA:9);
 'p': begin
      WRITELN(kirj,'    SIGMA      ','S-CHANNEL ',' ST-CROSS  ',
      ' SU-CROSS  ',' T-CHANNEL ',' U-CHANNEL ',' TU-CROSS');
      WRITELN(kirj,SIGMA:12,' ',S_OSA:10,' ',ST_OSA:10,
      ' ',SU_OSA:10,' ',TT_OSA:10,' ',UU_OSA:10,' ',TU_OSA:10);
      Writeln(kirj,'-----------------------------------------
    -----------------------------------');
      end
  else begin
       writeln(kirj,ENERGY,' ',SIGMA);
       If kirjval = 't' then writeln(ENERGIA,' ',SIGMA);end
  end;    {end of case-command}

evaihto:
ENERGIA := ENERGIA + EVALI;
UNTIL ENERGIA > EMAX;                  {END of REPEAT LOOP}


{REINPUT OF DATA}
if kirjval = 't' then close(kirj);
```

```
GotoXY(1,25);
if kirjval='n' then HighVideo;
write('do I STOP: yes=k, no=e: ');
readln(val);
if (val='k') or (val='K') then goto loppu;
writeln('Do you change energy (e), higgs or neutrino mass (h) or final');
writeln('state (l) or basic data (p) M(W1), M(n1), g, g(R)/g(L), eta, ksi ');
write('Select  e or h or l or p: ');
readln(val);
case val of
 'e', 'e': goto energiavali;
 'h', 'H': goto toinendata;
 'l', 'L': goto lopputila;
 'p', 'P': goto ensidata;
     else  goto loppu;
end;

loppu:
      if (kirjval = 'p') or (kirjval = 'n') then close(kirj);
      TextMode(AlkuTila);
END.
```

A.2 Program for Calculation of Traces

```
% PROGRAM FOR CALCULATION OF TRACES by REDUCE
% for same-signed vector boson production
% in electron-electron collisions

OFF ECHO; OFF NAT; OUT TTKANAVA;

FACTOR Z,S; % Z and S = Mandelstam's t and s

INDEX U1,U2,V1,V2,MU1,MU2,NU1,NU2;
MASS P1=0, K1=M1,K2=M2,P2=0;
MSHELL P1,P2,K1,K2;
VECTOR  Q1, Q2;

LET (P1.K1)=(M1**2-Z)/2,  (P2.K1)=(M1**2-U)/2, (P1.P2)=S/2, K2=P1+P2-K1;
LET U = SMM - S - Z, M1**2 * M2**2 = TMM;


PROCEDURE JALKI1(U1,U2,V1,V2,Q1,Q2);

((MU1.NU1)-(K1.MU1)*(K1.NU1)/M1**2)*((MU2.NU2)-(K2.MU2)*(K2.NU2)/M2**2)*
G(L,U1,Q1,U2,P2,V1,Q2,V2,P1);


PROCEDURE JALKI2(U1,U2,V1,V2);

((MU1.NU1)-(K1.MU1)*(K1.NU1)/M1**2)*((MU2.NU2)-(K2.MU2)*(K2.NU2)/M2**2)*
G(L,U1,U2,P2,V1,V2,P1);
```

```
PROCEDURE JALKI3(U1,U2,V1,V2,Q1,Q2);

((MU1.NU1)-(K1.MU1)*(K1.NU1)/M1**2)*((MU2.NU2)-(K2.MU2)*(K2.NU2)/M2**2)*
(V1.V2)*G(L,U1,U2,Q1,Q2);


PROCEDURE JALKI4(U1,U2,V1,V2,Q1,Q2);

((MU1.NU1)-(K1.MU1)*(K1.NU1)/M1**2)*((MU2.NU2)-(K2.MU2)*(K2.NU2)/M2**2)*
(V1.V2)*G(L,U1,U2,A,Q1,Q2);


LET
TK1TK11 = JALKI1(MU1,MU2,NU2,NU1,P1-K1,P1-K1),
TK1TK12 = JALKI2(MU1,MU2,NU2,NU1),

TK1TK21 = JALKI1(MU1,MU2,NU1,NU2,P1-K1,P1-K2),
TK1TK22 = JALKI2(MU1,MU2,NU1,NU2),

TK2TK11 = JALKI1(MU2,MU1,NU2,NU1,P1-K2,P1-K1),
TK2TK12 = JALKI2(MU2,MU1,NU2,NU1),

TK2TK21 = JALKI1(MU2,MU1,NU1,NU2,P1-K2,P1-K2),
TK2TK22 = JALKI2(MU2,MU1,NU1,NU2),

TS1 = JALKI3(MU1,MU2,NU1,NU2,P2,P1),
TS2 = JALKI4(MU1,MU2,NU1,NU2,P2,P1);

WRITE "TK1TK11 =",TK1TK11; WRITE "TK1TK12 =",TK1TK12;
WRITE "TK1TK21 =",TK1TK21; WRITE "TK1TK22 =",TK1TK22;
WRITE "TK2TK11 =",TK2TK11; WRITE "TK2TK12 =",TK2TK12;
WRITE "TK2TK21 =",TK2TK21; WRITE "TK2TK22 =",TK2TK22;
WRITE "TS1 = ",TS1; WRITE "TS2 = ",TS2;

SHUT TTKANAVA; ON NAT; ON ECHO;

