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Chiral Logarithmic Corrections to Heavy to Light Semileptonic Decays of the Type $H \rightarrow \pi e \nu$

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ABSTRACT

Using an effective chiral Lagrangian that describes the low-momentum interactions of mesons containing a single heavy quark with the pseudo Goldstone bosons, we calculate the chiral logarithmic corrections to transitions of the type $H \rightarrow \pi e \nu$ ($\pi \in \{\pi^{\pm}, \pi^0\}; H \in \{D, B\}$) and present improved estimates for the heavy-meson coupling g to the axial-vector Goldstone current based on recent experimental data. We find that the chiral logarithmic corrections reduce $|g|$ by $(20-30)\%$ relative to its leading-order value.

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1 Introduction

The investigation of processes involving heavy quarks has been a very active field over the last view years [1]. Recently, a synthesis of Chiral Perturbation Theory and the Heavy Quark Effective Theory has been developed which describes the strong interactions between light pseudo Goldstone bosons of low energy and mesons containing a single heavy quark [2, 3]. The corresponding effective interaction Lagrangian is given by

$$\begin{aligned}
\mathcal{L} \;=\; & \frac{f^2}{8}\mathrm{Tr}\left[\partial^\mu\Sigma\partial_\mu\Sigma^\dagger\right] + \lambda_0\mathrm{Tr}[m_q\Sigma + \Sigma^\dagger m_q] - \\
& -i\mathrm{Tr}\left[\bar{H}_a v_\mu \partial^\mu H_a\right] + \frac{i}{2}\mathrm{Tr}\left[\bar{H}_a v^\mu H_b\left(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger\right)_{ba}\right] + \\
& +i\frac{g}{2}\mathrm{Tr}\left[\bar{H}_a H_b \gamma^\mu\gamma_5\left(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger\right)_{ba}\right] + \\
& +\lambda_1\mathrm{Tr}\left[\bar{H}_a H_b\left(\xi m_q\xi + \xi^\dagger m_q\xi^\dagger\right)_{ba}\right] + \\
& +\lambda_1'\mathrm{Tr}\left[\bar{H}_a H_a\left(m_q\Sigma + \Sigma^\dagger m_q\right)_{bb}\right] + \\
& +\frac{\lambda_2}{m_Q}\mathrm{Tr}\left[\bar{H}_a\sigma_{\mu\nu}H_a\sigma^{\mu\nu}\right] + \ldots,
\end{aligned} \tag{1}$$

where the ellipsis denotes terms with more derivatives, more factors of the light-quark mass matrix

$$m_q = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} \tag{2}$$

or more factors of $1/m_Q$ associated with the heavy-quark spin symmetry violation. The 4×4 matrices H_a describe heavy mesons of four-velocity v with flavour content $\bar{q}_a Q$ ($Q \in \{c,b\}; q_1 = u, q_2 = d, q_3 = s$) and are combinations of the pseudoscalar and vector meson fields $P_a^{(Q)}$ and $P_{a\mu}^{*(Q)}$ (note that $P_{a\mu}^{*(Q)}v^\mu = 0$) [4, 5]

$$H_a^{(Q)} = \frac{(1+\not{v})}{2}\left[P_{a\mu}^{*(Q)}\gamma^\mu - P_a^{(Q)}\gamma_5\right]. \tag{3}$$

The hermitian conjugate fields are defined by

$$\bar{H}_a^{(Q)} = \gamma_0 H_a^{(Q)\dagger}\gamma_0. \tag{4}$$

Since factors of $\sqrt{m_P}$ have been absorbed in their definition, the heavy-meson fields have dimension 3/2. The pseudo Goldstone bosons are incorporated in the 3×3 unitary matrix

$$\Sigma = \xi^2 = \exp\left(2iM/f\right), \tag{5}$$

where

$$M = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}. \tag{6}$$

The Parameter f is related to the pion decay constant (see eq. (20)). Under $SU(3)_L \times SU(3)_R$ chiral symmetry, these fields transform as

