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QCD Multipole Expansion and Its Applications to Heavy-Quarkonium Decays[1]

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ABSTRACT

We give a review of the generalised formalism of QCD multipole expansion including constituent quark field quantization and its applications to heavy-quarkonium decays such as hadronic transitions in the $c\bar{c}$ and $b\bar{b}$ systems and radiative decays of J/ψ.

I. QCD Multipole Expansion

There are a lot of physically interesting hadronic processes containing soft-gluon emissions to which perturbative QCD is not applicable. For heavy-quarkonium systems, due to the smallness of their sizes, QCD multipole expansion can be a useful tool for treating the soft-gluon emission problem. Let a be the size of a heavy-quarkonium, k be the typical momentum of the emitted gluons. The expansion parameter in QCD multipole expansion is ka, therefore the expansion is good if ka is small even the QCD coupling constant g_s is large. For instance, in the hadronic transition process $\Upsilon' \to \Upsilon \quad \pi+\pi$, the pions are converted from at least two gluons emitted by b or $\bar{b}$ and the total gluon-energy is roughly $M_{\Upsilon'} - M_{\Upsilon} = 560MeV$, so that the gluons are soft. The size of Υ is $a \approx 0.2fm^1$, therefore the expansion paremeter is roughly $ka \sim 0.3$ which shows that QCD multipole expansion is powerful in calculating hadronic transition rates. Actually from the experience in nuclear physics and the formula in classical electrodynamics[2] we expect that the expansion may work well even when $ka \sim 1$ due to the rapid convergence of the coefficients in the expansion. Applications of QCD multipole expansion to hadronic transitions in heavy-quarkonium systems has been studied by many authors[3-7]. A systematical gauge invariant formalism of QCD multipole expansion in terms of consititutent quark field is given by Yan[6]. This is generalized to include constituent quark field quantization and to incorparate electroweak interactions in Ref.8, which can be applied to study soft-gluon emission processes of heavy- quarkonium systems with quark flavor changing or quark pair annihilation. In the following, we will give a brief review of the formalism given in Ref.8.

Let us consider a system consisting of a heavy quark Q and its antiquark $\overline{Q}$. (Generalization to a system with different quarks is trival.) The fundamental Lagrangian of QCD is

$$\mathcal{L}_{QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \overline{\psi}\left[\gamma^\mu(i\partial_\mu - g_s\frac{\lambda_a}{2}A^a_\mu) - m\right]\psi + \mathcal{L}_{g.f.} + \mathcal{L}_{FP} \ , \qquad (1)$$

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where ψ is the field of Q , $A^a_\mu(a = 1,...,8)$ is the gluon field, λ_a is the Gell-Mann matrix, $\mathcal{L}_{g.f.}$ and $\mathcal{L}_{FP}$ are the gauge fixing term and the gauge conpensation term, respectively. Following Yan[6], we can obtain from (1) the gauge invariant effective Lagrangian for the heavy quark system suitable for studying soft-gluon emissions

$$L_Q = \int d^3x\overline{\Psi}\left[\gamma^\mu(i\partial_\mu - g_s\frac{\lambda_a}{2}A^{a'}_\mu) - m\right]\Psi - \frac{1}{2}\int d^3x d^3y \sum_{a=0}^{8}\overline{\Psi}(\vec{x},t)\gamma^0\frac{1}{2}\lambda_a\Psi(\vec{x},t)$$
$$\times\left[\delta_{a0}V_1(|\vec{x}-\vec{y}|) + (1-\delta_{a0})V_2(|\vec{x}-\vec{y}|)\right]\overline{\Psi}(\vec{x},t)\gamma^0\frac{1}{2}\lambda_a\Psi(\vec{x},t) \ , \qquad (2)$$

in which $\frac{1}{2}\lambda_0 \equiv 1$, V_1 and V_2 are static potentials related to the interactions between Q and $\overline{Q}$ in color singlet and color octet states, $\Psi(\vec{x},t)$ and $A^{a'}_\mu(\vec{x},t)$ are related to the original $\psi(x)$ and $A^a_\mu(x)$ by

$$\Psi(\vec{x},t) = U^{-1}(\vec{x},t)\psi(x) \ , \qquad (3)$$

$$\frac{1}{2}\lambda_a A^{a'}_\mu(\vec{x},t) = U^{-1}(\vec{x},t)\frac{1}{2}\lambda_a A^a_\mu(x)U(\vec{x},t) - \frac{i}{g_s}U^{-1}(\vec{x},t)\partial_\mu U(\vec{x},t) \ , \qquad (4)$$

with $U(\vec{x},t)$ defined by

$$U(\vec{x},t) = P\, exp\, ig_s\int_{\vec{X}}^{\vec{x}} d\vec{x}'\cdot\lambda_a\vec{A}^a(\vec{x}',t) \ , \qquad (5)$$

where $\vec{X}$ is the coordinate of the center-of-mass of Q and $\overline{Q}$, P is the path ordering operation and the line integral is along the straight line segment connecting two ends. In most soft-gluon emission processes, the emitted gluon energy is much smaller than the quarkonium masses. We shall neglect the small recoil of the quarkonium due to the emission and regard $\vec{X}$ as a constant. We see from (2) that it is the field Ψ , not ψ , that represents Q and $\overline{Q}$ interacting via the static potential. Therefore Ψ is regarded as the constituent quark field in the potential model of heavy quarkoia.