END;
```

Figures

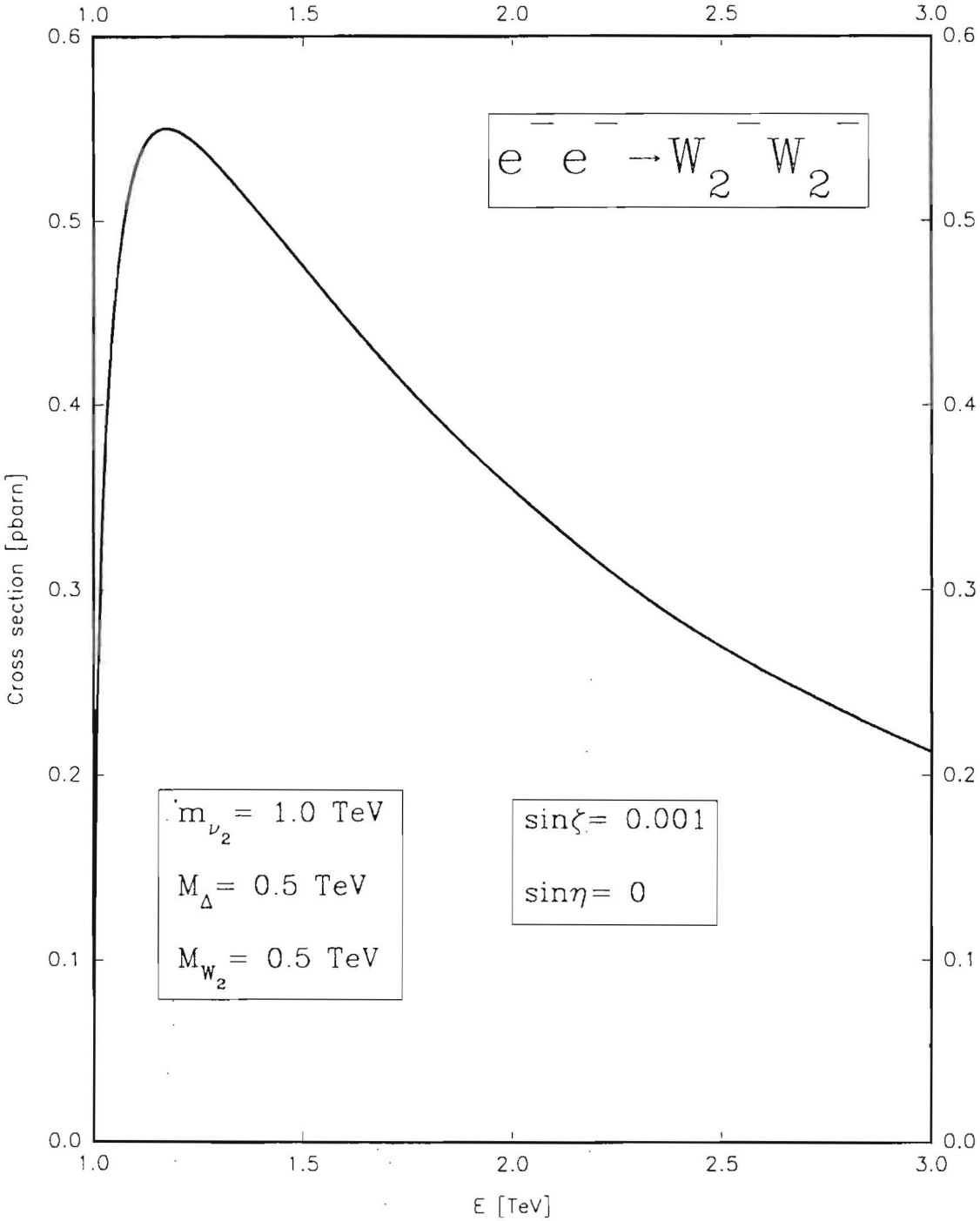

Figure 1

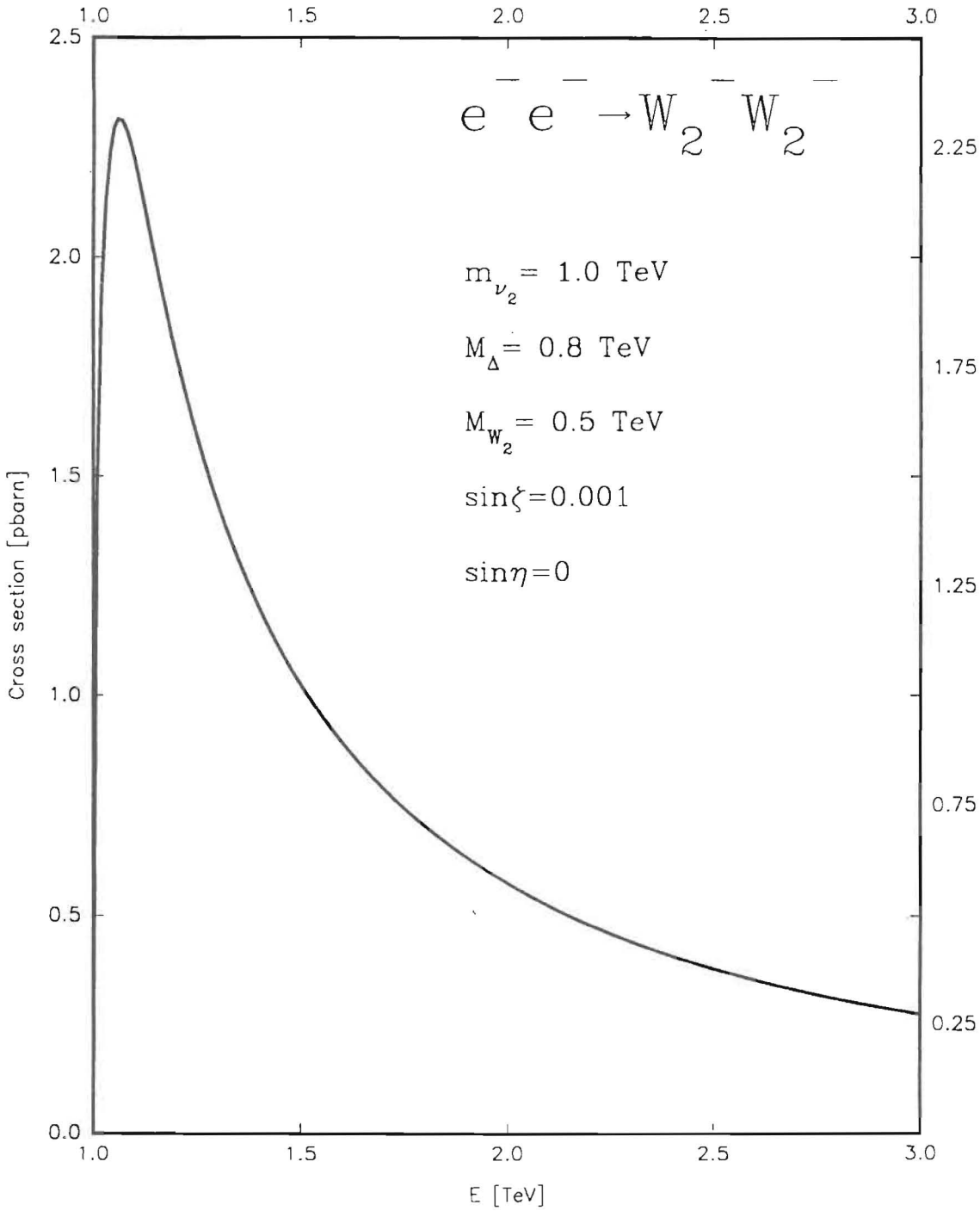
$e^- e^- \to W_2^- W_2^-$
m_{ν_2} = 1.0 TeV
M_Δ = 0.8 TeV
M_{W_2} = 0.5 TeV
$\sin\zeta$=0.001
$\sin\eta$=0
Cross section [pbarn]
E [TeV]
1.0
1.5
2.0
2.5
3.0
0.0
0.5
0.25
0.75
1.25
1.75
2.25

Figure 2

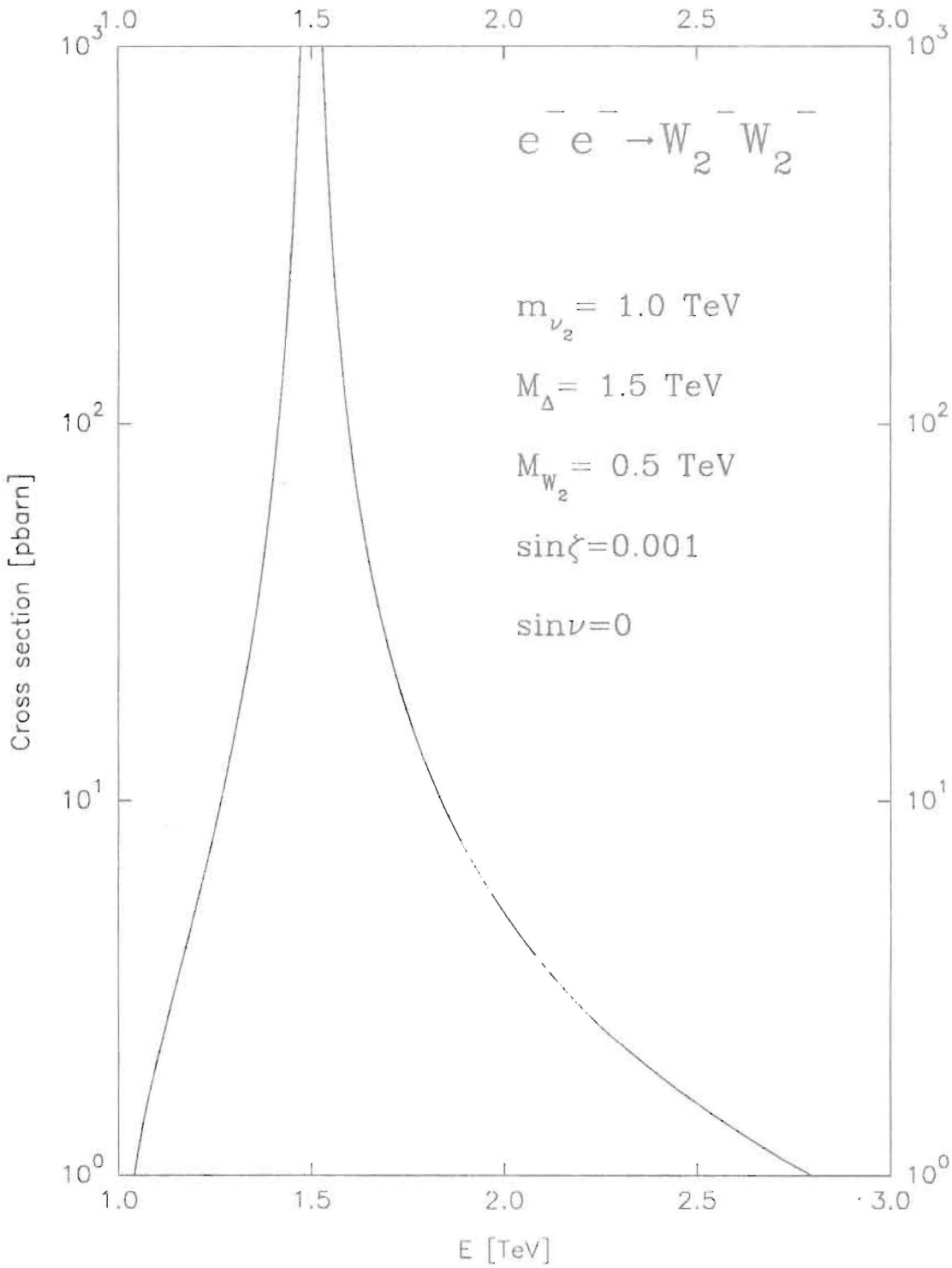
