The Higgs Contribution to the W-Boson Pair Production in e^+e^- Annihilation in the Left-Right Model

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Abstract

We investigate the neutral higgs contributions to the pair production of light (W_1) and heavy (W_2) charged gauge bosons of the left-right symmetric electroweak model in electron-positron annihilations. These contributions were neglected in our previous studies as they were assumed to be small as compared with the contributions of the leading gauge boson and neutrino exchange processes. The higgs contributions are shown to be of the same order of magnitude as the chirality flip part of the Majorana neutrino exchange channel (they cancel each other at high energies). Our numerical examples concern with the process $e^+e^- \to W_1 W_2$ where the contribution of the higgses is more important than in the pair production of equal mass weak bosons. We show that depending on the value of $W_L - W_R$ mixing angle the cross section of this process may be in the experimentally accessible range of 1 fbarn provided the Z_2 pole lies in the energy range of the process.

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Chapter 1

Introduction

Many attempts have been made to extend the standard electroweak model to a theory possessing a left-right symmetry. Among the schemes considered most attractive is perhaps the original model of Pati, Salam and Mohapatra [8], which is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$. In this model left- and righ-handed fermion fields accommodate doublets of $SU(2)_L$ and $SU(2)_R$ symmetry, respectively. The breaking of the gauge symmetry, following the chain $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$, can be arranged by introducing to the theory a bidoublet higgs field Φ transforming as $\Phi = (\mathbf{2}, \mathbf{2}, 0)$ and a "right-handed" triplet field Δ_R transforming as $\Delta_R = (\mathbf{1}, \mathbf{3}, 2)$. The triplet higgs having both $SU(2)_R$ and $U(1)_{B-L}$ charge takes care of the first step of the symmetry breaking. Its vacuum expectation value sets the mass scale of the "right-handed" gauge bosons, which is according to experiments most probably above 0.5 TeV. One could in the minimal left-right symmetric model (MLRSM) use instead of the triplet also a field transforming as a doublet under $SU(2)_R$ and having a non-vanishing $B - L$. The triplet field has, however, an extra benefit which makes it a more natural choice. It couples to $|\Delta L| = 2$ lepton currents through the Yukawa coupling $ih_R \Psi_R^T C \tau_2 \Delta_R \Psi_R$ giving rise to Majorana mass terms for right-handed neutrinos. This leads to the see-saw mechanism of neutrino masses according to which there are in each fermion family two Majorana neutrinos, one very light and another very heavy. In the name of left-right symmetry one could introduce also a "left-handed" triplet $\Delta_L = (\mathbf{3}, \mathbf{1}, 2)$ which would yield a small Majorana mass term for the left-handed neutrinos (the VEV of Δ_L is strongly constrained by the phenomenological parameter ρ).

So far there are no direct experimental evidences of the left-right symmetric model; no right-handed interactions have been discovered and experimental data does not make difference between Dirac and Majorana neutrinos. It will be one of the challenges of the next generation accelerators to look for evidences of the left-right symmetry in various high-energy processes.

In our previous papers [5],[6] we considered the pair production of the charged gauge bosons of the MLRSM in e^+e^- collisions in the energy range of the planned new linear colliders CLIC, NLC, TESLA and JLC [10]. We especially paid attention to the dependence of the cross sections on the mass of the heavy right-handed neutrino in order to find out if one could shed light to the fundamental question of the neutrino mass by experimentally studying these processes. The lowest-order contributions relevant in this respect arise from the exchange of the neutrinos in the t-channel and from the exchange of the neutral members of the bidoublet higgses in the s-channel.

In our previous works [5], [6] we didn't take into account the neutral higgs contributions. This is a justified approximation as long as higgses are much heavier than the "right-handed" gauge bosons. If these higgses were light their contribution to the process $e^+e^- \rightarrow W_1W_2$ would not be negligible and might together with the neutrino mass flip term in some cases form even the dominant contribution. In the case of the processes $e^+e^- \rightarrow W_1W_1, W_2W_2$ one can always safely neglect the higgs contributions in the energy range of the next generation accelerators. Of course, they play an important role at high energies where they garantie the good asymptotic behaviour of the theory.

The subject of this paper is to complete our previous studies of W boson pair production in the left-right symmetric model by including the higgs effects. The rich higgs sector of the MLRSM has been investigated by several authors [9],[4], [2],[3]. There are two complex electrically neutral members in the higgs bidoublet Φ and another two in the triplets Δ_L and Δ_R, that is, altogether eight neutral real degrees of freedom. Two of them are eaten by the neutral gauge bosons. Various scenarios for the physical neutral fields have been considered and phenomenologically analyzed in ref. [2]. We will derive cross sections for the the W-pair production in the case of general higgs structure and apply the results to a phenomenologically allowed simple scenario of higgs mass eigenstates to computate the the cross section of $W_1^- W_2^+$. In this process the neutrino mass flip and higgs contributions could dominate being proportional to the $(m_{\nu_2}\eta)^2$, where η is the mixing angle of the neutrinos and m_{ν_2} is the mass of the heavy neutrino. Due to the model relation $\eta \approx \sqrt{m_{\nu_1}/m_{\nu_2}}$ this demands, however, an ultrahigh neutrino mass, as we shall see. In our computation the simplest scenario, where only two from the possible six higgses mediate the process in the s-channel and the both directly are mass eigenstates, will be used. The more complicated schemes are not supposed to change the order of magnitude of the cross section.

Chapter 2

Formulas

2.1 Calculation of vertices

The matter of the MLRSM contains one left-handed and one right-handed doublet of the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ for the electron family,

$$\Psi_L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L = (2,0,-1), \quad \Psi_R = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_R = (0,2,-1), \tag{2.1}$$

and similarly for other families. In order to generate masses for fermions one requires at least one Higgs bidoublet of the form

$$\Phi = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix} = (2,2,0), \tag{2.2}$$

whose vacuum expectation value (VEV) is given by

$$< \Phi >= \frac{1}{\sqrt{2}} \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix}. \tag{2.3}$$

Here K_1 and K_2 are in general complex, but if we assume that the explicit CP violation is small we can take them real [2]. In order to break the symmetry to the electromagnetic group additional higgs multiplets with $B - L \neq 0$ are needed. To introduce at the same time Majorana mass terms for the neutrinos we add to the theory the triplets

$$\begin{aligned} \Delta_L &= \begin{pmatrix} \delta_L^+ & \sqrt{2}\delta_L^{++} \\ \sqrt{2}\delta_L^0 & -\delta_L^+ \end{pmatrix} = (3,1,2), \\ \Delta_R &= \begin{pmatrix} \delta_R^+ & \sqrt{2}\delta_R^{++} \\ \sqrt{2}\delta_R^0 & -\delta_R^+ \end{pmatrix} = (1,3,2) \end{aligned} \tag{2.4}$$

with the VEV's given by

$$< \Delta_{L,R} >= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}. \tag{2.5}$$

There are all together seven vector bosons: $W_{L,R}^{\pm} = \frac{1}{\sqrt{2}}(V_{L,R}^1 \pm iV_{L,R}^2)$, $V_{L,R}^3$, and B. We define the physical states of the bosons by the equations

$$\begin{pmatrix} W_L^{\pm} \\ W_R^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\zeta & -\sin\zeta \\ \sin\zeta & \cos\zeta \end{pmatrix} \begin{pmatrix} W_1^{\pm} \\ W_2^{\pm} \end{pmatrix}, \tag{2.6}$$

$$\begin{pmatrix} V_L^3 \\ V_R^3 \\ B \end{pmatrix} = (R_{ij}) \begin{pmatrix} Z_1 \\ Z_2 \\ Z_3 = \gamma \end{pmatrix} \qquad (i = L, R, B). \tag{2.7}$$

The left-handed and the right-handed neutrinos are related to the mass eigenstate Majorana neutrinos ν_1 and ν_2:

$$\begin{aligned} \nu_L &= \tfrac{1}{2}(1-\gamma^5)\,(\nu_1 \cos\eta - \nu_2 \sin\eta)\,, \\ \nu_R &= \tfrac{1}{2}(1+\gamma^5)\,(\nu_1 \sin\eta + \nu_2 \cos\eta)\,. \end{aligned} \tag{2.8}$$

We will calculate the pair production of the charged weak bosons in the processes $e^-e^+ \to W_l^- W_{l'}^+$, $l, l' = 1, 2$. In the lowest order, there are three different types of diagrams associated with these prosesses (see Figure 2.1): vector boson (photon, Z_1 and Z_2) exchange in the s-channel, neutral higgs exchange (ϕ_j^{0r} and ϕ_j^{0i}) in the s-channel, and neutrino exchange (ν_1 and ν_2) in the t-channel. The parts of the Lagrangian involved are the following:
the electron contribution to the neutral current interaction

$$\mathcal{L}_{nc}^{e} = \overline{e}\gamma^\mu (G_V^j - \gamma^5 G_A^j) e Z_{j\mu}; \tag{2.9}$$

the charged current interaction

$$\begin{aligned} \mathcal{L}_{cc} &= \sum_{jl} \overline{\nu}_j \left(G_{lj}^L \gamma^\mu (1-\gamma_5) + G_{lj}^R \gamma^\mu (1+\gamma_5) \right) e\, W_{l\mu,-}^\dagger \quad + h.c. && (2.10) \\ &= \sum_{jl} \overline{\nu}_j\, \Gamma_{lj}^\mu\, e\, W_{l\mu,-}^\dagger \quad + h.c.; && (2.11) \end{aligned}$$

the ZWW interaction (denoting $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$ etc.)