$$\begin{aligned} \vec{H}^T &\rightarrow \vec{H}^T U^\dagger \\ \xi &\rightarrow L\xi U^\dagger = U\xi R^\dagger \\ \Sigma &\rightarrow L\Sigma R^\dagger, \end{aligned} \tag{7}$$

where $L \in SU(3)_L$, $R \in SU(3)_R$ and U is the usual 3×3 unitary matrix that is introduced to transform matter fields in a chiral Lagrangian. Note that U is a complicated non-linear function of the matrices L, R and the pseudo Goldstone fields and is, therefore, in general, space-time dependent.

Under the heavy-quark spin symmetry group, we have

$$\begin{aligned} H_a &\rightarrow SH_a \\ \bar{H}_a &\rightarrow \bar{H}_a S^{-1}, \end{aligned} \tag{8}$$

where $S \in SU(2)_v$.

In order to study semileptonic decays of the type $H \rightarrow \pi e \nu$, we have to calculate the hadronic matrix elements of the left-handed QCD current

$$L_a^\nu = \bar{q}_a \gamma^\nu (1 - \gamma_5) Q, \tag{9}$$

which can be represented in the chiral theory as [2]

$$L_a^\nu = \left(\frac{i\alpha}{2}\right) \mathrm{Tr}\left[\gamma^\nu(1-\gamma_5)H_b^{(Q)}\right] \left(\xi^\dagger\right)_{ba} + \ldots \tag{10}$$

Here the ellipsis denotes terms with derivatives, factors of m_q or factors of $1/m_Q$.

In a recent analysis [6], the coupling constant g in eq. (1) has been estimated by using experimental data [7] for the decay $D^0 \rightarrow \pi^- e^+ \nu_e$. These estimates are improved in the present paper by taking into account the logarithmic corrections arising from one-loop diagrams in chiral perturbation theory. In the following discussion, we take the transition $D^0 \rightarrow \pi^- e^+ \nu_e$ as an example of heavy to light semileptonic decays of the type $H \rightarrow \pi e \nu$ ($\pi \in \{\pi^\pm, \pi^0\}; H \in \{D, B\}$). The corresponding results can be generalized in a straightforward way for the treatment of B decays as well. Note that g is independent of the heavy-quark flavour to leading order in Λ_{QCD}/m_Q due to the heavy-quark flavour symmetry.

2 The Decay $D^0 \to \pi^- e^+ \nu_e$ at Leading Order in the Effective Theory

Before calculating the logarithmic corrections from the one-loop diagrams depicted in Fig. 1, let us briefly recall the leading-order results. Parametrizing the hadronic matrix element needed for the transition $D^0 \to \pi^- e^+ \nu_e$ through

$$\left\langle \pi^-(p_\pi) \left| \bar{d}\gamma^\nu(1-\gamma_5)c \right| D^0(p_D) \right\rangle = f_+(q^2)(p_D + p_\pi)^\nu + f_-(q^2)(p_D - p_\pi)^\nu, \quad (11)$$

we find for the form factors $f_\pm(q^2)$ at small values for $v \cdot p_\pi$ [2, 6]:

$$f_+ + f_- = -\frac{\alpha}{\sqrt{m_D} f}\left[1 - g\frac{p_\pi \cdot v}{\Delta + p_\pi \cdot v}\right] \quad (12)$$

$$f_+ - f_- = -\frac{\alpha\sqrt{m_D}}{f}\left[\frac{g}{\Delta + p_\pi \cdot v}\right]. \quad (13)$$

Here we have used eqns. (1) and (10). The parameter α of the weak current in (10) is related to the decay constant of the "heavy" D meson through the leading-order relation [2]

$$\alpha = \sqrt{m_D} f_D \quad (14)$$

and $\Delta = m_{D^*} - m_D$ denotes the mass difference between the D^* and D mesons. The logarithmic dependence of α on the heavy-quark mass arising from perturbative QCD [2, 8, 9] has been absorbed in the decay constant f_D.