$e^- e^- \to W_2^- W_2^-$
m_{ν_2} = 1.0 TeV
M_Δ = 1.5 TeV
M_{W_2} = 0.5 TeV
$\sin\zeta$=0.001
$\sin\nu$=0
Cross section [pbarn]
E [TeV]
1.0
1.5
2.0
2.5
3.0
10^0
10^1
10^2
10^3

Figure 3

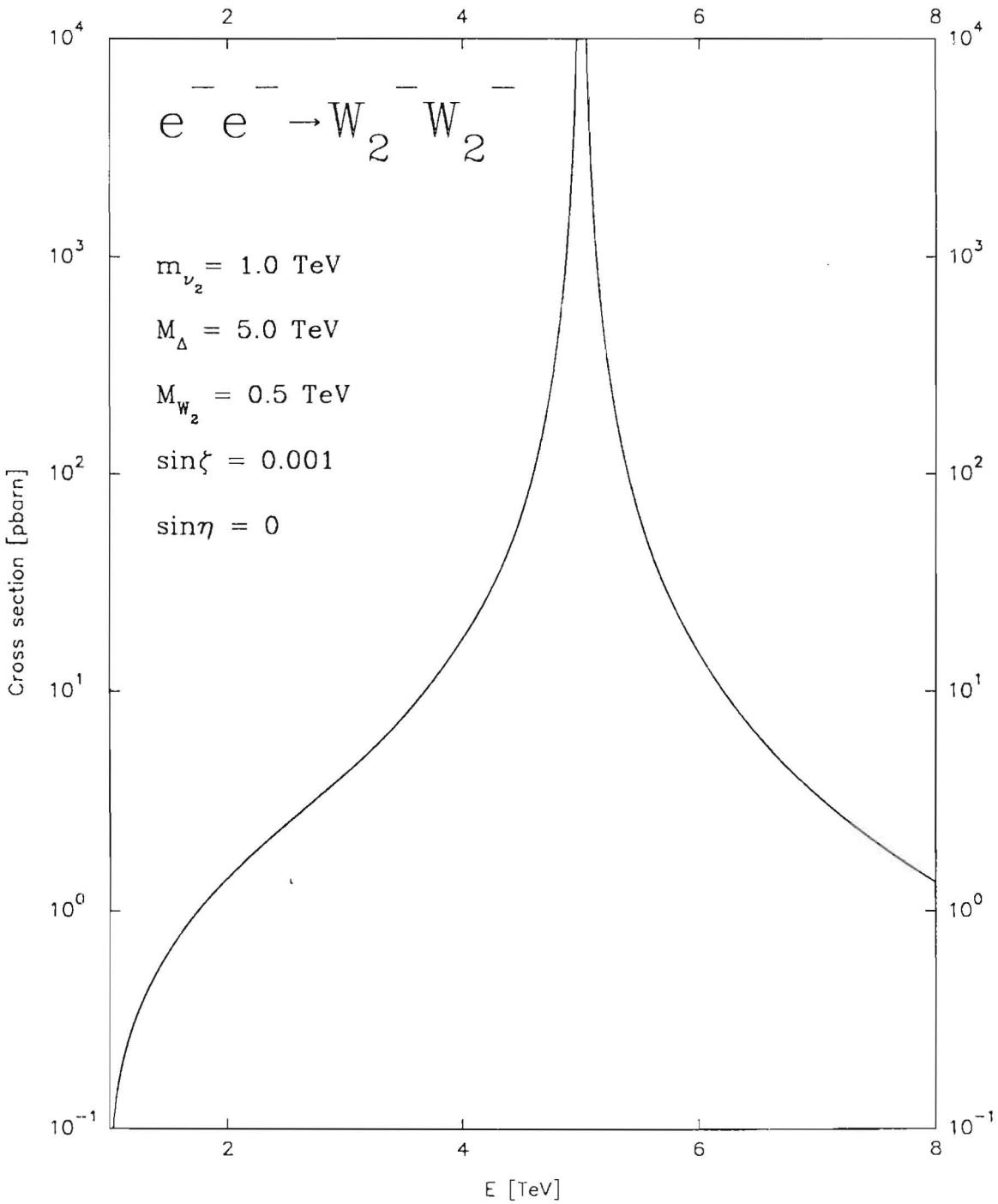
$e^- e^- \to W_2^- W_2^-$
m_{ν_2} = 1.0 TeV
M_Δ = 5.0 TeV
M_{W_2} = 0.5 TeV
$\sin\zeta$ = 0.001
