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GAUGE INVARIANT BILOCAL EFFECTIVE LAGRANGIAN IN QCD-LIKE THEORIES[1]

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ABSTRACT

Gauge invariant bilocal effective Lagrangian for mesons is formally derived from the first principles of QCD. The coefficients of the bilocal meson fields in the effective Lagrangian are parametrized in terms of Green's functions of gluons.

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I. Introduction

The study of the effective Lagrangian for low lying mesons has received much attention in recently years not only for its power of studying low energy physics of hadrons [1]-[3] but also for its advantage of providing a convenient tool of studying the physics at TeV scale such as precise tests of the electroweak theory including possible contributions from new physics at higher energies [4]-[5] and the study of strongly interacting electroweak symmetry breaking mechanisms [6]. At low energies, it is sufficient to consider only the lightest pseudoscalar mesons which are regarded as pseudo-Goldstone-bosons due to spontaneous chiral symmetry breaking and describe them by local fields. The general form of the effective Lagrangian for these local pseudoscalar meson fields can be known from symmetry arguments [1]. To improve this description, it has been tried recently to construct effective Lagrangians in terms of bilocal fields [7]-[9] which may be extended to higher energy scales and reflect more detailed properties of QCD. Moreover, for higher energies, other light mesons like the vector and axial-vector mesons may also be relevant which are not directly related to chiral symmetry breaking. It seems that simple symmetry argument is not powerful enough for studying the form of such an effective Lagrangian. Therefore understanding the relation between such an effective Lagrangian and its underlying fundamental dynamics is of both theoretical and practical interest.

In this paper, we study this relation in QCD-like gauge theories. We start from the fundamental Lagrangian of the QCD and formally integrate out the gluon and quark fields to obtain the general form of the effective Lagrangian for bilocal meson fields. To construct gauge invariant bilocal composite quark operators understood

as the mesons, we transform the quark and gluon fields by a non-integratable phase factor and the transformed quark field is related to the constituent quark [10]. Due to the self-interactions of the gluons, the integration over gluon fields cannot be explicitly carried out. We formally express this integration through the gluon Green's functions which are not explicitly know. A formal effective Lagrangian for the transformed quark fields is then obtained. We then propose a method of introducing gauge invariant bilocal auxiliary fields for the gauge invariant bilocal composite quark operators, and with this the quark fields can be integrated out to obtain the formal effective Lagrangian for the gauge invariant bilocal auxiliary fields describing various kinds of mesons. In this method the coefficients of the bilocal fields in the effective Lagrangian can be related to the gluon Green's functions through a certain dynamical approach to the bilocal-field integration, which we take to be loop expansion in this paper. We shall show that the coefficients of the bilocal fields in the effective Lagrangian can all be expressed in terms of certain extented gluon Green's functions defined in Sec.II. The effective Lagrangian so obtained is only formal but its form is general in the sense that no approximation has been made so far as only the bilocal meson fields are considered. To realize this effective Lagrangian and make phenomenological predictions, one needs to develop proper approximations for evaluating the gluon Green's functions. If we simply truncate the effective Lagrangian for quarks in such a way that all higher dimensional quark operators beyond the four-quark interactions are dropped, the effective Lagrangian for the bilocal meson fields reduces to the form considered in Ref.[9]. Therefore one can at least model the bilocal functions as what Holdom did [9] which may lead to successful phenomenological consequences. The present form of the effective Lagrangian admits a more general study of the problem.

This paper is organized as follows. In Sec.II, we introduce the non-integratable phase factor to the quark and gluon fields, and formulate the generating functional which we need in the derivation of the gauge invariant bilocal effective Lagrangian. The physical meaning of the transformed quark field is also discussed. The integration over the transformed gluon field is formally expressed in terms of the gluon Green's functions. In Sec.III, we develop a general method of introducing gauge invariant bilocal auxiliary fields which can be applied to theories with arbitrary multi-fermion interactions. With this method, the quark field can be easily integrated out and we obtained the effective Lagrangian for the gauge invariant bilocal auxiliary fields. The form of the effective Lagrangian is determined order by order in the standard loop expansion in which the coefficients in the effective Lagrangian are expressed in terms of the gluon Green's functions in the underlying theory. In Sec.IV, we calculate the effective action of the gauge invariant bilocal theory and study the vacuum stability condition in this theory from which we can see the relation between the present theory and the approach by Holdom et.al.[8][9] Sec.V is a concluding remark, and some technical details are given in the Appendices.

II. Gauge Invariant Bilocal Composite Quark Operators and the Generating Functional

Consider a QCD-type gauge theory with $SU(N_c)$ local gauge symmetry. Let B^a_μ $(a = 1, 2, \cdots, N_c^2 - 1)$ be the gauge field, ψ^i_α be the fermion field with colour index α $(\alpha = 1, 2, \cdots, N_c)$ and flavour index i $(i = 1, 2, \cdots, N_f)$. From now on, we simply call ψ^i_α the "quark-field" and B^a_μ the "gluon-field" for convenience. The

Lagrangian is

$$\mathcal{L} = -\frac{1}{4}B^a_{\mu\nu}B^{a\mu\nu} - \overline{\psi}(i\partial\!\!\!/ - g\not{B})\psi \quad , \tag{1}$$

where λ_a's are Gell-Mann matrices,$B_\mu \equiv \frac{\lambda_a}{2}B^a_\mu$, and

$$B^a_{\mu\nu} \equiv \partial_\mu B^a_\nu - \partial_\nu B^a_\mu - g f_{abc} B^b_\mu B^c_\nu \quad ,$$

in which f_{abc} is the $SU(N_c)$ structure constant.

In order to construct the gauge invariant formalism, we introduce a non-integrable phase factor

$$U(x_1, x_2) \equiv P \; e^{-ig\int_{x_1}^{x_2} A_\mu(y)dy^\mu} \quad , \tag{2}$$

where P is a path-ordering operator, $A_\mu \equiv \frac{\lambda_a}{2}A^a_\mu$ is an external field. In (2), the path and the external field A^a_μ are not specified yet. We then transform ψ and B_μ into new fields Ψ and $\mathcal{B}_\mu$ as follows

$$\begin{aligned}
\psi(x) &\rightarrow \Psi(x, x_0) = U(x_0, x)\psi(x) \quad , \\
\overline{\psi}(x) &\rightarrow \overline{\Psi}(x, x_0) = \overline{\psi}(x)U(x, x_0) \quad , \\
B_\mu(x) &\rightarrow \mathcal{B}_\mu(x, x_0) = U(x_0, x)B_\mu(x)U(x, x_0) + \frac{i}{g}[\frac{\partial}{\partial x^\mu}U(x_0, x)]U(x, x_0) \quad ,
\end{aligned} \tag{3}$$

We can parametrize the space-time points on the path by $y^\mu(x_0, x, s)$ with $y^\mu(x_0, x, 0) = x^\mu$, $y^\mu(x_0, x, 1) = x_0^\mu$. Then $\mathcal{B}_\mu(x.x_0)$ can be expressed as (cf. Apppendix A)

$$\begin{aligned}
\mathcal{B}_\mu(x, x_0) &= U(x_0, x)[B_\mu(x) - A_\mu(x)]U(x, x_0) - \int_0^1 ds \frac{\partial y^\nu(x_0, x, s)}{\partial x^\mu}\frac{\partial y^\lambda(x_0, x, s)}{\partial s} \times \\
&\times U(x_0, y(x_0, x, s))A_{\lambda\nu}(y(x_0, x, s))U(y(x_0, x, s), x)U(x, x_0) \quad ,
\end{aligned} \tag{4}$$

in which $A_{\lambda\nu}(y)$ is defined as

$$A_{\lambda\nu}(y) \equiv \partial_\lambda A_\nu(y) - \partial_\nu A_\lambda(y) + ig[A_\lambda(y), A_\nu(y)] \quad . \tag{5}$$

We then introduce the external sources $J_S, J_P, J_V^\mu, J_A^\mu$ for the composite quark operators $-\overline{\Psi}\Psi, i\overline{\Psi}\gamma_5\Psi, \overline{\Psi}\gamma_\mu\Psi$, and $\overline{\Psi}\gamma_\mu\gamma_5\Psi$, respectively, and construct the generating functional for the Green's functions. Since A_μ is an external field, the functional measure is unchanged after the transformation (3), i.e. $D\psi D\overline{\psi}DB_\mu = D\Psi D\overline{\Psi}D\mathcal{B}_\mu$. So that

$$\begin{aligned}
Z[J] &= e^{iW[J]} \\
&= \int D\psi D\overline{\psi}DB_\mu exp i\int d^4x\{\mathcal{L}(\psi, \overline{\psi}, B_\mu) + \overline{\psi}(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + \not{J}_V + \not{J}_A\gamma_5)\psi\} \\
&= \int D\Psi D\overline{\Psi}D\mathcal{B}_\mu exp i\int d^4x\{\mathcal{L}(\Psi, \overline{\Psi}, \mathcal{B}_\mu) + \overline{\Psi}(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + \not{J}_V + \not{J}_A\gamma_5)\Psi\} \quad ,
\end{aligned} \tag{6}$$

The usual Fadeev-Popov gauge-fixing procedure should be further imposed in (6). Let the gauge-fixing condition be

$$F^a(\mathcal{B}_\mu(x, x_0)) = 0 \tag{7}$$

with the requirement

$$\frac{\lambda_a}{2}F^a(V(x_0)\mathcal{B}_\mu(x, x_0)V^\dagger(x_0)) = V(x_0)\frac{\lambda_a}{2}F^a(\mathcal{B}_\mu(x, x_0))V^\dagger(x_0) \quad , \tag{8}$$

where $V(x_0)$ is a colour local gauge transformation matrix at the space-time point x_0. (8) is crucial in the following discussions. With (4), the gauge-fixing condition (7) reads

$$\begin{aligned}
&F^a\{U(x_0, x)[B_\mu(x) - A_\mu(x)]U(x, x_0) - \int_0^1 ds \frac{\partial y^\nu(x_0, x, s)}{\partial x^\mu}\frac{\partial y^\lambda(x_0, x, s)}{\partial s} \times \\
&\times U(x_0, y(x_0, x, s))A_{\lambda\nu}(y(x_0, x, s))U(y(x_0, x, s), x)U(x, x_0)\} = 0 \quad .
\end{aligned} \tag{9}$$