$$\mathcal{L}_{ZWW} = \frac{i}{2} \sum_{j,k,l} e_{jkl} \left(W_{k\mu\nu}^+ W_l^{-\mu} Z_j^\nu - W_{l\mu\nu}^- W_k^{+\mu} Z_j^\nu + W_{k\mu}^+ W_{l\nu}^- Z_j^{\mu\nu} \right); \tag{2.12}$$

the Yukawa interactions (denoting $\tilde{\Phi} = \sigma^2 \Phi^* \sigma^2$)

$$\mathcal{L}_{Yu} = f\overline{\Psi}_R \Phi \Psi_L + g\overline{\Psi}_R \tilde{\Phi} \Psi_L + i h_L \Psi_L^T C \tau_2 \Delta_L \Psi_L + i h_R \Psi_R^T C \tau_2 \Delta_R \Psi_R + h.c.; \tag{2.13}$$

the kinetic energy of the higgs doublet

$$\mathcal{L}_{kin}^{\phi} = \mathrm{Tr}\{(D_\mu \Phi)^\dagger (D^\mu \Phi)\} \tag{2.14}$$

with

$$D_\mu \Phi = \partial_\mu \Phi - \frac{i}{2}(g_L \vec{\tau} \cdot \vec{V}_{L\mu} \Phi - g_R \Phi \vec{\tau} \cdot \vec{V}_{R\mu}) \tag{2.15}$$

where the Cartesian components of the vector bosons are used. In these equations we have defined

$$\Gamma_{lj}^\mu = G_{Llj}\, \gamma^\mu (1-\gamma_5) + G_{Rlj}\, \gamma^\mu (1+\gamma_5), \tag{2.16}$$

$$\begin{aligned} G_{Llj} &= \frac{g_L}{2\sqrt{2}} \begin{pmatrix} \cos\eta\cos\zeta & \sin\eta\cos\zeta \\ -\cos\eta\sin\zeta & -\sin\eta\sin\zeta \end{pmatrix}, \\ G_{Rlj} &= \frac{g_R}{2\sqrt{2}} \begin{pmatrix} -\sin\eta\sin\zeta & \cos\eta\sin\zeta \\ -\sin\eta\cos\zeta & \cos\eta\cos\zeta \end{pmatrix}, \end{aligned} \tag{2.17}$$

$$e_{jll'} = \begin{cases} (\cos\zeta)^2 g_L R_{Lj} + (\sin\zeta)^2 g_R R_{Rj}\,, & l = l' = 1 \\ (\sin\zeta)^2 g_L R_{Lj} + (\cos\zeta)^2 g_R R_{Rj}\,, & l = l' = 2 \\ \sin\zeta\cos\zeta\,(g_R R_{Rj} - g_L R_{Lj}), & l \neq l'. \end{cases} \tag{2.18}$$

In our previous work [5] we have already calculated the contributions to the W-pair productions from the Z-boson and the neutrino exchanges. Now we are going to derive the contribution from the higgs exchange channels. In order to find the appropriate couplings we have to expand the

kinetic energy of the higgs bidoublet and to collect the terms where the higgses interact with the W-bosons. Defining the real higgses according to

$$\phi_j^0 = \frac{1}{\sqrt{2}}(\phi_j^{0r} + i\phi_j^{0i}) \tag{2.19}$$

we obtain

$$\begin{aligned}
\mathcal{L}_{\phi WW} &= \frac{1}{2}(g_L^2 W_L^- W_L^+ + g_R^2 W_R^- W_R^+)(K_1\phi_1^{0r} + K_2\phi_2^{0r}) \\
&- \frac{1}{2} g_L g_R [(W_L^- W_R^+ + W_R^- W_L^+)(K_2\phi_1^{0r} + K_1\phi_2^{0r}) \\
&- i(W_L^- W_R^+ - W_R^- W_L^+)(K_2\phi_1^{0i} - K_1\phi_2^{0i})].
\end{aligned} \tag{2.20}$$

We note that the last term of our equation is missing in eq. (2.13) of [4]. In terms of the mass eigenstates of the W-bosons this is rewritten as

$$\begin{aligned}
2\mathcal{L}_{\phi WW} &= \{[(g_L\cos\zeta)^2 + (g_R\sin\zeta)^2]W_1^- W_1^+ + [(g_L\sin\zeta)^2 + (g_R\cos\zeta)^2]W_2^- W_2^+ \\
&+ (g_R^2 - g_L^2)(W_1^- W_2^+ + W_2^- W_1^+)\}(K_1\phi_1^{0r} + K_2\phi_2^{0r}) \\
&- g_L g_R\{[\sin 2\zeta(W_1^- W_1^+ - W_2^- W_2^+) + \cos 2\zeta(W_1^- W_2^+ + W_2^- W_1^+)](K_2\phi_1^{0r} + K_1\phi_2^{0r}) \\
&- 2i(W_1^- W_2^+ - W_2^- W_1^+)(K_2\phi_1^{0i} - K_1\phi_2^{0i})\}.
\end{aligned} \tag{2.21}$$

The form of the $ee\phi$ interaction can be inferred from the Yukawa Lagrangian and it reads

$$\mathcal{L}_{ee\phi} = \frac{\bar{e}e}{\sqrt{2}}[f\phi_2^{0r} + g\phi_1^{0r}] + i\frac{\bar{e}\gamma_5 e}{\sqrt{2}}[f\phi_2^{0i} - g\phi_1^{0i}]. \tag{2.22}$$

In Fig 2.2 the Feynman rules for the relevant vertices derived from the above couplings are given.

2.2 Amplitudes

Let us now consider the amplitudes for the processes (l, $l' = 1, 2$)

$$e^- e^+ \to W_l^- W_{l'}^+. \tag{2.23}$$

The amplitude corresponding to all diagrams of Figure 2.1 has the decomposition

$$T = T(Z_1) + T(Z_2) + T(\gamma = Z_3) + T(\nu_1) + T(\nu_2) + \sum T(\phi_j). \tag{2.24}$$

From the vertices given in Figure 2.1 we obtain

$$T_{\mu'\mu}(Z_j) = \frac{i\, e_{jl'l}(G_{jV} + \gamma_5 G_{jA})}{s - M_{Z_j}^2 + i\,\Gamma_{Z_j} M_{Z_j}} F_{\mu'\mu}(k', k), \tag{2.25}$$

$$F_{\mu'\mu}(k', k) = g_{\mu'\mu}(\not{k} - \not{k}') + \gamma_{\mu'}(2k' + k)_\mu - \gamma_\mu(2k + k')_{\mu'}, \tag{2.26}$$

$$T_{\mu'\mu}(\nu_k) = T_{\mu'\mu}^L(\nu_k) + T_{\mu'\mu}^R(\nu_k), \tag{2.27}$$

$$T_{\mu'\mu}^L(\nu_k)_{l'l} = \frac{-2i(1+\gamma_5)}{t - m_{\nu_k}^2}[G_{LlLl'k}\,\gamma_{\mu'}(\not{p} - \not{k})\gamma_\mu + G_{RlLl'k}\, m_{\nu_k}\gamma_{\mu'}\gamma_\mu], \tag{2.28}$$

$$T_{\mu'\mu}^R(\nu_k)_{l'l} = \frac{-2i(1-\gamma_5)}{t - m_{\nu_k}^2}[G_{RlRl'k}\,\gamma_{\mu'}(\not{p} - \not{k})\gamma_\mu + G_{LlRl'k}\, m_{\nu_k}\gamma_{\mu'}\gamma_\mu], \tag{2.29}$$

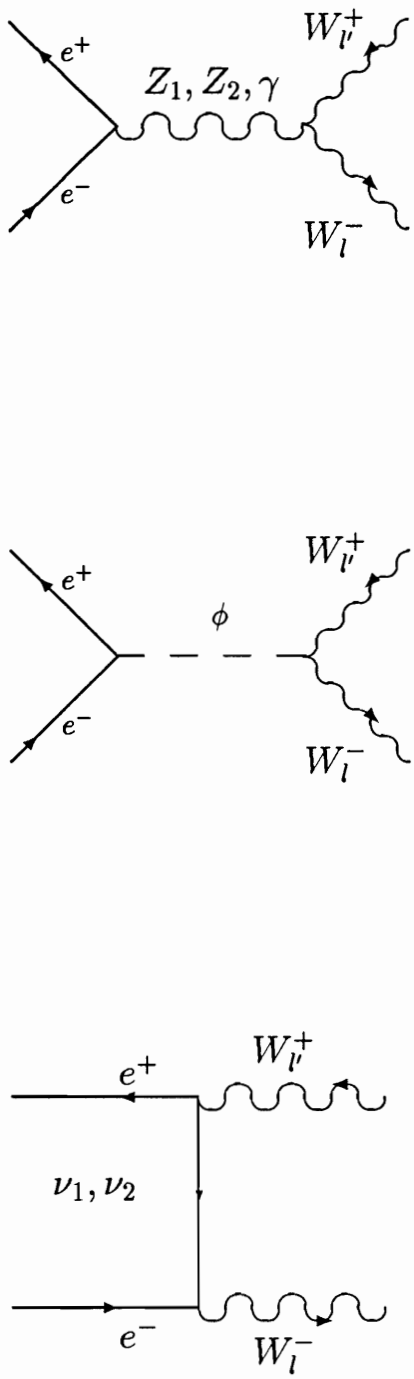
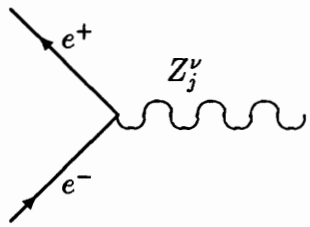
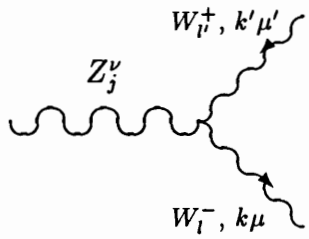


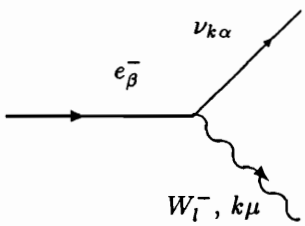
Figure 2.1: Feynman graphs for $e^+e^- \to W_l^- W_{l'}^+$



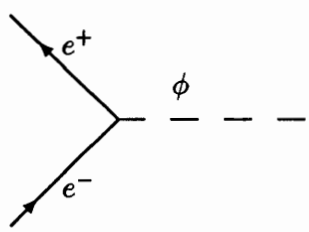
$$= i\gamma_\nu(G_{jV} - G_{jA}\gamma_5)$$



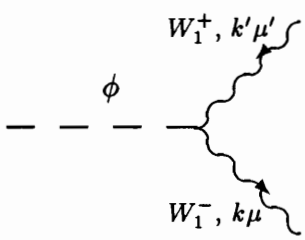
$$= -i\, e_{jll'}\, [g_{\mu'\mu}(k-k')_\nu + g_{\nu\mu'}(2k'+k)_\mu - g_{\mu\nu}(2k+k')_{\mu'}]$$