$\sin\eta$ = 0
Cross section [pbarn]
E [TeV]
10^4
10^3
10^2
10^1
10^0
10^{-1}
2
4
6
8

Figure 4

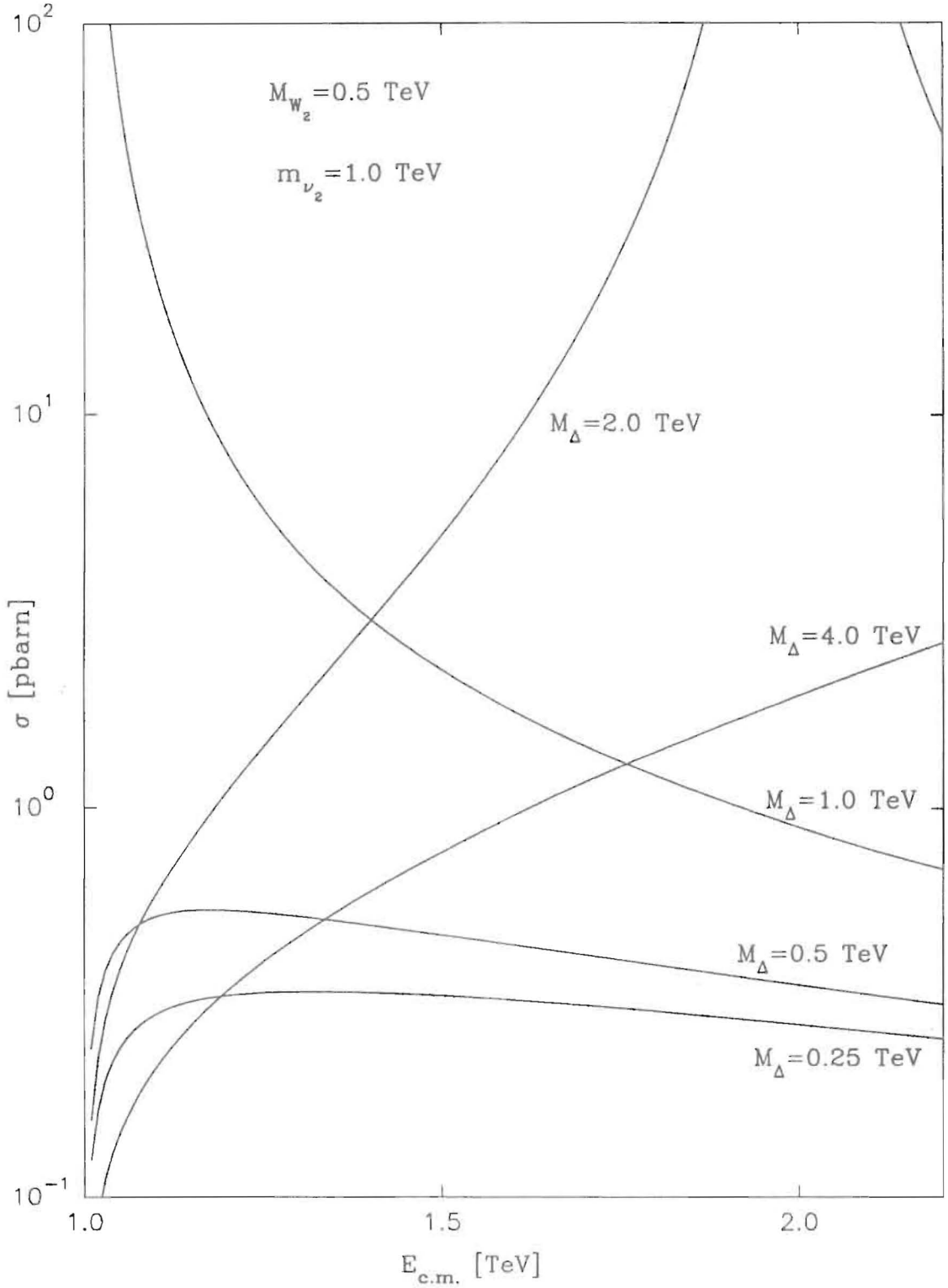
10^2
10^1
10^0
10^{-1}
M_{W_2}=0.5 TeV
m_{ν_2}=1.0 TeV
M_Δ=2.0 TeV
M_Δ=4.0 TeV
M_Δ=1.0 TeV
M_Δ=0.5 TeV
M_Δ=0.25 TeV
σ [pbarn]
1.0
1.5
2.0
$E_{c.m.}$ [TeV]

Figure 5

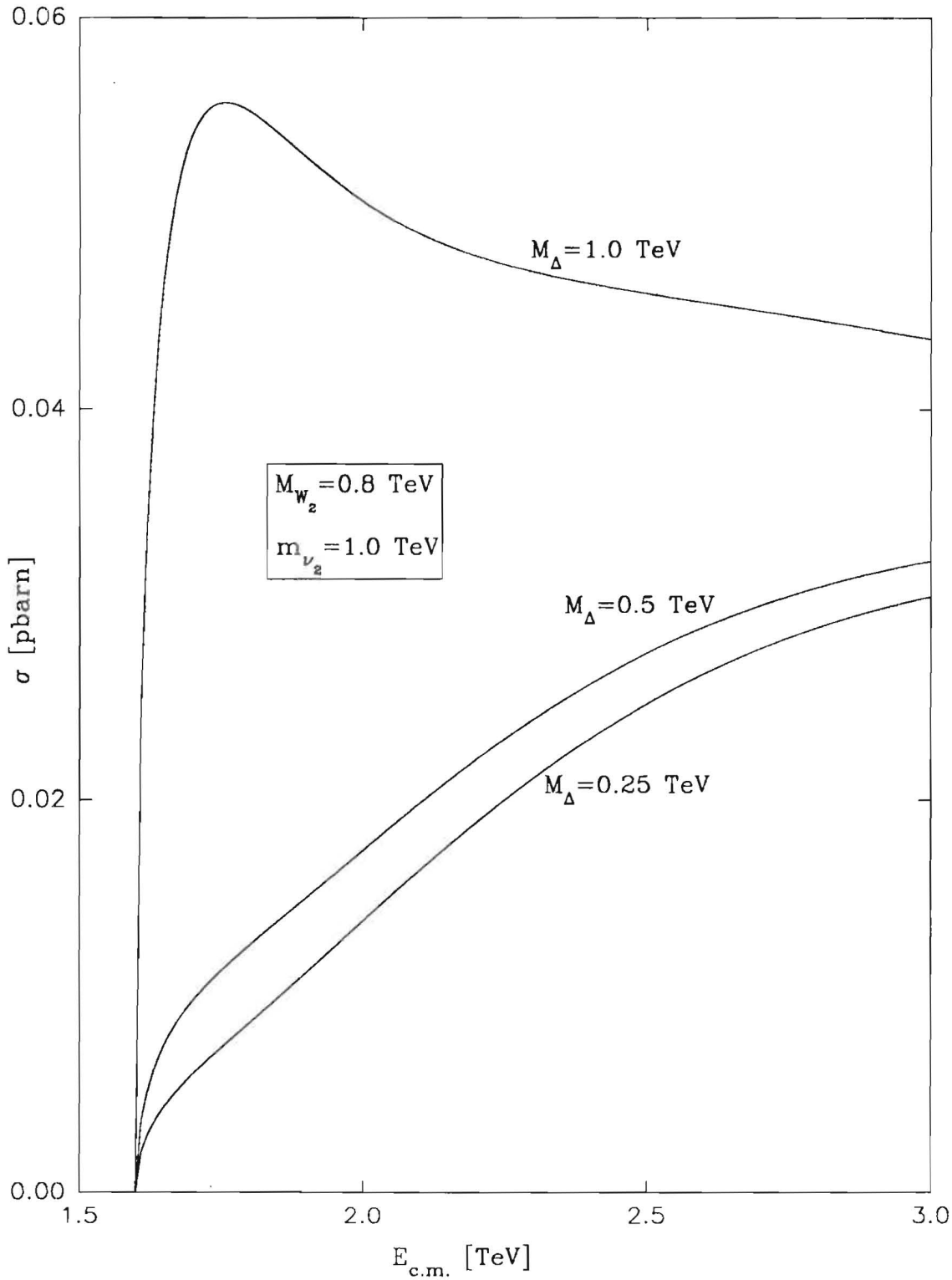
0.06