We see that (9) restricts A_μ to be a function of B_μ, which depends on the choice of the path from x_0 to x. Furthermore,if we take the gauge transformation of A_μ to

be

$$A_\mu(x) \to A'_\mu(x) = V(x)A_\mu(x)V^\dagger(x) + \frac{i}{g}[\partial_\mu V(x)]V^\dagger(x) \quad , \tag{10}$$

it will leads to the consequence that (9) keeps the same form after making any gauge transformation $V(x)$ as it shoud be, provided (8) is satisfied. This can be seen as follows. First we note that the gauge transformation of $U(x_1, x_2)$ is

$$U(x_1, x_2) \to U'(x_1, x_2) = V(x_1)U(x_1, x_2)V^\dagger(x_2) \quad , \tag{11}$$

Then we can easily see that with (10) the gauge transformation of $\mathcal{B}_\mu(x, x_0)$ is

$$\mathcal{B}_\mu(x, x_0) \to \mathcal{B}'_\mu(x, x_0) = V(x_0)\mathcal{B}_\mu(x, x_0)V^\dagger(x_0) \quad , \tag{12}$$

which looks like a global transformation and thus $F(\mathcal{B}_\mu)(x, x_0) \equiv \frac{\lambda_a}{2}F^a(\mathcal{B}_\mu(x, x_0))$ transforms as

$$F(\mathcal{B}_\mu(x, x_0)) \to F'(\mathcal{B}'_\mu(x, x_0)) = V(x_0)F(\mathcal{B}_\mu(x, x_0))V^\dagger(x_0) \quad , \tag{13}$$

in which we have used the property (8). (13) means that the gauge-fixing condition (7) or (9) keeps unchanged under the transformation $V(x)$. Eq.(10) has another advantage that the gauge transformations of $\Psi(x, x_0)$ and $\overline{\Psi}(x, x_0)$ are

$$\begin{aligned} \Psi(x, x_0) &\to \Psi'(x, x_0) = V(x_0)\Psi(x, x_0) \quad , \\ \overline{\Psi}(x, x_0) &\to \overline{\Psi}'(x, x_0) = \overline{\Psi}(x, x_0)V^\dagger(x_0) \quad . \end{aligned} \tag{14}$$

So that bilocal composite quark operators of the form $\overline{\Psi}(x_1, x_0)\Gamma\Psi(x_2, x_0)$ (where Γ is a certain matrix characterizing the flavour, spin and parity) are gauge invariant. Now we do the Fadeev-Popov gauge-fixing procedure. Define $\Delta_F(\mathcal{B})$ satisfying

$$\Delta_F(\mathcal{B}_\mu) \int DV \;\; \delta(F^a(\mathcal{B}^V_\mu(x, x_0)) = 1 \tag{15}$$

where

$$\mathcal{B}^V_\mu(x, x_0) \equiv V(x)\mathcal{B}_\mu(x, x_0)V^\dagger(x) + \frac{i}{g}[\partial_\mu V(x)]V^\dagger(x) \quad . \tag{16}$$

This $\Delta_F(\mathcal{B})$ is evidently invariant under $\mathcal{B}_\mu \to \mathcal{B}^V_\mu$, i.e.

$$\Delta_F(\mathcal{B}^V_\mu) = \Delta_F(\mathcal{B}_\mu) \quad . \tag{17}$$

Plugging (15) into (6), we get

$$\begin{aligned} Z[J] &= e^{iW[J]} \\ &= \int DVD\Psi D\overline{\Psi}D\mathcal{B}_\mu \Delta_F(\mathcal{B}_\mu)\delta(F^a(\mathcal{B}^V_\mu)) \times \\ &\quad expi\int d^4x\{\mathcal{L}(\Psi, \overline{\Psi}, \mathcal{B}_\mu) + \overline{\Psi}(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + \not{J}_V + \not{J}_A\gamma_5)\Psi\} \\ &= \int DVD\Psi^{V^{-1}}D\overline{\Psi}^{V^{-1}}D\mathcal{B}^{V^{-1}}_\mu \Delta_F(\mathcal{B}^{V^{-1}}_\mu)\delta(F^a(\mathcal{B}_\mu)) \times \\ &\quad expi\int d^4x\{\mathcal{L}(\Psi^{V^{-1}}, \overline{\Psi}^{V^{-1}}, \mathcal{B}^{V^{-1}}_\mu) + \overline{\Psi}^{V^{-1}}(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + \not{J}_V + \not{J}_A\gamma_5)\Psi^{V^{-1}}\} \\ &= \int DVD\Psi D\overline{\Psi}D\mathcal{B}_\mu \Delta_F(\mathcal{B}_\mu)\delta(F^a(\mathcal{B}_\mu)) \times \\ &\quad expi\int d^4x\{\mathcal{L}(\Psi, \overline{\Psi}, \mathcal{B}_\mu) + \overline{\Psi}(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + \not{J}_V + \not{J}_A\gamma_5)\Psi\} \\ &= const. \lim_{\xi\to 0}\int D\Psi D\overline{\Psi}D\mathcal{B}_\mu \Delta_F(\mathcal{B}_\mu) \times \\ &\quad expi\int d^4x\{\mathcal{L}(\Psi, \overline{\Psi}, \mathcal{B}_\mu) - \frac{1}{2\xi}[F^a(\mathcal{B}_\mu)]^2 + \overline{\Psi}(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + \not{J}_V + \not{J}_A\gamma_5)\Psi\} \end{aligned} \tag{18}$$

This is the well-defined generating functional for Green's functions that we need in the derivation [11]. Note that, in the gauge fixing procedure, we only insert (15) which is exactly unity into the path integral. Although different gauge fixing functions F^a lead to different gauge compensations $\Delta_F(\mathcal{B}_\mu)$, they give the same generating functional $Z[J]$.

So far the path between x and x_0 and the gauging-fixing function F^a are still not specified. We can make a suitable choice of them to simplify the calculations and the physical interpretation. We take the path to be a straight line connecting x and x_0, and the gauge-fixing function $F^a(B_\mu)$ to be the fix-point gauge [12]

$$F^a(B_\mu) = (x^\mu - x_0^\mu)B_\mu^a(x, x_0) \quad . \tag{19}$$

The straight-line path can be parametrized by

$$y^\mu(x_0, x, s) = x^\mu + f(s)(x_0^\mu - x^\mu) \quad , \tag{20}$$

where $f(s)$ is an analytic function of s in the region $0 \leq s \leq 1$ with $f(0) = 0$ and $f(1) = 1$. With this choice, (4) becomes

$$\begin{aligned} B_\mu(x, x_0) &= U(x_0, x)[B_\mu(x) - A_\mu(x)]U(x, x_0) - \int_0^1 ds[1 - f(s)]\frac{df}{ds}(x_0^\nu - x^\nu) \times \\ &\times U(x_0, y(x_0, x, s))A_{\nu\mu}(y(x_0, x, s))U(y(x_0, x, s), x)U(x, x_0) \quad , \end{aligned} \tag{21}$$

Multiplying (21) by $(x^\mu - x_0^\mu)$, we see from (19) and (7) that the arbitrariness of x_0 leads to

$$A_\mu(x) = B_\mu(x) \quad . \tag{22}$$

This makes the calculation very simple. Furthermore, in Ref.[10], when considering the static interaction between heavy quarks, the transformed quark field Ψ with a straight-line path in the spatial direction and the choice (22), can be interpretated as the constituent quark field. Now our straight-line path differs from the path in Ref.[10] only by a Lorentz transformation. Therefore our Ψ can be seen as a relativistic generalization of the constituent quark field defined in Ref.[10]. Thus the composite operator $\overline{\Psi}(x_1, x_0)\Gamma\Psi(x_2, x_0)$ can be regarded as gauge invariant bilocal

meson fields in the naive constituent quark model.

Having all these,we can go ahead to derive the gauge invariant effective Lagrangian for quarks. Consider the integration over DB_μ keeping $D\Psi D\overline{\Psi}$ unintegrated. Then $Q_a^\mu \equiv \overline{\Psi}\frac{\lambda_a}{2}\gamma^\mu\Psi$ can be regarded as an external source in the DB_μ integration. Since $Z[J]$ is the generating functional for Green's functions, we have

$$\begin{aligned} &\int DB_\mu e^{\int d^4x[\mathcal{L}_{QCD}(B) + Q_a^\mu B_\mu^a]} = \\ &= exp\sum_{n=2}^{\infty}\int d^4x_1 \cdots d^4x_n \frac{i^n g^n}{n!} G_{\mu_1\cdots\mu_n}^{a_1\cdots a_n}(x_1, \cdots, x_n) Q_{a_1}^{\mu_1}(x_1, x_0) \cdots Q_{a_n}^{\mu_n}(x_n, x_0) \quad , \end{aligned} \tag{23}$$

where $\mathcal{L}_{QCD}(B)$ is the gluon kinetic energy term $-\frac{1}{4}B_{\mu\nu}^a B^{a\mu\nu}$, and $G_{\mu_1\cdots\mu_n}^{a_1\cdots a_n}$ is the full n-point Green's function of the B_μ^a field with Ψ,$\overline{\Psi}$ treated as external fields. The form of $G_{\mu_1\cdots\mu_n}^{a_1\cdots a_n}$ is not known yet, so that (23) is only a formal expression. By means of the $SU(N_c)$ algebra

$$\begin{aligned} &(\frac{\lambda_a}{2})_{\alpha\beta}(\frac{\lambda_a}{2})_{\gamma\delta} = \frac{1}{2}\delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{1}{2N_c}\delta_{\alpha\beta}\delta_{\gamma\delta} \\ &[\frac{\lambda^a}{2}, \frac{\lambda^b}{2}] = if_{abc}\frac{\lambda^c}{2} \quad , \quad \{\frac{\lambda^a}{2}, \frac{\lambda^b}{2}\} = d_{abc}\frac{\lambda^c}{2} + \delta_{ab}\frac{1}{N_c} \quad , \\ &tr_c(\frac{\lambda_a}{2}\frac{\lambda_b}{2}) = \frac{1}{2}\delta_{ab} \quad , \end{aligned} \tag{24}$$

we can diagonalize the colour indices of the operators and have

$$\begin{aligned} &G_{\mu_1\cdots\mu_n}^{a_1\cdots a_n}(x_1, \cdots, x_n)[\overline{\Psi}_{\alpha_1}^{i_1}(x_1)(\frac{\lambda_{a_1}}{2})_{\alpha_1\beta_1}\gamma^{\mu_1}\Psi_{\beta_1}^{i_1}(x_1)] \cdots [\overline{\Psi}_{\alpha_n}^{i_n}(x_n)(\frac{\lambda_{a_n}}{2})_{\alpha_n\beta_n}\gamma^{\mu_n}\Psi_{\beta_n}^{i_n}(x_n)] \\ &= \int d^4x_1' \cdots d^4x_n' \overline{G}_{\rho_1\cdots\rho_n}^{\sigma_1\cdots\sigma_n}(x_1, x_1', \cdots, x_n, x_n')\overline{\Psi}_{\alpha_1}^{\sigma_1}(x_1, x_0)\Psi_{\alpha_1}^{\rho_1}(x_1', x_0) \cdots \\ &\cdots \overline{\Psi}_{\alpha_n}^{\sigma_n}(x_n, x_0)\Psi_{\alpha_n}^{\rho_n}(x_n', x_0) \end{aligned} \tag{25}$$

where $\sigma(\rho)$ is a short notation combining the flavour index i and the Lorents index μ, $\overline{G}_{\rho_1\cdots\rho_n}^{\sigma_1\cdots\sigma_n}(x_1, x_1', \cdots, x_n, x_n')$ is a generalized Green's function containing 2n space-

time points. The specific forms of $\overline{G}^{\sigma_1\sigma_2}_{\rho_1\rho_2}$ and $\overline{G}^{\sigma_1\sigma_2\sigma_3}_{\rho_1\rho_2\rho_3}$ are given in Appendix B. Thus the generating functional (18) can be written as

$$\begin{aligned} Z[J] &= e^{iW[J]} \\ &= \int D\Psi D\overline{\Psi} exp i\{\int d^4x\overline{\Psi}(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + V\!\!\!/ + A\!\!\!/\gamma_5)\Psi + \\ &\quad + \sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x_n d^4x'_1\cdots d^4x'_n \frac{i^n g^n}{n!}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x'_1,\cdots,x_n,x'_n)\times \\ &\quad \times\overline{\Psi}^{\sigma_1}_{\alpha_1}(x_1,x_0)\Psi^{\rho_1}_{\alpha_1}(x'_1,x_0)\cdots\overline{\Psi}^{\sigma_n}_{\alpha_n}(x_n,x_0)\Psi^{\rho_n}_{\alpha_n}(x'_n,x_0)\} \quad . \end{aligned} \tag{26}$$

In (26),the bilocal composite quark operators $\overline{\Psi}^{\sigma_1}_{\alpha_1}(x_1,x_0)\Psi^{\rho_1}_{\alpha_1}(x'_1,x_0)\cdots\overline{\Psi}^{\sigma_n}_{\alpha_n}(x_n,x_0)$ $\Psi^{\rho_n}_{\alpha_n}(x'_n,x_0)$ are colour singlet and gauge invariant. The generating functional (26) is our basis of constructing the gauge invariant bilocal effective Lagrangian for mesons.

III. Gauge Invariant Bilocal Auxiliary Fields and the Effective Lagrangian for Mesons

Usually auxiliary fields are introduced in the study of theories with quartic-field interactions. The advantage of introducing the auxiliary field is that it provide a convenient way of integrating out the fundamental field and leaving over the information of the quadratic composite-field operator.Here we generalize this idea to theories with complicated interactions like (26).

We introduce the bilocal auxiliary field $\chi_{\sigma\rho}(x,x')$ satisfying the following relation

$$\begin{aligned} &\int D\chi_{\sigma\rho} e^{i\{K[\chi]+\int d^4xd^4x' g\chi_{\sigma\rho}(x,x')\overline{\Psi}^{\sigma}_{\alpha}(x,x_0)\Psi^{\rho}_{\alpha}(x',x_0)\}} \\ &= exp i\{\sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x_n d^4x'_1\cdots d^4x'_n \frac{i^n g^n}{n!}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x'_1,\cdots,x_n,x'_n)\times \\ &\quad \times\overline{\Psi}^{\sigma_1}_{\alpha_1}(x_1,x_0)\Psi^{\rho_1}_{\alpha_1}(x'_1,x_0)\cdots\overline{\Psi}^{\sigma_n}_{\alpha_n}(x_n,x_0)\Psi^{\rho_n}_{\alpha_n}(x'_n,x_0)\} \quad , \end{aligned} \tag{27}$$

where $K[\chi]$ is a functional of χ, which is not known yet. Our task is to develop a method to determine $K[\chi]$. Once this is done, we can simplify the expression (26) by using (27) and get

$$\begin{aligned} Z[J] &= e^{iW[J]} \\ &= \int D\chi D\Psi D\overline{\Psi} exp i\{K[\chi] + \int d^4xd^4x'\overline{\Psi}^{\sigma}_{\alpha}(x,x_0)\times \\ &\quad \times[(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + V\!\!\!/ + A\!\!\!/\gamma_5)_{\sigma\rho}\delta^4(x-x') + g\chi_{\sigma\rho}(x,x')]\Psi^{\rho}_{\alpha}(x',x_0)\} \end{aligned} \tag{28}$$

We see that the quark field appears quadratically in (28), so that it can be easily integrated out. Thus we have

$$Z[J] = \int D\chi e^{i\{K[\chi]-iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + V\!\!\!/ + A\!\!\!/\gamma_5 + g\chi)\}} \quad , \tag{29}$$

The effective Lagrangian for the χ-field is then defined by

$$\int d^4xd^4x'\mathcal{L}_{eff}(\chi) = K[\chi] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + V\!\!\!/ + A\!\!\!/\gamma_5 + g\chi) \quad . \tag{30}$$

We know that a change of the path between x and x_0 will change the definition of the transformed quark and gluon fields, but the generating functional $Z[J]$ and the Green's functions $\overline{G}$ are independent of the choice of the path. Therefore from (29) we see that the form of the effective Lagrangian $\mathcal{L}_{eff}(\chi)$ defined in (30) is independent of the choice of the path.

The transformation property of $\chi_{\sigma\rho}(x,x')$ can be seen as follows. If the theory is massless, it respect a chiral symmetry $U(N_f)_L\times U(N_f)_R$. We can further choose the external sources to make the theory to respect local $U(N_f)_L\times U(N_f)_R$ symmetry.Let $L(x)$ and $R(x)$ be the transformation of $U(N_f)_L$ and $U(N_f)_R$, respectively. The

transformations of the quark,gluon, and external sources are

$$
\begin{aligned}
\psi(x) &\rightarrow \psi'(x) = [R(x)P_R + L(x)P_L]\psi(x) \quad , \\
\overline{\psi}(x) &\rightarrow \overline{\psi}'(x) = \overline{\psi}(x)[R(x)P_R + L(x)P_L] \quad , \\
B^a_\mu(x) &\rightarrow B^{a\prime}_\mu(x) = B^a_\mu(x) \quad , \\
-J_S(x) + iJ_P(x) &\rightarrow L(x)[-J_S(x) + iJ_P(x)]R^\dagger(x) \quad , \\
-J_S(x) - iJ_P(x) &\rightarrow R(x)[-J_S(x) - iJ_P(x)]L^\dagger(x) \quad , \\
J^\mu_V(x) + J^\mu_A(x) &\rightarrow R(x)[J^\mu_V(x) + J^\mu_A(x)]R^\dagger(x) - i[\partial^\mu R(x)]R^\dagger(x) \quad , \\
J^\mu_V(x) - J^\mu_A(x) &\rightarrow L(x)[J^\mu_V(x) - J^\mu_A(x)]L^\dagger(x) - i[\partial^\mu L(x)]L^\dagger(x) \quad ,
\end{aligned}
\tag{31}
$$

Since $U(x,x_0)$ is independent of $L(x)$ and $R(x)$,we have

$$
\begin{aligned}
\Psi(x) &\rightarrow \Psi'(x) = [R(x)P_R + L(x)P_L]\Psi(x) \quad , \\
\overline{\Psi}(x) &\rightarrow \overline{\Psi}'(x) = \overline{\Psi}(x)[R(x)P_R + L(x)P_L] \quad , \\
\mathcal{B}^a_\mu(x,x_0) &\rightarrow \mathcal{B}^{a\prime}_\mu(x,x_0) = \mathcal{B}^a_\mu(x,x_0) \quad ,
\end{aligned}
\tag{32}
$$

Then from (27) we see that

$$
\chi(x,x') \rightarrow \chi'(x,x') = [R(x)P_L + L(x)P_R]\chi(x,x')[R^\dagger(x')P_R + L^\dagger(x')P_L] \quad . \tag{33}
$$

The form (30) is specially useful when we gauge an $SU(2)_L \times U(1)$ subgroup of $U(N_f)_L \times U(N_f)_R$ to incorporate the theory with electroweak interactions. We see from (31) that this can be done by simply identifying the $SU(2)_L \times U(1)$ part of J^μ_V and J^μ_A in (30) with the electroweak gauge fields.

We can further expand $\chi(x,x')$ in the γ-matrices

$$
\chi(x,x') = \chi_S(x,x') + i\chi_P(x,x')\gamma_5 + \chi^\mu_V(x,x')\gamma_\mu +
$$

$$
+\chi^\mu_A(x,x')\gamma_\mu\gamma_5 + \chi^{\mu\nu}_T(x,x')\sigma_{\mu\nu} \quad . \tag{34}
$$

Substituting (34) into (33), we get

$$
\begin{aligned}
\chi_S(x,x') + i\chi_P(x,x') &\rightarrow L(x)[\chi_S(x,x') + i\chi_P(x,x')]R^\dagger(x') \quad , \\
\chi_S(x,x') - i\chi_P(x,x') &\rightarrow R(x)[\chi_S(x,x') - i\chi_P(x,x')]L^\dagger(x') \quad , \\
\chi^\mu_V(x,x') + \chi^\mu_A(x,x') &\rightarrow R(x)[\chi^\mu_V(x,x') + \chi^\mu_A(x,x')]R^\dagger(x') \quad , \\
\chi^\mu_V(x,x') - \chi^\mu_A(x,x') &\rightarrow L(x)[\chi^\mu_V(x,x') - \chi^\mu_A(x,x')]L^\dagger(x') \quad , \\
\chi^{\mu\nu}_T(x,x') + \frac{i}{2}\epsilon^{\mu\nu\lambda\kappa}\chi_{T\,\lambda\kappa}(x,x') &\rightarrow L(x)[\chi^{\mu\nu}_T(x,x') + \frac{i}{2}\epsilon^{\mu\nu\lambda\kappa}\chi_{T\,\lambda\kappa}(x,x')]R^\dagger(x') \quad , \\
\chi^{\mu\nu}_T(x,x') - \frac{i}{2}\epsilon^{\mu\nu\lambda\kappa}\chi_{T\,\lambda\kappa}(x,x') &\rightarrow R(x)[\chi^{\mu\nu}_T(x,x') - \frac{i}{2}\epsilon^{\mu\nu\lambda\kappa}\chi_{T\,\lambda\kappa}(x,x')]L^\dagger(x') \quad ,
\end{aligned}
\tag{35}
$$

These χ_S,χ_P,χ^μ_V,χ^μ_A, $\chi^{\mu\nu}_T$ is to be regarded as effective meson fields with various spin and parity.

We can expand the functional $K[\chi]$ in powers series of the χ_A fields (A stands for S,P,V,A,T and the flavour index, i.e.

$$
K[\chi] = \sum_{n=1}^{\infty} \int dX_1 \cdots dX_n \frac{1}{n!} K_{A_1 \cdots A_n}(X_1, \cdots, X_n)\chi_{A_1}(X_1) \cdots \chi_{A_n}(X_n) \quad . \tag{36}
$$

Here we use a short notation X for the bilocal variables (x,x'). What we are going to do is to determine the relation between the coefficients $K_{A_1 \cdots A_n}(X_1, \cdots, X_n)$ in the effective Lagrangian for χ_A and the extended gluon Green's functions $\overline{G}^{\sigma_1 \cdots \sigma_n}_{\rho_1 \cdots \rho_n}(X_1, \cdots, X_n)$ in the underlying theory. This can be done by comparing the two sides of (27) provided the functional integration on the left-hand-side (L.H.S.) of (27) is carried out. In this paper, we take the standard loop expansion [13] to do the functional integration on the L.H.S. of (27), and determine $K_{A_1 \cdots A_n}(X_1, \cdots, X_n)$ order by order

in loop expansion. This is only for easily illustrating the formalism. Of course one can take other methods to do the integration.

Let $Q^A \equiv [\overline{\Psi}(x,x_0)\Psi(x',x_0)]^A$. (27) can be written as

$$\begin{aligned}\overline{Z}[Q] &= e^{i\overline{W}[Q]} = \int \mathcal{D}e^{i\{K[\chi]+\int d^4x g Q^A(X)\chi_A(X)\}} \\ &= exp i\{\sum_{n=2}^{\infty}\int d^4X_1\cdots d^4X_n \frac{i^n g^n}{n!}\overline{G}_{A_1\cdots A_n}(X_1,\cdots,X_n)Q^{A_1}(X_1)\cdots Q^{A_n}(X_n)\} \quad .\end{aligned} \tag{37}$$

We see that $\overline{W}[Q]$ is just the generating functional of the extended connected Green's functions of gluons $\overline{G}_{A_1\cdots A_n}(X_1,\cdots,X_n)$. Next we define the classical field $\overline{\chi}_A(X)$

$$\overline{\chi}_A(X) \equiv \frac{\delta\overline{W}[Q]}{\delta Q^A(X)} = \sum_{n=2}^{\infty}\frac{i^{n-1}}{(n-1)!}\overline{G}_{A_1\cdots A_{n-1}}(X_1\cdots,X_{n-1})Q^{A_1}(X_1)\cdots Q^{A_{n-1}}(X_{n-1}). \tag{38}$$

Then the effective action $\overline{\Gamma}[\overline{\chi}]$ is obtained by the Legendre transformation

$$\overline{\Gamma}[\overline{\chi}] = \overline{W}[Q] - \int d^4X Q^A(X)\overline{\chi}_A(X) \quad , \tag{39}$$

in which $\overline{W}[Q]$ and $\overline{\chi}_A(X)$ are all known (cf. (37) and (38)). According to the rule of loop expansion given in Ref.[13], we have

$$K[\overline{\chi}] + \frac{i}{2}Trln(\frac{\delta^2 K[\overline{\chi}]}{\delta\overline{\chi}_{A_1}(X_1)\delta\overline{\chi}_{A_2}(X_2)}) + \overline{\Gamma}_2 = \overline{\Gamma}[\overline{\chi}] \quad , \tag{40}$$

in which $\overline{\Gamma}_2$ is the sum of all one-particle- irreducible (1PI) multi-loop vacuum diagrams with the propagator $\frac{\delta^2 K[\overline{\chi}]}{\delta\overline{\chi}_{A_1}(X_1)\delta\overline{\chi}_{A_2}(X_2)}$ and the vertices

$$K_{int}[\chi,\overline{\chi}] = \sum_{n=3}^{\infty}\int d^4X_1\cdots d^4X_n\frac{1}{n!}K^{A_1\cdots A_n}(X_1,\cdots,X_n;\overline{\chi})\chi_{A_1}(X_1)\cdots\chi_{A_n}(X_n) \quad ,$$

where $K^{A_1\cdots A_n}(X_1,\cdots,X_n;\overline{\chi})$ is the expansion coefficient of $K[\chi+\overline{\chi}]$. On the L.H.S. of (40), the first term is the tree level contribution to $\overline{\Gamma}[\overline{\chi}]$, the second term is the

one-loop contribution, and the third term is the higher loop contributions. We denote the loop expansion of $K[\chi]$ by

$$K[\chi] = \sum_{l=0}^{\infty}K_l[\chi] \quad . \tag{41}$$

Substituting (41) into the L.H.S. of (40), we can determine $K[\chi]$ order by order in loop expansion.

At tree level, we get from (40)

$$K_0[\overline{\chi}] = \overline{\Gamma}[\overline{\chi}] \quad , \tag{42}$$

or

$$\begin{aligned} &K_0^A(X) = 0 \\ &K_0^{A_1\cdots A_n}(X_1,\cdots,X_n) = \overline{\Gamma}^{A_1\cdots A_n}(X_1,\cdots,X_n) \quad , \qquad n\geq 2 \quad . \end{aligned} \tag{43}$$

where $\overline{\Gamma}^{A_1\cdots A_n}(X_1,\cdots,X_n)$ is the n-point 1PI vertex function generated from $\overline{\Gamma}[\overline{\chi}]$.

To one-loop level, (40) is

$$K_0[\overline{\chi}] + K_1[\overline{\chi}] + \frac{i}{2}Trln(\frac{\delta^2 K_0[\overline{\chi}]}{\delta\overline{\chi}_{A_1}(X_1)\delta\overline{\chi}_{A_2}(X_2)}) = \overline{\Gamma}[\overline{\chi}] \quad .$$

This leads to

$$K_1[\overline{\chi}] = -\frac{i}{2}Trln(\frac{\delta^2 K_0[\overline{\chi}]}{\delta\overline{\chi}_{A_1}(X_1)\delta\overline{\chi}_{A_2}(X_2)}) \quad , \tag{44}$$

in which,from (42) and (43),

$$\begin{aligned}\frac{i}{2}Trln(\frac{\delta^2 K_0[\overline{\chi}]}{\delta\overline{\chi}_{A_1}(X_1)\delta\overline{\chi}_{A_2}(X_2)}) &= \overline{\Gamma}^{A_1A_2}(X_1,X_2) + \sum_{n=3}^{\infty}\int d^4X_1'\cdots d^4X_{n-2}'\times \\ &\times\frac{1}{(n-2)!}\overline{\Gamma}^{A_1A_2A_1'\cdots A_{n-2}'}(X_1,X_2,X_1',\cdots,X_{n-2}')\overline{\chi}_{A_1}(X_1')\cdots\overline{\chi}_{A_{n-2}}(X_{n-2}')\end{aligned} \tag{45}$$

Substituting (45) into (44) and expanding the logarithm in powers of $\overline{\chi}_A$, we obtain

$$
\begin{aligned}
&K_1^A(X) = -\frac{1}{2}\int d^4X_1' d^4X_2' \overline{\Gamma}^{AA_1'A_2'}(X, X_1', X_2')\overline{G}_{A_1'A_2'}(X_1', X_2') \quad , \\
&K_1^{A_1A_2}(X_1, X_2) = -\int d^4X_1' d^4X_2' \overline{\Gamma}^{A_1A_2A_1'A_2'}(X_1, X_2, X_1', X_2')\overline{G}_{A_1'A_2'}(X_1', X_2') - \\
&-\frac{1}{2}\int d^4X_1' d^4X_2' d^4X_3' d^4X_4' \overline{\Gamma}^{A_1A_1'A_2'}(X_1, X_1', X_2')\overline{\Gamma}^{A_2A_3'A_4'}(X_2, X_3', X_4') \times \\
&\times \overline{G}_{A_1'A_3'}(X_1', X_3')\overline{G}_{A_2'A_4'}(X_2', X_4') \quad , \qquad (46) \\
&\cdots\cdots
\end{aligned}
$$

This procedure can be carried on to determine $K_l^{A_1\cdots A_n}(X_1, \cdots, X_n)$ for arbitrary l. Substituting the determined $K[\chi]$ into (30), we get the expression for the effective Lagrangian $\mathcal{L}_{eff}(\chi)$ in which all the coefficients of $\chi_{A_1}\cdots\chi_{A_n}$ are expressed in terms of $\overline{\Gamma}$'s and $\overline{G}$'s. In Appendix C, we shall give more explicit expressions for $K_0^{A_1A_2}$ and $K_0^{A_1A_2A_3}$ in terms of the original Green's functions of gluons.

IV. The Effective Action

Given the effective Lagrangian $\mathcal{L}_{eff}(\chi)$ in (30), the calculation of various physical quantities concerns the calculation of the following generating functional

$$
\begin{aligned}
Z[J, I] = e^{iW[J,I]} \;=\; &\int \mathcal{D}\chi exp i\{K[\chi] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + A\!\!\!/_V + A\!\!\!/_A\gamma_5 + g\chi) + \\
&+\int d^4X \;\; I^A(X)\chi_A(X)\} \quad , \qquad (47)
\end{aligned}
$$

where $I^A(X)$ is an external source coupling to the $\chi_A(X)$ field.

The generating functional $Z[J]$ defined in (6) is related to $Z[J, I]$ by $Z[J, 0] = Z[J]$. Now we define the classical field

$$
\hat{\chi}_A(X) \equiv \frac{\delta W[J, I]}{\delta I^A(X)} \quad , \qquad (48)
$$

and the partial Legendre transformation

$$
\tilde{\Gamma}[J, \hat{\chi}] = W[J, I] - \int d^4X \;\; I^A(X)\hat{\chi}_A(X) \quad . \qquad (49)
$$

It is easy to see that

$$
\frac{\delta\tilde{\Gamma}[J, \hat{\chi}]}{\delta\hat{\chi}_A(X)} = -I^A(X) \quad . \qquad (50)
$$

Hence

$$
W[J] = W[J, 0] = \tilde{\Gamma}[J, \hat{\chi}]\,|_{\frac{\delta\tilde{\Gamma}}{\delta\hat{\chi}_A}=0} \quad . \qquad (51)
$$

Therefore what we need is to calculate $\tilde{\Gamma}[J, \hat{\chi}]$. When we turn off $I^A(X)$, (50) becomes

$$
\frac{\delta\tilde{\Gamma}[J, \hat{\chi}]}{\delta\hat{\chi}_A} = 0 \quad , \qquad (52)
$$

which reflects the stability of the physical vacuum.

The standard loop expansion formula for $\tilde{\Gamma}[J, \hat{\chi}]$ has been given in Ref.[13], which is

$$
\begin{aligned}
&\tilde{\Gamma}[J, \hat{\chi}] = K[\hat{\chi}] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + A\!\!\!/_V + A\!\!\!/_A\gamma_5 + g\hat{\chi}) + \\
&+\frac{i}{2}Trln\{\frac{\delta^2}{\delta\hat{\chi}_{A_1}(X_1)\delta\hat{\chi}_{A_2}(X_2)}[K[\hat{\chi}] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + A\!\!\!/_V + A\!\!\!/_A\gamma_5 + g\hat{\chi})]\} + \\
&+\tilde{\Gamma}_2 \quad , \qquad (53
\end{aligned}
$$

where the first two terms come from the tree level contributions, the third term comes from the one-loop contribution, and $\tilde{\Gamma}_2$ represents higher loop contributions containing the sum over all higher loop 1PI vacuum diagrams with the propagator $\frac{\delta^2}{\delta\hat{\chi}_{A_1}(X_1)\delta\hat{\chi}_{A_2}(X_2)}[K[\hat{\chi}] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + A\!\!\!/_V + A\!\!\!/_A\gamma_5 + g\hat{\chi})]$ and vertices $\frac{\delta^n}{\delta\hat{\chi}_{A_1}(X_1)\cdots\delta\hat{\chi}_{A_n}(X_n)}[K[\hat{\chi}] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + A\!\!\!/_V + A\!\!\!/_A\gamma_5 + g\hat{\chi})]$ $(n\geq 3)$.

Let us first look at the tree level contributions,

$$\tilde{\Gamma}_0[J,\hat{\chi}] = K_0[\hat{\chi}] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi}) \quad . \tag{54}$$

Turning off $I^A(X)$, (52) now reads

$$\frac{\delta K_0[\hat{\chi}]}{\delta\hat{\chi}_A(X)} - i[(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi})^{-1}(X)]^A = 0 \quad , \tag{55}$$

or precisely

$$\begin{aligned}
&\sum_{n=2}^{\infty}\frac{i}{(n-1)!}\int d^4x_1 d^4x_1' \cdots d^4x_{n-1}d^4x_{n-1}'\overline{\Gamma}^{\sigma\rho\sigma_1\rho_1\cdots\sigma_{n-1}\rho_{n-1}}(x,x',x_1,x_1',\cdots,x_{n-1},x_{n-1}') \times \\
&\times\hat{\chi}_{\sigma_1\rho_1}(x_1,x_1')\cdots\hat{\chi}_{\sigma_{n-1}\rho_{n-1}}(x_{n-1},x_{n-1}') + \\
&+[(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi})^{-1}(x,x')]^{\sigma\rho} = 0 \quad .
\end{aligned} \tag{56}$$

This is the general stability condition that the classical field $\hat{\chi}_{\sigma\rho}(x,x')$ should satisfy. Up to one loop,

$$\begin{aligned}
&\tilde{\Gamma}_0[J,\hat{\chi}] + \tilde{\Gamma}_1[J,\hat{\chi}] = K_0[\hat{\chi}] + K_1[\hat{\chi}] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi}) + \\
&+\frac{i}{2}Trln\{\frac{\delta^2}{\delta\hat{\chi}_{A_1}(X_1)\delta\hat{\chi}_{A_2}(X_2)}[K_0[\hat{\chi}] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi})]\} + \\
&= K_0[\hat{\chi}] - iTrln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi}) - \frac{i}{2}Trln(\frac{\delta^2K_0[\hat{\chi}]}{\delta\hat{\chi}_{A_1}(X_1)\delta\hat{\chi}_{A_2}(X_2)}) + \\
&+\frac{i}{2}Trln\{\frac{\delta^2K_0[\hat{\chi}]}{\delta\hat{\chi}_{A_1}(X_1)\delta\hat{\chi}_{A_2}(X_2)} - i\frac{\delta^2Trln(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi})}{\delta\hat{\chi}_{A_1}(X_1)\delta\hat{\chi}_{A_2}(X_2)}\} \quad .
\end{aligned} \tag{57}$$

Substituting (57) into (52) we get the stability condition with 1-loop correction to (56).

Eq.(56) is rather complicated. Let us consider its solution in a simple approximation. We truncate (56) by neglecting the $n\geq 3$ Green's functions in (23). Then

(56) reduces to

$$\begin{aligned}
&i\int d^4x_1d^4x_1'\overline{\Gamma}^{\sigma\rho\sigma_1\rho_1}(x,x',x_1,x_1')\hat{\chi}_{\sigma_1\rho_1}(x_1,x_1') + \\
&+[(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi})^{-1}(x,x')]^{\sigma\rho} = 0 \quad .
\end{aligned} \tag{58}$$

We shall show that this is equivalent to the commonly used Schwinger-Dyson equation in the improved ladder approximation[9], or the so-called Hartree-Fock approximation[14]. This can be seen as follows. Let

$$\tilde{\chi}^{\sigma\rho}(x,x') \equiv -i\int d^4x_1d^4x_1'\overline{\Gamma}^{\sigma\rho\sigma_1\rho_1}(x,x',x_1,x_1')\hat{\chi}_{\sigma_1\rho_1}(x_1,x_1') \quad . \tag{59}$$

or

$$\hat{\chi}_{\sigma\rho}(x,x') = i\int d^4x_1d^4x_1'\overline{G}_{\sigma\rho\sigma_1\rho_1}(x,x',x_1,x_1')\tilde{\chi}^{\sigma_1\rho_1}(x_1,x_1') \quad . \tag{60}$$

Then (58) can be written as

$$\tilde{\chi}^{-1}_{\sigma\rho}(x,x') = (i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 + g\hat{\chi})_{\sigma\rho}(x,x') \quad . \tag{61}$$

Here we have used the permutational symmetry in $\overline{G}_{\sigma_1\rho_1\cdots\sigma_n\rho_n}(x_1,x_1',\cdots,x_n,x_n')$ which leads to $\tilde{\chi}^{\sigma\rho}(x,x') = \tilde{\chi}^{\rho\sigma}(x,x')$. Using (60),we can write (61) as

$$\begin{aligned}
&(i\partial\!\!\!/ - J_S + iJ_P\gamma_5 + v\!\!\!/_V + a\!\!\!/_A\gamma_5 - \tilde{\chi}^{-1})_{\sigma\rho}(x,x') \\
&= -ig\int d^4x_1d^4x_1'\overline{G}_{\sigma\rho\sigma_1\rho_1}(x,x',x_1,x_1')\tilde{\chi}^{\sigma_1\rho_1}(x_1,x_1') \\
&= i\frac{g}{2}G_{\mu\nu}(x,x')(\gamma^\mu)_{\sigma\rho_1}(\gamma^\nu)_{\sigma_1\rho}\tilde{\chi}^{\rho_1\sigma_1}(x',x) + \\
&+i\frac{g}{2N_c}\int d^4x_1G_{\mu\nu}(x,x_1)(\gamma^\mu)_{\sigma\rho}(\gamma^\nu)_{\sigma_1\rho_1}\tilde{\chi}^{\sigma_1\rho_1}(x_1,x_1)\delta(x-x') \quad .
\end{aligned} \tag{62}$$

In the vacuum state, $-J_S + iJ_P\gamma_5 = -m$, $J_V^\mu = J_A^\mu = 0$. (62) becomes

$$\begin{aligned}
(i\partial\!\!\!/ - m - \tilde{\chi}^{-1})_{\sigma\rho}(x,x') \;=\; & i\frac{g}{2}G_{\mu\nu}(x,x')[\gamma^\mu\tilde{\chi}(x',x)\gamma^\nu]_{\sigma\rho} + \\
& +i\frac{g}{2N_c}\delta(x-x')\int d^4x_1G_{\mu\nu}(x,x_1)(\gamma^\mu)_{\sigma\rho}tr[\gamma^\nu\tilde{\chi}(x_1,x_1)] \quad .
\end{aligned} \tag{63}$$

The trace in the last term on the R.H.S. vanishes in the ordinary formalism with translational invariance. This is just the Schwinger-Dyson equation in the improved ladder approximation considered in Ref.[9]. Based on the analysis of this equation Holdom et.al.[8][9] proposed a model for the bilocal function which leads to successful phenomenological consequences. Our more general equation (55) or (56) admits further improvements.

Our tree level results (54)-(56) may have another advantage.

Let us rescale the bilocal field by $g\chi \to \chi$. Then eq.(27) becomes

$$\begin{aligned} & ln \int \mathcal{D}\chi e^{i\{K[g^{-1}\chi]+\int d^4x d^4x' \chi_{\sigma\rho}(x,x')\overline{\Psi}^{\sigma}_{\alpha}(x,x_0)\Psi^{\rho}_{\alpha}(x',x_0)\}} \\ = \; & i\{\sum_{n=2}^{\infty}\int d^4x_1\cdots d^4x_n d^4x'_1\cdots d^4x'_n \frac{i^n g^n}{n!}\overline{G}^{\sigma_1\cdots\sigma_n}_{\rho_1\cdots\rho_n}(x_1,x'_1,\cdots,x_n,x'_n)\times \\ & \times\overline{\Psi}^{\sigma_1}_{\alpha_1}(x_1,x_0)\Psi^{\rho_1}_{\alpha_1}(x'_1,x_0)\cdots\overline{\Psi}^{\sigma_n}_{\alpha_n}(x_n,x_0)\Psi^{\rho_n}_{\alpha_n}(x'_n,x_0)\} \quad . \end{aligned} \tag{64}$$

Consider the large-N_c limit. We take the self-consistent limit that χ and g^2N_c keep finite as $N_c \to \infty$. It can be seen from the large-N_c argument[15] that the R.H.S. of (64) is proportional to N_c in the $N_c \to \infty$ limit. On the L.H.S. of (64), the second term in the exponential is proportional to N_c, so that the leading tem in $K[g^{-1}\chi]$ should also be proportional to N_c to make (64) consistent. Therefore the leading terms in the exponential function on the L.H.S. of (64) contain an overall factor N_c, while other terms are smaller by powers of $1/N_c$. This overall factor always goes with the Planck constant as $N_c/\hbar$. Therefore in the loop expansion, loop contributions are smaller than the tree level contribution by at least a power of $1/N_c$. Thus in the $N_c \to \infty$ limit, the tree level results (54)-(56) are exact, so that the large-N_c limit calculation is greatly simplified. This is reminicent of the auxiliary field formulation

of the Gross-Neveu model[16] which makes the large-N'limit calculation very simple.

V. Conclusions

In this paper, we have developed a method of deriving the general form of the effective Lagrangian for gauge invariant bilocal auxiliary fields describing mesons with various spins by formally integrating out the quark and gluon fields in a QCD-like gauge theory. The integration over the gluon field is not carried out explicitly but is formally expressed in terms of the yet unknown n-point Green's functions of gluons. The integration over the quark field is achieved by developing a general method of introducing gauge invariant bilocal auxiliary fields $\chi(x,x')$ (cf.(27)). In the approximation neglecting all the gluon Green's functions with $n\geq 3$, $\chi(x,x')$ is just like the auxiliary field usually used in the bilocal four-fermion theories, and it represents the bilocal quark operator of the type $\overline{\Psi}(x,x_0)\Gamma\Psi(x',x_0)$ (Γ is a spin and flavour matrix) which is interpreted as the meson field in the naive constituent quark model. When more gluon Green's functions are taken into account, $\chi(x,x')$ contains higher dimensional composite quark operators as well. This goes beyond the naive constituent quark model. The obtained effective Lagrangian for χ is of the form (30) with $K[\chi]$ of the form (36) in which the coefficients $K^{A_1\cdots A_n}(X_1,\cdots,X_n)$ are related, in principle, to the gluon Green's functions. The determination of the relation between $K^{A_1\cdots A_n}(X_1,\cdots,X_n)$ and the gluon Green's functions requires carrying out the $\mathcal{D}\chi$ integration in (27). In this paper, we take the standard loop expansion to carray out the $\mathcal{D}\chi$ integration in which the coefficients $K^{A_1\cdots A_n}(X_1,\cdots,X_n)$ are determined loop by loop expressed in terms of the gluon Green's functions shown in (43),(46),etc. A more explicit expression is given in Appendix C.

Although the present formulation is only formal, it does give the general form and the physical interpretation of the gauge invariant bilocal effective Lagrangian including mesons with various spins. It also provides a possibility of really calculating the effective Lagrangian and making phenomenological predictions if a proper approximation for calculating the gluon Green's functions can be developed. As we have shown in Sec.IV that if we neglect all the $n \geq 3$ gluon Green's functions, the vacuum stability condition at tree level just reduces to the improved ladder approximation of the Schwinger-Dyson equation considered by Holdom in Ref.[9]. So that at least one can do the same modeling in this approximation as what was done by Holdom et.al.[8][9] which can lead to rather successful phenomenological consequences. Our general formula admits further improvements. One may either develop new approximations to calculate the gluon Green's functions and make predictions, or express various physical observables in terms of the gluon Green's so that one can inversely study the properties of the gluon Green's functions from the data of the observables. We have also shown that taking a certain limit in $N_c \to \infty$, our tree level formulae can be the exact formulae in the lage-N_c limit which greatly simplifies the calculation.

Appendix A

We derive here the relation (4) in the text. Let us parametrize the path between x^μ and x_0^μ by $y^\mu(x_0, x, s)$ with $y^\mu(x_0, x, 0) = x^\mu$, $y^\mu(x_0, x, 1) = x_0^\mu$. The non-integratable factor $U(x_0, x)$ defined in (2) can then be written as

$$\begin{aligned} U(x_0, x) &= P e^{-ig \int_x^{x_0} dy_\mu A^\mu(y)} \\ &= P e^{-ig \int_0^1 ds \frac{\partial y_\mu(x_0, x, s)}{\partial s} A^\mu(y(x_0, x, s))} \quad . \end{aligned} \tag{A.1}$$

Thus

$$\begin{aligned} & \partial_\mu^x U(x_0, x) \\ = & -ig \int_0^1 ds U(x_0, y(x_0, y(x_0, x, s)) \{ \partial_\mu^x [\frac{\partial y_\nu(x_0, x, s)}{\partial s} A^\mu(y(x_0, x, s)) \} U(y(x_0, x, s), x) \\ = & -ig \int_0^1 ds U(x_0, y(x_0, y(x_0, x, s)) [\frac{\partial^2 y_\nu(x_0, x, s)}{\partial x^\mu \partial s} A^\nu(y(x_0, x, s)) + \\ & + \frac{\partial y_\nu(x_0, x, s)}{\partial s} A_{\nu,\omega}(y(x_0, x, s)) \frac{\partial y^\omega(x_0, x, s)}{\partial x^\mu}] U(y(x_0, x, s), x) \quad , \end{aligned} \tag{A.2}$$

where

$$A_{\nu,\omega}(y) \equiv \frac{\partial A_\nu(y)}{\partial y^\omega} \quad , \tag{A.3}$$

Now

$$\begin{aligned} & ig \int_0^1 ds \frac{\partial y_\nu(x_0, x, s)}{\partial x^\mu} \frac{d}{ds} [U(x_0, y(x_0, y(x_0, x, s)) A^\nu(y(x_0, x, s)) U(y(x_0, x, s), x)] \\ & = ig [\frac{\partial y_\nu(x_0, x, s)}{\partial x^\mu} U(x_0, y(x_0, y(x_0, x, s)) A^\nu(y(x_0, x, s)) U(y(x_0, x, s), x)] \, |_{s=0}^{s=1} - \\ & -ig \int_0^1 ds \frac{\partial^2 y_\nu(x_0, x, s)}{\partial s \partial x^\mu} U(x_0, y(x_0, y(x_0, x, s)) A^\nu(y(x_0, x, s)) U(y(x_0, x, s), x)] \end{aligned}$$

$$= -igU(x_0,x)A_\mu(x) - ig\int_0^1 ds \frac{\partial^2 y_\nu(x_0,x,s)}{\partial s \partial x^\mu} \times$$
$$\times U(x_0, y(x_0, y(x_0,x,s))A^\nu(y(x_0,x,s))U(y(x_0,x,s),x) \quad . \tag{A.4}$$

Therefore (A2) can be written as

$$\partial^x_\mu U(x_0,x)$$
$$= igU(x_0,x)A_\mu(x) + ig\int_0^1 ds\{\frac{\partial y_\nu(x_0,x,s)}{\partial x^\mu} \times$$
$$\times \frac{d}{ds}[U(x_0, y(x_0, y(x_0,x,s))A^\nu(y(x_0,x,s))U(y(x_0,x,s),x)] -$$
$$-\frac{\partial y_\nu(x_0,x,s)}{\partial s}\frac{\partial y^\omega(x_0,x,s)}{\partial x^\mu}U(x_0, y(x_0, y(x_0,x,s))A_{\nu,\omega}(y(x_0,x,s))U(y(x_0,x,s),x)\}$$
$$= igU(x_0,x)A_\mu(x) + ig\int_0^1 ds \frac{\partial y^\nu(x_0,x,s)}{\partial x^\mu}\frac{\partial y^\omega(x_0,x,s)}{\partial s} \times$$
$$\times U(x_0, y(x_0, y(x_0,x,s))\{A_{\omega,\nu}(y(x_0\, x,s)) - A_{\nu,\omega}(y(x_0,x,s)) +$$
$$+ig[A_\omega(y(x_0,x,s)), A_\nu(y(x_0,x,s))]\}U(y(x_0,x,s),x)$$
$$= igU(x_0,x)A_\mu(x) + ig\int_0^1 ds \frac{\partial y^\nu(x_0,x,s)}{\partial x^\mu}\frac{\partial y^\omega(x_0,x,s)}{\partial s} \times$$
$$\times U(x_0, y(x_0, y(x_0,x,s))A_{\omega\nu}(y(x_0,x,s))U(y(x_0,x,s),x) \quad . \tag{A.5}$$

With this we have

$$B_\mu(x,x_0) \equiv \mathbf{U}(x_0,x)B_\mu(x)\mathbf{U}(x,x_0) + \frac{1}{g}[i\partial_{\mu x}U(x_0,x)]U(x,x_0)$$
$$= U(x_0,x)[B_\mu(x) - A_\mu(x)]U(x,x_0) - \int_0^1 ds \frac{\partial y^\nu(x_0,x,s)}{\partial x^\mu}\frac{\partial y^\lambda(x_0,x,s)}{\partial s} \times$$
$$\times U(x_0, y(x_0,x,s))A_{\lambda\nu}(y(x_0,x,s))U(y(x_0,x,s),x)U(x,x_0) \quad , \tag{A.6}$$

where

$$A_{\omega\nu}(y) \equiv \partial_\omega A_\nu(y) - \partial_\nu A_\omega(y) + ig[A_\omega(y), A_\nu(y)] \quad . \tag{A.7}$$

(A.6) is just eq.(4) in the text.

Appendix B

Here we calculate the 2-point and 3-point extended Green's functions $\overline{G}_{\sigma_1\rho_1\sigma_2\rho_2}(x_1, x_1', x_2, x_2')$, $\overline{G}_{\sigma_1\rho_1\sigma_2\rho_2\sigma_3\rho_3}(x_1, x_1', x_2, x_2', x_3, x_3')$.

From the colour symmetry, the 2-point Green's functions can be written as

$$G^{a_1a_2}_{\mu_1\mu_2}(x_1,x_2) = \delta^{a_1a_2}G_{\mu_1\mu_2}(x_1,x_2) \quad . \tag{B.1}$$

Then

$$G^{a_1a_2}_{\mu_1\mu_2}(x_1,x_2)[\overline{\Psi}^{i_1}_{\alpha_1}(x_1)(\frac{\lambda_{a_1}}{2})_{\alpha_1\beta_1}\gamma^{\mu_1}\Psi^{i_1}_{\beta_1}(x_1)][\overline{\Psi}^{i_2}_{\alpha_2}(x_2)(\frac{\lambda_{a_2}}{2})_{\alpha_2\beta_2}\gamma^{\mu_2}\Psi^{i_2}_{\beta_2}(x_2)]$$
$$= \frac{1}{2}G_{\mu_1\mu_2}(x_1,x_2)[\overline{\Psi}^{i_1}_{\alpha_1}(x_1)\gamma^{\mu_1}\Psi^{i_1}_{\beta_1}(x_1)][\overline{\Psi}^{i_2}_{\alpha_2}(x_2)\gamma^{\mu_2}\Psi^{i_2}_{\beta_2}(x_2)] \times$$
$$\times(\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_1} - \frac{1}{N_c}\delta_{\alpha_1\beta_1}\delta_{\alpha_2\beta_2})$$
$$= \int d^4x_1' d^4x_2' G_{\mu_1\mu_2}(x_1,x_2)[-\frac{1}{2}(\gamma^{\mu_1})_{\sigma_1\rho_2}(\gamma^{\mu_2})_{\sigma_2\rho_1}\delta(x_1'-x_2)\delta(x_2'-x_1) -$$
$$-\frac{1}{2N_c}(\gamma^{\mu_1})_{\sigma_1\rho_1}(\gamma^{\mu_2})_{\sigma_2\rho_2}\delta(x_1'-x_1)\delta(x_2'-x_2)] \times$$
$$\times\overline{\Psi}^{\sigma_1}_{\alpha_1}(x_1,x_0)\Psi^{\rho_1}_{\alpha_1}(x_1',x_0)\overline{\Psi}^{\sigma_2}_{\alpha_2}(x_2,x_0)\Psi^{\rho_2}_{\alpha_2}(x_2',x_0) \quad , \tag{B.2}$$

where i is the flavour index, α, β are colour indices, σ, ρ are combined Lorents and flavour indices. Hence the extended 2-point Green's functions in (25) is

$$\overline{G}^{\sigma_1\sigma_2}_{\rho_1\rho_2}(x_1,x_1',x_2,x_2') = -\frac{1}{2}G_{\mu_1\mu_2}(x_1,x_2)[(\gamma^{\mu_1})_{\sigma_1\rho_2}(\gamma^{\mu_2})_{\sigma_2\rho_1}\delta(x_1'-x_2)\delta(x_2'-x_1) +$$
$$+\frac{1}{N_c}(\gamma^{\mu_1})_{\sigma_1\rho_1}(\gamma^{\mu_2})_{\sigma_2\rho_2}\delta(x_1'-x_1)\delta(x_2'-x_2)] \quad . \tag{B.3}$$

For the 3-point function,we need the $SU(N_c)$ relations

$$
\begin{aligned}
f_{abc} &= -2i([\frac{\lambda_a}{2},\frac{\lambda_b}{2}]\frac{\lambda_c}{2}) = \frac{i}{4}(\lambda_b)_{\alpha\beta}(\lambda_a)_{\beta\gamma}(\lambda_c)_{\gamma\alpha} - \frac{i}{4}(\lambda_a)_{\alpha\beta}(\lambda_b)_{\beta\gamma}(\lambda_c)_{\gamma\alpha} \quad , \\
d_{abc} &= 2(\{\frac{\lambda_a}{2},\frac{\lambda_b}{2}\}\frac{\lambda_c}{2}) = \frac{1}{4}(\lambda_b)_{\alpha\beta}(\lambda_a)_{\beta\gamma}(\lambda_c)_{\gamma\alpha} + \frac{1}{4}(\lambda_a)_{\alpha\beta}(\lambda_b)_{\beta\gamma}(\lambda_c)_{\gamma\alpha} \quad ,
\end{aligned}
\tag{B.4}
$$

and

$$
\begin{aligned}
&f_{abc}(\lambda_a)_{\alpha_1\beta_1}(\lambda_b)_{\beta_2}(\lambda_c)_{\alpha_3\beta_3} = 2i(\delta_{\alpha_3\beta_2}\delta_{\alpha_2\beta_1}\delta_{\alpha_1\beta_3} - \delta_{\alpha_3\beta_1}\delta_{\alpha_1\beta_2}\delta_{\alpha_{12}\beta_3}) \quad , \\
&d_{abc}(\lambda_a)_{\alpha_1\beta_1}(\lambda_b)_{\beta_2}(\lambda_c)_{\alpha_3\beta_3} = 2(\delta_{\alpha_3\beta_2}\delta_{\alpha_2\beta_1}\delta_{\alpha_1\beta_3} + \delta_{\alpha_3\beta_1}\delta_{\alpha_1\beta_2}\delta_{\alpha_2\beta_3} - \\
&-\frac{2}{N_c}\delta_{\alpha_1\beta 1}\delta_{\alpha_2\beta 3}\delta_{\alpha_3\beta 2} - \frac{2}{N_c}\delta_{\alpha_1\beta 3}\delta_{\alpha_2\beta 2}\delta_{\alpha_3\beta 1} - \frac{2}{N_c}\delta_{\alpha_1\beta 2}\delta_{\alpha_2\beta 1}\delta_{\alpha_3\beta 3} + \frac{4}{N_c^2}\delta_{\alpha_1\beta 1}\delta_{\alpha_2\beta 2}\delta_{\alpha_3\beta 3}) \quad .
\end{aligned}
\tag{B.5}
$$

From the colour symmetry, the 3-point Geen's function can be written as

$$
G^{a_1a_2a_3}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3) = f_{a_1a_2a_3}G^{(0)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3) + d_{a_1a_2a_3}G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3) \quad . \tag{B.6}
$$

Combine (B4) (B5) and (B6),we get

$$
\begin{aligned}
&\overline{G}^{\sigma_1\sigma_2\sigma_3}_{\rho_1\rho_2\rho_3}(x_1,x_1',x_2,x_2',x_3,x_3') \\
&= \frac{i}{4}G^{(0)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)[\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_3}\gamma^{\mu_3}_{\sigma_3\rho_1}\delta(x_1'-x_3)\delta(x_2'-x_1)\delta(x_3'-x_2) - \\
&-\gamma^{\mu_1}_{\sigma_1\rho_3}\gamma^{\mu_2}_{\sigma_2\rho_1}\gamma^{\mu_3}_{\sigma_3\rho_2}\delta(x_1'-x_2)\delta(x_2'-x_3)\delta(x_3'-x_1)] \\
&+\frac{1}{4}G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)[\gamma^{\mu_1}_{\sigma_1\rho_3}\gamma^{\mu_2}_{\sigma_2\rho_1}\gamma^{\mu_3}_{\sigma_3\rho_2}\delta(x_1'-x_2)\delta(x_2'-x_3)\delta(x_3'-x_1) + \\
&+\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_3}\gamma^{\mu_3}_{\sigma_3\rho_1}\delta(x_1'-x_3)\delta(x_2'-x_1)\delta(x_3'-x_2) + \\
&+\frac{2}{N_c}\gamma^{\mu_1}_{\sigma_1\rho_1}\gamma^{\mu_2}_{\sigma_2\rho_3}\gamma^{\mu_3}_{\sigma_3\rho_2}\delta(x_1'-x_1)\delta(x_2'-x_3)\delta(x_3'-x_2) + \\
&+\frac{2}{N_c}\gamma^{\mu_1}_{\sigma_1\rho_3}\gamma^{\mu_2}_{\sigma_2\rho_2}\gamma^{\mu_3}_{\sigma_3 rho_1}\delta(x_1'-x_3)\delta(x_2'-x_2)\delta(x_3'-x_1) + \\
&+\frac{2}{N_c}\gamma^{\mu_1}_{\sigma_1\rho_2}\gamma^{\mu_2}_{\sigma_2\rho_1}\gamma^{\mu_3}_{\sigma_3\rho_3}\delta(x_1'-x_2)\delta(x_2'-x_1)\delta(x_3'-x_3) + \\
&+\frac{4}{N_c^2}\gamma^{\mu_1}_{\sigma_1\rho_1}\gamma^{\mu_2}_{\sigma_2\rho_2}\gamma^{\mu_3}_{\sigma_3\rho_3}\delta(x_1'-x_1)\delta(x_2'-x_2)\delta(x_3'-x_3)] \quad .
\end{aligned}
\tag{B.7}
$$

Appendix C

Now we give the explicit expressions for $K_0^{\sigma_1\rho_1\sigma_2\rho_2}$ and $K_0^{\sigma_1\rho_1\sigma_2\rho_2\sigma_3\rho_3}$ in terms of the original gluon Green's functions. For convinience, we take the covariant gauge $\partial_\mu B^\mu = 0$. Due to translational invariance and colour symmetry, the 2-point Green's function can be written as

$$
G^{a_1a_2}_{\mu_1\mu_2}(x_1,x_2) = \delta_{a_1a_2}G_{\mu_1\mu_2}(x_1-x_2) \quad . \tag{C.1}
$$

We define $C_1^{\sigma_1\rho_1\sigma_2\rho_2}$ and $C_2^{\sigma_1\rho_1\sigma_2\rho_2}$ by

$$
\begin{aligned}
&G_{\mu_1\mu_2}(-x)(\gamma^{\mu_1})_{\sigma_1\rho_2}(\gamma^{\mu_2})_{\sigma_2\rho_1}C_1^{\sigma_2\rho_2\sigma_3\rho_3}(x) = -2\delta_{\sigma_1\sigma_3}\delta_{\rho_1\rho_3} \quad , \\
&[(\gamma_\mu)_{\sigma_1\rho_2}(\gamma^\mu)_{\sigma_2\rho_1} + \frac{\delta(0)}{N_c}\int d^4x\gamma_\mu)_{\sigma_1\rho_1}(\gamma^\mu)_{\sigma_2\rho_2}]C_2^{\sigma_2\rho_2\sigma_3\rho_3} = -\frac{1}{N_c}(\gamma_\mu)_{\sigma_1\rho_1}(\gamma^\mu)_{\sigma_2\rho_2}C_1^{\sigma_2\rho_2\sigma_3\rho_3} \quad .
\end{aligned}
\tag{C.2}
$$

Then elementary calculations show that

$$
\begin{aligned}
&\int d^4x_1d^4x_1'd^4x_2d^4x_2'K_0^{\sigma_1\rho_1\sigma_2\rho_2}(x_1,x_1',x_2,x_2')\chi_{\sigma_1\rho_1}(x_1,x_1')\chi_{\sigma_2\rho_2}(x_2,x_2') \\
&= \int d^4x_1d^4x_2[C_1^{\sigma_1\rho_1\sigma_2\rho_2}(x_1-x_2)\chi_{\sigma_1\rho_1}(x_1,x_2)\chi_{\sigma_2\rho_2}(x_2,x_1) + \\
&+C_2^{\sigma_1\rho_1\sigma_2\rho_2}\chi_{\sigma_1\rho_1}(x_1,x_1)\chi_{\sigma_2\rho_2}(x_2,x_2)] \quad ,
\end{aligned}
\tag{C.3}
$$

$$
\begin{aligned}
&\int d^4x_1d^4x_1'd^4x_2d^4x_2'd^4x_3d^4x_3'K_0^{\sigma_1\rho_1\sigma_2\rho_2\sigma_3\rho_3}(x_1,x_1',x_2,x_2',x_3,x_3') \times \\
&\times\chi_{\sigma_1\rho_1}(x_1,x_1')\chi_{\sigma_2\rho_2}(x_2,x_2')\chi_{\sigma_3\rho_3}(x_3,x_3') \\
&= \int d^4x_1d^4x_2d^4x_3\{[\frac{i}{2}G^{(0)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3) - \frac{1}{2}G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)] \times
\end{aligned}
$$

$$
\begin{aligned}
&\times(\gamma^{\mu_1})_{\epsilon_1\delta_3}(\gamma^{\mu_2})_{\epsilon_2\delta_1}(\gamma^{\mu_3})_{\epsilon_3\delta_2}C_1^{\sigma_1\rho_1\epsilon_1\delta_1}(x_1-x_2)C_1^{\sigma_2\rho_2\epsilon_2\delta_2}(x_2-x_3)C_1^{\sigma_3\rho_3\epsilon_3\delta_3}(x_3-x_1)\times\\
&\times\chi_{\sigma_1\rho_1}(x_1,x_2)\chi_{\sigma_2\rho_2}(x_2,x_3)\chi_{\sigma_3\rho_3}(x_3,x_1)+\\
&+3[(\frac{i}{2}G^{(0)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)-\frac{1}{2}G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3))(\gamma^{\mu_1})_{\epsilon_1\delta_3}(\gamma^{\mu_2})_{\epsilon_2\delta_1}(\gamma^{\mu_3})_{\epsilon_3\delta_2}\\
&-\frac{\delta(0)}{2N_c}\int d^4y_1G^{(1)}_{\mu_1\mu_2\mu_3}(y_1,x_2,x_3))(\gamma^{\mu_1})_{\epsilon_1\delta_1}(\gamma^{\mu_2})_{\epsilon_2\delta_3}(\gamma^{\mu_3})_{\epsilon_3\delta_2}]\times\\
&\times C_2^{\sigma_1\rho_1\epsilon_1\delta_1}C_1^{\sigma_2\rho_2\epsilon_2\delta_2}(x_2-x_3)C_1^{\sigma_3\rho_3\epsilon_3\delta_3}(x_3-x_2)\chi_{\sigma_1\rho_1}(x_1,x_1)\chi_{\sigma_2\rho_2}(x_2,x_3)\chi_{\sigma_3\rho_3}(x_3,x_2)+\\
&+[-\frac{1}{2N_c}G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)(3(\gamma^{\mu_1})_{\epsilon_1\delta_1}(\gamma^{\mu_2})_{\epsilon_2\delta_3}(\gamma^{\mu_3})_{\epsilon_3\delta_2}+\\
&+\frac{2}{N_c}(\gamma^{\mu_1})_{\epsilon_1\delta_1}(\gamma^{\mu_2}_{\nu})_{\epsilon_2\delta_2}(\gamma^{\mu_3})_{\epsilon_3\delta_3})C_1^{\sigma_1\rho_1\epsilon_1\delta_1}(0)-\\
&-\frac{3}{N_c}(G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)(\gamma^{\mu_1})_{\epsilon_1\delta_2}(\gamma^{\mu_2})_{\epsilon_2\delta_1}(\gamma^{\mu_3})_{\epsilon_3\delta_3}+\frac{\delta(0)}{N_c}\int d^4yG^{(1)}_{\mu_1\mu_2\mu_3}(y,x_2,x_3)\times\\
&\times(\gamma^{\mu_1})_{\epsilon_1\delta_1}(\gamma^{\mu_2})_{\epsilon_2\delta_2}(\gamma^{\mu_3})_{\epsilon_3\delta_3})\times\\
&\times C_2^{\sigma_1\rho_1\epsilon_1\delta_1}]C_1^{\sigma_2\rho_2\epsilon_2\delta_2}(0)C_1^{\sigma_3\rho_3\epsilon_3\delta_3}(0)\chi_{\sigma_1\rho_1}(x_1,x_1)\chi_{\sigma_2\rho_2}(x_2,x_2)\chi_{\sigma_3\rho_3}(x_3,x_3)\}\quad . \qquad (C.4)
\end{aligned}
$$

where $G^{(0)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)$ and $G^{(1)}_{\mu_1\mu_2\mu_3}(x_1,x_2,x_3)$ are defined in (B).

In general, the $K_0^{\sigma_1\rho_1\cdots\sigma_n\rho_n}$ term must be of the form

$$
\begin{aligned}
&\int d^4x_1d^4x_1'\cdots d^4x_nd^4x_n'K_0^{\sigma_1\rho_1\cdots\sigma_n\rho_n}(x_1,x_1',\cdots,x_n,x_n')\chi_{\sigma_1\rho_1}(x_1,x_1')\cdots\chi_{\sigma_n\rho_n}(x_n,x_n')\\
=\ &\int d^4x_1d^4x_2d^4x_3\cdots d^4x_{n-2}d^4x_{n-1}d^4x_n\times\\
&\overline{C}^{\sigma_1\rho_1\cdots\sigma_n\rho_n}_{(1)}(x_1,x_2,x_3,\cdots,x_{n-2},x_{n-1},x_n)\chi^{\sigma_1\rho_1}(x_1,x_2)\chi^{\sigma_2\rho_2}(x_2,x_3)\chi^{\sigma_3\rho_3}(x_3,x_4)\cdots\\
&\cdots\chi^{\sigma_{n-2}\rho_{n-2}}(x_{n-2},x_{n-1})\chi^{\sigma_{n-1}\rho_{n-1}}(x_{n-1},x_n)\chi^{\sigma_n\rho_n}(x_n,x_1)+\\
&+\overline{C}^{\sigma_1\rho_1\cdots\sigma_n\rho_n}_{(2)}(x_1,x_2,x_3,\cdots,x_{n-2},x_{n-1},x_n)\chi^{\sigma_1\rho_1}(x_1,x_1)\chi^{\sigma_2\rho_2}(x_2,x_3)\chi^{\sigma_3\rho_3}(x_3,x_4)\cdots\\
&\cdots\chi^{\sigma_{n-2}\rho_{n-2}}(x_{n-2},x_{n-1})\chi^{\sigma_{n-1}\rho_{n-1}}(x_{n-1},x_n)\chi^{\sigma_n\rho_n}(x_n,x_2)+\\
&+\cdots\cdots\\
&\overline{C}^{\sigma_1\rho_1\cdots\sigma_n\rho_n}_{(n-1)}(x_1,x_2,x_3,\cdots,x_{n-2},x_{n-1},x_n)\chi^{\sigma_1\rho_1}(x_1,x_1)\chi^{\sigma_2\rho_2}(x_2,x_2)\chi^{\sigma_3\rho_3}(x_3,x_3)\cdots\\
&\cdots\chi^{\sigma_{n-2}\rho_{n-2}}(x_{n-2},x_{n-2})\chi^{\sigma_{n-1}\rho_{n-1}}(x_{n-1},x_n)\chi^{\sigma_n\rho_n}(x_n,x_{n-1})+\\
&\overline{C}^{\sigma_1\rho_1\cdots\sigma_n\rho_n}_{(n)}(x_1,x_2,x_3,\cdots,x_{n-2},x_{n-1},x_n)\chi^{\sigma_1\rho_1}(x_1,x_1)\chi^{\sigma_2\rho_2}(x_2,x_2)\chi^{\sigma_3\rho_3}(x_3,x_3)\cdots\\
&\cdots\chi^{\sigma_{n-2}\rho_{n-2}}(x_{n-2},x_{n-2})\chi^{\sigma_{n-1}\rho_{n-1}}(x_{n-1},x_{n-1})\chi^{\sigma_n\rho_n}(x_n,x_n)]\quad ,
\end{aligned}
$$

(C.5)

in which $\overline{C}^{\sigma_1\rho_1\cdots\sigma_n\rho_n}_{(n)}(x_1,x_2,x_3,\cdots,x_{n-2},x_{n-1},x_n)$ is some coefficients which only depend on the gluon Green's functions.

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