If we neglect the masses of the leptons in the final state, then only the form factor $f_+(q^2)$ contributes to the decay $D^0 \to \pi^- e^+ \nu_e$. In the paper by Casalbuoni et al. [6], the form factor $f_+(0)$ is related to $f_+(q^2_{max} = (m_D - M_\pi)^2)$, which can be obtained from (12) and (13), by assuming a single pole dependence for $f_+(q^2)$:

$$f_+(q^2) = m_1^2 \frac{f_+(0)}{m_1^2 - q^2}, \quad (15)$$

where m_1 is the mass of the $D^*(2010)$ state in the present case. With the help of this relation, the parameter g in eq. (1) describing the heavy-meson coupling to the axial-vector Goldstone current can be estimated by using the value of $|f_+(0)|$ extracted from the experimental data [6]:

$$g_{l.o.} = -\left(\frac{\Delta + M_\pi}{m_D - M_\pi}\right)\left[\frac{2f}{f_D}\frac{m_1^2}{[m_1^2 - (m_D - M_\pi)^2]} f_+(0) + 1\right]. \quad (16)$$

Applying this procedure, Casalbuoni et al. find

$$g_{l.o.} = \begin{cases} 0.48 \pm 0.15 & (f_+(0) < 0) \\ -0.80 \pm 0.15 & (f_+(0) > 0). \end{cases} \quad (17)$$

3 Chiral Perturbation Theory for the Decay $D^0 \to \pi^- e^+ \nu_e$ at the One-Loop Level

Using the interaction Lagrangian specified in eq. (1), we are in a position to calculate chiral logarithms in the effective theory. These corrections break the $SU(3)_V$ light-quark flavour symmetry and allow to take into account the light-quark flavour dependence both of the meson decay constants f, f_D and of the mass splitting Δ. Following refs. [12, 13, 14], we neglect the up and down quark masses ($m_u = m_d = 0$) in comparison to the strange quark mass ($m_s \neq 0$) and use the Gell-Mann – Okubo formula to express M_η^2 in terms of M_K^2:

$$M_\eta^2 \approx \frac{4}{3} M_K^2. \tag{18}$$

Note that in recent publications logarithmic $SU(3)_V$-breaking corrections have already been calculated in chiral perturbation theory for the ratios f_{D_s}/f_D, B_{B_s}/B_B [13] and for the Isgur-Wise function [14].

In order to evaluate the one-loop diagrams in the effective theory, we use dimensional regularization and calculate in $\mathcal{D} = 4 - 2\epsilon$ space-time dimensions, where the coupling constants are modified in the following way:

$$\begin{aligned} g &\longrightarrow \mu^0 g \\ f &\longrightarrow \mu^{-\epsilon} f \\ \alpha &\longrightarrow \mu^{-\epsilon} \alpha. \end{aligned} \tag{19}$$

Here μ denotes the renormalization scale which is introduced to make the couplings g, f and α dimensionless in $\mathcal{D} \neq 4$ dimensions. The parameters f and α can be expressed in terms of the physical $\pi^\pm$ and D_a meson decay constants, f_π and f_{D_a}, respectively. Evaluating the corresponding one-loop diagrams, we find for the logarithmic corrections to f_π [10, 11]

$$f = f_\pi (1 + \chi), \tag{20}$$

where χ is given by

$$\chi = \frac{M_K^2}{16\pi^2 f_\pi^2} \ln\left(\frac{M_K^2}{\mu^2}\right). \tag{21}$$