0.04
0.02
0.00
1.5
2.0
2.5
3.0
σ [pbarn]
E c.m. [TeV]
M Δ =1.0 TeV
M Δ =0.5 TeV
M Δ =0.25 TeV
M W 2 =0.8 TeV
m ν 2 =1.0 TeV

Figure 6

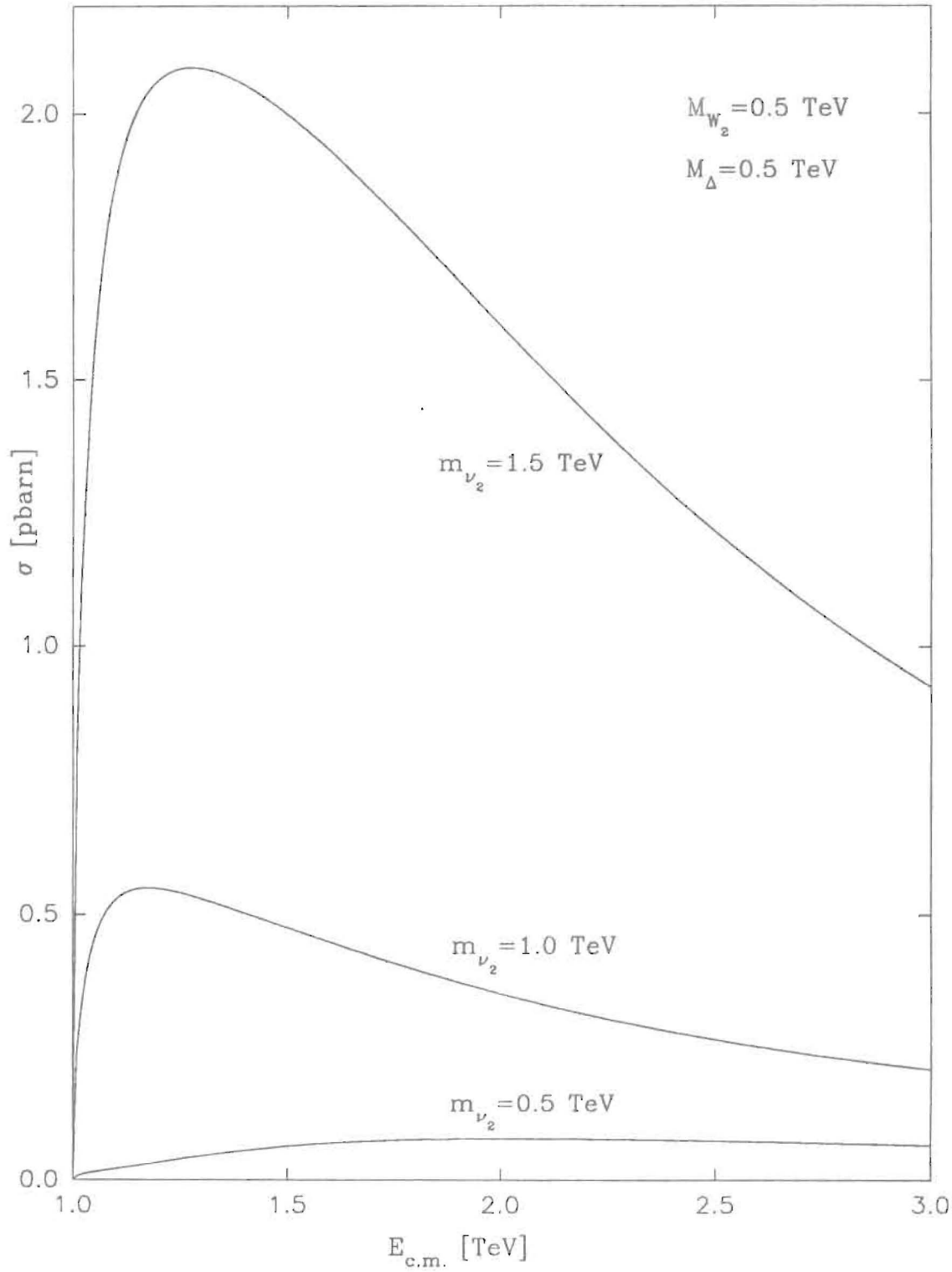
M_{W_2}=0.5 TeV
M_Δ=0.5 TeV
m_{ν_2}=1.5 TeV
m_{ν_2}=1.0 TeV
m_{ν_2}=0.5 TeV
σ [pbarn]
$E_{c.m.}$ [TeV]
0.0
0.5
1.0
1.5
2.0
1.0
1.5
2.0
2.5
3.0

Figure 7