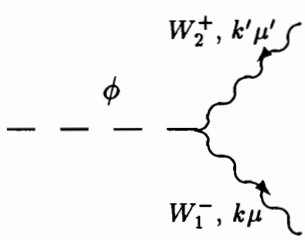
$$= i(\Gamma_{lk}^\mu)_{\alpha\beta}$$



$$= \begin{cases} i\frac{g}{\sqrt{2}} & \text{if } \phi = \phi_1^{0r} \\ i\frac{f}{\sqrt{2}} & \text{if } \phi = \phi_2^{0r} \\ +\frac{g}{\sqrt{2}}\gamma_5 & \text{if } \phi = \phi_1^{0i} \\ -\frac{f}{\sqrt{2}}\gamma_5 & \text{if } \phi = \phi_2^{0i} \end{cases}$$



$$= \begin{cases} \frac{i}{2}[(g_L^2\cos^2\zeta + g_R^2\sin^2\zeta)K_1 - g_L g_R K_2 \sin 2\zeta] & \text{if } \phi = \phi_1^{0r} \\ \frac{i}{2}[(g_L^2\cos^2\zeta + g_R^2\sin^2\zeta)K_2 - g_L g_R K_1 \sin 2\zeta] & \text{if } \phi = \phi_2^{0r} \end{cases}$$



$$= \begin{cases} \frac{i}{4}(\delta g^2 K_1 \sin 2\zeta - 2g_L g_R K_2 \cos 2\zeta) & \text{if } \phi = \phi_1^{0r} \\ \frac{i}{4}(\delta g^2 K_2 \sin 2\zeta - 2g_L g_R K_1 \cos 2\zeta) & \text{if } \phi = \phi_2^{0r} \\ -\frac{1}{2}g_L g_R K_2 & \text{if } \phi = \phi_1^{0i} \\ +\frac{1}{2}g_L g_R K_1 & \text{if } \phi = \phi_2^{0i} \end{cases}$$

Figure 2.2: Feynman rules

$$\gamma^0\Big(T_{\nu'\nu}(Z_j)_{l'l}\Big)^\dagger\gamma^0 = -\frac{i\,e_{jl'l}(G_{jV}+\gamma_5 G_{jA})}{s-M_{Z_j}^2-i\,\Gamma_{Z_j}M_{Z_j}}F_{\nu'\nu}(k',k), \tag{2.30}$$

$$\gamma^0\Big(T_{\nu'\nu}(\nu_k)_{l'l}\Big)^\dagger\gamma^0 = -T^L_{\nu\nu'}(\nu_k)_{ll'} - T^R_{\nu\nu'}(\nu_k)_{ll'}. \tag{2.31}$$

Here we have used the abbreviations [6]

$$\begin{aligned} G_{LlLl'k} &= G_{Lkl}G_{Lkl'}\,, \qquad & G_{LlRl'k} &= G_{Lkl}G_{Rkl'}\,, \\ G_{RlLl'k} &= G_{Rkl}G_{Lkl'}\,, \qquad & G_{RlRl'k} &= G_{Rkl}G_{Rkl'}\,, \end{aligned} \tag{2.32}$$

and their expressions are given in Table 2.1 and in Table 2.2, which we have taken from [7].

The amplitudes above have been given in unitary gauge. The vertices containing higgs in Figure 2.1 are, however, given for shifted fields, which, in general, are not mass eigenstates. The vertices in unitary gauge or in renormalizabel gauge R_ζ vary according to the scenario used for the model parameters. Before going into detailed models we change our higgs basis to a more convenient one by defining [2]

$$\begin{aligned} \phi^0_+ &= -k_2\phi^0_1 + k_1\phi^{0\dagger}_2, \\ \phi^0_- &= k_1\phi^0_1 + k_2\phi^{0\dagger}_2, \end{aligned} \tag{2.33}$$

with the inverse transformation

$$\begin{aligned} \phi^0_1 &= -k_2\phi^0_+ + k_1\phi^0_-, \\ \phi^0_2 &= k_1\phi^{0\,\dagger}_+ + k_2\phi^{0\,\dagger}_-. \end{aligned} \tag{2.34}$$

Here $k_j = K_j/K_+$ and $K_+ = \sqrt{K_1^2+K_2^2}$. The new basis is connected to flavour. The field ϕ^0_+ has flavour-changing couplings with fermions in contrast to the field ϕ^0_- which conserves the flavour. Thus, loosly speaking, the real component of the ϕ^0_- corresponds to the standard model higgs boson and the imaginary component is the massles Goldstone field absorbed by the light Z-boson. In order to avoid the flavour-changing couplings from conflicting with experiment the mass eigenstates containing significant mixtures of the ϕ^0_+ must have a large mass.

The higgses in R_ζ gauge are obtained from the shifted fields by a rotation and therefore the high-energy limit of the higgs exchange amplitude can immediately be written. From Figure 2.1 we obtain after a little algebra the following expressions for the processes $e^-e^+ \to W_1^-W_2^+$ and $e^-e^+ \to W_1^-W_1^+$ in the limit of $s \gg$ any M^2_ϕ in our flavour-diagonal basis:

$$-\,isT_{W_1W_2} = \sum_{j=+,-}[T_{12}(\phi^{0r}_j) + T_{12}(\phi^{0i}_j)] \tag{2.35}$$

and

$$-\,isT_{W_1W_1} = \sum_{j=+,-}T_{11}(\phi^{0r}_j), \tag{2.36}$$

where

$$T_{12}(\phi^{0r}_+) = g_Lg_R[m_D/2 - \frac{K_1K_2}{K_+^2}m_e]\cos 2\zeta, \tag{2.37}$$

$$T_{12}(\phi^{0r}_-) = m_e[(g_L^2-g_R^2) + g_Lg_R\frac{K_1K_2}{K_+^2}\cos 2\zeta], \tag{2.38}$$

$$T_{12}(\phi^{0i}_+) = -g_Lg_R[m_D/2 - \frac{K_1K_2}{K_+^2}m_e]\gamma_5, \tag{2.39}$$

$$T_{12}(\phi_-^{0i}) = -g_L g_R m_e \frac{K_1 K_2}{K_+^2}\gamma_5, \tag{2.40}$$

$$T_{11}(\phi_+^{0r}) = g_L g_R[m_D/2 - \frac{K_1 K_2}{K_+^2} m_e]\sin 2\zeta, \tag{2.41}$$

$$T_{11}(\phi_-^{0r}) = -m_e[(g_L^2\cos^2\zeta + g_R^2\sin^2\zeta)/2 - g_L g_R \frac{K_1 K_2}{K_+^2}\sin 2\zeta]. \tag{2.42}$$

Here we have utilized the fact that in the MLRSM the electron mass is given by

$$m_e = \frac{fK_2 + gK_1}{\sqrt{2}} \tag{2.43}$$

and that the neutrino has a Dirac mass given by

$$m_D = \frac{fK_1 + gK_2}{\sqrt{2}}. \tag{2.44}$$

The latter one can be expressed in terms of the mass eigenvalues m_i and the mixing angle η of the neutrinos:

$$m_D = \frac{1}{2}(m_1 + m_2)\sin 2\eta. \tag{2.45}$$

We note that here and anywhere in this paper the masses m_i refer to the eigenvalues of the neutrino mass matrix; hence the mass of the neutrino ν_2 is $m_{\nu_2} = -m_2$ in this model.

By using the amplitudes derived above one can demonstrate the good high-energy behaviour of the theory as far as the pair production of W-bosons is concerned. We note that the amplitudes at high energies are linear functions of the electron and the neutrino masses. In addition, they depend on the gauge coupling contants and on the mixing angles, but they do not involve any other model parameters. In the following we drop the terms which are proportional to the electron mass.

Deshpande et al [2] have found that the real and the imaginary components of the higgses do not mix and they have shown that the would-be Goldstone bosons are formed of the imaginary components in realistic scenarios. In the simplest scheme [4] with $K_2 = 0$ and $v_R \gg v_L$, ϕ_1^{0i} is eaten, whereas ϕ_2^{0r} and ϕ_2^{0i} are physical mass eigenstate particles. The state ϕ_1^{0r} mixes with the real component of δ_R^0, but we have got rid of this state by approximating $m_e = 0$, as can be seen from equations containing ϕ_-^0 (see eq.(2.33)). Under these simplifying conditions it is useful to present the amplitudes in the unitary gauge. For higgs exchanges they read

$$T^{\phi}_{W_1W_2} = \frac{i}{4} g_L g_R g_{\mu\mu'}(m_1 + m_2)\sin 2\eta \left(\frac{\cos 2\zeta}{s - M_{2r}^2} - \frac{\gamma_5}{s - M_{2i}^2}\right) \tag{2.46}$$

and

$$T^{\phi}_{W_1W_1} = -\frac{i}{4} g_L g_R g_{\mu\mu'}(m_1 + m_2)\sin 2\eta \frac{\sin 2\zeta}{s - M_{2r}^2}, \tag{2.47}$$

where M_{2r} and M_{2i} refer to the masses of the higgses ϕ_2^{0r} and ϕ_2^{0i}, respectively.

For comparison we give the neutrino exchange amplitudes corresponding to the mass part of the neutrino propagators and which we call the mass flip amplitudes. For the reaction $e^-e^+ \to W_1^- W_2^+$ the mass flip amplitude is given by

$$T^{\nu-flip}_{W_1W_2} = -\frac{i}{8} g_L g_R \sin 2\eta \gamma_\mu \gamma_{\mu'}(\cos 2\zeta - \gamma_5)\left(\frac{m_1}{t - m_1^2} + \frac{m_2}{t - m_2^2}\right) \tag{2.48}$$

and for the reaction $e^-e^+ \to W_1^- W_1^+$ by

$$T^{\nu-flip}_{W_1W_1} = -\frac{i}{8} g_L g_R \sin 2\eta \sin 2\zeta \gamma_\mu \gamma_{\mu'}\left(\frac{m_1}{t - m_1^2} + \frac{m_2}{t - m_2^2}\right). \tag{2.49}$$

	$G_{LlRl'j}/G_LG_R = 8\,G^L_{jl}\,G^R_{jl'}/g_Lg_R$	
	$j=1$	$j=2$
$l=l'=1$	$-\sin\eta\cos\eta\sin\zeta\cos\zeta$	$sin\eta\cos\eta\sin\zeta\cos\zeta$
$l=l'=2$	$\sin\eta\cos\eta\sin\zeta\cos\zeta$	$-\sin\eta\cos\eta\sin\zeta\cos\zeta$
$l=1,\,l'=2$	$-\sin\eta\cos\eta\cos^2\zeta$	$\sin\eta\cos\eta\cos^2\zeta$
$l=2,\,l'=1$	$\sin\eta\cos\eta\sin^2\zeta$	$-\sin\eta\cos\eta\sin^2\zeta$

Table 2.1: The $G_{LlRl'j}$-coefficients.

	$G_{LlLl'j}/G^2_L = 8\,G^L_{jl}\,G^L_{jl'}/g^2_L$		$G_{RlRl'j}/G^2_R = 8\,G^R_{jl}\,G^R_{jl'}/g^2_R$	
	$j=1$	$j=2$	$j=1$	$j=2$
$l=l'=1$	$\cos^2\eta\cos^2\zeta$	$\sin^2\eta\cos^2\zeta$	$\sin^2\eta\sin^2\zeta$	$\cos^2\eta\sin^2\zeta$
$l=l'=2$	$\cos^2\eta\sin^2\zeta$	$\sin^2\eta\sin^2\zeta$	$\sin^2\eta\cos^2\zeta$	$\cos^2\eta\cos^2\zeta$
$l\neq l'$	$-\cos^2\eta\sin\zeta\cos\zeta$	$-\sin^2\eta\sin\zeta\cos\zeta$	$\sin^2\eta\sin\zeta\cos\zeta$	$\cos^2\eta\sin\zeta\cos\zeta$

Table 2.2: The $G_{LlLl'j}$ and $G_{RlRl'j}$-coefficients.

2.3 Cross sections

The general form of the amplitude for the processes $e^-e^+ \to W^+_l W^-_{l'}$ including the spin factors is given by

$$M_{ss'aa'} = \overline{v}_{s'}(p')T_{\mu'\mu}u_s(p)\epsilon^{\prime *\mu'}_{a'}(k)\epsilon^{\mu}_{a}(k). \tag{2.50}$$

After squaring the amplitude performing the spin summations we obtain

$$\sum_{spins}|M|^2 \equiv \sum_{ss'}\sum_{aa'}|M_{s's'a'a}|^2 = (g^{\mu'\nu'} - \frac{k'^{\mu'}k'^{\nu'}}{M'^2})(g^{\mu\nu} - \frac{k^{\mu}k^{\nu}}{M^2})\,\mathrm{Tr}\{\not{p}\,[\gamma^0 T^{\dagger}_{\nu'\nu}\gamma^0]\,\not{p}'\,T_{\mu'\mu}\}. \tag{2.51}$$

The total cross section for the production of a pair of unpolarized W-bosons in a collision of unpolarized electrons and positrons is then given by the formula

$$\sigma_{e^+e^-\to W^+_{l'}W^-_l} = \frac{1}{16\pi s^2}\int_{t_{min}}^{t_{max}}\langle\Sigma|M|^2\rangle dt, \tag{2.52}$$

with

$$\langle\Sigma|M|^2\rangle = \frac{1}{4}\sum_{spins}|M|^2. \tag{2.53}$$

In our previous paper [5] we derived the contribution of the boson and neutrino exchange channels and obtained

$$
\begin{aligned}
&< \Sigma|M|^2 >^{Z,\nu} \\
&= 2\left[\left(\sum_{j=1}^{3}\frac{se_{jl'l}G_{jV}(s-M_{Z_j}^2)}{(s-M_{Z_j}^2)^2+\Gamma_{Z_j}^2M_{Z_j}^2}\right)^2+\left(\sum_{j=1}^{3}\frac{se_{jl'l}G_{jV}\Gamma_{Z_j}M_{Z_j}}{(s-M_{Z_j}^2)^2+\Gamma_{Z_j}^2M_{Z_j}^2}\right)^2\right. \\
&\quad\left.+\left(\sum_{j=1}^{3}\frac{se_{jl'l}G_{jA}(s-M_{Z_j}^2)}{(s-M_{Z_j}^2)^2+\Gamma_{Z_j}^2M_{Z_j}^2}\right)^2+\left(\sum_{j=1}^{3}\frac{se_{jl'l}G_{jA}\Gamma_{Z_j}M_{Z_j}}{(s-M_{Z_j}^2)^2+\Gamma_{Z_j}^2M_{Z_j}^2}\right)^2\right]A_{LR}(s,t,u) \\
&+8\left[\left(\sum_{j=1}^{3}\frac{se_{jl'l}(G_{jV}+G_{jA})(s-M_{Z_j}^2)}{(s-M_{Z_j}^2)^2+\Gamma_{Z_j}^2M_{Z_j}^2}\right)\left(\sum_{k=1}^{2}\frac{t(G_L)_{l'k}(G_L)_{lk}}{t-m_{\nu_k}^2}\right)\right. \\
&\quad\left.+\left(\sum_{j=1}^{3}\frac{se_{jl'l}(G_{jV}-G_{jA})(s-M_{Z_j}^2)}{(s-M_{Z_j}^2)^2+\Gamma_{Z_j}^2M_{Z_j}^2}\right)\left(\sum_{k=1}^{2}\frac{t(G_R)_{l'k}(G_R)_{lk}}{t-m_{\nu_k}^2}\right)\right]I_{LR}(s,t,u) \\
&\quad+16\left[\left(\sum_{k=1}^{2}\frac{t(G_L)_{l'k}(G_L)_{lk}}{t-m_{\nu_k}^2}\right)^2+\left(\sum_{k=1}^{2}\frac{t(G_R)_{l'k}(G_R)_{lk}}{t-m_{\nu_k}^2}\right)^2\right]E_{LR}(s,t,u) \\
&\quad+16\left[\left(\sum_{k=1}^{2}\frac{(G_L)_{l'k}(G_R)_{lk}m_{\nu_k}}{t-m_{\nu_k}^2}\right)^2+\left(\sum_{k=1}^{2}\frac{(G_R)_{l'k}(G_L)_{lk}m_{\nu_k}}{t-m_{\nu_k}^2}\right)^2\right]E'_{LR}(s,t,u).
\end{aligned}
\tag{2.54}
$$

The functions A_{LR}, I_{LR}, E_{LR}, and E'_{LR}, which essentially are the traces appearing in (2.51), are given by

$$
\begin{aligned}
A_{LR}(s,t,u) &= \left(\frac{ut}{M'^2M^2}-1\right)\left(\frac{1}{4}-\frac{M'^2+M^2}{2s}+\frac{8M'^2M^2+(M'^2+M^2)^2}{4s^2}\right) \\
&\quad+2\frac{(M'^2+M^2)}{s}\left(\frac{(s-M'^2-M^2)^2}{4M'^2M^2}-1\right), \\
I_{LR}(s,t,u) &= \left(\frac{ut}{M'^2M^2}-1\right)\left(\frac{1}{4}-\frac{M'^2+M^2}{4s}-\frac{M'^2M^2}{st}\right)-\left(1+\frac{M'^4+M^4}{2M'^2M^2}\right) \\
&\quad+\left(\frac{M'^2+M^2}{2M'^2M^2}\right)s+\frac{M'^2+M^2}{t}, \\
E_{LR}(s,t,u) &= \left(\frac{ut}{M'^2M^2}-1\right)\left(\frac{1}{4}+\frac{M'^2M^2}{t^2}\right)+\frac{1}{2}\left(\frac{1}{M'^2}+\frac{1}{M^2}\right)s, \\
E'_{LR}(s,t,u) &= \frac{1}{2}\left[M'^2\left(\frac{ut}{M'^4}-1\right)+M^2\left(\frac{ut}{M^4}-1\right)+2s\left(\frac{t^2}{4M'^2M^2}+1\right)\right].
\end{aligned}
\tag{2.55}
$$

The last line of eq (2.54) consists of the contribution from the neutrino flip term.

We will now derive the contributions arising from the higgs exchanges in the s-channel. Substituting (2.46) into (2.51) we obtain the expression

$$
(k;\mu,\nu)(k';\mu',\nu')g^{\mu\mu'}g^{\nu\nu'}\mathrm{Tr}(\not p\,\not p') = \frac{s}{2b}[8b+(s-a)^2] \equiv 8H_{LR}(s), \tag{2.56}
$$

where $a = M^2+M'^2$, $b = (MM')^2$, and M and M' are the masses of the W-bosons in the final state. Here we have also introduced the notation

$$
(k;\mu,\nu) = g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}. \tag{2.57}
$$

The higgs exchange amplitude interferes only with the flip part of the neutrino exchange amplitude, because in the other amplitudes the electron helisities do not match. The interference yields the factor

$$(k;\mu,\nu)(k';\mu',\nu')g^{\nu\nu'}\mathrm{Tr}(\not p\ \not p'\gamma_{\mu'}\gamma_\mu) = \frac{s}{b}[4b-(s-a)^2] \equiv 8J_{LR}(s,t). \tag{2.58}$$

We have defined above new Lorentz-invariant functions $H_{LR}(s)$ and $J_{LR(s,t)}$ in resemblance with our earlier definition of the function $E'_{LR}(s,t,u)$,

$$\begin{aligned}(k;\mu,\nu)(k';\mu',\nu')\mathrm{Tr}(\not p\gamma_\nu\gamma_{\nu'}\ \not p'\gamma_{\mu'}\gamma_\mu) &= \frac{2}{b}[(s-2a)t^2-2a(s-a)t+2b(2s-a)] \\ &\equiv 8E'_{LR}(s,t,u) \end{aligned} \tag{2.59}$$

The squaring of the higgs exchange amplitude yields an expression with H_{LR} as an overall factor and the interference of the higgs exchange and the neutrino exchange amplitudes yields an expression with J_{LR} as an overall factor. In our special scenario of the physical higgses these amplitudes lead in the case of the process $e^+e^- \to W_1^- W_2^+$ to

$$\begin{aligned} <\Sigma|M|^2>^{\phi}_{e^+e^-\to W_1^-W_2^+} &= \left(\frac{g_L g_R}{4}\right)^2 (m_1+m_2)^2 \sin^2 2\eta\, 2H_{LR}(s) \\ &\times \left[\frac{\cos^2 2\zeta}{(s-M_{2r}^2)^2}+\frac{1}{(s-M_{2i}^2)^2}\right], \end{aligned} \tag{2.60}$$

$$\begin{aligned} <\Sigma|M|^2>^{\nu-flip}_{e^+e^-\to W_1^-W_2^+} &= \left(\frac{g_L g_R}{4}\right)^2 \sin^2 2\eta(\cos^2 2\zeta+1)\tfrac{1}{2}\, E'_{LR}(s,t,u) \\ &\times \left[\frac{m_1}{t-m_1^2}+\frac{m_2}{t-m_2^2}\right]^2, \end{aligned} \tag{2.61}$$

$$\begin{aligned} <\Sigma|M|^2>^{\phi,\nu-flip}_{e^+e^-\to W_1^-W_2^+} &= -2\left(\frac{g_L g_R}{4}\right)^2 (m_1+m_2)\sin^2 2\eta\, J_{LR}(s,t) \\ &\times \left[\frac{\cos^2 2\zeta}{s-M_{2r}^2}+\frac{1}{s-M_{2i}^2}\right]\left[\frac{m_1}{t-m_1^2}+\frac{m_2}{t-m_2^2}\right]. \end{aligned} \tag{2.62}$$

One can easily convince oneself of the cancellation of these three contributions to the total cross section when $s \to \infty$ by noticing that in this limit the limits of the t integration are $-s$ and 0.

Chapter 3

Numerical results and discussion

The gauge boson and neutrino exchange contributions to the W-pair production was worked out in detail in [5] and [6]. The experimental input [1] used to fix the various parameters of the model in those computations is listed in Table 3. In this work we used the same data and the same computer code as earlier to obtain the numerical results from the gauge boson and neutrino exchange channels, to which we then added the contribution from the higgs exchange channel. In the following we will consider the features of the latter calculation.

Deshpande et al [2] have recently pointed out that the phenomenology of the higgs sector depends crucially on the magnitude of the three coupling constants β_i in the general higgs potential (see Appendix). These constants belong to the terms, which can symbolically be written as $\mathrm{Tr}(\Phi\Delta\Phi\Delta)$ where Δ appropriately refers to $\Delta_{R,L}$, ${\Delta_{R,L}}^\dagger$ and Φ to Φ, $\tilde{\Phi}$, $\Phi^\dagger$, $\tilde{\Phi}^\dagger$. If the β's do not vanish, the mass scales of the model are connected by the see-saw relation, which can be shortly written as $v_L = \gamma K_+^2/v_R$, where γ is a certain linear function of the couplings β_i. If we want to have neutrinos as Majorana particles and preserve the see-saw mechanism and at the same time keep the extra higgs particles and gauge bosons light enough to be accesible for the next generation accelerators, we have according to the analysis of Deshpande et al only one possibility. We have to fine-tune the γ at least to the order of 10^{-7}. If we constrain β's to vanish by some symmetry beyond the MLRSM, the mass scales v_L and v_R are disconnected, but there remains a remnant see-saw relation, which is most naturally satisfied by the condition $v_L = 0$.

Deshpande et al [2] have given the mass matrices of the higgses in the case $\beta_i = v_L = 0$. In these scenarios ϕ_-^{0i} and δ_R^{0i} are Goldstone fields and ϕ_+^{0i} and δ_L^{0i} are physical states. The state ϕ_+^{0i} must be heavy, about 10 TeV or more, otherwise the phenomenological constraints on flavour-changing neutral currents are violated. As to the W_1W_2-pair production, the ϕ_+^{0i} is the only higgs of the imaginary sector which contributes and it is now a mass eigenstate. Thus in order to calculate the cross section we can simply replace ϕ_2^{0i} with ϕ_+^{0i} in the formulas (2.60) and (2.62).

M_{W_1}	=	80.14 GeV
M_{Z_1}	=	91.175 GeV
Γ_{Z_1}	=	2.487 GeV
Γ_{Z_2}	=	$\Gamma_{Z_1}(M_{Z_2}/M_{Z_1})$
$\sin^2\theta_{WS}$	=	0.2327
g_v^2	=	0.0012 ± 0.0003
g_a^2	=	0.2492 ± 0.0011

Table 3.1: The experimental data used in the computation.

The mass matrix of the real sector of the neutral higgs fields is more complicated and we give it in the Appendix. It includes nine coupling constants of the higgs potential. Gunion et al [4] have presented a simple model, which can be retrieved by constraining the couplings $\alpha_2 = \lambda_4 = K_2 = 0$. Such a model is according to [2] phenomenologically acceptable. The conditon $K_2 = 0$, however, would mean that the right-handed and the left-handed vector bosons do not mix. We relax this condition by allowing a tiny mixture and write the corresponding mass matrix in the flavour-diagonal basis (ϕ_-^{0r}, ϕ_+^{0r}, δ_R^{or}, δ_L^{or}) as follows:

$$\begin{pmatrix} M_{11} + 8\Lambda\epsilon^2 & 4\Lambda\epsilon & M_{13} + \Delta\epsilon^2 & 0 \\ 4\Lambda\epsilon & M_{22} & \Delta\epsilon & 0 \\ M_{13} + \Delta\epsilon^2 & \Delta\epsilon & M_{33} & 0 \\ 0 & 0 & 0 & M_{44} \end{pmatrix}, \tag{3.1}$$

where in the notation of ref. [2]

$$\begin{cases} M_{11} = 2K_1{\lambda_1}^2 \\ M_{13} = \alpha_1 K_1 v_R \\ M_{22} = 2\Lambda + \alpha_3 v_R^2/2 \\ M_{33} = 2\rho_1 v_R^2 \\ M_{44} = (\rho_3 - 2\rho_1) v_R^2/2 \\ \Lambda = K_1^2(2\lambda_2 + \lambda_3) \\ \Delta = K_1 \alpha_3 v_R \\ \epsilon = K_2/K_1. \end{cases} \tag{3.2}$$

From this we see that δ_L^{or} has decoupled into a physical state. Substituting $K_2 = 0$ makes ϕ_+^{0r} decouple, too. The other physical states h^0 and H^0 are mixings of ϕ_-^{0r} and δ_R^{0r} and h^0 corresponds to the higgs in the standard model. The MLRSM with $g_L = g_R = g$ provide us with simple relations between the VEV's and the boson masses. Taking $K_2 = 0 = v_L$ we have

$$M_{W_L}^2 = g^2 K_1^2/4, \tag{3.3}$$

$$M_{W_R}^2 = g^2(K_1^2 + 2v_R^2)/4, \tag{3.4}$$

$$M_{Z_1}^2 = g^2 \frac{g^2 + 2g'^2}{g^2 + g'^2} K_1^2/4, \tag{3.5}$$

$$M_{Z_2}^2 = (g^2 + g'^2) v_R^2. \tag{3.6}$$

Here g' is coupling constant associated with the group $U(1)_{B-L}$. Abandoning the condition $K_2 = 0$, the mixing angle ζ of the charged bosons is given by

$$\tan 2\zeta = \frac{4M_{W_L}^2}{M_{W_R}^2} \frac{K_1 K_2}{K_+^2}. \tag{3.7}$$

Giving g its standard model value 0.65 we can see from these equations that $M_{W_R} \approx$ 0.5-1.5 TeV means $v_R \approx$ 1 - 3 TeV. The mass of the lighter charged boson $M_{W_L} \approx 0.08$ TeV is consistent with $K_1 \approx 0.25$ TeV. Equation (3.7) shows that the previously quoted experimental uper limit 0.003 of ζ is consistent with the $\epsilon \equiv K_2/K_1 \approx 0.25$, if $M_{W_R} = 0.5$ TeV, but the flavour-changing interactions might require a much lower value for ϵ. One has to require $M_+^2 = M_{22} \geq 10$ TeV. Assuming the coefficients λ of the order of one, one must set the Deshpande parameter $\alpha_3 = 2$ - 5 to keep the value of v_R in the range of 1 - 3 TeV. The parameter ρ_1 is also better to have its value in the range 2 - 5 in order to have M_{33} large enough to ensure a light h^0. Then because the ρ_3 must be greater than $2\rho_1$, its value increases, however, to the range 4 - 10. It could be more natural that all the parameters were of the order of one, but then M_{W_R} were too heavy to be seen in the linear colliders of the near future. Having the parameters in the above-mentioned ranges we obtain the following mass spectrum ($K_2 = 0$):

$$\begin{cases} M_h^2 \approx M_{11} - M_{13}^2/M_{33}, \\ M_{+r}^2 = M_{22}, \\ M_H^2 \approx M_{33} + M_{13}^2/M_{33}. \end{cases} \tag{3.8}$$

We can apply for these higgses the formulas (2.60) and (2.62) as the flavour-diagonal basis and the real-imaginary component basis are identical in the case $K_2 = 0$ (see eq. (2.33)).

A non-vanishing K_2 causes the mixing between the fields ϕ_+^{0r}, h^0, and H^0. The mixing between the fields ϕ_+^{0r} and h^0 is up to the leading term in ϵ given by

$$\frac{[(M_{11} - M_{13} - M_h^2)\Delta - 8M_{13}\Lambda]\epsilon^2}{(M_{+r}^2 - M_h^2)\sqrt{M_{13}^2 + (M_{11} - M_h^2)^2}} \approx \epsilon^2(\Delta + 8\Lambda)/M_{22} \approx 0.05\epsilon^2. \tag{3.9}$$

Because ϵ^2 must be at most about 0.06, the pair production of the light heavy W through h_0 exchange would be negligible in the next generation colliders. We will neglect it in the following.

The smallness of the higgs exchange and neutrino flip contributions is due to the relation between the mixing angle of the neutrinos and their masses

$$\tan\eta \approx \sqrt{\frac{m_{\nu_1}}{m_{\nu_2}}}, \tag{3.10}$$

which has its origin in the see-saw mechanism of the neutrino mass generation. In our previous work [5] we followed a model independent approach in that we didn't fix ourselves to the simple see-saw model realized in the MLRSM. We thus kept neutrino masses and their mixing angle independent variables, in contrast with eq. (3.10). We didn't specify the elements of the model beyond the MLRSM, which would be responsible of the violation of the relation (3.10); extra neutral fermions and/or higgses. Hence the cross section formulas derived didn't show a good high-energy behaviour as the contributions of these unspecified particles were overlooked.

In the present paper we restrict ourselves to the MLRSM, where the relation (3.10) is valid. Phenomenologically the mixing angle η is not strictly bounded, but the see-saw mechanism with the experimental upper bound (about 10 eV) of the electron neutrino mass gives $\eta \approx 10^{-5}$ for $m_{\nu_2} = 100$ GeV. The amplitudes (2.46) and (2.47) have an overall factor $(m_{\nu_1} - m_{\nu_2})\sin\eta$ which determines the order of magnitude of the cross sections. According to eq. (3.10), the contribution from the higgses has as an overall factor $m_{\nu_1}m_{\nu_2}$ and we would expect the cross section to increase with the m_{ν_2} above some value of the m_{ν_2}.

In Figures 1 and 2 we present the total cross section of the process $e^+e^- \to W_1^- W_2^+$ as a function of the CM-energy E_{CM} for three values of the heavy neutrino mass $m_{\nu_2} \equiv m$ applying the relation (3.10). In both of the figures we have taken $M_{W_2} = 0.5$ TeV and used for the mass of the two higgses the value 10 TeV. In Fig. 1 we have assumed $M_{Z_2} = 1$ TeV, which causes a strong resonance the cross section exceeding the value 3 fbarn at the peak. In Fig. 2 we have $M_{Z_2} = 0.5$ TeV, so that the process takes place above the Z_2 pole. Then the cross section is quite small, of the order of 0.01 fbarn. The contributions arising from the higgses and the mass flip term of the neutrinos are totally negligible. In the region $E_{CM} \approx 1$ TeV and for $m_{\nu_2} = 1$ TeV, the contribution from neutrino mass flip is about $4.3 \cdot 10^{-11}$ pbarn, the pure higgs contribution about $2.3 \cdot 10^{-14}$ pbarn, and the contribution from their interference about $1.5 \cdot 10^{-12}$ pbarn.

Our numerical results show that as the m_{ν_2} increases to 100 TeV, the contributions from the boson and neutrino exchanges, excluding the mass flip term, reach a constant value, which depend remarkably on the collision energy and is about 0.020 fbarn for $E_{CM} = 1$ TeV. The other contributions behave in a very simple way above 10 TeV, too. The neutrino mass flip contribution is inversily proportional to the cubic power of the m_{ν_2}, the pure higgs contribution is directly and the cross term is inversily proportional to m_{ν_2}. From this we can deduce that the enormous mass of 10^{10} TeV were needed for ν_2 for the higgs contribution to be on the level of 1 fbarn. This does not mean that the higgs exchange processes were totally unimportant, because at high energies their contribution increases relative to the other channels as does the mass flip term of the neutrinos. We illustrate their role in Figures 3 - 5, where the mass of 5 TeV has been assumed for the both higgses.

In Fig. 3 we present separately the contributions of the neutrino mass flip term, higgs exchange terms and their interference as a function of the CM-energy in the case $m_{\nu_2} = 1$ TeV. Also plotted is the total cross section σ whose contributions in practice come from the gauge boson and neutrino

non-flip exchanges. One can see that the higgs and the neutrino mass flip terms are totally negligible at this energy range. In the figure their interference is constructive, because the energy lies below the masses of the higgses.

In Figures 4 - 5 we have abandoned the relation (3.10) and use for illustration for the neutrino mixing angle the value $\sin\eta = 0.01$. In Fig. 4 at the energy 2.5 TeV for $m_{\nu_2} = 1$ TeV the contributions from the neutrino mass flip term, the cross term, and the pure higgs term are 2.8, 2.2 and 0.5 fbarn, respectively. The behaviour of the cross section as a function of the m_{ν_2} is changed, because the interrelation of the neutrino mass and the mixing angle η is now different from eq. (3.10). The effect is that the cross sections depend more strongly on the neutrino mass m_{ν_2}. In Fig. 5 we illustrate the situation when the masses of the higgses are below the threshold of the process. We assume that both of the higgses have mass of 0.5 TeV and the neutrino mixing angle has the value 0.01. We note that actually one of the mediating higgses should contain a tiny mixture of a standard model like light higgs. This scalar most probably lies in the region of 100 - 300 GeV, but its contribution is quite negligible because of the smallness of the mixing angle η.

We do not consider here the processes $e^+e^- \to W_1^- W_1^+$ or $e^+e^- \to W_2^- W_2^+$. In our previous paper [5] we found out that the neutrino mass flip term is negligible and the contribution from the higgses is at best of the same order of magnitude, as one can convince oneself from the formulas of the amplitudes. We also note that these cross sections are insensitive to the value of the neutrino mixing angle η when this is below 0.01.

As a conclusion we can say that the process $e^+e^- \to W_1^- W_2^+$ might be measurable in the near future, if the Z_2 resonance lies in the energy region of the colliders. Then its total cross section would be of order of 1 fbarn given the phenomenological constraints on the MLRSM and assuming neutrinos to be the Majorana particles. The higgs exchange channels are totally unimportant in the energies nearby 1 TeV, but they are needed to balance the exchange channel of the neutrinos in the high-energy limit.

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Appendix A

Higgs potential and mass matrix of neutral components

In order to clarify the notation we give the most general form of the Higgs potential derived by Deshpande et al [2], which is invariant under the left-right symmetry $\Delta_R \leftrightarrow \Delta_L$, $\Phi \leftrightarrow \Phi^\dagger$:

$$
\begin{aligned}
V = \; & - \mu_1^2[\mathrm{Tr}(\Phi^\dagger\Phi)] - \mu_2^2[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)] - \mu_3^2[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)] \\
& + \lambda_1[\mathrm{Tr}(\Phi\Phi^\dagger)]^2 + \lambda_2\{[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)]^2 + [\mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)]^2\} + \lambda_3[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger)\mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)] \\
& + \lambda_4\{\mathrm{Tr}(\Phi\Phi^\dagger)[\mathrm{Tr}(\tilde{\Phi}\Phi^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)]\} + \rho_1\{[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)]^2 + [\mathrm{Tr}(\Delta_R\Delta_R^\dagger)]^2\} \\
& + \rho_2[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger)] + \rho_3[\mathrm{Tr}(\Delta_L\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger)] \\
& + \rho_4[\mathrm{Tr}(\Delta_L\Delta_L)\mathrm{Tr}(\Delta_R^\dagger\Delta_R^\dagger) + \mathrm{Tr}(\Delta_L^\dagger\Delta_L^\dagger)\mathrm{Tr}(\Delta_R\Delta_R)] \\
& + \alpha_1\{\mathrm{Tr}(\Phi\Phi^\dagger)[\mathrm{Tr}(\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Delta_R\Delta_R^\dagger)]\} \\
& + \alpha_2[\mathrm{Tr}(\Phi\tilde{\Phi}^\dagger)\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\Phi^\dagger\tilde{\Phi})\mathrm{Tr}(\Delta_L\Delta_L^\dagger)] \\
& + \alpha_2^*[\mathrm{Tr}(\Phi^\dagger\tilde{\Phi})\mathrm{Tr}(\Delta_R\Delta_R^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Phi)\mathrm{Tr}(\Delta_L\Delta_L^\dagger)] \\
& + \alpha_3[\mathrm{Tr}(\Phi\Phi^\dagger\Delta_L\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Phi\Delta_R\Delta_R^\dagger)] + \beta_1[\mathrm{Tr}(\Phi\Delta_R\Phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Delta_L\Phi\Delta_R^\dagger)] \\
& + \beta_2[\mathrm{Tr}(\tilde{\Phi}\Delta_R\Phi^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\tilde{\Phi}^\dagger\Delta_L\Phi\Delta_R^\dagger)] + \beta_3[\mathrm{Tr}(\Phi\Delta_R\tilde{\Phi}^\dagger\Delta_L^\dagger) + \mathrm{Tr}(\Phi^\dagger\Delta_L\tilde{\Phi}\Delta_R^\dagger)].
\end{aligned}
\tag{A.1}
$$

All terms in the potential are self-conjugate exept for the α_2 term. So α_2 is the only parameter which may be complex. We have already taken all the other couplings in this work real and in order to avoid explicit CP violation α_2 should be taken to be real, too. The mass matrix of the higgses has been presented by Deshpande et al in this case. We give their result for the neutral higgses in the flavour-diagonal basis assuming the simplifying condition $v_L = 0$. The symmetric block corresponding to the real components of the higgses is in the $(\phi_-^{0r}, \phi_+^{0r}, \delta_R^{or}, \delta_L^{or})$ basis:

$$
\begin{aligned}
M_{11} &= 2\lambda_1 K_+^2 + 8K_1^2K_2^2(2\lambda_2 + \lambda_3)/K_+^2 + 8K_1K_2\lambda_4, && \text{(A.2)} \\
M_{12} &= 4K_1K_2K_-^2(2\lambda_2 + \lambda_3)/K_+^2 + 2\lambda_4K_-^2, && \text{(A.3)} \\
M_{13} &= \alpha_1 v_R K_+ + K_2 v_R(4\alpha_2K_1 + \alpha_3K_2)/K_+, && \text{(A.4)} \\
M_{22} &= (4\lambda_2 + 2\lambda_3)K_-^4/K_+^2 + \alpha_3 v_R^2 K_+^2/(2K_-^2), && \text{(A.5)} \\
M_{23} &= v_R(2\alpha_2K_-^2 + \alpha_3K_1K_2)/K_+, && \text{(A.6)} \\
M_{24} &= v_R(\beta_1K_1 + 2\beta_3K_2)K_+/(2K_1), && \text{(A.7)} \\
M_{33} &= 2\rho_1 v_R^2, && \text{(A.8)} \\
M_{44} &= -v_R^2(2\rho_1 - \rho_3)/2, && \text{(A.9)} \\
M_{14} &= M_{34} = 0, && \text{(A.10)}
\end{aligned}
$$

and the symmetric block corresponding to the imaginary components of the higgses is in the $(\phi_-^{0i}, \phi_+^{0i}, \delta_R^{oi}, \delta_L^{oi})$ basis:

$$
\begin{aligned}
M_{22} &= -2K_+^2(2\lambda_2 - \lambda_3) + \alpha_3 v_R^2 K_+^2/(2K_-^2), && \text{(A.11)}\\
M_{24} &= v_R(\beta_1 K_1 + 2\beta_3 K_2)K_+/(2K_1), && \text{(A.12)}\\
M_{44} &= -v_R^2(2\rho_1 - \rho_3)/2, && \text{(A.13)}\\
M_{11} &= M_{12} = M_{13} = M_{14} = M_{23} = M_{33} = M_{34} = 0. && \text{(A.14)}
\end{aligned}
$$

In these equations $K_\pm = \sqrt{K_1^2 \pm K_2^2}$, and K_1 and K_2 refer to the vacuum expectation values of the matrix Φ as in Chapter 2.

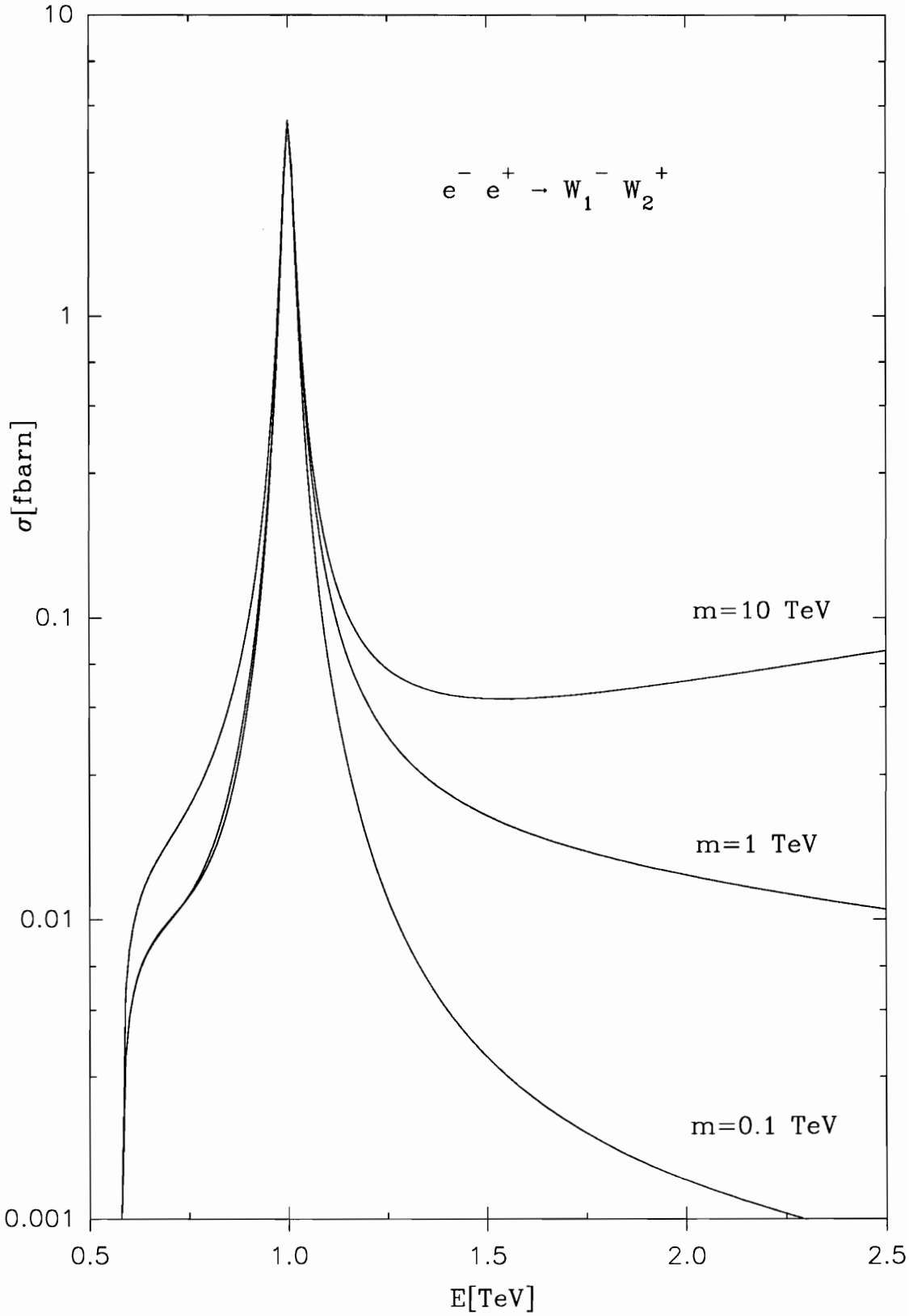
10
1
0.1
0.01
0.001
σ[fbarn]
0.5
1.0
1.5
2.0
2.5
E[TeV]
e⁻ e⁺ → W₁⁻ W₂⁺
m=10 TeV
m=1 TeV
m=0.1 TeV

Figure 1

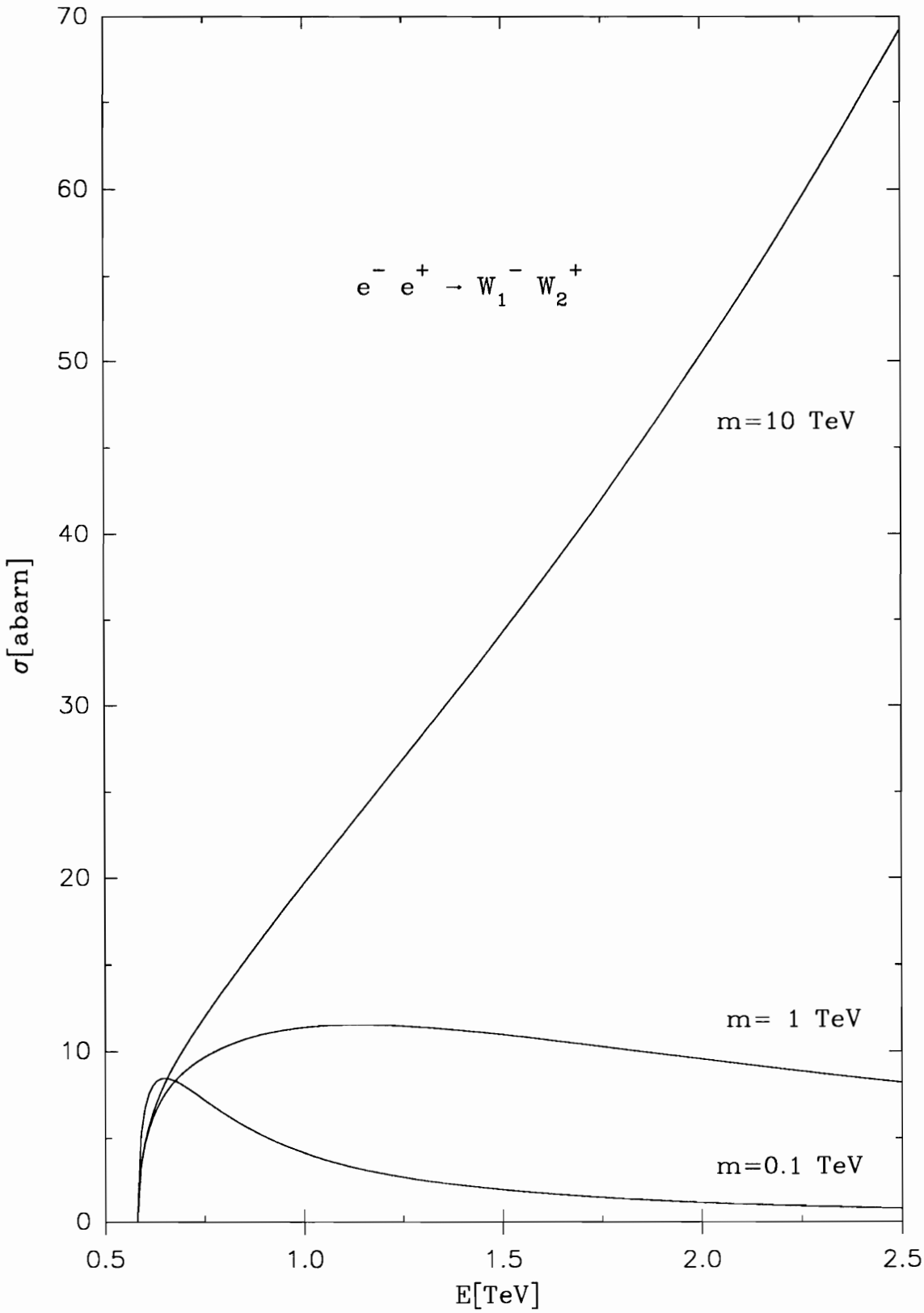
70
60
50
40
30
20
10
0
σ[abarn]
0.5
1.0
1.5
2.0
2.5
E[TeV]
e⁻ e⁺ → W₁⁻ W₂⁺
m=10 TeV
m= 1 TeV
m=0.1 TeV