We then generalize (2) to include electroweak interactions and Ψ field quantization. We add the electromagnetic interaction

$$\mathcal{L}_{e.m.} = -e\overline{\psi}\gamma^\mu\mathcal{Q}\mathcal{A}_\mu\psi \ , \qquad (6)$$

and weak interactions

$$\mathcal{L}_W = -\frac{g}{\sqrt{2}}\overline{\psi}\gamma^\mu\frac{1-\gamma_5}{2}(t_+W^+_\mu + t_-W^-_\mu)\psi -$$
$$-\frac{g}{\cos\theta_W}\overline{\psi}\gamma^\mu\left(\frac{1-\gamma_5}{2}t_3 - \sin^2\theta_W\mathcal{Q}\right)Z_\mu\psi \ , \qquad (7)$$

to (1) . Here e is the electromagnetic coupling constant, $\mathcal{Q}$ is the electric charge operator of the quark ($\mathcal{Q} = 2/3$ for $c-$quark, $\mathcal{Q} = -1/3$ for $b-$quark), $\mathcal{A}_\mu$ is the

photon field, g is the $SU(2)$ coupling constant, θ_W is the Weinberg angle, $W_\mu^\pm$, Z_μ are the intermediate vector boson fields and $t_\pm$, t_3 are $SU(2)$ generators. The total Lagrangian is then

$$\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{e.m.} + \mathcal{L}_W \ . \tag{8}$$

Since the electroweak sector is independent of the QCD sector, the ψ's in (6) and (7) can be transormed directly into Ψ's by (3) , and thus the total effective Lagrangian is

$$L_Q = L_{QCD}^{(0)} + L_{QCD}^{(1)} + L_{e.m.} + L_W \ , \tag{9}$$

where

$$\begin{aligned} L_{QCD}^{(0)} &= \int d^3x \overline{\Psi}(\vec{x},t)(i\ \partial\!\!\!/ - m)\Psi(\vec{x},t) - \frac{1}{2}\int d^3x_1 d^3x_2 \\ &\sum_{a=0}^{8} \overline{\Psi}(\vec{x}_1,t)\gamma^a \frac{\lambda_a}{2}\Psi(\vec{x}_1,t)\Big[\delta_{a0}V_1(|\vec{x}_1-\vec{x}_2|)+ \\ &+ (1-\delta_{a0})V_2(|\vec{x}_1-\vec{x}_2|)\Big]\overline{\Psi}(\vec{x}_2,t)\gamma^0\frac{\lambda_a}{2}\Psi(\vec{x}_2,t) \ , \end{aligned} \tag{10}$$

$$L_{QCD}^{(1)} = -g_s \int d^3x \overline{\Psi}(\vec{x},t)\gamma^\mu \frac{\lambda_a}{2} A_\mu^{a'}(\vec{x},t)\Psi(\vec{x},t) \ , \tag{11}$$

$$L_{e.m.} = -e\int d^3x \overline{\Psi}(\vec{x},t)\gamma^\mu \mathcal{Q} A_\mu(x)\Psi(\vec{x},t) \ , \tag{12}$$

$$\begin{aligned} L_W = -\int d^3x \Big[&\frac{g}{\sqrt{2}}\overline{\Psi}(\vec{x},t)\gamma^\mu \frac{1-\gamma_5}{2}\left(t_+W_\mu^+(x) + t_-W_\mu^-(x)\right)\Psi(\vec{x},t) + \\ &+ \frac{g}{\cos\theta_W}\overline{\Psi}(\vec{x},t)\gamma^\mu\left(\frac{1-\gamma_5}{2}t_3 - \sin^2\theta_W \mathcal{Q}\right)Z_\mu(x)\Psi(\vec{x},t)\Big] \ . \end{aligned} \tag{13}$$

Expanding $A_\mu^{a'}(\vec{x},t)$ in powers of $(\vec{x}-\vec{X})$ via the procedure given in Ref.6 , we get the multipole expansion

$$\begin{aligned} A_0^{a'}(\vec{x},t) &= A_0^a(\vec{x},t) - (\vec{x}-\vec{X})\cdot \vec{E}^a(\vec{X},t) + \cdots \ , \\ \vec{A}^{a'}(\vec{x},t) &= -\tfrac{1}{2}(\vec{x}-\vec{X})\times \vec{B}^a(\vec{X},t) + \cdots \ , \end{aligned} \tag{14}$$

where $\vec{E}^a$ and $\vec{B}^a$ are the color electric and magnetic fields. In (9)-(13) we have separated L_{QCD} into two parts. The part $L_{QCD}^{(0)}$ gives the bound state properties of the $Q\overline{Q}$ system, while $L_{QCD}^{(1)}$ gives multipole gluon emissions.