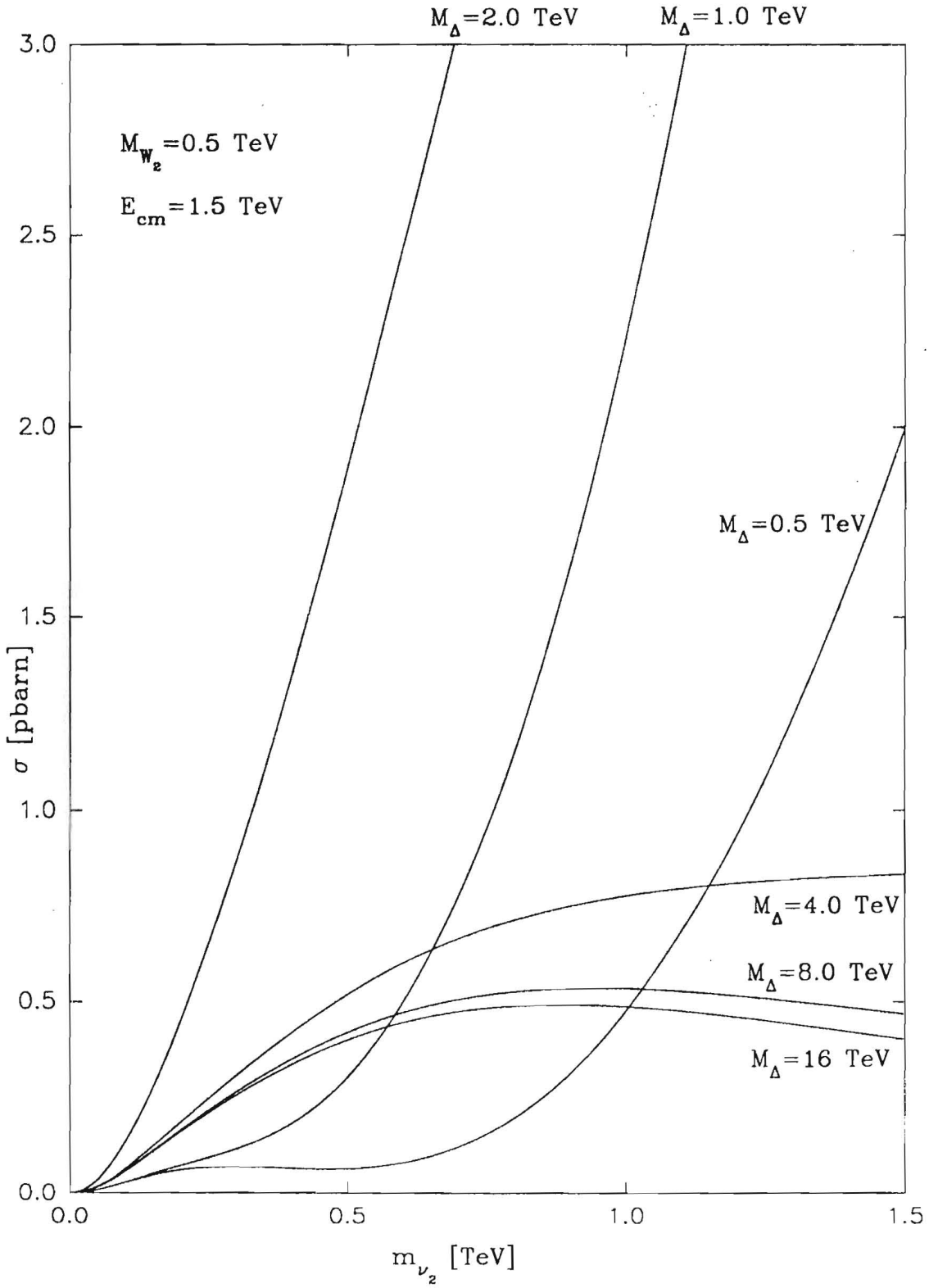
M_Δ=2.0 TeV
M_Δ=1.0 TeV
M_{W_2}=0.5 TeV
E_{cm}=1.5 TeV
M_Δ=0.5 TeV
M_Δ=4.0 TeV
M_Δ=8.0 TeV
M_Δ=16 TeV
σ [pbarn]
m_{ν_2} [TeV]
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Figure 8

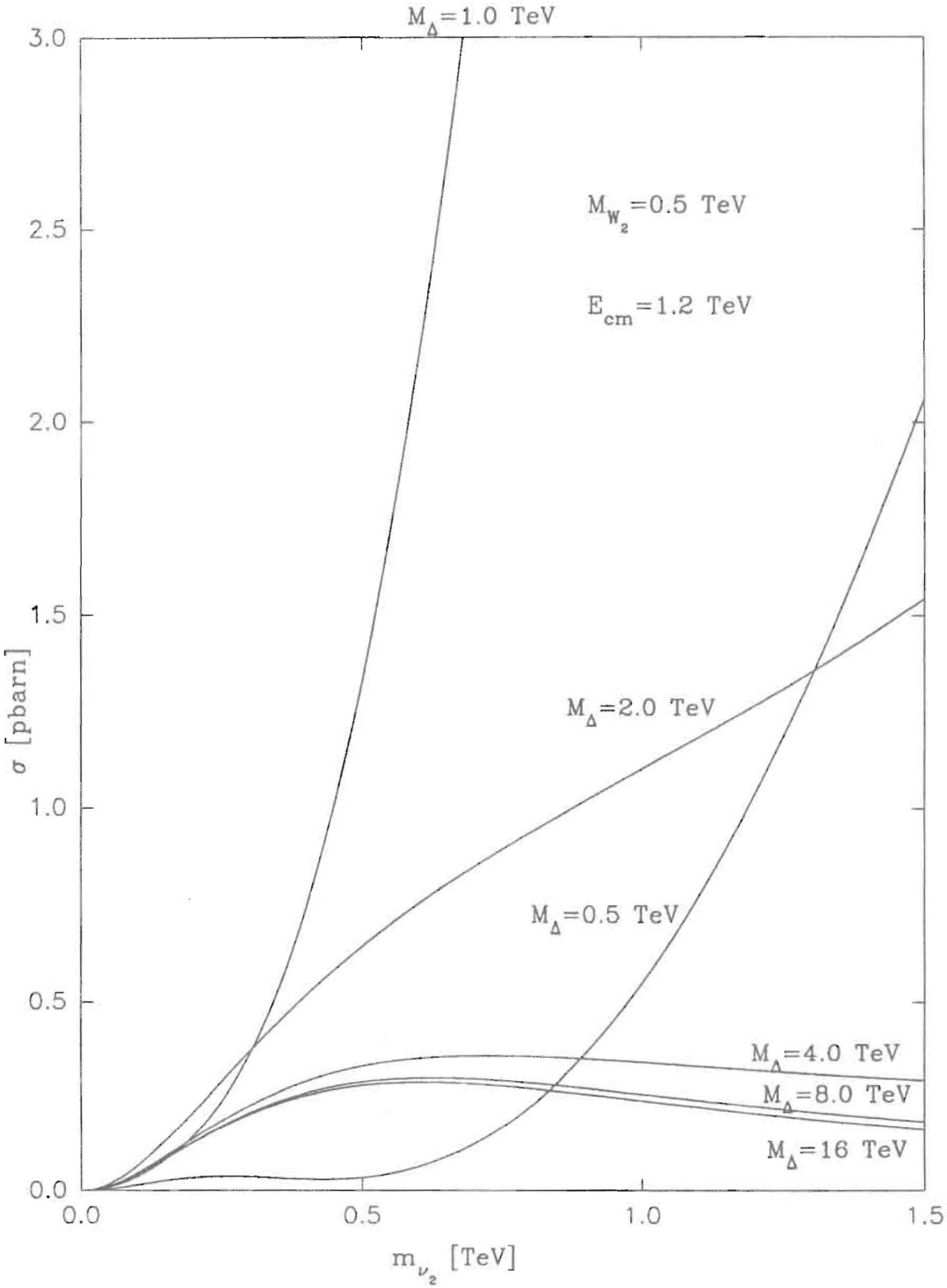
M_Δ=1.0 TeV
M_{W_2}=0.5 TeV
E_{cm}=1.2 TeV
M_Δ=2.0 TeV
M_Δ=0.5 TeV
M_Δ=4.0 TeV