On the other hand, the leading-order relation (14) between α and f_D is modified at the chiral one-loop level through logarithmic corrections as follows:

$$\alpha = \begin{cases} \sqrt{m_D} f_{D_{1/2}} \left[1 + \left(\frac{11}{18} + \frac{11}{6} g^2\right)\chi\right] \\ \sqrt{m_D} f_{D_3} \left[1 + \left(\frac{13}{9} + \frac{13}{3} g^2\right)\chi\right], \end{cases} \tag{22}$$

where the corresponding Feynman diagrams are given in ref. [13]. Note that (22) gives the same result for f_{D_s}/f_D as that presented by Grinstein et al. in ref. [13]. The chiral logarithmic corrections to the wavefunction renormalization of the heavy-meson fields are obtained by calculating the usual one-loop self-energy diagrams (see, e.g., refs. [12, 13, 14]):

$$\sqrt{Z_a} = \begin{cases} 1 - \frac{11}{6} g^2 \chi & \text{for } a \in \{1,2\} \\ 1 - \frac{13}{3} g^2 \chi & \text{for } a = 3, \end{cases} \tag{23}$$

where Z_a is defined by $P_a^{unren} = \sqrt{Z_a} P_a^{ren}$. Note that due to the heavy-quark spin symmetry, the wavefunction renormalizations for the P_a and P_a^* fields are equal. The same diagrams which renormalize the heavy-meson wavefunctions lead also to logarithmic corrections to the $P_a^* - P_a$ mass difference:

$$\Delta = \begin{cases} \Delta_{1/2} \left(1 + \frac{11}{9} g^2 \chi\right) \\ \Delta_3 \left(1 + \frac{26}{9} g^2 \chi\right), \end{cases} \tag{24}$$

where Δ denotes the leading-order mass splitting. At the chiral one-loop level, the pion wavefunction receives logarithmic corrections as well and the result being analogous to eq. (23) is given by

$$\sqrt{Z_\pi} = 1 + \frac{1}{3} \chi. \tag{25}$$

Expressing now the leading-order form factors (12) and (13) for the transition matrix element (11) in terms of the renormalized "physical" quantities given in eqns. (20), (22) and (24), we get at small values for $p_\pi \cdot v$

$$\begin{aligned} &\left\langle \pi^-(p_\pi) \left| \bar{d} \gamma^\nu (1 - \gamma_5) c \right| D^0(v) \right\rangle_{l.o.}^{ren} = \\ &= -\frac{m_D f_{D_1}}{f_\pi} \left[v^\nu + g \left(\frac{p_\pi^\nu - (p_\pi \cdot v) v^\nu}{p_\pi \cdot v + \Delta_2} \right) \right] \left[1 + \left(\frac{11}{6} g^2 - \frac{7}{18} \right) \chi \right] + \\ &+ \frac{m_D f_{D_1}}{f_\pi} g \left(\frac{p_\pi^\nu - (p_\pi \cdot v) v^\nu}{p_\pi \cdot v + \Delta_2} \right) \frac{11}{9} \left(\frac{\Delta_2}{p_\pi \cdot v + \Delta_2} \right) g^2 \chi. \end{aligned} \tag{26}$$

On the other hand, taking into account the corrections to the π^- and D^0 wavefunctions and calculating the one-loop diagrams shown in Fig. 1, we find

$$\begin{aligned} \left\langle \pi^-(p_\pi) \left| \bar{d} \gamma^\nu (1 - \gamma_5) c \right| D^0(v) \right\rangle_{1-loop} &= \frac{m_D f_D}{f} \left[v^\nu \left(\frac{11}{6} g^2 - \frac{7}{18} \right) + \right. \\ &\left. + g \left(\frac{p_\pi^\nu - (p_\pi \cdot v) v^\nu}{p_\pi \cdot v + \Delta} \right) \left\{ \frac{11}{18} + \frac{37}{18} g^2 + \frac{11}{9} g^2 \left(\frac{3 p_\pi \cdot v + 2\Delta}{p_\pi \cdot v + \Delta} \right) \right\} \right] \chi. \end{aligned} \tag{27}$$