In non-relativistic approximation, the Hamiltonian of the system can be written as

$$H = H_{QCD}^{(0)} + H_{QCD}^{(1)} + H_{e.m.} + H_W \ , \tag{15}$$

where

$$\begin{aligned} H_{QCD}^{(0)} &= \int d^3x_1 d^3x_2 \Psi^\dagger(\vec{x}_1,t)\Psi(\vec{x}_1,t)\hat{H}\Psi^\dagger(\vec{x}_2,t)\Psi(\vec{x}_2,t) \ , \\ H_{QCD}^{(1)} &= -L_{QCD}^{(1)}, \quad H_{e.m.} = -L_{e.m.}, \quad H_W = -L_W \ , \end{aligned} \tag{16}$$

and

$$\hat{H} \equiv \frac{1}{2m}(\partial_1^2 + \partial_2^2) + V_1(|\vec{x}_1 - \vec{x}_2|) + \sum_{a=1}^{8}\frac{\lambda_a}{2}\frac{\overline{\lambda}_a}{2}V_2(|\vec{x}_1-\vec{x}_2|) + 2m \ , \tag{17}$$

is just the quantum mechanical Hamiltonian of the $Q\overline{Q}$ system.

The canonical momentum $\pi(\vec{x},t)$ conjugate to $\Psi(\vec{x},t)$ is now

$$\pi(\vec{x},t) = \frac{\delta\mathcal{L}}{\delta\dot{\Psi}(\vec{x},t)} = \frac{\delta\mathcal{L}_{QCD}^{(0)}}{\delta\dot{\Psi}(\vec{x},t)} = i\Psi^\dagger(\vec{x},t) \ . \tag{18}$$

Therefore the canonical quantization of the Ψ field is given by

$$\begin{aligned} &\{\Psi(\vec{x},t),\Psi^\dagger(\vec{x}',t)\} = \delta^3(\vec{x}-\vec{x}') \ , \\ &\textit{others anticommute} \ . \end{aligned} \tag{19}$$

With this equal-time communication relation, we can evaluate the $S-$matrix elements for soft-gluon emission processes.

A gauge invariant color singlet $Q\overline{Q}$ bound state with momentum $\vec{k}$ (momentum of the c.o.m.), a set of spatial quantum number λ and spin quantum number σ , can be expressed as

$$|\lambda,\sigma,\vec{k}> = \frac{1}{\sqrt{3}}\int d^3x_1 d^3x_2 f_\lambda(\vec{x}_1-\vec{x}_2)\overline{\Psi}_\alpha(\vec{x}_1,t)\Gamma_\sigma\Psi_\alpha(\vec{x}_2,t)e^{i\vec{k}\cdot\vec{X}}\,|0> \ , \tag{20}$$

where $\alpha(\alpha = 1,2,3)$ is the color index of the quark, $f_\lambda(\vec{x}_1-\vec{x}_2)$ is the spatial wave function of the bound state and Γ_σ is the matrix determing the addition of spins of Q and $\overline{Q}$. This state is normalized as

$$\begin{aligned} &\int \tfrac{d^3k}{(2\pi)^3}\,|\lambda,\sigma,\vec{k}><\lambda,\sigma,\vec{k}| = 1 \ , \\ &\textstyle\sum_{\lambda,\sigma}\int \tfrac{d^3k}{(2\pi)^3}\,|\lambda,\sigma,\vec{k}><\lambda,\sigma,\vec{k}| = 1 \ . \end{aligned} \tag{21}$$

In the following calculations, we shall take $H_{QCD}^{(0)}$ as the zeroth order Hamiltonian and regard $H_{QCD}^{(1)} + H_{e.m.} + H_W$ as a perturbation. This is different from the conventional pertuabation in quantum field theory since $H_{QCD}^{(0)}$ is not the free quark Hamiltonian. We shall use the following shortened natations:

$$H_0 \equiv H_{QCD}^{(0)} \ , \quad H_{int} \equiv H_{QCD}^{(1)} + H_{e.m.} + H_W \ , \tag{22}$$

and

$$\begin{aligned} H_{QCD}^{(1)} &= H_1 + H_2 \ , \\ H_1 &\equiv Q_a A_0^a(\vec{X},t) \ , \\ H_2 &\equiv -\vec{d}_a\cdot\vec{E}^a(\vec{X},t) - \vec{m}_a\cdot\vec{B}^a(\vec{X},t) + \cdots \ , \end{aligned} \tag{23}$$

where

$$
\begin{aligned}
Q_a &\equiv g_s \int d^3x \Psi^\dagger(\vec{x},t)\tfrac{\lambda_a}{2}\Psi(\vec{x},t) \ , \\
\vec{d}_a &\equiv g_s \int d^3x(\vec{x}-\vec{X})\Psi^\dagger(\vec{x},t)\tfrac{\lambda_a}{2}\Psi(\vec{x},t) \ , \\
\vec{m}_a &\equiv \tfrac{1}{2} g_s \int d^3x(\vec{x}-\vec{X}) \times \Psi^\dagger(\vec{x},t)\, \vec{\gamma}\, \tfrac{\lambda_a}{2}\Psi(\vec{x},t) \ ,
\end{aligned}
\tag{24}
$$

are the color charge, color electric dipole moment, and color magnetic dipole moment of the $Q\overline{Q}$ system, respectively. Here we have separated $H^{(1)}_{QCD}$ into $H_1 + H_2$. In soft-gluon emission processes, H_1 is not small, so that we should treat H_1 to all orders.

Let $| i >$ and $| f >$ be the initial and final states. To n−th order, the S−matrix element is

$$
< f \mid S^{(n)} \mid i > = < f \mid \int_{-\infty}^{+\infty} dt_1 \int_{-\infty}^{t_1} dt_2 \cdots \int_{-\infty}^{t_{n-1}} dt_n
$$

$$
H_{int}(t_1) H_{int}(t_2) \cdots H_{int}(t_2) \mid i > \ . \tag{25}
$$

Taking into account H_1 contributions to all orders and $(H_2 + H_{e.m.} + H_m)$ to n−th order, for color singlet $| i >$ and $| f >$ the S−matrix element can be written as

$$
< f \mid S \mid i > = -i2\pi\delta(E_f + \omega_f - E_i) < f \mid (H_2 + H_{e.m.} + H_W) \frac{1}{E_i - H_0 + i\partial_0 - H_1}
$$

$$
\times \cdots \times \frac{1}{E_i - H_0 + i\partial_0 - H_1}(H_2 + H_{e.m.} + H_W) \mid i > \ , \tag{26}
$$

where E_i and E_f are energies of the $Q\overline{Q}$ systems in the initial and final states, respectively, and ω_f is the energy emitted gauge fields in the final states. In (26) , it is understood that the resolvent $(E_i - H_0 + i\partial_0)^{-1}$ is the shortened notation of $(E_i - H_0 + i\partial_0 + i\epsilon)^{-1}$ with $\epsilon \to +0$. This is our general formula for the S−matrix element. It can be evaluated by using the equal-time commutation relation (19).

Eq.(26) can be further generalized to include coupled-channel effects. Suppose H_{QPC} is a Hamiltonian which creats light quark pair from the vacuum. Let

$$
H_{pair} \equiv H_{QPC} + H^\dagger_{QPC} \ . \tag{27}
$$

It can be shown that the generalized S−matrix element including coupled-channel effects is[9]

$$
< f \mid S \mid i > = -i2\pi\delta(E_f + \omega_f - E_i) < f \mid (H_2 + H_{e.m.} + H_W + H_{pair}) \times
$$

$$
\times \frac{1}{E_i - H_0 + i\partial_0 - H_1} \cdots \frac{1}{E_i - H_0 + i\partial_0 - H_1} \times
$$

$$
\times (H_2 + H_{e.m.} + H_W + H_{pair}) \mid i > \ . \tag{28}
$$

II. Application to Hadronic Transitions in Heavy-quarkonium Systems

Consider hadrinic transition processes

$$
\Phi_i \to \Phi_f + h \ , \tag{29}
$$

where h denotes light hadrons, Φ_i and Φ_f are the initial- and final-state quarkonia, respectively. The diagram for (29) is shown in Fig.1. It contains two steps: (a)multipole gluon-emissions from the heavy- quarks, (b)conversion of the emitted gluons into h—hadronization. Different transition processes may be dominated by different multipole gluon-emissions. From the general formula (26) , we can derive the formula for transition amplitudes[8], e.g. the $E1 - E1$ amplitude is

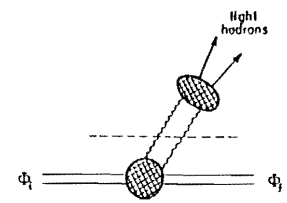

Fig.1. Amplitude for hadronic transition between heavy quarkonium states Φ_i and Φ_f.

$$
\mathcal{M}_{E_1-E_1} = i\frac{g_E^2}{6} \sum_{KL} \frac{< \Phi_f \mid x_k \mid KL >< KL \mid x_l \mid \Phi_i >}{E_i - E_{KL}} < h \mid E^a_k E^a_l \mid 0 > \ , \tag{30}
$$

where $| KL >$ and E_{KL} are the state-vector and energy eigenvalue of the intermediate state with principal quantum number K and angular momentum quantum number L , respectively. Note that (2) is an effective Lagrangian. Considering that the heave-quark may have an anomalous magnetic moment, we distinguish here the electric coupling constant g_E and the magnetic coupling constant g_M in the effective Lagrangian instead of taking simply a single g_s. The first factor related to Φ_i , Φ_f in (30) represents the multipole gluon-emission which can be calculated by taking a certain potential model for the quarkonium. The intermediate state contains a soft gluon and a $Q\overline{Q}$ pair in color octet. Such a three-body bound state is difficult to study from the first principles of QCD. We can take a proper model to imitate this state, for example, the vibrational state of the quark-confining-string model[10] or the hybrid state in the bag model[11]. They all give similar results[7,12]. The second factor related to h in (30) represents the hadronization (conversion of gluons into h) which needs further considerations.

1. $\Phi(n_i^3S_1) \to \Phi(n_f^3S_1) + \pi + \pi$ *Transitions.*

This is dominated by $E_1 - E_1$ multipole gluon-emission. The evaluation of the multipole gluon-emission factor in (30) is straightforward. The hadronization factor is considered as follows.

From PCAC and soft-pion approach, we can write

$$
\frac{g_E^2}{6} < \pi^\alpha(q_1),\ \pi^{\beta(q_2)} \mid E^a_k E^a_l \mid 0 > = \frac{\delta_{\alpha\beta}}{(2\omega_1 2\omega_2)^{1/2}} C_1 \delta_{kl} q_1^\mu q_{2\mu} + \cdots \ , \tag{31}
$$

With (28) we can further take into account the coupled-channel corrections to (30)[9] . In TABLE I we list the predicted rates for $\Upsilon' \to \Upsilon\pi\pi$, $\Upsilon'' \to \Upsilon'\pi^+\pi^-$ and $\Upsilon'' \to \Upsilon\pi^+\pi^-$ in the coupled-channel theory[9] together with the corresponding experimental data. The phase $e^{i\theta}$ of C_1 is taken to be the bounds $cos\theta = -1$ and $cos\theta = -0.676$ determined from the data of $d\Gamma(\psi' \to J/\psi\pi\pi)/dM_{\pi\pi}$. We see that they are in good agreement. The comparison of the predicted $M_{\pi\pi}$ distributions in the coupled-channel theory with the experiments is shown in Fig.2 and Fig.3. We see that for $\Upsilon' \to \Upsilon\pi\pi$ the agreement is excellent. For $\Upsilon'' \to \Upsilon\pi^+\pi^-$ the CLEO data[15] shows a two-peak structure while the coupled-channel theory cannot account for this. The extra peak in the low $M_{\pi\pi}$ region might be related to certain undiscovered state, so it is interesting. Voloshin[16] and Truong[17] suggested a four-quark state having nearly the same mass as Υ'' and coupling strongly to $\Upsilon''\pi$ and $\Upsilon\pi$. However, quantitative calculation shows that the low $M_{\pi\pi}$ peak is not at the right position[18], and so far there is no positive report of possible evidence for the existence for the four-quark state. At present, this discrepency is still an interesting open question requiring furtrer investigation.

TABLE I. Comparison of the calculated rates in the coupled-channel theory with the experiments. The phase of C_1 is taken to be the bounds $\cos\theta = -0.676$, $\cos\theta = -1$ determined from the data of $d\Gamma(\psi' \to J/\psi\pi\pi)/dM_{\pi\pi}$. The experimental values are obtained from the data of total widths and branching ratios in Ref.13.

	Theoretical rates(KeV)		Experimental rates
	$\cos\theta = -0.676$	$\cos\theta = -1$	(KeV)
$\Upsilon' \to \Upsilon\ \pi\ \pi$	13	14	12.0 ± 3.3
$\Upsilon'' \to \Upsilon'\pi^+\pi^-$	1.0	1.1	0.9 ± 0.3
$\Upsilon'' \to \Upsilon\pi^+\pi^-$	0.3	0.1	0.6 ± 0.3

2. $\psi''(3770) \to J/\psi + \pi^+ + \pi^-$

$\psi''(3770)$ is understood as mainly the 1^3D_1 state of $c\bar{c}$ lying above the charm threshold. This transition is also dominated by $E_1 - E_1$ multipole gluon- emission. For hadronic transition of $D-$state quarkonium, a second rank tensor term should also be considered in (31), i.e.[7]

$$\frac{g_E^2}{6} < \pi^\alpha(q_1),\ \pi^\beta(q_2) \mid E_k^a E_l^a \mid 0 > = \frac{\delta_{\alpha\beta}}{[(2\omega_1)(2\omega_2)]^{1/2}} \Big[C_1 \delta_{kl} q_1^\mu q_{2\mu} + $$

$$+ C_2 (q_{1k} q_{2l} + q_{1l} q_{2k} - \frac{2}{3}\delta_{kl}\ \vec{q}_1 \cdot \vec{q}_2) \Big] + \cdots \ . \qquad (34)$$

There is no experimental input to determine C_2 yet. Theoretically, the two-gluon approximation gives[7]

$$\mid C_2 \mid \approx 3 \mid C_1 \mid \ . \qquad (35)$$

where ω_1 and ω_2 are, respectively, the energies of the two pions, and C_1 is an unknown constant. We can take the well measured rate $\Gamma_{expt.}(\psi' \to J/\psi\pi\pi)$ as input to determine C_1 . Then (30) can be used to predict other $\Phi_i(n_i^3S_1) \to \Phi_f(n_f^3S_1) + \pi + \pi$ transition rates.

We can try another simple approximation. From the behavior of the kinematics, we can approximately take $< \pi(q_1),\ \pi(q_2) \mid E_k^a E_l^a \mid 0 >$ to be proportional to the two-gluon emission matrix, i.e.

$$< \pi(q_1),\ \pi(q_2) \mid E_k^a E_l^a \mid 0 > \approx const. < g(q_1),\ g(q_2) \mid E_k^a E_l^a \mid 0 > \ . \qquad (32)$$

The matrix element on the right-hand-side can be evaluated straightforwardly, and the constant factor can be absorbed into the redefinition of g_E which is to be determined by imposing $\Gamma_{expt.}(\psi' \to J/\psi\pi\pi)$ as input. Taking the Cornell Coulomb plus linear potential model[1], the determined $\alpha_E \equiv g_E^2/(4\pi)$ is

$$\alpha_E = 0.54 \ . \qquad (33)$$

As far as the energy dependence of the amplitude is concerned (but not angular distributions), the approxmation (32) is reasonable provided the phase-space in the transition process is large enough so that the $m_\pi \neq 0$ effect is negligible. Numerical calculatuon shows that for transitions with large enough phase-space, the rates obtained from (31) and (32) are very close to each other[7]. For transitions with small phase-space, certain $m_\pi \neq 0$ correction should be taken into account.

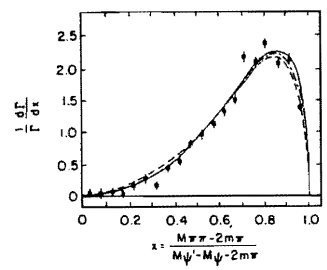

Fig.2. Comparison of the predicted curve of $d\Gamma(\Upsilon' \to \Upsilon\pi\pi)/dM_{\pi\pi}$ with the ARGUS data[14]. The solid and dash-dotted lines stand for $\cos\theta = -1$ and $\cos\theta = -0.676$, respectively. The dashed line is the naive single-channel result for comparison.

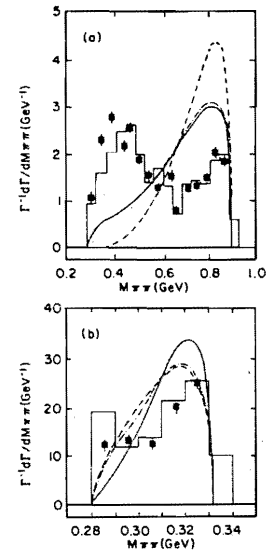

Fig.3. Comparison of the predicted curves of $d\Gamma(\Upsilon'' \to \Upsilon\pi^+\pi^-)/dM_{\pi\pi}$ and $d\Gamma(\Upsilon'' \to \Upsilon'\pi^+\pi^-)/dM_{\pi\pi}$ with the CLEO data[15]. The solid and dash-dotted lines stand for $\cos\theta = -1$ and $\cos\theta = -0.676$, respectively. The dashed line is the naive single-channel result for comparison. (a)$\Upsilon'' \to \Upsilon\pi^+\pi^-$, (b)$\Upsilon'' \to \Upsilon'\pi^+\pi^-$.

We expect that $| C_2 |$ is of the same order of magnitude as $| C_1 |$.

From the examination of the leptonic widths of $\psi''(3770)$ and $\psi'(3686)$ and from the point of view of the coupled-channel theory, these two states may be seen as mixtures of the 1^3D_1 and 2^3S_1 states of $c\bar{c}$, i.e. $\psi' = \psi(2S)\cos\theta + \psi(1D)\sin\theta$, $\psi'' = -\psi(2S)\sin\theta + \psi(1D)\cos\theta$. Taking the data of leptonic widths as input, the determined mixing angle is $\theta = -10°$ in the Cornell Coulomb plus linear potential model[12]. The theoretically predicted rate for $\psi''(3770) \to J/\psi + \pi^+ + \pi^-$ is[19]

$$\Gamma(\psi'' \to J/\psi\pi^+\pi^-) = \begin{cases} 107 KeV & (| C_2 | = 3 | C_1 |) \\ 20 KeV & (| C_2 | = | C_1 |) \end{cases} \quad if\ \theta = -10° \ . \tag{36}$$

This is to be compared with the recent MARK III data[20]

$$\Gamma(\psi'' \to J/\psi\pi^+\pi^-) = \begin{cases} 37 \pm 17 \pm 8\ KeV & from\ the\ directly \\ & measured\ \sigma_{\psi''}\ ^{21}\ , \\ 55 \pm 23 \pm 11\ KeV & from\ the\ measured \\ & \sigma(e^+e^- \to \psi'' \to D\overline{D})\ ^{22}\ . \end{cases} \tag{37}$$

The theory gives the right order of magnitude of the rate. We expect similar experiment at BEPC to improve the statistics.

3. *Search for $^1P_1$ State of Heavy Quarkonia*

The mass of $^1P_1$ states of heavy quarkonia may give important information about the spin-spin interaction between heavy quarks. Therefore the search for $^1P_1$ states of $c\bar{c}$ and $b\bar{b}$ are of great interest. There can be several ways of reaching the $^1P_1$ states, namely

(a) direct $\psi(^1P_1)$ production in $p\bar{p}$ annihilation[23], and the rate is measured by $\Gamma(\psi(^1P_1) \to p\bar{p})B(\psi(^1P_1) \to J/\psi\pi\pi)B(J/\psi \to e^+e^-)$,

(b) indirect $\psi(^1P_1)$ production in e^+e^- collision[24] and the crucial rate is $\Gamma(\psi' \to \psi(^1P_1)\pi^0)$,

(c) indirect $\Upsilon(^1P_1)$ producton in e^+e^- collision[7,15] and the crucial rate is $\Gamma(\Upsilon'' \to \Upsilon(^1P_1)\pi\pi)$.

They all concern the hadronic transition rates dominated by $E_1 - M_1$ gluon-emissions. For the single π^0 transition (b) the hadronization matrix element can be evaluated by using the Gross-Treiman-Wilczek relation[25] . For the $\pi\pi$ transions (a) and (c) there is no proper input datum for determing the unknown constant in the soft-pion approach like (31), so that we use the two-gluon approach with phenomenological $\alpha_E \equiv g_E^2/(4\pi)$ and $\alpha_M \equiv g_M^2/(4\pi)$. There is no reliable determination of α_M , so we estimate theoretically that the reasonable value of α_M is $\alpha_E \leq \alpha_M \leq (2-3)\alpha_E$. With this we predict[7]

$$\Gamma(\psi(^1P_1) \to p\bar{p})B(\psi(^1P_1) \to J/\psi\pi\pi)B(J/\psi \to e^+e^-) \sim 0.1 eV \ , \tag{38}$$

$$B(\psi' \to \psi(^1P_1)\pi^0) = 5(\alpha_M/\alpha_E) \times 10^{-4} \ , \tag{39}$$

$$B(\Upsilon'' \to \Upsilon\ ^1P_1)\pi^+\pi^-) = 4(\alpha_M/\alpha_E) \times 10^{-4} \ . \tag{40}$$

The result (38) is of the same order of magnitude as the preliminary R704 Collaboration data at ISR[23] . The branching ratios (39),(40) are slightly lower than the Crystal Ball[24] and CLEO[15] limits, so that the $\psi(^1P_1)$ and $\Upsilon(^1P_1)$ states are hopeful to be found at BEPC and CESR, respectively.

4. *Other hadronic Transitions*

The $\pi\pi$ transition rates between $\Upsilon(2^3P_J)$ and $\Upsilon(1^3P_J)$ states and $\Gamma(\Upsilon' \to \Upsilon\eta)$,$\Gamma(\Upsilon'' \to \Upsilon\eta)$ have also been predicted by the above theory. Further experimental data of these transition rates will give more tests of the theory.

III.Application to Radiative Decays

Perturbative QCD does not give succesful predictions for the rate of $J/\psi \to \gamma + \eta$[26] and the direct photon spectrum in $J/\psi \to \gamma + hadrons$[27]. The reason is obvious. In these processes, the light hadron is converted by at least two gluons and the typical momentum of each gluon is roughly

$$k \sim M_{J/\psi}/4 \sim 770 MeV \ , \tag{41}$$

which is too small for perturbative QCD to be good. We know, from our experience of studying hadronic transitions, that QCD multipole expansion works well for calculating emission of gluons in the few handred MeV range. In the following, we shall show the results obtained from QCD multipole expansion.

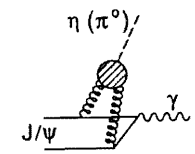

Fig.4. Dominant diagram for $J/\psi \to \gamma + \eta$.

1. $J/\psi \to \gamma + \eta$

It is shown from the general formula (26) that $J/\psi \to \gamma + \eta$ is dominated by the diagram shown in Fig.4[8] which describes that a virtual hadronic transition $J/\psi \to \psi(n^3S_1) + \eta$ first takes place and then the c and $\bar{c}$ in $\psi(n^3S_1)$ annihilate into a photon. This picture is different from that in perturbative QCD in the sanse that the c and $\bar{c}$ in the intermediate states are all in bound states instead of being free quarks. The hadronic transition $J/\psi \to \psi(n_3S_1) + \eta$ is dominated by $E_1 - M_2$ multipole gluon emissions, eq.(34). The evaluation of the present hadronization matrix element $< \eta \mid E_j^a \nabla_j B_l^a \mid 0 >$ needs more consideration. The operator can be written as

$$E_j^a \nabla_j B_l^a = \nabla_j(E_j^a B_l^a) - (\nabla_j E_j^a) B_l^a \ . \tag{42}$$

Voloshin and Zakhalov[5] argued that the second term on the right-hand-side is smaller than the first term and they suggested the approximation

$$< \eta \mid E_j^a \nabla_j B_l^a \mid 0 > \approx i q_{\eta j} < \eta \mid E_j^a B_l^a \mid 0 > \ , \tag{43}$$

which can be related to the Gross-Treiman-Wilczek formula[25]. With this approximation we obtain[8]

$$\Gamma(J/\psi \to \gamma\eta) = \frac{1}{12\pi}\left(\frac{\alpha_M}{\alpha_E}\right)\left(\frac{\alpha_E}{\alpha_s}\right)^2 \frac{M_{J/\psi}^2 - m_\eta^2}{M_{J/\psi}^2} |\vec{q}_\eta|^2 \cdot$$

$$\cdot \left(\frac{2eQ}{3\sqrt{6}m_c}\right)^2 \left(\frac{4\pi^2}{3\sqrt{6}} f_\pi m_\eta^2 \sum_n h_{10n0}^{111}\right)^2 , \tag{44}$$

in which α_s is the QCD coupling constant at the scale m_η ,

$$h_{n_i l_i nl}^{11L} \equiv \sum_K \frac{< R_{nl} \mid r \mid R'_{KL} >< R'_{KL} \mid r \mid R_{n_i l_i} >}{(M_{J/\psi} - E_n - \omega_\eta)(M_{J/\psi} - E'_{KL})} f_{nl}(\vec{0}) , \tag{45}$$

where $R_{n_i l_i}$, R_{nl}, R'_{KL} are, respectively, the radial wave functions of the initial, final and intermediate states in the hadronic transition process, and $f_{nl}(\vec{0})$ is the wave function at the origin of the $\psi(n^3S_1)$ state. The currently accepted value of α_s is close to $\alpha_E = 0.54$. Thus we take $\alpha_s \approx \alpha_E$. The $f_{n0}(\vec{0})$'s can be determined by the data of leptonic widths. Our result is[8]

$$\Gamma(J/\psi \to \gamma\eta) = 0.020\left(\frac{\alpha_M}{\alpha_E}\right) KeV , \tag{46}$$

and the corresponding branching ratio is

$$B(J/\psi \to \gamma\eta) = \begin{cases} 2.9 \times 10^{-4} , & \alpha_M = \alpha_E , \\ 8.6 \times 10^{-4} , & \alpha_M = 3\alpha_E . \end{cases} \tag{47}$$

This is to be compared with the experimental value[13]

$$B(J/\psi \to \gamma\eta) = (8.6 \pm 0.8) \times 10^{-4} . \tag{48}$$

There are some uncertain in (46) ; namely, (α_M/α_E) is not well determined, (α_E/α_s) is not precisely known, and we do not know how good the approximation (43) is. To eliminate these uncertainties, let us consider another $E_1 - M_2$ transition process $\psi' \to J/\psi + \eta$. It has been calculated in Ref.7 with the same uncertainties and the result is

$$\Gamma(\psi' \to J/\psi\,\eta) = \frac{2}{243\pi}\left(\frac{\alpha_M}{\alpha_E}\right)\left(\frac{\alpha_E}{\alpha_s}\right)^2 \left(\frac{1}{3m_c}\right)^2 \cdot$$

$$\cdot |\vec{q}_\eta\,(\psi' \to J/\psi\,\eta)|^3 \left(\frac{\pi^2}{\sqrt{3}} f_\pi m_\eta^2 f_{2010}^{111}\right)^2 , \tag{49}$$

where

$$f_{n_i l_i nl}^{P_i PL} \equiv \sum_K \frac{< R^{nl} \mid r^P \mid R'_{KL} >< R'_{KL} \mid r^{P_i} \mid R_{n_i l_i} >}{M_{\psi'} - E'_{KL}} . \tag{50}$$

Now we take the ratio

$$R_\eta \equiv \frac{\Gamma(J/\psi \to \gamma\eta)}{\Gamma(\psi' \to J/\psi\,\eta)}$$

$$= \frac{(4/81)(eQ)^2[(M_\psi^2 - m_\eta^2)/M_\eta^2]\,|\vec{q}_\eta\,(J/\psi \to \gamma\eta)|^2 |\sum_n h_{10n0}^{111}|^2}{(2/243)\,|\vec{q}_\eta\,(\psi' \to J/\psi\,\eta)|^3 |f_{2010}^{111}|^2} . \tag{51}$$

The uncertainties in the two processes cancel each other in R_η , therefore R_η just test the soft-gluon emission dynamics in our approach. Our theoretical result is

$$R_\eta\,|_{theory} = 0.012 . \tag{52}$$

It is in agreement with the corresponding experimental value[13]

$$R_\eta\,|_{expt.} = 0.009 \pm 0.005 . \tag{53}$$

2. Direct Photon Spectrum in the Inclusive Radiative Decay $J/\psi \to \gamma + hadrons$.

Conventional theoretical studies of inclusive radictive decays of J/ψ and Υ are based on perturbative QCD calculations[28]. In Born approximation, the hadrons are represented by two free gluons. Let Γ_{tot} and M be the total width and mass of $J/\psi(\Upsilon)$, respectively, ω_γ be the energy of the photon, $x \equiv 2\omega_\gamma/M$. The direct photon spectrum is expressed as

$$\frac{1}{\Gamma_{tot}} \frac{d\Gamma(J/\psi(\Upsilon) \to \gamma + hadrons)}{dx} . \tag{54}$$

For Υ, the perturbative QCD calculated direct photon spectrum is in good agreement with experiment, while for J/ψ , perturbative QCD gives too hard distribution[27] (cf. the dashed curve in Fig.5). The disagreement between theory and experiment in the range $x < 0.8$ is expected since in this region there are a lot of resonances which have not be taken into account in the Born approximation. In the range $x > 0.8$ the disagreement does imply the shortcoming of the simple perturbative approach. This is understandable because, as is shown in (41), when $x \simeq 1$ the typical gluon momentum is only $k \sim M_{J/\psi}/4 \sim 770MeV$. Similar to the case of $J/\psi \to \gamma + \eta$, we can also calculate the direct photon spectrum in $J/\psi \to \gamma + g + g$ by using QCD multipole expansion. Now the c and $\bar{c}$ in the intermediate states are all in bound states, and in this sense the approach is nonperturbative. This bound- state contribution to the intermediate states significantly suppresses the large x distribution[30] and the result is dipicted by the solid curve in Fig.5. In the $x > 0.8$ region the theoretical curve is very close to the experiment. The $x < 0.8$ region is more complicated to study not only because of the resonances but also because of that the gluons are harder so that QCD multipole expansion is not so reliable in this region.

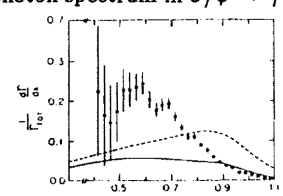

Fig.5. The direct photon spectrum in $J/\psi \to \gamma + g + g$. The solid curve is the result of the present theory. The dashed curve is the result of leading-order perturbative QCD(Ref.27). The experimental data is from Ref.27.

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