M_Δ=8.0 TeV
M_Δ=16 TeV
σ [pbarn]
m_{ν_2} [TeV]
0.0
0.5
1.0
1.5
2.0
2.5
3.0

Figure 9

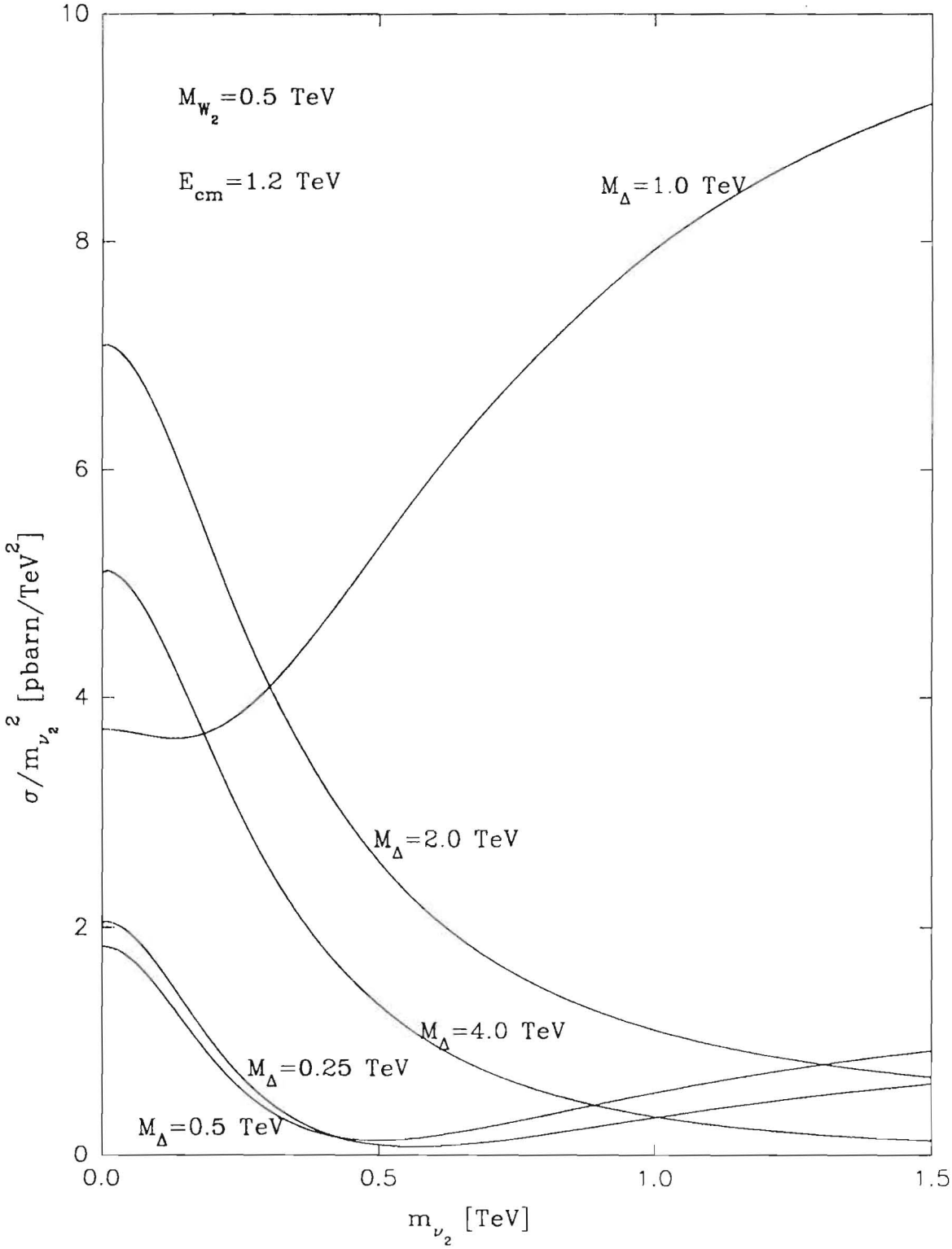
M_{W_2}=0.5 TeV
E_{cm}=1.2 TeV
M_Δ=1.0 TeV
M_Δ=2.0 TeV
M_Δ=4.0 TeV
M_Δ=0.25 TeV
M_Δ=0.5 TeV
$\sigma/m_{\nu_2}^2$ [pbarn/TeV^2]
m_{ν_2} [TeV]
0
2
4
6
8
10
0.0
0.5
1.0
1.5

Figure 10

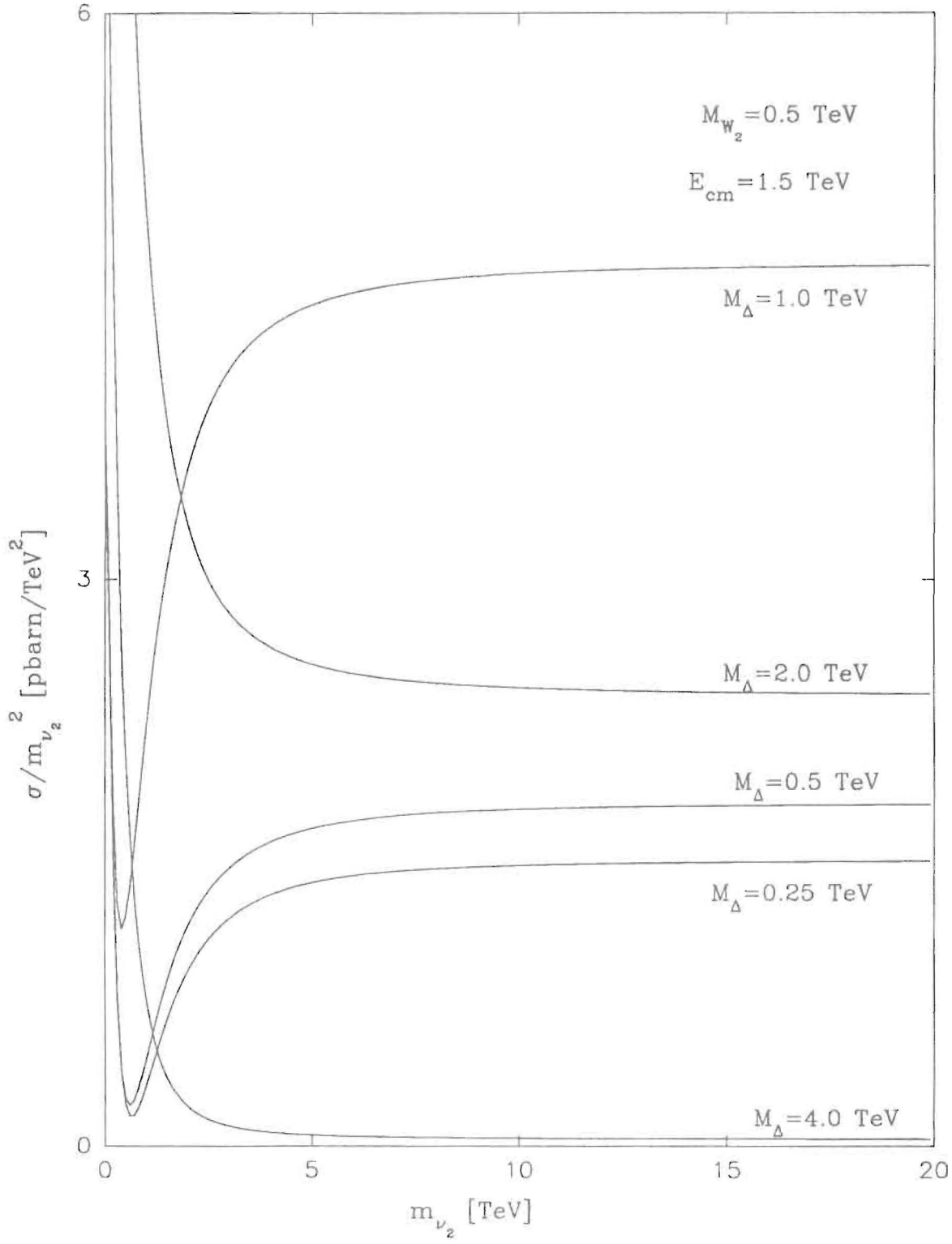