Therefore, the hadronic amplitude for the decay $D^0 \to \pi^- e^+ \nu_e$ including logarithmic corrections arising from chiral one-loop diagrams is given by

$$\begin{aligned}
&\langle \pi^-(p_\pi) \left| \bar{d}\gamma^\nu (1-\gamma_5) c \right| D^0(v) \rangle = \\
&= -\frac{m_D f_{D_1}}{f_\pi} \left[v^\nu + g \left(\frac{p_\pi^\nu - (p_\pi \cdot v) v^\nu}{p_\pi \cdot v + \Delta_2} \right) \left\{ 1 - \left(1 + \frac{35}{9} g^2 \right) \chi \right\} \right] . \qquad (28)
\end{aligned}$$

Introducing the "renormalized" coupling constant g^{ren} through

$$g^{ren} = g \left[1 - \left(1 + \frac{35}{9} g^2 \right) \chi \right] , \qquad (29)$$

this matrix element can be written as

$$\langle \pi^-(p_\pi) \left| \bar{d}\gamma^\nu (1-\gamma_5) c \right| D^0(v) \rangle = -\frac{m_D f_{D_1}}{f_\pi} \left[v^\nu + g^{ren} \left(\frac{p_\pi^\nu - (p_\pi \cdot v) v^\nu}{p_\pi \cdot v + \Delta_2} \right) \right] . \qquad (30)$$

Note that g^{ren} describes the chiral logarithmic corrections to the $D^{*+} D^0 \pi^+$ vertex:

$$\mathcal{T}(D^{*+} \to D^0 \pi^+) = \frac{2(\epsilon \cdot p_\pi)}{f_\pi} g \left[1 - \left(1 + \frac{35}{9} g^2 \right) \chi \right] , \qquad (31)$$

where ϵ denotes the polarization vector of the D^{*+} meson. The result of eq. (31) is obtained by calculating the one-loop diagrams depicted in Fig. 2 and by taking into account both the D^{*+}, D^0, π^+ meson wavefunction renormalizations and eq. (20).

Applying now the procedure presented by Casalbuoni et al. in ref. [6], we find the equation

$$\begin{aligned}
& g \left[1 - \left(1 + \frac{35}{9} g^2 \right) \frac{M_K^2}{16\pi^2 f_\pi^2} \ln \left(\frac{M_K^2}{\mu^2} \right) \right] = \\
& = - \left(\frac{\Delta_2 + M_\pi}{m_D - M_\pi} \right) \left[\frac{2 f_\pi}{f_{D_1}} \frac{m_1^2}{[m_1^2 - (m_D - M_\pi)^2]} f_+(0) + 1 \right] = \\
& = g_{l.o.}^{ren} , \qquad (32)
\end{aligned}$$

where $g_{l.o.}^{ren}$ corresponds to the leading-order result (16). The numerical values for $g_{l.o.}$ given in eq. (17) are modified through eq. (32) at the renormalization scale $\mu = 1$ GeV as follows:

$$\begin{array}{lclll}
g_{l.o.} & = & 0.48 & (f_+(0) < 0) \longrightarrow g & = 0.40 \\
g_{l.o.} & = & -0.80 & (f_+(0) > 0) \longrightarrow g & = -0.61.
\end{array} \qquad (33)$$

The dependence of these results on the renormalization scale μ is plotted in Fig. 3 and changes the values for $|g|$ by about 5% within the range 0.8 GeV $\leq \mu \leq 1.2$ GeV.

4 Conclusions

In summary, we have found that the chiral logarithmic corrections arising from one-loop diagrams in chiral perturbation theory reduce the values for $|g|$ determined from the experimental data for the semileptonic decay $D^0 \to \pi^- e^+ \nu_e$ [6, 7] by $(20 - 30)\%$ relative to the leading-order results. These corrections break the $SU(3)_V$ symmetry and introduce a light-quark flavour dependence of the meson decay constants and of the $P_a^* - P_a$ mass difference into the calculation.