Figure 2

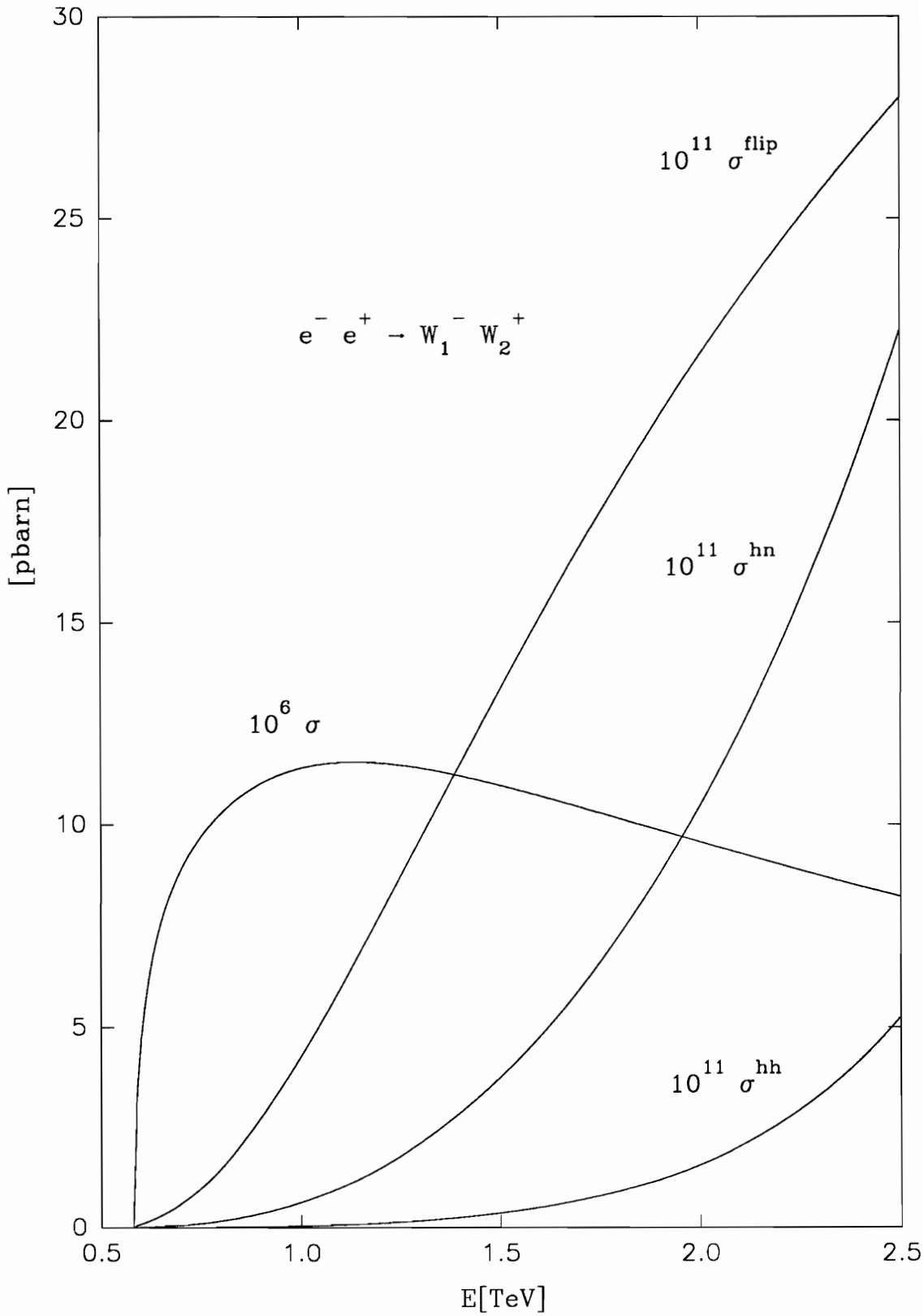
30
25
20
15
10
5
0
[pbarn]
0.5
1.0
1.5
2.0
2.5
E[TeV]
$e^- e^+ \to W_1^- W_2^+$
$10^{11}\ \sigma^{flip}$
$10^{11}\ \sigma^{hn}$
$10^{6}\ \sigma$
$10^{11}\ \sigma^{hh}$

Figure 3

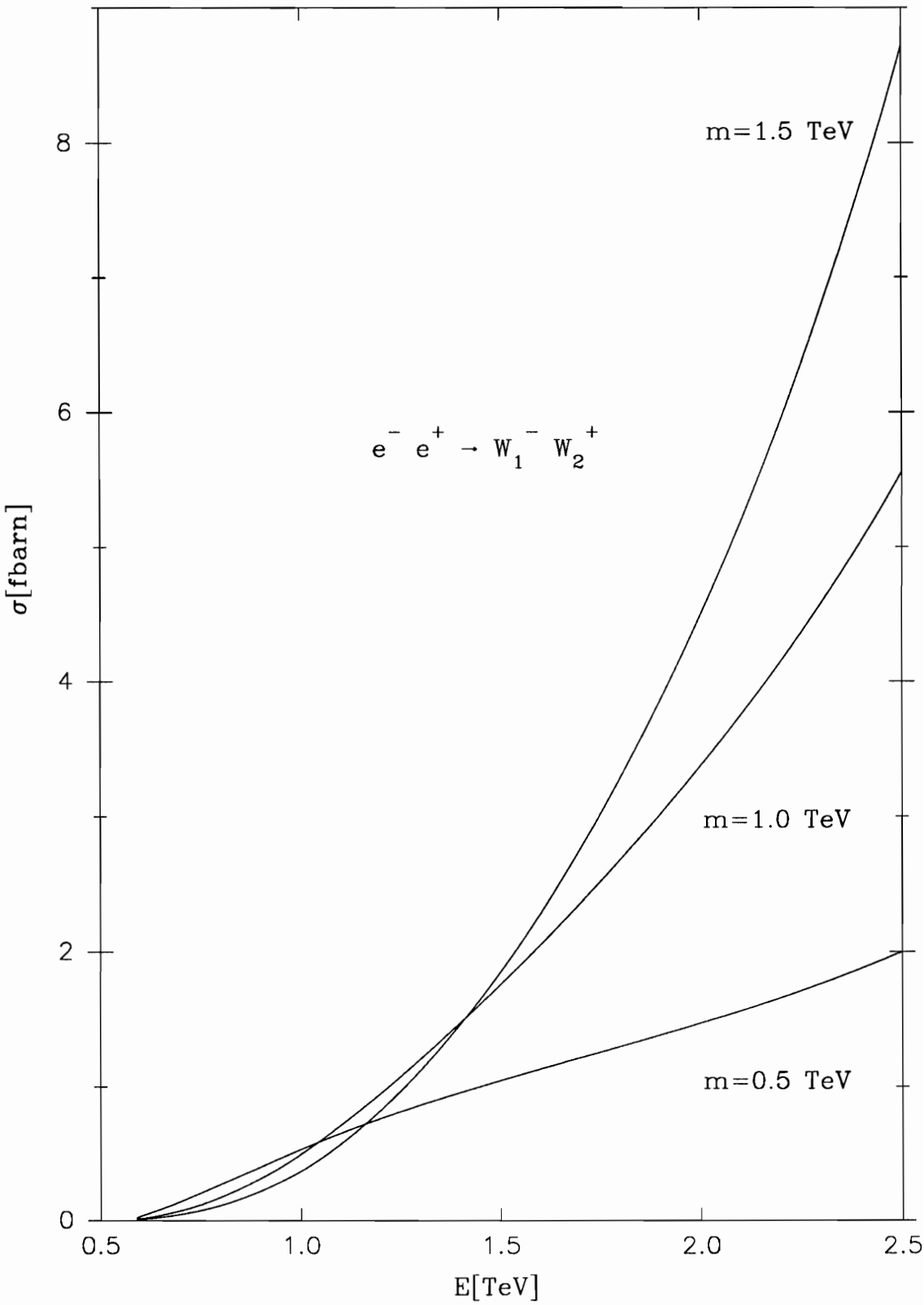
m=1.5 TeV
m=1.0 TeV
m=0.5 TeV
$e^- e^+ \rightarrow W_1^- W_2^+$
σ[fbarn]
E[TeV]
0
2
4
6
8
0.5
1.0
1.5
2.0
2.5

Figure 4

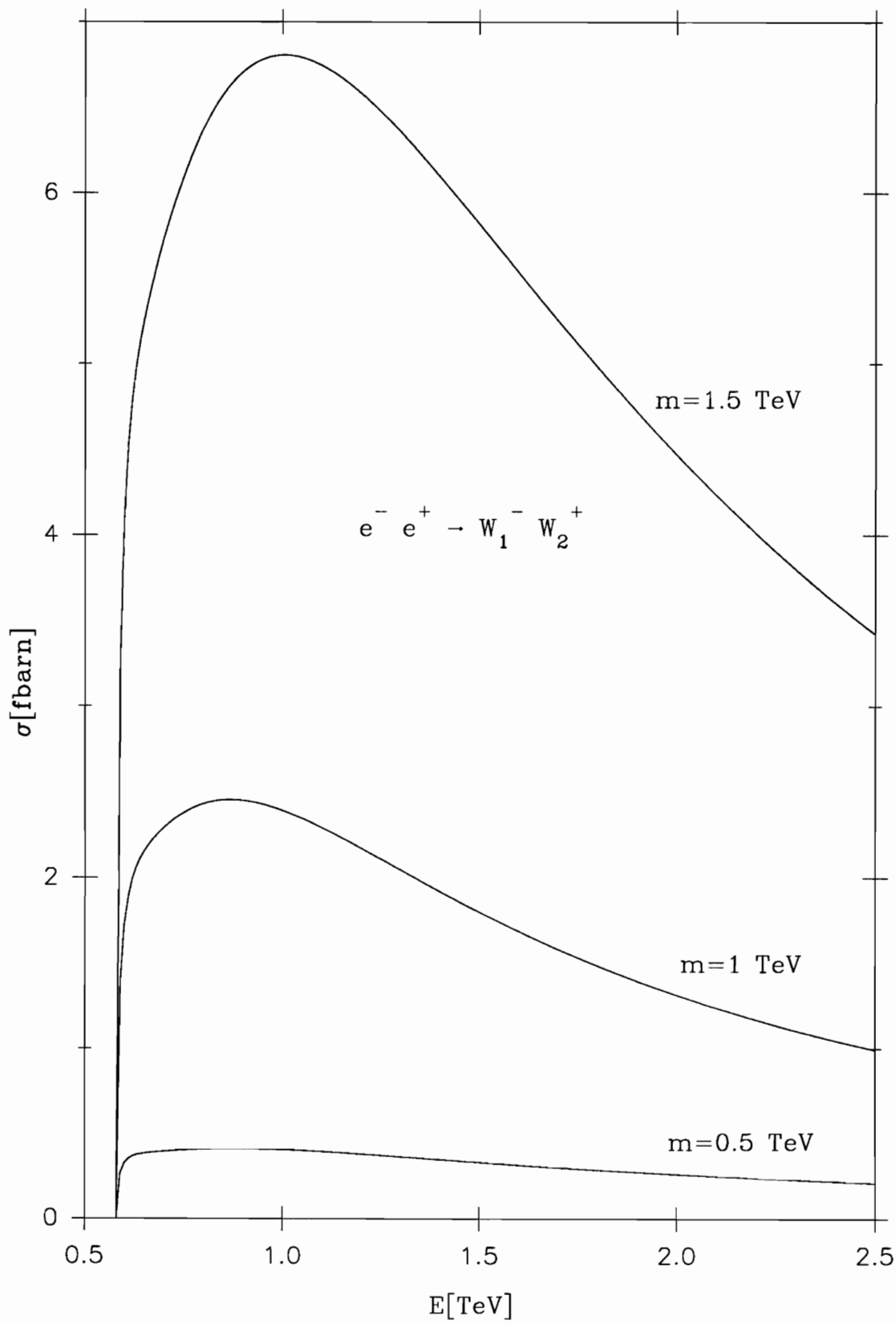

Figure 5