6
3
0
0
5
10
15
20
M_{W_2}=0.5 TeV
E_{cm}=1.5 TeV
M_Δ=1.0 TeV
M_Δ=2.0 TeV
M_Δ=0.5 TeV
M_Δ=0.25 TeV
M_Δ=4.0 TeV
$\sigma/m_{\nu_2}^2$ [pbarn/TeV^2]
m_{ν_2} [TeV]

Figure 11

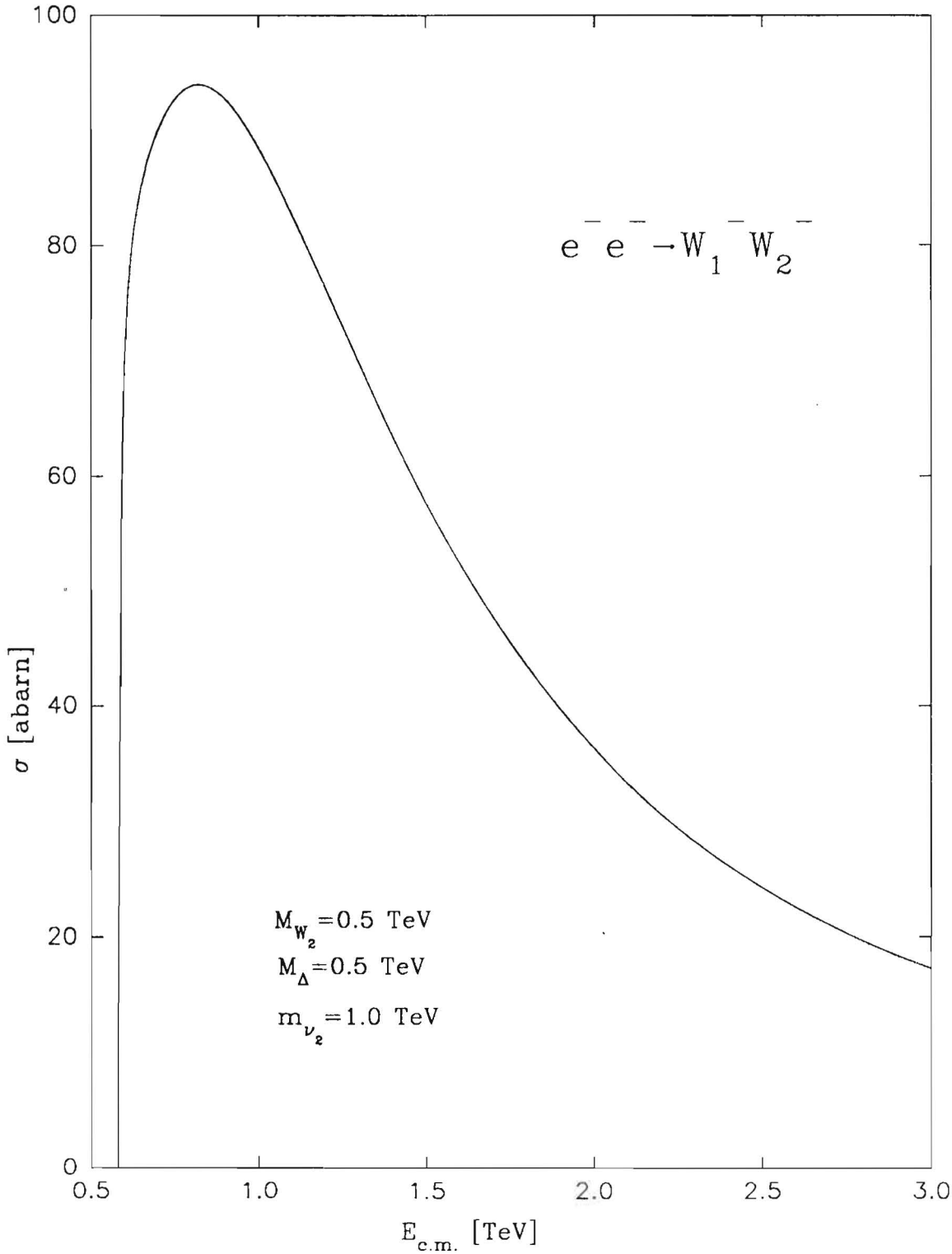
100
80
60
40
20
0
0.5
1.0
1.5
2.0
2.5
3.0
σ [abarn]
E c.m. [TeV]
e⁻e⁻→W₁⁻W₂⁻
M W₂ =0.5 TeV
M Δ =0.5 TeV
m ν₂ =1.0 TeV

Figure 12