Note that the chiral one-loop corrections are quite large, because the strange quark mass is not very small in comparison to the chiral symmetry breaking scale, and not due to a breakdown in the derivative expansion of the effective chiral theory. Therefore, the next-order $SU(3)_V$-breaking corrections, which behave as M_K^2, could modify our results to some extent. Unfortunately, a quantitative treatment of these additional corrections is not possible at present [12, 13, 14].

The situation concerning the analysis of the corrections in $1/m_Q$ and in higher derivatives is similar to these next-order $SU(3)_V$-breaking corrections and requires the introduction of new coupling constants, which are unknown at present. Therefore, until estimates of these couplings are available, only the chiral logarithmic corrections calculated in this paper can be taken into account to the determination of the coupling g from the experimental data for the decay $D^0 \to \pi^- e^+ \nu_e$.

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Figure Captions

Figure 1: One-loop diagrams contributing to $D^0 \to \pi^- e^+ \nu_e$. The dot represents vertices in the chiral Lagrangian (1) and the square represents insertions of the weak current (10). Dashed lines denote pseudo Goldstone bosons. (Wavefunction renormalization graphs are not shown.)

Figure 2: One-loop diagrams contributing to $D^{*+} \to D^0 \pi^+$. (Wavefunction renormalization graphs are not shown.)

Figure 3: The dependence of the coupling $|g|$ determined from eq. (32) on the renormalization scale μ.

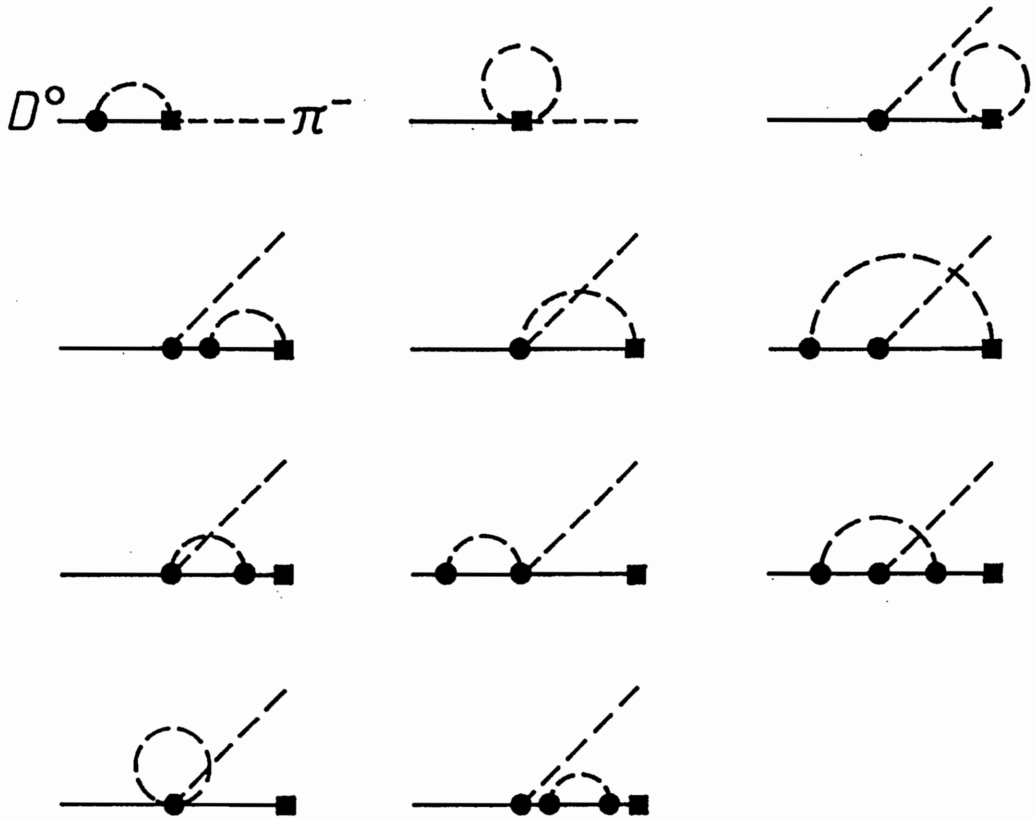

Figure 1:

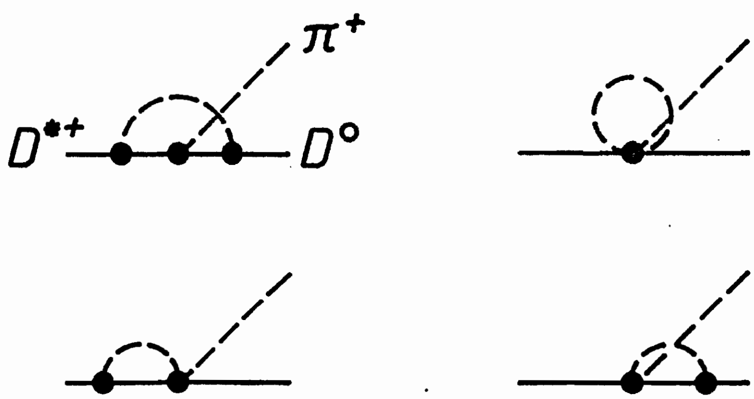

Figure 2:

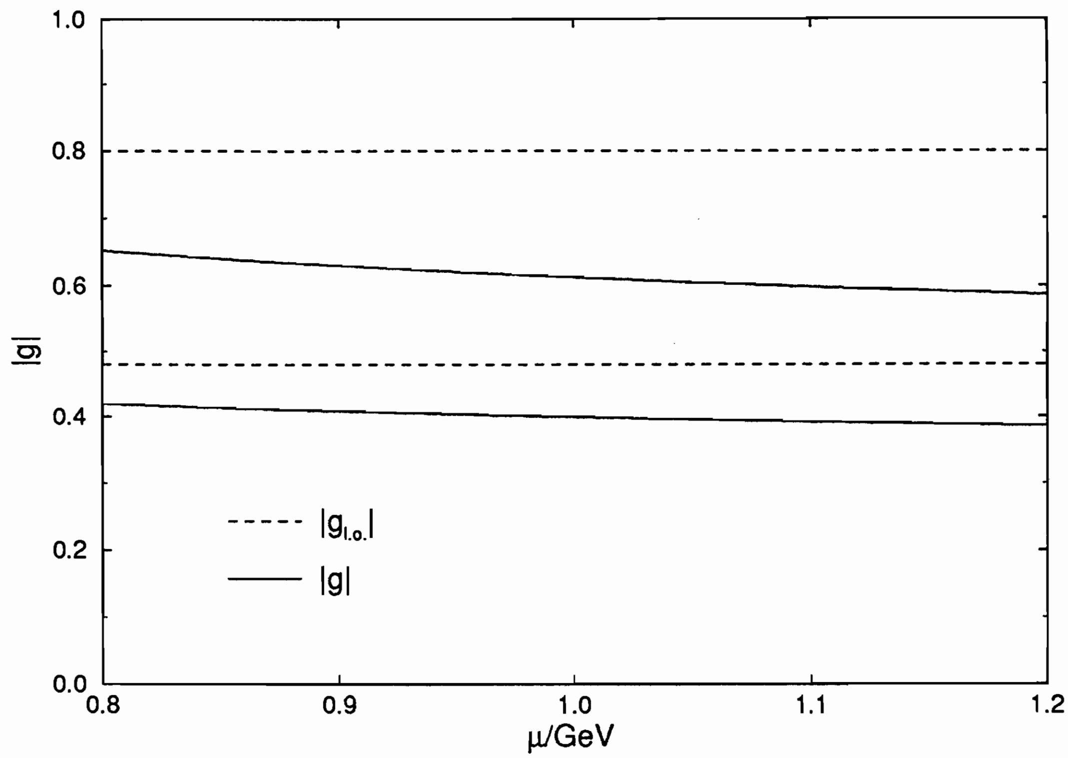

Figure 3: