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Quantum Coherence and Skyrmions in Bilayer Quantum Hall System

Z. F. EZAWA

Department of Physics, Tohoku University, Sendai 980, Japan

ABSTRACT

We analyze quantum coherence in a bilayer quantum Hall system. We use a bosonic Chern-Simons gauge theory with the lowest Landau level projection taken into account. Although the kinetic energy term is quenched in the Hamiltonian, the dynamics arises since the X and Y components of the guiding center $\boldsymbol{X} = (X, Y)$ do not commute. In the case of the bilayer system the dynamics is governed by the $W_\infty \times SU(2)$ algebra. We emphasize that the fractional quantum Hall state is a condensed but not a coherent phase of composite bosons. It follows from this ground state property and the W_∞ algebra that the fractional quantum Hall system is incompressible. In a certain bilayer quantum Hall system the ground state is a coherent state of the SU(2) component of the composite boson field, which is the CP^1 field. Skyrmions are topological excitations in this coherent mode. A systematic method is presented to calculate the current and static correlation functions. It is argued that Skyrmion excitations are detectable by measuring the Hall current distribution. We construct the Landau-Ginzburg theory of the coherent mode. The coherent mode is a superfluid mode in the vanishing limit of the tunneling interaction.

1. INTRODUCTION

Entirely new physics is possible in the 2-dimensional space due to its intrinsic topologi structure [1,2]. For instance, an electron may turn into a boson by making a charge-fl composite in external magetic field, which we call *composite boson*. As a result electrons m condense into an incompressible fluid without making Cooper pairs. This is the fractio quantum Hall (QH) state [3]. In a certain bilayer QH system, an interlayer coherer develops spontaneously [4] and Josephson- like phenomena occur [4,5]. Skyrmions [6] topological solitons in this coherent mode [7,8]. It is most convenient to use the boso Chern-Simons (CS) gauge theory to formulate the composite-boson picture.

However, to make a consistent theory of the QH effect it is necessary to make lowest-Landau-level (LLL) projection [9], which has so far been used in the single-mo approximation (SMA) of the first quantized theory of electrons [9,8]. It is our aim to analy the quantum coherence in a bilayer QH system based on a bosonic CS gauge theory w the LLL projection made. We have presented elsewhere a brief report of this work [10]. the present paper we provide a full account of our results, which we summarize as follo (a) The dynamics of the bilayer system is governed by the $W_\infty \times SU(2)$ algebra. (b) T ground state is an eigenstate of the in-phase density operator. (c) The incompressibility the fractional QH state follows from this ground state property and the W_∞ algebra. (d) T U(1) phase symmetry is not broken spontaneously in spite of bose condensation since QH state is not a coherent state of composite bosons. (e) In the so-called Halperin $(m, m,$ phase the QH state is a coherent state of the SU(2) component of the composite boson fie which is the CP^1 field. (f) Skyrmions are coherent excitations of the CP^1 field in this pha (g) A systematic method is presented to calculate the current and the static correlati functions. (h) Skyrmions are detectable by measuring the Hall current distribution. (i) T coherent mode is a superfluid mode in the vanishing limit of the tunneling interacti Almost all these results can be taken over to the monolayer QH system with spin degr of freedom by replacing the capacitance energy with the Zeemann energy. In this c

r analysis presents a field theoretical proof that the monolayer QH system is a quantum romagnet with Skyrmions as topological excitations in spin texture.

This paper is composed as follows. In section II we start with the Hamiltonian of the layer QH system. Assigning "up" and "down" pseudospins to the electrons belonging the upper and lower layers, respectively, we introduce the pseudospin SU(2) structure to the bilayer system. Low-energy excitations are described in terms of the pseudospin xture. We then formulate the LLL projection in the electron field theory. We consider e case in which the magnetic energy greatly exceeds thermal and potential energies. It reasonable to assume that all electrons are confined within the lowest Landau level if ey can be accommodated there. However, in order to make a consistent theory, it is ecessary to suppress level mixings by making the LLL projection. Indeed, without the LL projection the potentials kick out electrons out of the lowest Landau level however nall its probability is. The LLL projection is achieved by quenching the kinetic term and eezing out the relative coordinate in the potential. Although the kinetic energy term is sent in the projected system, the dynamics arises because the X and Y coordinates of the uiding center $\boldsymbol{X} = (X, Y)$ do not commute. This noncommutativity leads to the magnetic anslation group. It generates W_∞ group in the monolayer system and the $W_\infty \times SU(2)$ oup in the bilayer system.

In section III we introduce composite bosons based on the CS gauge theory. A composite son is obtained by attaching an odd number of flux to an electron. It is a peculiar feature the 2-dimensional system that the statistics of a particle can be altered by attaching a ux to it [2]. Thus, an electron bound to an odd number of flux turns into a boson and ndergoes bose condensation in an appropriate circumstance, as makes the fractional QH ate [3]. However, they are not genuine bosons; they are *hardcore bosons* with the exclusion inciple implemented. In the bilayer system, physics is very different in the so-called alperin (m_1, m_2, n) phase and the (m, m, m) phase [4]. In this paper we are concerned ly about the (m, m, m) phase, since this is the phase where the interlayer coherence evelops spontaneously. In this phase the composite boson field is naturally decomposed

into two fields: The U(1) field is associated with the total density (in-field density), and the CP^1 field with the density difference (out-of-phase density). The Skyrmion excitation is a coherent excitation of the CP^1 field carrying the Pontryagin number [6]. We also formulate the LLL projection in the composite boson theory.

In section IV we analyze the (m, m, m) phase of the bilayer QH system in a semiclassical approximation of the bosonic CS theory. We first determine the classical ground state by minimizing the energy of the classical Hamiltonian. It describes a uniform distribution of electrons, which is realized only at the magic filling factor $\nu = 1/m$. We then determine the ground state by considering quantum corrections due to perturbative fluctuations around it. In this paper we ignore nonperturbative fluctuations generating topological vortices. Then, there exist no in-phase density fluctuations in the lowest Landau level, as implies the incompressibility of the system [4]. However, the out-of-phase density fluctuation has a gapless mode confined within the lowest Landau level, which leads to a spontaneous development of a novel interlayer coherence. We also study the Skyrmion classical field.

In section V we analyze the bilayer QH system by using the algebraic structure of the LLL-projected operators and a few additional properties of the ground state. The key properties of the ground state are the following: (A) It is an eigenstate of the in-phase density mode. (B) It is a coherent state of the out-of-phase density mode. The property (A) makes the QH system very unique. Its origin is the LLL projection because such a ground state is realized only in the absence of the kinetic term. It is remarkable that the W_∞ algebra with no central extension has only the trivial vacuum sector [11] leading to the incompressibility of the system, as is consistent with the above perturbative result. We also present a systematic method to calculate static correlation functions. Then, evaluating the Coulomb energy of the pseudospin texture, we derive the effective Hamiltonian for the interlayer coherent mode. The mode is shown to be a superfluid mode in the vanishing limit of the tunneling strength.

In section VI we evaluate the Hall current, the supercurrent and the tunneling current in the pseudospin texture. A special care is needed for the analysis of the Hall current.

This is because the electric current vanishes identically, when it acts on any state in the lowest Landau level. Actually the current flows via a Landau level mixing [9]. We present a systematic way to evaluate electric currents on the QH state. We point out that Skyrmion excitations would be detectable by measuring the Hall current distribution. We also show that the interlayer coherent mode is a superfluid mode in the vanishing limit of the tunneling interaction.

Section VII is devoted for discussions.

2. BILAYER QUANTUM HALL SYSTEM

A. Pseudospin Texture

We consider a bilayer electron system in strong magnetic field. We denote the electron field at the layer $\alpha(=1,2)$ by $\psi_\alpha(\boldsymbol{x})$. The field theoretical Hamiltonian is given by

$$H = \frac{1}{2M}\sum_{\alpha=1}^{2}\int d^2x\psi_\alpha^\dagger(\boldsymbol{x})(P_x^2+P_y^2)\psi_\alpha(\boldsymbol{x}) + H_C + H_T, \tag{2.1}$$

where P_x and P_y are the covariant derivative, $P_k = -i\hbar\partial_k + (e/c)A_k^{\rm ext}$, with the external magnetic field taken in the symmetric gauge, $A_k^{\rm ext} = \frac{1}{2}B\varepsilon_{kj}x_j$. The Coulomb interaction term H_C is

$$H_C = \frac{1}{2}\sum_{\alpha,\beta=1}^{2}\int d^2xd^2yV_{\alpha\beta}(\boldsymbol{x}-\boldsymbol{y})\rho^\alpha(\boldsymbol{x})\rho^\beta(\boldsymbol{y}), \tag{2.2}$$

with $\rho^\alpha(\boldsymbol{x}) \equiv \psi_\alpha(\boldsymbol{x})^\dagger\psi_\alpha(\boldsymbol{x})$ and

$$V_{\alpha\beta}(\boldsymbol{x}-\boldsymbol{y}) = \frac{e^2}{\varepsilon}\frac{1}{\sqrt{|\boldsymbol{x}-\boldsymbol{y}|^2+d_{\alpha\beta}^2}}, \tag{2.3}$$

where $d_{11} = d_{22} = 0$ and $d_{12} = d$ is the interlayer distance. We may rewrite it as

$$H_C = \frac{1}{2}\int d^2xd^2yV_+(\boldsymbol{x}-\boldsymbol{y})\rho(\boldsymbol{x})\rho(\boldsymbol{y}) + 2\int d^2xd^2yV_-(\boldsymbol{x}-\boldsymbol{y})S^3(\boldsymbol{x})S^3(\boldsymbol{y}), \tag{2.4}$$

where $V_\pm = \frac{1}{2}(V_{11}\pm V_{12})$. The tunneling term H_T is

$$H_T = -\lambda\int d^2x(\psi_1^\dagger\psi_2+\psi_2^\dagger\psi_1). \tag{2.5}$$

The tunneling interaction induces the energy gap $\Delta_{SAS} = 2\lambda$ between the symmetric a antisymmetric states. The bilayer system has a much richer phase structure than the mor layer system because we can control the interlayer distance d and the tunneling streng λ.

We assign "up" and "down" pseudospins to the electrons belonging to the upper a lower layers, respectively. The pseudospin densities are $S^a(\boldsymbol{x}) = \frac{1}{2}\Psi^\dagger\tau^a\Psi$, where τ^a is t Pauli matrix and Ψ is the SU(2) field, $\Psi^T = (\psi_1,\psi_2)$. In particular, $S^3(\boldsymbol{x}) = \frac{1}{2}(\rho^1-\rho^2)$ w $\rho^\alpha = \psi_\alpha^\dagger\psi_\alpha$. The total density is $\rho(\boldsymbol{x}) = \rho^1+\rho^2$. They satisfy the commutation relatio $[S^a(\boldsymbol{x}),S^b(\boldsymbol{y})] = i\delta(\boldsymbol{x}-\boldsymbol{y})\varepsilon_{abc}S^c(\boldsymbol{x})$ and $[\rho(\boldsymbol{x}),S^a(\boldsymbol{y})] = 0$. We call ρ the *in-phase density*, a S^3 the *out-of-phase density*. The pseudospin operator generates a local SU(2) transformatic $e^{-i\mathcal{O}}$, with

$$\mathcal{O} = \sum_{a=1}^{3}\int d^2xf^a(\boldsymbol{x})S^a(\boldsymbol{x}), \tag{2}$$

where $f^a(\boldsymbol{x})$ is a real function. It acts on the SU(2) field as

$$\Psi(\boldsymbol{x}) \quad\rightarrow\quad e^{-i\mathcal{O}}\Psi(\boldsymbol{x})e^{i\mathcal{O}} = \exp[i\sum f^a(\boldsymbol{x})\frac{\tau^a}{2}]\Psi(\boldsymbol{x}). \tag{2}$$

Since it is not a symmetry of the system (2.1), the state $|\Phi\rangle = e^{i\mathcal{O}}|g\rangle$ is an excited state w $|g\rangle$ the ground state. In this paper we mainly consider the pseudospin texture describ by this state.

The monolayer system with spin degrees of freedom has the global SU(2) symmet as well as the U(1) symmetry, where the SU(2) symmetry is broken down to the SO symmetry by the Zeemann energy. On the other hand, the bilayer case has the sa symmetry structure, where the SU(2) symmetry is broken down to the SO(2) symmet by the capacitance energy. This is seen in the Coulomb term (2.4), where the potent V_+ part has the global SU(2) invariance and the V_- part (capacitance term) has the glob SO(2) invariance. The bilayer system has the tunneling interaction (2.5) additionally, whi may be rewritten as $H_T = -2\lambda\int d^2xS^1(\boldsymbol{x})$. It breaks the SO(2) symmetry explicitly but or weakly provided that λ is sufficiently small.

B. LLL Projection and $W_\infty \times SU(2)$ Density Algebra

When the magnetic field is strong enough, the magnetic energy greatly exceeds thermal d potential energies. It is reasonable to assume that electrons are confined within the west Landau level. To make a consistent theory, it is necessary to make the LLL projection quenching the kinetic term [9].

For this purpose we decompose the electron coordinate $\boldsymbol{x} = (x, y)$ into the center-of-mass ordinate $\boldsymbol{X}$ called the guiding center and the relative coordinate $\boldsymbol{R}$, where $\boldsymbol{x} = \boldsymbol{X} + \boldsymbol{R}$ with $= (-\ell_B^2 P_y/\hbar, \ell_B^2 P_x/\hbar)$; $\ell_B \equiv \sqrt{\hbar c/eB}$ is the magnetic length. They satisfy $[X, Y] = -i\ell_B^2$ and $_x, R_y] = i\ell_B^2$. We may define two sets of independent harmonic oscillators,

$$a \equiv \frac{\ell_B}{\sqrt{2}\hbar}(P_x + iP_y) = -\frac{i}{\sqrt{2}}\left(z + \frac{\partial}{\partial z^*}\right), \quad a^\dagger \equiv \frac{\ell_B}{\sqrt{2}\hbar}(P_x - iP_y) = \frac{i}{\sqrt{2}}\left(z^* - \frac{\partial}{\partial z}\right), \tag{2.8}$$

d

$$b \equiv \frac{1}{\sqrt{2}\ell_B}(X - iY) = \frac{1}{\sqrt{2}}\left(z^* + \frac{\partial}{\partial z}\right), \quad b^\dagger \equiv \frac{1}{\sqrt{2}\ell_B}(X + iY) = \frac{1}{\sqrt{2}}\left(z - \frac{\partial}{\partial z^*}\right). \tag{2.9}$$

ey satisfy $[a, a^\dagger] = [b, b^\dagger] = 1$ and $[a, b] = [a^\dagger, b] = 0$. We have introduced the complex mber z by

$$z = \frac{1}{2\ell_B}(x + iy), \qquad \frac{\partial}{\partial z} = \ell_B\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right). \tag{2.10}$$

nce the excitation of the relative coordinate causes a transition to an upper Landau level, is necessary to freeze out this degree of freedom.

The LLL projection is to quench the kinetic energy term in the Hamiltonian, which is written as

$$H_K = \hbar\omega_c \sum_{\alpha=1}^{2} \int d^2x \psi_\alpha^\dagger a^\dagger a \psi_\alpha + \frac{N}{2}\hbar\omega_c, \tag{2.11}$$

here N is the number of electrons in the system, $\int d^2x\rho(\boldsymbol{x}) = N$, and $\omega_c \equiv eB/Mc$ is the clotron frequency. In order to confine all the electrons in the lowest Landau level, we quire the LLL condition,

$$a(\boldsymbol{x})\psi_\alpha(\boldsymbol{x})|\hat{\Psi}\rangle \propto (P_x + iP_y)\psi_\alpha(\boldsymbol{x})|\hat{\Psi}\rangle = 0, \tag{2.12}$$

for the states $|\hat{\Psi}\rangle$. All the states satisfying this condition constitute a Hilbert space.

After the LLL projection, although the kinetic term does not exist in the Hamiltonian, the system develops dynamically because the X and Y coordinates of the guiding center $\boldsymbol{X} = (X, Y)$ do not commute, $[X, Y] = -i\ell_B^2$. It is easy to verify [9],

$$[e^{i\boldsymbol{q}\boldsymbol{X}}, e^{i\boldsymbol{p}\boldsymbol{X}}] = 2ie^{i(\boldsymbol{q}+\boldsymbol{p})\boldsymbol{X}} \sin[\ell_B^2 \frac{\boldsymbol{q} \wedge \boldsymbol{p}}{2}], \tag{2.13}$$

which is called the magnetic translation algebra. Due to this algebra the dynamics arises even in the absence of the kinetic term.

The smeared density operator is $O = \int d^2\boldsymbol{x} f(\boldsymbol{x})\rho(\boldsymbol{x})$. The state $O|\hat{\Psi}\rangle$ does not belong to the lowest Landau level even if $|\hat{\Psi}\rangle$ does, because

$$a\psi_\alpha(\boldsymbol{x})O|\hat{\Psi}\rangle = af(\boldsymbol{x})\psi_\alpha(\boldsymbol{x})|\hat{\Psi}\rangle \neq 0. \tag{2.14}$$

We denote the LLL projection of the operator O and the c-number function $f(\boldsymbol{x})$ by $\hat{O}$ and $\hat{f}(\boldsymbol{x})$, respectively. Since the projected quantity does not involve the operator $a^\dagger$, we have

$$a\psi_\alpha(\boldsymbol{x})\hat{O}|\hat{\Psi}\rangle = \hat{f}(\boldsymbol{x})a\psi_\alpha(\boldsymbol{x})|\hat{\Psi}\rangle = 0. \tag{2.15}$$

It is clear that the state $\hat{O}|\hat{\Psi}\rangle$ belongs to the lowest Landau level.

We make the LLL projection in a systematic way. We first make a Fourier transformation of $f(\boldsymbol{x})$,

$$f(\boldsymbol{x}) = \int \frac{d^2q}{2\pi} e^{i\boldsymbol{x}\boldsymbol{q}} f_{\boldsymbol{q}}. \tag{2.16}$$

The problem of the LLL projection is reduced to that of the plane wave $e^{i\boldsymbol{x}\boldsymbol{q}}$. We make normal ordering with respect to a and $a^\dagger$ as

$$e^{i\boldsymbol{q}\boldsymbol{x}} = \exp[-\frac{\ell_B^2}{4}\boldsymbol{q}^2]\exp[\frac{\ell_B}{\sqrt{2}}qa^\dagger]e^{i\boldsymbol{q}\boldsymbol{X}}\exp[-\frac{\ell_B}{\sqrt{2}}q^*a], \tag{2.17}$$

where $q = q_x + iq_y$. The LLL projection is to quench the operators a and $a^\dagger$. Hence,

$$\hat{f}(\boldsymbol{x}) = \int \frac{d^2q}{2\pi} \exp[-\frac{\ell_B^2}{4}\boldsymbol{q}^2] e^{i\boldsymbol{q}\boldsymbol{X}} f_{\boldsymbol{q}}, \tag{2.18}$$

and

$$\hat{O} = \int d^2x \hat{f}(\boldsymbol{x})\rho(\boldsymbol{x}) = \int d^2q f_{-\boldsymbol{q}} \hat{\rho}_{\boldsymbol{q}}. \tag{2.19}$$

Here, we have defined the LLL projected density operator by

$$\hat{\rho}_{\boldsymbol{q}} \equiv \exp[-\frac{\ell_B^2}{4}\boldsymbol{q}^2] \int \frac{d^2\boldsymbol{x}}{2\pi} e^{-i\boldsymbol{q}\boldsymbol{X}} \rho(\boldsymbol{x}). \tag{2.20}$$

We define similarly the LLL projection of S^a,

$$\hat{S}^a_{\boldsymbol{q}} \equiv \exp[-\frac{\ell_B^2}{4}\boldsymbol{q}^2] \int \frac{d^2\boldsymbol{x}}{2\pi} e^{-i\boldsymbol{q}\boldsymbol{X}} S^a(\boldsymbol{x}). \tag{2.21}$$

These operators satisfy the algebra,

$$[\hat{\rho}_{\boldsymbol{p}}, \hat{\rho}_{\boldsymbol{q}}] = \frac{i}{\pi} \hat{\rho}_{\boldsymbol{p}+\boldsymbol{q}} \sin[\ell_B^2 \frac{\boldsymbol{p}\wedge\boldsymbol{q}}{2}] \exp[\frac{\ell_B^2}{2}\boldsymbol{p}\boldsymbol{q}], \tag{2.22}$$

$$[\hat{S}^a_{\boldsymbol{p}}, \hat{\rho}_{\boldsymbol{q}}] = \frac{i}{\pi} \hat{S}^a_{\boldsymbol{p}+\boldsymbol{q}} \sin[\ell_B^2 \frac{\boldsymbol{p}\wedge\boldsymbol{q}}{2}] \exp[\frac{\ell_B^2}{2}\boldsymbol{p}\boldsymbol{q}], \tag{2.23}$$

$$\begin{aligned}[\hat{S}^a_{\boldsymbol{p}}, \hat{S}^b_{\boldsymbol{q}}] = &\frac{i}{2\pi} \varepsilon^{abc} \hat{S}^c_{\boldsymbol{p}+\boldsymbol{q}} \cos[\ell_B^2 \frac{\boldsymbol{p}\wedge\boldsymbol{q}}{2}] \exp[\frac{\ell_B^2}{2}\boldsymbol{p}\boldsymbol{q}] \\ &+ \frac{i}{4\pi} \delta^{ab} \hat{\rho}_{\boldsymbol{p}+\boldsymbol{q}} \sin[\ell_B^2 \frac{\boldsymbol{p}\wedge\boldsymbol{q}}{2}] \exp[\frac{\ell_B^2}{2}\boldsymbol{p}\boldsymbol{q}],\end{aligned} \tag{2.24}$$

which is isomorphic to the $W_\infty \times SU(2)$ algebra. We call it the density algebra.

The algebra follows from the following operator products,

$$\rho(\boldsymbol{x})\rho(\boldsymbol{y}) = \rho(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y}) + :\rho(\boldsymbol{x})\rho(\boldsymbol{y}):, \tag{2.25}$$

$$S^a(\boldsymbol{x})\rho(\boldsymbol{y}) = S^a(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y}) + :S^a(\boldsymbol{x})\rho(\boldsymbol{y}):, \tag{2.26}$$

$$S^a(\boldsymbol{x})S^b(\boldsymbol{y}) = \frac{i}{2}\varepsilon^{abc} S^c(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y}) + \frac{1}{4}\delta^{ab}\rho(\boldsymbol{x})\delta(\boldsymbol{x}-\boldsymbol{y}) + :S^a(\boldsymbol{x})S^b(\boldsymbol{y}):, \tag{2.27}$$

and the magnetic translation algebra (2.13). In the derivation we required the normal-ordered terms to vanish for $\boldsymbol{x} = \boldsymbol{y}$. The Pauli exclusion principle ($\psi_\alpha(\boldsymbol{x})^2 = 0$) implies that $:\rho(\boldsymbol{x})\rho(\boldsymbol{x}) := 2\psi_1^\dagger(\boldsymbol{x})\psi_2^\dagger(\boldsymbol{x})\psi_1(\boldsymbol{x})\psi_2(\boldsymbol{x})$. Here, the product $\psi_1(\boldsymbol{x})\psi_2(\boldsymbol{x})$ needs not vanish since two electrons belong to different layers. However, we have required it to vanish as well.

This is certainly the case in the vanishing limit of the interlayer distance ($d = 0$), where t two layers collapse into a single layer and the layer index loses its meaning. When $d \neq$ although this "hardcore" condition is not required, it is realized dynamically due to t strong Coulomb repulsive force between the electrons. Indeed, the wave function of t bilayer QH state has this hardcore property; see the wave function (3.3)

III. COMPOSITE BOSONS

A. Chern-Simons Gauge Theory

To reveal the existence of various phases in the bilayer system it is convenient to use t Chern-Simons gauge theory n_α. It is a field theory of composite electrons. We define t composite-electron field ϕ_α by an operator phase transformation,

$$\phi_\alpha(\boldsymbol{x}) = e^{i\Theta_\alpha(\boldsymbol{x})}\psi_\alpha(\boldsymbol{x}). \tag{3}$$

The phase field Θ_α are defined in the basis $|\{\boldsymbol{x}^1_r\};\{\boldsymbol{x}^2_p\}\rangle$ with the electron positions diag nalized as

$$\begin{aligned}\Theta_1(\boldsymbol{x})|\{\boldsymbol{x}^1_r\};\{\boldsymbol{x}^2_p\}\rangle &= (m_1 \sum \theta(\boldsymbol{x}-\boldsymbol{x}^1_r) + n \sum \theta(\boldsymbol{x}-\boldsymbol{x}^2_s))|\{\boldsymbol{x}^1_r\};\{\boldsymbol{x}^2_p\}\rangle, \\ \Theta_2(\boldsymbol{x})|\{\boldsymbol{x}^1_r\};\{\boldsymbol{x}^2_p\}\rangle &= (n \sum \theta(\boldsymbol{x}-\boldsymbol{x}^1_r) + m_2 \sum \theta(\boldsymbol{x}-\boldsymbol{x}^2_s))|\{\boldsymbol{x}^1_r\};\{\boldsymbol{x}^2_p\}\rangle,\end{aligned} \tag{3}$$

where m_α and n are integers. Here, m_α is associated with the statistics between compos electrons within layer α, while n with the *relative* statistics between composite electrons the different layers; Each of them gives a relative angular momentum to a pair of electrons that they never come to the same xy position [4]. When we choose m_α to be odd, compos electrons are called composite bosons since they become bosonic.

The property of the system depends crucially on these parameters: It turns out [that the QH states are characterized by the three integers (m_1, m_2, n). The wave function composite bosons in the (m_1, m_2, n) phase is

$$\Psi[z, z^*] = \check{\Lambda}[z] \prod |z^1_r - z^1_s|^{m_1} \prod |z^2_p - z^2_q|^{m_2} \prod |z^1_r - z^2_p|^n e^{-\sum |z^1_r|^2 - \sum |z^2_p|^2}, \tag{3}$$

here $\check{\lambda}[z]$ is an arbitrary analytic function symmetric in z_r^1 and also in z_p^2. This is obtained] by solving the LLL condition (3.25) we mention later. The Halperin wave function [12] simply a singular phase transformation of (3.3). It is observed manifestly in this wave nction that no two composite bosons come to the same xy position, even if they belong to e different layers. It is also observed that the parameters m_α and n measure the strength of rrelations between composite bosons which are determined dynamically by the Coulomb teraction. We can realize the (m_1, m_2, n) phase by tuning the system parameters such as e electron densities and the interlayer distance. In particular, we expect the (m, m, m) ase to appear when the magnetic length and the interlayer distance are of the same order $_B \approx d$).

In term of the composite boson field the Hamiltonian (2.1) reads

$$H = \frac{1}{2M}\sum_\alpha \int d^2x \phi_\alpha^\dagger(\boldsymbol{x})[(\check{P}_x^\alpha)^2 + (\check{P}_y^\alpha)^2]\phi_\alpha(\boldsymbol{x}) + \frac{1}{2}\sum_{\alpha,\beta}\int d^2x d^2y V_{\alpha\beta}(\boldsymbol{x}-\boldsymbol{y})\rho^\alpha(\boldsymbol{x})\rho^\beta(\boldsymbol{y}), \quad (3.4)$$

here $\rho^\alpha = \psi_\alpha^\dagger\psi_\alpha = \phi_\alpha^\dagger\phi_\alpha$ and $\check{P}_k^\alpha$ is the covariant momentum incorporating the Chern-mons field defined by $C_k^\alpha(\boldsymbol{x}) = \hbar c\partial_k\Theta^\alpha(\boldsymbol{x})$,

$$\check{P}_k^\alpha = -i\hbar\partial_k + \mathcal{A}_x^\alpha, \qquad \mathcal{A}_k^\alpha = \frac{1}{c}(C_k^\alpha + eA_k^{\rm ext}). \quad (3.5)$$

follows from (3.2) that

$$\varepsilon_{jk}\partial_j C_k^1 = 2\pi\hbar c(m_1\rho^1 + n\rho^2), \qquad \varepsilon_{jk}\partial_j C_k^2 = 2\pi\hbar c(n\rho^1 + m_2\rho^2). \quad (3.6)$$

his set of equations is the constraint condition which determines the Chern-Simons field $_k^\alpha$ in terms of the density ρ^α.

Although the electron Hamiltonian is invariant under the global SU(2) transformation, e composite-boson Hamiltonian (3.4) is no longer so unless $m_1 = m_2 = n$ due to the Chern-mons fields C_k^α in the covariant momentum. The existence of the (m_1, m_2, n) phases is dden in the original electron theory, where these phases must be realized by way of ontaneous breakdown of the SU(2) symmetry. On the contrary, in the composite boson eory each phase is described by its own Lagrangian containing the symmetry-breaking rameters explicitly.

It should be noted that, although we can construct the pseudospin density $\check{S}^a(\boldsymbol{x})$ out of the composite boson fields, $\check{S}^1(\boldsymbol{x})$ and $\check{S}^2(\boldsymbol{x})$ are not the same objects as the ones in the original electron theory. The exceptional case is the (m, m, m) phase, where all the operators $S^a(\boldsymbol{x})$ are identical to the ones in the original electron theory. This is because two equations in (3.2) become identical. There exists only one phase field $\Theta \equiv \Theta_1 = \Theta_2$, and hence only one Chern-Simons field $C_k \equiv C_k^1 = C_k^2$. This is the basic reason why an interlayer coherence develops spontaneously in this phase and leads to novel physics intrinsic to the bilayer Hall system [4].

B. CP^1 Field and O(3) Sigma Field

To analyze the pseudospin texture in the (m, m, m) phase, it is convenient to introduce the CP^1 field and the O(3) sigma field. We may decompose the composite boson field ϕ_α into the two fields ϕ and n_α,

$$\phi_\alpha(\boldsymbol{x}) = \phi(\boldsymbol{x})n_\alpha(\boldsymbol{x}), \qquad \phi(\boldsymbol{x}) = e^{i\chi(\boldsymbol{x})}\sqrt{\rho(\boldsymbol{x})}. \quad (3.7)$$

We substitute (3.7) into the density operator $\rho(\boldsymbol{x}) = \sum_\alpha \phi_\alpha^\dagger(\boldsymbol{x})\phi_\alpha(\boldsymbol{x}) = \phi^\dagger(\boldsymbol{x})\phi(\boldsymbol{x})$, and find that $\boldsymbol{n}^\dagger(\boldsymbol{x})\boldsymbol{n}(\boldsymbol{x}) = 1$, where $\boldsymbol{n}^T = (n_1, n_2)$. The pseudospin generators are expressed as

$$S^a(\boldsymbol{x}) = \boldsymbol{n}^\dagger(\boldsymbol{x})\tau^a\boldsymbol{n}(\boldsymbol{x})\rho(\boldsymbol{x}). \quad (3.8)$$

We count the number of the real fields in the decomposition (3.7). The composite boson ϕ_α has four real fields in total, and the U(1) field ϕ has two real fields. Hence, the two-component complex field n_α has only two real fields. Such a field is the CP^1 field, and the O(3) sigma field is given by $\boldsymbol{n}^\dagger(\boldsymbol{x})\tau^a\boldsymbol{n}(\boldsymbol{x})$ as a composite field [6].

The overall phase of the CP^1 field is unphysical, since the CP^1 field is defined merely as a projective field by using a set of patches to cover the entire two-dimensional space [6]. When we set

$$n_1(\boldsymbol{x}) = e^{i\varphi_1(\boldsymbol{x})}\sqrt{\frac{1+\sigma(\boldsymbol{x})}{2}}, \qquad n_2(\boldsymbol{x}) = e^{i\varphi_2(\boldsymbol{x})}\sqrt{\frac{1-\sigma(\boldsymbol{x})}{2}}, \quad (3.9)$$

only the difference $\varphi(\boldsymbol{x}) = \varphi_1(\boldsymbol{x}) - \varphi_2(\boldsymbol{x})$ is a physical field. When we make a perturbative analysis about the classical ground state $\varphi_\alpha(\boldsymbol{x}) = \sigma(\boldsymbol{x}) = 0$, we may carry out calculations in one patch by setting

$$n_1(\boldsymbol{x}) = e^{i\varphi(\boldsymbol{x})/2}\sqrt{\frac{1+\sigma(\boldsymbol{x})}{2}}, \qquad n_2(\boldsymbol{x}) = e^{-i\varphi(\boldsymbol{x})/2}\sqrt{\frac{1-\sigma(\boldsymbol{x})}{2}}. \tag{3.10}$$

Regarding $\chi(\boldsymbol{x})$, $\delta\rho(\boldsymbol{x}) = \rho(\boldsymbol{x}) - 2\rho_0$, $\varphi(\boldsymbol{x})$ and $\sigma(\boldsymbol{x})$ as the fluctuation fields, we obtain

$$\begin{aligned}\phi_1 &= \sqrt{\rho_0}(1 + \frac{\delta\rho}{4\rho_0} + \frac{\sigma}{2} + i\chi + i\frac{\varphi}{2}) + \cdots,\\ \phi_2 &= \sqrt{\rho_0}(1 + \frac{\delta\rho}{4\rho_0} + \frac{\sigma}{2} - i\chi - i\frac{\varphi}{2}) + \cdots.\end{aligned} \tag{3.11}$$

The canonical commutation relations $[\phi_\alpha(\boldsymbol{x}), \phi_\beta^\dagger(\boldsymbol{y})] = \delta_{\alpha\beta}\delta(\boldsymbol{x}-\boldsymbol{y})$ are realized by

$$[\delta\rho(\boldsymbol{x}), \chi(\boldsymbol{y})] = i\delta(\boldsymbol{x}-\boldsymbol{y}) \tag{3.12}$$

and

$$[\rho_0\sigma(\boldsymbol{x}), \varphi(\boldsymbol{y})] = i\delta(\boldsymbol{x}-\boldsymbol{y}). \tag{3.13}$$

The phase difference φ is the conjugate field of the out-of-phase density σ, just the phase χ is the conjugate field of the in-phase density ρ.

The parametrization (3.10) is not appropriate for a discussion of the Skyrmion classical field. If we identify the phase $\varphi(\boldsymbol{x})$ as the azimuthal angle $\theta(\boldsymbol{x})$, the CP^1 field (3.10) becomes singular at the Skyrmion center where $\sigma(\boldsymbol{x}) = 1$. We should rather set

$$n_1(\boldsymbol{x}) = \sqrt{\frac{1+\sigma(\boldsymbol{x})}{2}}, \qquad n_2(\boldsymbol{x}) = e^{-i\theta(\boldsymbol{x})}\sqrt{\frac{1-\sigma(\boldsymbol{x})}{2}}, \tag{3.14}$$

with $\varphi(\boldsymbol{x}) = \theta(\boldsymbol{x})$. In this way there exists a phase ambiguity in the CP^1 field. On the contrary, there is no such ambiguity in the pseudospin fields $S^a(\boldsymbol{x})$ which depend only on the phase difference $\varphi = \varphi_1 - \varphi_2$.

C. Coherent State

The key property of the (m, m, m) phase is that the ground state is an eigenstate of the density operator ρ and a coherent state of the CP^1 field, as we discuss in section V A. Hence, we are interested in such a state that

$$\rho(\boldsymbol{x})|g\rangle = 2\rho_0|g\rangle \tag{3.}$$

and

$$n_1(\boldsymbol{x})|g\rangle = \sqrt{\frac{1}{2}}e^{i\varphi_0/2}|g\rangle, \qquad n_2(\boldsymbol{x})|g\rangle = \sqrt{\frac{1}{2}}e^{-i\varphi_0/2}|g\rangle. \tag{3.}$$

The pseudospin field reads

$$\langle g|S^1(\boldsymbol{x})|g\rangle = \rho_0\cos\varphi_0, \quad \langle g|S^2(\boldsymbol{x})|g\rangle = \rho_0\sin\varphi_0, \quad \langle g|S^3(\boldsymbol{x})|g\rangle = 0. \tag{3.}$$

Without loss of generality we may choose $\varphi_0 = 0$,

$$\boldsymbol{n}(\boldsymbol{x})|g_0\rangle = \boldsymbol{n}_0|g_0\rangle, \qquad \boldsymbol{n}_0 = \sqrt{\frac{1}{2}}\begin{pmatrix}1\\1\end{pmatrix}, \tag{3.}$$

and

$$s_0^a \equiv \frac{1}{\rho_0}\langle g_0|S^a(\boldsymbol{x})|g_0\rangle = \delta^{a1}. \tag{3.}$$

Note that the ground state $|g_0\rangle$ is different from the one in the monolayer case with sp degrees of freedom, where $\langle g|S^3(\boldsymbol{x})|g\rangle = 2\rho_0$, due to the difference between the Zeema energy and the capacitance energy.

A pseudospin texture is generated by performing an SU(2) transformation as $|\Phi\rangle$ $e^{iQ}|g_0\rangle$ with (2.6). We may parametrize,

$$\begin{aligned}s^1(\boldsymbol{x}) &\equiv \frac{1}{\rho_0}\langle\Phi|S^1(\boldsymbol{x})|\Phi\rangle = \sqrt{1-\sigma^2(\boldsymbol{x})}\cos\varphi(\boldsymbol{x}),\\ s^2(\boldsymbol{x}) &\equiv \frac{1}{\rho_0}\langle\Phi|S^2(\boldsymbol{x})|\Phi\rangle = \sqrt{1-\sigma^2(\boldsymbol{x})}\sin\varphi(\boldsymbol{x}),\\ s^3(\boldsymbol{x}) &\equiv \frac{1}{\rho_0}\langle\Phi|S^3(\boldsymbol{x})|\Phi\rangle = \sigma(\boldsymbol{x}).\end{aligned} \tag{3.}$$

The field $\boldsymbol{s}(\boldsymbol{x}) = (s^1, s^2, s^3)$ is the classical nonlinear O(3) sigma field subject to $\sum_a(s^a(\boldsymbol{x}))^2 =$

Using the SU(2) transformation (2.6) explicitly in the state $|\Phi\rangle = e^{iQ}|g_0\rangle$, we may expr s^a explicitly in terms of f^a,

$$s^a(\boldsymbol{x}) = \frac{1}{\rho_0}\langle\Phi|S^a(\boldsymbol{x})|\Phi\rangle = s_0^a - \varepsilon^{abc}f^b(\boldsymbol{x})s_0^c + \cdots, \tag{3.}$$

th (3.19), where the dots $\cdots$ denote higher terms in f^a. Comparing this with (3.20), we n relate the function f^a to the fields σ and φ. We have

$$s^1(\boldsymbol{x}) = 1, \qquad s^2(\boldsymbol{x}) = \varphi = -f^3, \qquad s^3(\boldsymbol{x}) = \sigma = f^2, \tag{3.22}$$

to the first order in f^a.

The pseudospin texture is classified by the Pontryagin number, $Q = \int d^2x Q_0(\boldsymbol{x})$, whose nsity have some equivalent representations [6],

$$Q_0(\boldsymbol{x}) = \frac{1}{8\pi}\varepsilon_{abc}\varepsilon_{ij}s_a\partial^i s_b\partial^j s_c = \frac{1}{4\pi}\varepsilon_{ij}\partial_i\varphi\partial_j\sigma = \frac{1}{2\pi}\varepsilon_{ij}\partial_i K_j, \tag{3.23}$$

here $K_j = -i\sum_\alpha n_\alpha^*\partial_j n_\alpha$ is the auxiliary field associated with the CP^1 field. When a eudospin texture has a nonzero Pontryagin number, it cannot be deformed into a trivial eudospin texture (the classical ground state) by a continuous deformation. The pseu- spin texture denotes a topological soliton which has the topological stability: It describes yrmions [6].

D. LLL Projection in CS Theory

We can make the LLL projection also in the bosonic CS theory. The LLL projection enchs the kinetic term in the Hamiltonian (3.4). The kinetic term is

$$H_K = \frac{1}{2M}\sum_\alpha \int d^2x\phi_\alpha^\dagger(\boldsymbol{x})(\check{P}_x^\alpha - i\check{P}_y^\alpha)(\check{P}_x^\alpha + i\check{P}_y^\alpha)\phi_\alpha(\boldsymbol{x}) + \frac{N}{2}\hbar\omega_c, \tag{3.24}$$

here $\check{P}_j^\alpha$ is the covariant momentum (3.5). The LLL condition reads

$$(\check{P}_x^\alpha + i\check{P}_y^\alpha)\phi_\alpha(\boldsymbol{x})|\hat{\Phi}\rangle = 0, \qquad \text{for} \quad \alpha = 1, 2, \tag{3.25}$$

corresponds to (2.12). It is easy to see [4] that the wave function of the state $|\hat{\Phi}\rangle$ is given the function (3.3) by solving (3.25). In the (m, m, m) phase the LLL condition reads

$$(\check{P}_x + i\check{P}_y)\phi_\alpha(\boldsymbol{x})|\hat{\Phi}\rangle = 0, \qquad \text{for} \quad \alpha = 1, 2. \tag{3.26}$$

mposite bosons are hardcore bosons. We postulate the exclusion principle for the U(1) ld ϕ in (3.7), as requires $\phi(\boldsymbol{x})^2 = 0$. Then, all the formulas for the LLL projection in

the electron theory hold as they are in the composite boson theory when we replace the quantities of the electron by the checked quantities of the composite boson. In particular, the LLL projected density and SU(2) generators satisfy the $W_\infty \times SU(2)$ algebra (2.22) ~ (2.24) also in the Chern-Simons formalism.

IV. SEMICLASSICAL ANALYSIS

A. Classical ground state

We analyze the (m, m, m) phase of the bilayer QH system in a semiclassical approximation based on the composite boson picture. The previous work [4] is insufficient since it predicts an isolated Goldstone mode which is physically unacceptable [5].

We first determine the classical ground state by minimizing the energy of the classical Hamiltonian. In this phase the LLL condition implies two classical equations,

$$(\check{P}_x + i\check{P}_y)\phi_\alpha(\boldsymbol{x}) = 0, \qquad \alpha = 1, 2 \tag{4.1}$$

with $\check{P}_k = -i\hbar\partial_k + \mathcal{A}_k$ and $\mathcal{A}_k = \frac{1}{c}(C_k + eA_k^{\text{ext}})$, while the CS constraint (3.6) yields one equation,

$$\varepsilon_{jk}\partial_j C_k(\boldsymbol{x}) = 2\pi\hbar c m\rho(\boldsymbol{x}), \tag{4.2}$$

where $\rho \equiv \rho^1 + \rho^2$ is the total density. The Coulomb energy is minimized by $\rho^1 = \rho^2 = \rho_0$, where ρ_0 is the uniform background density in each layer.

The classical ground state is given by the uniform distribution of composite bosons,

$$\phi^\alpha(\boldsymbol{x}) = \sqrt{\rho_0^\alpha}e^{i\chi_0^\alpha}, \qquad c\mathcal{A}_k = C_k^\alpha + eA_k^{\text{ext}} = 0, \tag{4.3}$$

with a fixed angle χ_0^α, since it satisfies the LLL condition (4.1) and minimizes the Coulomb term. Substituting this solution into the CS constraint constraint (4.2), we find $eB = 4\pi\hbar c m\rho_0$, or $\nu \equiv 4\pi\hbar c\rho_0/eB = 1/m$. The classical ground state (4.3) is possible only at this filling factor [4]. It should be emphasized that the densities ρ_0^α are arbitrary in each

layers though their sum is fixed. In the absence of the tunneling interaction, each of the classical ground states is the vacuum of a different sector of the Hamiltonian. However, the tunneling interaction mixes all of them. Consequently, the Hilbert space contains states having arbitrary uniform density distributions ρ_0^α with $\rho_0^1 + \rho_0^2 = 2\rho_0$ at the filling factor $\nu = 1/m$. In the next subsection, by considering quantum fluctuations, we show that the densities $\rho^\alpha(\boldsymbol{x})$ may change locally. This results in the arbitrariness in the out-of-phase density $S^3(\boldsymbol{x})$, as is the origin of the interlayer coherence.

This should be contrasted with the classical ground state in the (m_1, m_2, n) phase. Here, since the CS constraint contains two equations as in (3.6) rather than one, the classical ground state is uniquely determined: The densities in each layers are fixed as

$$\rho_0^1 = \frac{m_2 - n}{m_1 + m_2 - 2n} 2\rho_0, \qquad \rho_0^2 = \frac{m_1 - n}{m_1 + m_2 - 2n} 2\rho_0. \tag{4.4}$$

and there is no room for the interlayer coherence to emerge [4]. It is realized at the filling factor $\nu = (m_1 + m_2 - 2n)/(m_1 m_2 - n^2)$ with $m_1 m_2 \neq n^2$.

In the (m, m, m) phase it is convenient to parametrize the bosonized electron field $\phi_\alpha(\boldsymbol{x})$ by using the CP^1 field $n_\alpha(\boldsymbol{x})$ as in (3.7), or

$$\phi_\alpha(\boldsymbol{x}) = \phi(\boldsymbol{x}) n_\alpha(\boldsymbol{x}), \qquad \phi(\boldsymbol{x}) = e^{i\chi(\boldsymbol{x})}\sqrt{\rho(\boldsymbol{x})}. \tag{4.5}$$

Here, $\phi(\boldsymbol{x})$ and $n_\alpha(\boldsymbol{x})$ are independent fields. Corresponding to the classical ground state (4.3) a set of coherent states is defined by

$$\phi(\boldsymbol{x})|\alpha\rangle = \sqrt{2\rho_0} e^{i\alpha}|\alpha\rangle, \tag{4.6}$$

for the field $\phi(\boldsymbol{x})$, and

$$n_1(\boldsymbol{x})|\varphi_0\rangle = \sqrt{\frac{1}{2}} e^{i\varphi_0/2}|\varphi_0\rangle, \qquad n_2(\boldsymbol{x})|\varphi_0\rangle = \sqrt{\frac{1}{2}} e^{-i\varphi_0/2}|\varphi_0\rangle. \tag{4.7}$$

for the CP^1 field $n_\alpha(\boldsymbol{x})$. The mean-field ground state is given by $|\alpha\rangle \otimes |\varphi_0\rangle$.

The mean-field ground state is a coherent state of the composite bosons and contain an indefinite number of particles. This contradicts the fact that the system contains a fixed

number of electrons, which we have assumed to derive the Hamiltonian; see (2.11) a (3.24). We can construct a state having a fixed number of particles as follows. From (4 we obtain

$$\langle\alpha|\phi(\boldsymbol{x})|\beta\rangle = e^{i\alpha}\sqrt{2\rho_0}\langle\alpha|\beta\rangle, \tag{4}$$

with $\langle\alpha|\beta\rangle = 2\pi\delta(\alpha - \beta)$. We have such a relation between two vacua $|\alpha\rangle$ and $|0\rangle$ that

$$|\alpha\rangle = e^{i\alpha Q}|0\rangle, \qquad Q = \int d^2x \rho(\boldsymbol{x}), \tag{4}$$

where Q is the number operator. The gauge-invariant quantities do not depend on t gauge parameter α. We may construct the state

$$|0\rangle_N = \sum_\alpha e^{-i\alpha N}|\alpha\rangle = \sum_\alpha e^{i\alpha(Q-N)}|0\rangle = \delta(Q - N)|0\rangle, \tag{4.1}$$

where $\sum_\alpha = (2\pi)^{-1}\int_0^{2\pi} d\alpha$. The state has a fixed number N. Furthermore, using (4.8) v find

$$_N\langle 0|\phi(\boldsymbol{x})|0\rangle_N = \sum_{\alpha\beta} e^{i(\alpha-\beta)}\langle\alpha|\phi(\boldsymbol{x})|\beta\rangle = \sqrt{\rho_0}\sum_\alpha e^{i\alpha} = 0, \tag{4.1}$$

and hence the global phase symmetry is restored on $|0\rangle_N$.

The true ground state is given by $|g\rangle = |0\rangle_N \otimes |\varphi_0\rangle$, which has a fixed number of electron Nevertheless, the electron number in each layer is arbitrary. The QH state is a coherent sta of the CP^1 field in the (m, m, m) phase.

B. Perturbative Analysis

We analyze a quantum correction around the classical ground state (4.3). Any fluct ations violating the LLL condition (3.26) induce excitations across the Landau level ga Thus, we analyze the LLL condition or equivalently the kinetic Hamiltonian perturbative We express the composite boson field $\phi_\alpha(\boldsymbol{x})$ in terms of the U(1) field $\phi(\boldsymbol{x})$ and the CP^1 fie $n_\alpha(\boldsymbol{x})$. Since the CS constraint (3.6) reads $\varepsilon_{jk}\partial_j C_k(\boldsymbol{x}) = 2\pi\hbar c m \phi^\dagger(\boldsymbol{x})\phi(\boldsymbol{x})$, the CS field $C_k($ depends only on the U(1) field $\phi(\boldsymbol{x})$: It commutes with the CP^1 field $n_\alpha(\boldsymbol{x})$.

We consider a perturbative fluctuation around the classical ground state by setting

$$\phi(\boldsymbol{x}) = \sqrt{2\rho_0}e^{i\chi(\boldsymbol{x})}\Big(1 + \frac{\delta\rho(\boldsymbol{x})}{4\rho_0} + \cdots\Big), \tag{4.12}$$

nd

$$n_1(\boldsymbol{x}) = \sqrt{\frac{1}{2}}e^{i\varphi(\boldsymbol{x})/2}\Big(1 + \frac{\sigma(\boldsymbol{x})}{2} + \cdots\Big),$$
$$n_2(\boldsymbol{x}) = \sqrt{\frac{1}{2}}e^{-i\varphi(\boldsymbol{x})/2}\Big(1 - \frac{\sigma(\boldsymbol{x})}{2} + \cdots\Big). \tag{4.13}$$

ubstituting the above expansions to $\check{P}_k\phi_\alpha$ and taking only the linear terms in the fluctuation elds we find

$$\check{P}_k\phi_\alpha = \frac{1}{\sqrt{2}}e^{\pm i\varphi/2}\check{P}_k\phi - i\hbar\sqrt{2\rho_0}e^{i\chi}\partial_k n_\alpha, \tag{4.14}$$

here $\check{P}_k\phi$ is given by

$$\check{P}_k\phi = e^{i\chi}(\sqrt{\rho_0}\hbar\partial_k\chi - \frac{i\hbar}{2\sqrt{\rho_0}}\partial_k\delta\rho + \sqrt{\rho_0}\mathcal{A}_k). \tag{4.15}$$

ne Fourier transformations of the fluctuation fields are

$$\delta\rho(\boldsymbol{x}) = \int\frac{d^2k}{2\pi}\delta\rho_k e^{i\boldsymbol{kx}}, \quad \chi(\boldsymbol{x}) = \int\frac{d^2k}{2\pi}\chi_k e^{i\boldsymbol{kx}}, \tag{4.16}$$

nd

$$\mathcal{A}_i(\boldsymbol{x}) = \varepsilon_{ij}2\pi im\hbar\int\frac{d^2k}{2\pi}\frac{k_j}{\boldsymbol{k}^2}\delta\rho_k e^{i\boldsymbol{kx}}, \tag{4.17}$$

y solving the CS constraint.

In the linear approximation the kinetic Hamiltonian is easily diagonalized. It is convenient to introduce the operators

$$\xi_k = \frac{1}{\sqrt{2}}\Big(\sqrt{G_k}\delta\rho_k + i\frac{1}{\sqrt{G_k}}\chi_k\Big),$$
$$\xi_k^\dagger = \frac{1}{\sqrt{2}}\Big(\sqrt{G_k}\delta\rho_k^\dagger - i\frac{1}{\sqrt{G_k}}\chi_k^\dagger\Big), \tag{4.18}$$

here

$$G_k = \frac{1}{4\rho_0}\Big(1 + \frac{2}{\ell_B^2\boldsymbol{k}^2}\Big), \tag{4.19}$$

and

$$\zeta_k = \sqrt{\frac{\rho_0}{2}}[\sigma_k + i\varphi_k], \qquad \zeta_k^\dagger = \sqrt{\frac{\rho_0}{2}}[\sigma_k^\dagger - i\varphi_k^\dagger]. \tag{4.20}$$

They obey $[\xi_k, \xi_l^\dagger] = \delta(\boldsymbol{k} - \boldsymbol{l})$, and $[\zeta_k, \zeta_l^\dagger] = \delta(\boldsymbol{k} - \boldsymbol{l})$. Substituting (4.14) into the kinetic term in (3.24), we obtain [4]

$$H = \int d^2kE_\xi(\boldsymbol{k})\xi_k^\dagger\xi_k + \int d^2kE_\zeta(\boldsymbol{k})\zeta_k^\dagger\zeta_k \tag{4.21}$$

with

$$E_\xi(\boldsymbol{k}) = \frac{\hbar^2\boldsymbol{k}^2}{2M} + \hbar\omega_c, \qquad E_\zeta(\boldsymbol{k}) = \frac{\hbar^2\boldsymbol{k}^2}{4M}. \tag{4.22}$$

The dispersion relation $E_\xi(\boldsymbol{k})$ is precisely the same as the one in the monolayer case [4]. The operator $\xi_k^\dagger$ creates excitations across the Landau level gap as in the monolayer case. Therefore, as far as ξ_k mode concerns, the Hilbert space contains only the "vacuum". There are also vortex sectors created nonperturbatively [13]. However, to create a vortex it is necessary to supply a nonzero Coulomb energy. Namely, the in-phase density fluctuation is a gapful mode. This implies the incompressibility of the fractional QH system.

We next analyze the ζ_k mode. It is the Goldstone mode associated with the spontaneous breakdown of the SO(2) symmetry. The dispersion relation $E_\zeta(\boldsymbol{k})$ implies that the out-of-phase density fluctuation has a gapless mode [4]. However, a caution is necessary. It is clear in the Hamiltonian (4.21) that no energy is necessary to excite the ζ_0 mode. Hence, the uniform electron density ρ_0^α in each layer is arbitrary though their sum is fixed ($\rho_0^1 + \rho_0^2 = 2\rho_0$), which is the property of the classical ground states. However, the dispersion relation has a peculiar form since any excitation created by $\zeta_k^\dagger$ with $\boldsymbol{k} \neq 0$ escapes the lowest Landau level. Because of this fact it has been criticized [14] that it would not be the coherent mode we are looking for. Indeed, it would describe an isolated Goldstone mode [4], as would imply an infinitely large pseudospin stiffness and the Meissner effect [15] which are physically

unacceptable [5,8]. In spite of these facts, we now show that the criticism [14] is not correct. The isolated Goldstone mode is merely a superficial observation: It reflects the fact that $\zeta_k^\dagger$ creates the plane wave e^{ikx} which does not belong to the lowest Landau level except for $k = 0$.

The LLL condition (3.26) is linearized and yields the condition

$$\frac{\partial}{\partial z^*}\zeta(x)|\Phi\rangle = 0 \tag{4.23}$$

for the $\zeta(x)$ mode. Just the original LLL condition (3.25) determines the wave functions of the composite bosons as in (3.3), this restricts the wave functions of the $\zeta(x)$ mode. Let $|0\rangle_\zeta$ be the vacuum of the $\zeta(x)$ mode, $\zeta(x)|0\rangle_\zeta = 0$. We may construct a state $|f\rangle$ whose wave function is an arbitrary analytic function $f(z)$,

$$|f\rangle = \int d^2x f(z)\zeta^\dagger(x)|0\rangle_\zeta. \tag{4.24}$$

Use the commutation relation $[\zeta(x), \zeta^\dagger(y)] = \delta(x-y)$ we have $\zeta(x)|f\rangle = f(z)|0\rangle_\zeta$. The LLL condition (3.26) implies that the state $|f\rangle$ belongs to the lowest Landau level. In general, the N-excitation state is given by

$$|f_N\rangle = \int \prod_r d^2x_r f_N(z_1, z_2, \cdots z_N)\zeta^\dagger(x_1)\zeta^\dagger(x_2)\cdots\zeta^\dagger(x_N)|0\rangle_\zeta, \tag{4.25}$$

where $f_N(z_1, z_2, \cdots z_N)$ is an analytic function of the N variables z_r symmetric in all of them. In due course the kinetic energy of the state $|f_N\rangle$ vanishes, as is clearly seen by rewriting the Hamiltonian (4.21) as

$$H_\zeta = \frac{\hbar^2}{4M}\int d^2k\zeta_k^\dagger(k_x - ik_y)(k_x + k_y)\zeta_k = -\frac{\hbar^2}{4M\ell_B^2}\int d^2x\zeta^\dagger(x)\frac{\partial}{\partial z}\frac{\partial}{\partial z^*}\zeta(x). \tag{4.26}$$

The density difference between the two layers is described locally by the operator

$$S^3(x) = \rho_0\sigma(x) = \sqrt{\frac{\rho_0}{2}}[\zeta(x) + \zeta^\dagger(x)]. \tag{4.27}$$

We find $\langle f|\sigma(x)|f\rangle = 0$ for one-excitation state since $\sigma(x)$ changes the number of excitations by one. We may consider a superposition of various states,

$$|\hat{\Phi}\rangle = |f_1\rangle + |f_2\rangle + \cdots, \tag{4.2}$$

for which we have $\langle\hat{\Phi}|\sigma(x)|\hat{\Phi}\rangle \neq 0$. The state $|\hat{\Phi}\rangle$ describes a local modulation of t pseudospin texture, $\langle\hat{\Phi}|S^a(x)|\hat{\Phi}\rangle = \rho_0 s^a(x) \neq 0$. Although it has the vanishing kine energy, it is not a classical solution of the LLL condition (4.1).

Any states $|\hat{\Phi}\rangle$ in the lowest Landau level have no kinetic energy. In evaluating t Coulomb energy we cannot use the Coulomb potential (2.4) naively. If we used it, t state (4.28) would acquire the capacitance energy only. It is necessary to make the L projection of the Coulomb potential; see the projected potential (5.3) we give later. T projected potential $\hat{V}_q$ contains the guiding center $\check{X}$ in place of the coordinate x, whe $\check{X}$ is a differential operator acting on the analytic function f_N in (4.25) nontrivially. Hen the state $|\hat{\Phi}\rangle$ acquires a nontrivial Coulomb energy $\langle\hat{\Phi}|\hat{H}_C|\hat{\Phi}\rangle$. It is possible to modula the pseudospin densities $\langle\hat{\Phi}|\hat{S}_q^a|\hat{\Phi}\rangle$ arbitrarily small so that the Coulomb energy becom arbitrary small. This implies the existence of a gapless mode in the out-of-phase dens fluctuation. However, we do not pursue this line of approach any more since it is at bes crude approximation. Indeed, when the projected operators are linearized in the fluctuati fields, they do not satisfy the density algebra (2.22) ~ (2.24), though they are the fundamen symmetry. In section V D we evaluate the Coulomb energy by making use of the dens algebra, where we derive the dispersion relation E_k in (5.64) for the gapless mode confin within the lowest Landau level.

C. Skyrmion Classical Configuration

We have studied the classical ground state and quantum fluctuations around it. In t (m, m, m) phase the classical ground state is a classical solution of the LLL condition (4 satisfying the CS constraint (4.2). We now show that there exist other classical solutio We may rewrite the LLL condition (4.1) as

$$\left(\frac{\partial}{\partial z^*} + z + i\frac{\ell_B}{c\hbar}(C_x + iC_y)\right)\phi_\alpha(x) = 0, \tag{4.2}$$

here use was made of (3.5) and (2.10). Since we are dealing with the classical field, e U(1) phase factor $e^{i\chi}$ in (3.7) may be aborbed into the CP^1 field $n_\alpha(x)$. Thus, we set $_\alpha(x) = \sqrt{\rho(x)}n_\alpha(x)$. We substitute it into (4.29) and divide it with $\sqrt{\rho(x)}$,

$$\Big(\frac{\partial}{\partial z^*} + \frac{\partial \ln\sqrt{\rho(x)}}{\partial z^*} + z + i\frac{\ell_B}{c\hbar}(C_x + iC_y)\Big)n_\alpha(x) = 0. \tag{4.30}$$

nce the identical equation holds for $\alpha = 1, 2$, by setting $n_1(x) = \omega(x)n_2(x)$, we find

$$\frac{\partial}{\partial z^*}\omega(x) = 0. \tag{4.31}$$

herefore, ω is an arbitrary analytic function. The CP^1 field $n_\alpha(x)$ is determined as

$$n_1(x) = \frac{\omega(z)}{\sqrt{1 + |\omega(z)|^2}}, \qquad n_2(x) = \frac{1}{\sqrt{1 + |\omega(z)|^2}}. \tag{4.32}$$

his field configuration has the most general form of the Skyrmion configuration [6]. Hence, e classical field $n_\alpha(x)$ describes Skyrmions.

An example is given by

$$\omega(z) = \frac{z + i\kappa}{z - i\kappa}, \tag{4.33}$$

here the positive constant κ describes its scale. It yields the sigma field,

$$s^1 = \frac{r^2 - \kappa^2}{r^2 + \kappa^2}, \quad s^2 = \frac{2\kappa x}{r^2 + \kappa^2}, \quad s^3 = \frac{2\kappa y}{r^2 + \kappa^2}. \tag{4.34}$$

has the Pontryagin number $Q = 1$. We find $s^a(x) \to \delta^{a1}$ as $r \to \infty$. Thus, it approaches the assical ground state (3.19) at infinity. The example (4.33) gives the simplest Skyrmion on e bilayer ground state $|g_0\rangle$.

Another example is given by

$$\omega(z) = \kappa z. \tag{4.35}$$

yields the sigma field s^a_M such as $(s^1_M, s^2_M, s^3_M) = (s^2, s^3, s^1)$ with (4.34). Due to the cyclic mmetry the present Skyrmion has the same Pontryagin number, $Q = 1$. We find $s^a_M(x) \to$

δ^{a3} as $r \to \infty$. This is the simplest Skyrmion on the ground state in the monolayer QH system with spin degrees of freedom.

We next analyze the density $\rho(x)$ in the presence of a Skyrmion excitation. We multiply n^*_α to (4.30) and sum over $\alpha = 1, 2$,

$$-i\frac{\ell_B}{c\hbar}(C_x + iC_y) = i\ell_B(K_x + iK_y) + \frac{\partial \ln\sqrt{\rho(x)}}{\partial z^*} + z. \tag{4.36}$$

Operating $\partial/\partial z$ to (4.36) we obtain

$$\frac{\ell_B^2}{c\hbar}\varepsilon_{ij}\partial_i C_j = \ell_B\varepsilon_{ij}\partial_i K_j + \nabla^2 \ln\sqrt{\rho(x)} + 1. \tag{4.37}$$

Using the CS constraint equation (4.2) we find

$$\rho(x) = 2\rho_0 + \nu Q_0(x) + \frac{\nu}{4\pi}\nabla^2 \ln\rho(x), \tag{4.38}$$

where $Q_0(x)$ is given by (3.23). When one analytic function $\omega(z)$ is chosen, a Skyrmion is given together with the Pontryagin number density $Q_0(x)$. Then, we may solve (4.38) to determine the in-phase density $\rho(x)$. Since the Skyrmion excitation is a small modification of the ground state value $2\rho_0$, it is convenient to rewrite (4.38) as

$$\Delta\rho(x) \equiv \rho(x) - 2\rho_0 = \nu Q_0(x) + \frac{\nu}{8\pi\rho_0}\nabla^2\Delta\rho(x). \tag{4.39}$$

We may solve this by iteration as

$$\Delta\rho(x) = \nu Q_0(x) + \frac{\nu^2}{8\pi\rho_0}\nabla^2 Q_0(x) + \frac{\nu^3}{(8\pi\rho_0)^2}\nabla^4 Q_0(x) + \cdots. \tag{4.40}$$

The electric charge of the simplest Skyrmion is $-e\nu$ since its electric charge is $\int d^2x\Delta\rho(x) = \nu$. The Skyrmion classical configuration describes an in-phase density modulation proportional to the Pontryagin number density in a sufficiently smooth pseudospin texture.

We make some comments: First, there exist no anti-Skyrmion classical configurations carrying the negative Pontryagin number. An anti-Skyrmion is not a solution of (4.1) but a solution of

$$(\check{P}_x - i\check{P}_y)\phi_\alpha(x) = 0. \tag{4.41}$$

It has a component not only in the lowest Landau level but also has components in higher Laudau levels. Second, since the Skyrmion (4.34) descreases very slowly asymptotically, the capacitance energy diverges. Third, when we modifies the Skyrmion configuration to make the capacitance energy finite, it acquires a kinetic energy of the order of $\hbar\omega_c$. Therefore, the classical Skyrmion solution is not a physical excitation. Here, we should mention that Skyrmion and anti-Skyrmions are generated as pseudospin textures $|\hat{\Phi}\rangle = e^{i\hat{\Theta}}|g_0\rangle$, which are free from all these defects, as we discuss in the next section.

V. ALGEBRAIC ANALYSIS

A. Ground State in Lowest Landau Level

In this section we analyze the bilayer QH system by using only the algebraic structure of the LLL-projected operators and a few additional properties of the ground state. We have already seen that the $W_\infty \times SU(2)$ algebra is the fundamental symmetry of the Hilbert space spanned by the states in the lowest Landau level. Thus, the ground state and excited states are characterized by representations of this algebra. In particular, the in-phase density fluctuation is governed by the W_∞ algebra in the bilayer system, just as the density fluctuation is so in the monolayer system [9,16]. We note that all the representations of the W_∞ algebra are already known [11]. Although there are some attempts [16] to understand the fractional QH system solely by the W_∞ algebra without considering the Coulomb interactions, it is clear that the physics is not determined by the algebra alone. (For instance there are many different phases in the spin theory all of which are governed by the SO(3) symmetry.) It is necessary to input the dynamical conditions into the system, which is supplied by the Hamiltonian.

Let us first review the monolayer system from the algebraic point of view. The Hamiltonian is the projected Coulomb term,

$$\hat{H}_C = \frac{1}{2}\int d^2x d^2y \hat{V}(\boldsymbol{x}-\boldsymbol{y})\rho(\boldsymbol{x})\rho(\boldsymbol{y}) = \pi \sum_{\alpha,\beta}\int d^2q V(\boldsymbol{q})\hat{\rho}^\dagger_{\boldsymbol{q}}\hat{\rho}_{\boldsymbol{q}}, \tag{5.1}$$

where $V(\boldsymbol{q})$ is just the Fourier transformation of the potential $V(\boldsymbol{x})$. In order to minim the Hamiltonian we can require the ground state to be an eigenstate of the density operat

$$\hat{\rho}_{\boldsymbol{q}}|g\rangle = 4\pi\rho_0\delta(\boldsymbol{q})|g\rangle. \tag{5}$$

This determines the vacuum of the vacuum sector of the W_∞ algebra. The vacuum sec is constructed as a Fock space on it. Now, there exists a striking algebraic result about t W_∞ algebra with no central extension [11]: It says that the vacuum sector is a trivial spa containing only the vacuum state. This agrees with the perturbative result about the mode given in the previous section; see (4.22). This does not mean that the monolay system is an empty system since there are also vortex sectors. The vortex sector must gi an inequivalent representation of the algebra, and the total Hilbert space is the sum of the superselection sectors with the superselection charge being the vorticity as in the Hig model [13,17].

We proceed to study the bilayer system. The Coulomb term (2.4) is projected as

$$\hat{H}_C = \pi\int d^2q V_+(\boldsymbol{q})\hat{\rho}_{-\boldsymbol{q}}\hat{\rho}_{\boldsymbol{q}} + 4\pi\int d^2q V_-(\boldsymbol{q})\hat{S}^3_{-\boldsymbol{q}}\hat{S}^3_{\boldsymbol{q}}, \tag{5}$$

where $\hat{\rho}_{-\boldsymbol{q}} = \hat{\rho}^\dagger_{\boldsymbol{q}}$ and $\hat{S}^a_{-\boldsymbol{q}} = \hat{S}^{a\dagger}_{\boldsymbol{q}}$. We call the potential V_+ part and the V_- part t SU(2)-invariant term and the SO(2)-invariant term, respectively. With respect to the SU(invariant term it is justified to require the same condition (5.2) as in the monolayer ca This is not the case with respect to the SO(2)-invariant term in the (m,m,m) phase, sir there exists a gapless mode in the out-of-phase density fluctuation; see (4.21). Hence, t ground-state cannot be an eigenstate of the out-of-phase density operator. We can requ only a weaker condition that it is a coherent state of the pseudospin operators,

$$\begin{aligned}
\rho_0\bar{s}^1_{\boldsymbol{q}} &\equiv \langle g|\hat{S}^1_{\boldsymbol{q}}|g\rangle = 2\pi\rho_0\cos\varphi_0\delta(\boldsymbol{q}),\\
\rho_0\bar{s}^2_{\boldsymbol{q}} &\equiv \langle g|\hat{S}^2_{\boldsymbol{q}}|g\rangle = 2\pi\rho_0\sin\varphi_0\delta(\boldsymbol{q}),\\
\rho_0\bar{s}^3_{\boldsymbol{q}} &\equiv \langle g|\hat{S}^3_{\boldsymbol{q}}|g\rangle = 0,
\end{aligned} \tag{5}$$

where φ_0 is a constant angle. Taking the Fourier transformation of (5.4) we obtain

$$\bar{s}^1(\boldsymbol{x}) = \cos\varphi_0, \qquad \bar{s}^2(\boldsymbol{x}) = \sin\varphi_0, \qquad \bar{s}^3(\boldsymbol{x}) = 0, \tag{5.5}$$

corresponds to (3.17). The ground state is infinitely degenerate. Any choice of the ground ate breaks the SO(2) symmetry spontaneously. When we choose $\varphi_0 = 0$ we find

$$\langle g_0|\hat{S}^a_{\boldsymbol{q}}|g_0\rangle = 2\pi\rho_0 s^a_0\delta(\boldsymbol{q}), \qquad s^a_0 = \delta^{a1}, \tag{5.6}$$

$\bar{s}^a(\boldsymbol{x}) = \delta^{a1}$, as corresponds to (3.19).

Although the ground state is fixed by (5.2) and (5.6), we cannot determine excited states the algebra alone since its representations are not yet explored. Therefore, we adopt a riational method. We consider a pseudospin texture $|\hat{\Phi}\rangle = e^{i\hat{\mathcal{O}}}|g_0\rangle$ generated from the ound state $|g_0\rangle$. Let us analyze the bilayer system, using this variational excited state and glecting the vortex sectors.

B. Coherent Excitations and Skyrmions

We first evaluate the pseudospin $\hat{S}^a_{\boldsymbol{q}}$ of the state $|\hat{\Phi}\rangle$,

$$\rho_0\bar{s}^a_{\boldsymbol{q}} \equiv \langle\hat{\Phi}|\hat{S}^a_{\boldsymbol{q}}|\hat{\Phi}\rangle = \langle g_0|\hat{S}^a_{\boldsymbol{q}}|g_0\rangle - i\langle g_0|[\hat{\mathcal{O}},\hat{S}^a_{\boldsymbol{q}}]|g_0\rangle + \cdots. \tag{5.7}$$

e term $\langle g_0|\hat{S}^a_{\boldsymbol{q}}|g_0\rangle$ is given by (5.6). The first order term involves

$$\langle g_0|[\hat{\mathcal{O}},\hat{S}^a_{\boldsymbol{q}}]|g_0\rangle = \int d^2k f^b_{-\boldsymbol{k}}\langle g_0|[\hat{S}^b_{\boldsymbol{k}},\hat{S}^a_{\boldsymbol{q}}]|g_0\rangle = -i\varepsilon^{abc}\rho_0 s^c_0 f^b_{\boldsymbol{q}}\exp[-\frac{\ell_B^2}{2}\boldsymbol{q}^2], \tag{5.8}$$

here we have used the density algebra (2.24), the ground-state conditions (5.2) and (5.6). us, denoting the Fourier transformation of $\bar{s}^a_{\boldsymbol{q}}$ by $\bar{s}^a(\boldsymbol{x})$, we obtain

$$\bar{s}^a(\boldsymbol{x}) = s^a_0 - \varepsilon^{abc}\exp[-\frac{\ell_B^2}{2}\nabla^2]f^b(\boldsymbol{x})s^c_0 + \cdots. \tag{5.9}$$

consider a limit where f^a is sufficiently smooth though f^a itself may not be small, $\nabla f^a| \ll 1$. In this limit we may ignore the exponential term. The second order term $,[\hat{\mathcal{O}},\hat{S}^a_{\boldsymbol{q}}]]$ involves the factor,

$$\langle g_0|[\hat{S}^c_{\boldsymbol{l}},[\hat{S}^b_{\boldsymbol{k}},\hat{S}^a_{\boldsymbol{p}}]]|g_0\rangle = \left(\frac{i}{2\pi}\right)^2\varepsilon^{cde}\varepsilon^{bad}\langle g_0|\hat{S}^e_{\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{p}}|g_0\rangle\exp[\frac{\ell_B^2}{2}(\boldsymbol{lk}+\boldsymbol{kp}+\boldsymbol{pl})], \tag{5.10}$$

where we have used the fact that the matrix element contains $\delta(\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{p})$. By ignoring the exponential factor the algebraic structure is the same as the standard SU(2) one. All the commutators appearing in (5.7) have precisely the same structure. Hence, summing up all the terms in (5.7), we find that

$$\bar{s}^a(\boldsymbol{x}) \cong s^a(\boldsymbol{x}) = \frac{1}{\rho_0}\langle\Phi|S^a(\boldsymbol{x})|\Phi\rangle, \tag{5.11}$$

as the dominant term in the derivative expansion in a smooth pseudospin texture. The projected state $|\hat{\Phi}\rangle$ has essentially the same pseudospin structure as the state $|\Phi\rangle$.

We next evaluate the density in the pseudospin texture,

$$\bar{\rho}_{\boldsymbol{q}} \equiv \langle\hat{\Phi}|\hat{\rho}_{\boldsymbol{q}}|\hat{\Phi}\rangle = \langle g_0|\hat{\rho}_{\boldsymbol{q}}|g_0\rangle + \Delta\bar{\rho}_{\boldsymbol{q}}, \tag{5.12}$$

where $\Delta\bar{\rho}_{\boldsymbol{q}}$ denotes the density modulation,

$$\Delta\bar{\rho}_{\boldsymbol{q}} = -i\langle g_0|[\hat{\mathcal{O}},\hat{\rho}_{\boldsymbol{q}}]|g_0\rangle - \frac{1}{2}\langle g_0|[\hat{\mathcal{O}},[\hat{\mathcal{O}},\hat{\rho}_{\boldsymbol{q}}]]|g_0\rangle + \cdots. \tag{5.13}$$

The first order correction term $\langle g_0|[\hat{\mathcal{O}},\hat{\rho}_{\boldsymbol{q}}]|g_0\rangle$ vanishes due to the ground-state condition (5.2). The second order term gives

$$\Delta\bar{\rho}_{\boldsymbol{q}} = -\frac{1}{2}\int d^2l f^b_{-\boldsymbol{l}}\int d^2k f^a_{-\boldsymbol{k}}\langle g_0|[\hat{S}^b_{\boldsymbol{l}},[\hat{S}^a_{\boldsymbol{k}},\hat{\rho}_{\boldsymbol{q}}]]|g_0\rangle. \tag{5.14}$$

We use the algebra (2.23) and (2.24) to find

$$\begin{aligned}\langle g_0|[\hat{S}^b_{\boldsymbol{l}},[\hat{S}^a_{\boldsymbol{k}},\hat{\rho}_{\boldsymbol{q}}]]|g_0\rangle = &\frac{1}{2\pi^2}\varepsilon^{abc}\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\cos[\ell_B^2\frac{\boldsymbol{l}\wedge(\boldsymbol{q}+\boldsymbol{k})}{2}]\\ &\times\exp[\frac{\ell_B^2}{2}(\boldsymbol{qk}+\boldsymbol{kl}+\boldsymbol{lq})]\langle g_0|\hat{S}^c_{\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{q}}|g_0\rangle,\end{aligned} \tag{5.15}$$

where $\langle g_0|\hat{S}^c_{\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{q}}|g_0\rangle = 2\pi\rho_0\delta^{c1}\delta(\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{q})$. Making the momentum expansion, we find

$$\Delta\bar{\rho}_{\boldsymbol{q}} = -\frac{\rho_0}{2}\varepsilon^{ab1}\varepsilon_{ij}\ell_B^2\int d^2k f^b_{\boldsymbol{q}+\boldsymbol{k}}f^a_{-\boldsymbol{k}}k_iq_j + \cdots. \tag{5.16}$$

Its Fourier transformation is

$$\Delta\bar{\rho}(\boldsymbol{x}) = \frac{\nu}{4\pi}\varepsilon_{ij}\partial_i f^2\partial_j f^3 + \cdots = \frac{\nu}{4\pi}\varepsilon_{ij}\partial_i\varphi\partial_j\sigma + \cdots = \nu Q_0(\boldsymbol{x}) + \cdots, \tag{5.17}$$

where $Q_0(x)$ is the Pontryagin number density (3.23). This formula is consistent with the formula (4.40) in the classical Skyrmion configuration. The SU(2) transformation thus generates a topological excitation carrying the Pontryagin number, that is a Skyrmion. The electron number of the simplest Skyrmion is $-\nu e$, while that of the simplest anti-Skyrmion is νe. It should be noted that there exist both Skyrmion and anti-Skyrmion textures after making the LLL projection.

C. Static Correlation Functions

Physically relevant correlation functions are those of densities $\hat{\rho}$ and $\hat{S}^a$. Let us consider a two point function $\langle g_0|\hat{S}^a_{\boldsymbol{p}}\hat{S}^b_{\boldsymbol{q}}|g_0\rangle$. It is a sum of $\langle g_0|[\hat{S}^a_{\boldsymbol{p}},\hat{S}^b_{\boldsymbol{q}}]|g_0\rangle$ and $\langle g_0|\{\hat{S}^a_{\boldsymbol{p}},\hat{S}^b_{\boldsymbol{q}}\}|g_0\rangle$. The commutator part is calculable by using the algebraic relations (2.22) ~ (2.24), but the anticommutator part cannot be handled in the same way.

In order to evaluate the anticommutator we use the formula,

$$\{\hat{\rho}_{\boldsymbol{p}},\hat{\rho}_{\boldsymbol{q}}\} = \frac{1}{\pi}\hat{\rho}_{\boldsymbol{p}+\boldsymbol{q}}\cos[\ell_B^2\frac{\boldsymbol{p}\wedge\boldsymbol{q}}{2}]\exp[\frac{\ell_B^2}{2}\boldsymbol{p}\boldsymbol{q}] + :\{\hat{\rho}_{\boldsymbol{p}},\hat{\rho}_{\boldsymbol{q}}\}:, \tag{5.18}$$

and

$$\begin{aligned}\{\hat{S}^a_{\boldsymbol{p}},\hat{S}^b_{\boldsymbol{q}}\} = &-\frac{1}{2\pi}\varepsilon^{abc}\hat{S}^c_{\boldsymbol{p}+\boldsymbol{q}}\sin[\ell_B^2\frac{\boldsymbol{p}\wedge\boldsymbol{q}}{2}]\exp[\frac{\ell_B^2}{2}\boldsymbol{p}\boldsymbol{q}]\\ &+\frac{1}{4\pi}\delta^{ab}\hat{\rho}_{\boldsymbol{p}+\boldsymbol{q}}\cos[\ell_B^2\frac{\boldsymbol{p}\wedge\boldsymbol{q}}{2}]\exp[\frac{\ell_B^2}{2}\boldsymbol{p}\boldsymbol{q}] + :\{\hat{S}^a_{\boldsymbol{p}},\hat{S}^b_{\boldsymbol{q}}\}:,\end{aligned} \tag{5.19}$$

which follow from the operator products (2.25) and (2.27). Here, the symbol $:\cdots:$ stands for the normal ordering. We evaluate the matrix element as follows.

Using (3.8) we write

$$S^a(x)S^b(y) = \rho(x)\rho(y)\boldsymbol{n}^\dagger(x)\frac{\tau^a}{2}\boldsymbol{n}(x)\boldsymbol{n}^\dagger(y)\frac{\tau^a}{2}\boldsymbol{n}(y). \tag{5.20}$$

Since $|g_0\rangle$ is a coherent state of the CP^1 field $\boldsymbol{n}$ as in (3.18), we obtain that

$$\langle g_0|:\{\hat{S}^a_{\boldsymbol{p}},\hat{S}^b_{\boldsymbol{q}}\}:|g_0\rangle = \frac{1}{4}\delta^{a1}\delta^{a2}\langle g_0|:\{\hat{\rho}_{\boldsymbol{p}},\hat{\rho}_{\boldsymbol{q}}\}:|g_0\rangle. \tag{5.21}$$

Since $|g_0\rangle$ is an eigenstate of the density operator as in (5.2), the matrix eleme $\langle g_0|\{\hat{\rho}_{\boldsymbol{p}},\hat{\rho}_{\boldsymbol{q}}\}|g_0\rangle$ is trivially calculated. We use (5.18) to find that

$$\langle g_0|:\{\hat{\rho}_{\boldsymbol{p}},\hat{\rho}_{\boldsymbol{q}}\}:|g_0\rangle = 8(2\pi)^2\rho_0^2\delta(\boldsymbol{p})\delta(\boldsymbol{q}) - 4\rho_0\delta(\boldsymbol{p}+\boldsymbol{q})\exp[-\frac{\ell_B^2}{2}\boldsymbol{p}^2]. \tag{5.2}$$

Combining these equations we obtain the matrix element of (5.19) as

$$\langle g_0|\{\hat{S}^a_{\boldsymbol{p}},\hat{S}^b_{\boldsymbol{q}}\}|g_0\rangle = \begin{cases} 2(2\pi)^2\rho_0^2\delta(\boldsymbol{p})\delta(\boldsymbol{q}) & \text{for a=b=1}\\ \rho_0\delta(\boldsymbol{p}+\boldsymbol{q})\exp[-\frac{\ell_B^2}{2}\boldsymbol{p}^2] & \text{for a=b=2,3}\\ 0 & \text{otherwise}\end{cases} \tag{5.2}$$

Note that this simple formula is for the specific choice of the ground state $|g_0\rangle$. For a gener ground state $|g\rangle$ involving a parameter φ_0 we derive a slightly complicated expression. V can calculate various correlation functions in this way. In the following sections we use th method extensively to evaluate the Coulomb energy and the current.

Here we make a comment. In calculating correlation functions we have used the fa that the ground state is an eigenstate of the density operator and a coherent state of the C field. If it were a coherent state of the composite boson field ϕ^α, we would have obtaine

$$\begin{aligned}\langle g_0|\{\hat{\rho}_{\boldsymbol{p}},\hat{\rho}_{\boldsymbol{q}}\}|g_0\rangle &= 4\delta(\boldsymbol{p}+\boldsymbol{q})\exp[-\frac{\ell_B^2}{2}\boldsymbol{p}^2] + 8(2\pi)^2\rho_0^2\delta(\boldsymbol{p})\delta(\boldsymbol{q}),\\ \langle g_0|\{\hat{S}^a_{\boldsymbol{p}},\hat{\rho}_{\boldsymbol{q}}\}|g_0\rangle &= 2\delta(\boldsymbol{p}+\boldsymbol{q})\exp[-\frac{\ell_B^2}{2}\boldsymbol{p}^2] + 4(2\pi)^2\delta^{a1}\rho_0^2\delta(\boldsymbol{p})\delta(\boldsymbol{q}),\\ \langle g_0|\{\hat{S}^a_{\boldsymbol{p}},\hat{S}^b_{\boldsymbol{q}}\}|g_0\rangle &= \delta^{ab}\rho_0\delta(\boldsymbol{p}+\boldsymbol{q})\exp[-\frac{\ell_B^2}{2}\boldsymbol{p}^2] + 2(2\pi)^2\rho_0^2\delta^{a1}\delta^{b1}\delta(\boldsymbol{p})\delta(\boldsymbol{q}),\end{aligned} \tag{5.2}$$

and the resulting physics would be very different.

D. Coulomb Energy

We evaluate the Coulomb energy ΔE_C of the pseudospin texture $|\hat{\Phi}\rangle = e^{i\hat{\phi}}|g_0\rangle$ as function of the pseudospin configuration $s^a(x)$. The Hamiltonian is given by (5.3), or

$$\hat{H}_C = \pi\int d^2q V_+(\boldsymbol{q})\hat{\rho}_{-\boldsymbol{q}}\hat{\rho}_{\boldsymbol{q}} + 4\pi\int d^2q V_-(\boldsymbol{q})\hat{S}^3_{-\boldsymbol{q}}\hat{S}^3_{\boldsymbol{q}}, \tag{5.2}$$

We analyze the SU(2)-invariant term $\hat{H}^+_C$ and the SO(2)-invariant term $\hat{H}^-_C$ separately. Befo giving details of calculations, let us present the result,

$$\Delta E_C = \frac{1}{2}\rho_E \sum_{a=1}^{2} \int d^2x[\partial_k s^a(\boldsymbol{x})]^2 + \frac{1}{2}\rho_A \int d^2x[\partial_k s^3(\boldsymbol{x})]^2$$
$$+\frac{e^2\rho_0^2}{2C}\int d^2x s^3(\boldsymbol{x})s^3(\boldsymbol{x}) + \frac{\nu^2}{2}\int d^2x d^2y V(\boldsymbol{x}-\boldsymbol{y})Q_0(\boldsymbol{x})Q_0(\boldsymbol{y}). \tag{5.26}$$

ere, C is a certain capacitance, and

$$\rho_E \equiv \rho_s^+ - \rho_s^-, \qquad \rho_A \equiv \rho_s^+ + \rho_s^-, \tag{5.27}$$

ith

$$\rho_s^\pm = \frac{\nu}{16\pi}\ell_B^2 \int \frac{d^2q}{2\pi} V_\pm(\boldsymbol{q})\boldsymbol{q}^2 \exp[-\frac{\ell_B^2}{2}\boldsymbol{q}^2]. \tag{5.28}$$

e formula (5.26) describes the Coulomb energy of the pseudospin texture described by $(\boldsymbol{x})$ in a sufficiently smooth limit such that $\ell_B \nabla s^a(\boldsymbol{x}) \ll 1$. The terms involving $(\partial_i s^a)^2$ present the energy necessary to modulate the pseudospin from the constant configuration, mely, they describe the stiffness of the pseudospin. The last term in (5.26) describes the oulomb energy of static Skyrmions carrying the Pontryagin number density $Q_0(\boldsymbol{x})$. It rees with the result [8] obtained based on the SMA in the first quantization scheme.

1. *SU(2)-invariant term*

We evaluate the SU(2)-invariant term $\hat{H}_C^+$. It is an important observation that the energy is e same for any constant pseudospin s^a. Hence, the energy depends only on the curvature m $\partial_k s^a$ and the value of s^a itself is irrelevant. In the lowest order approximation, therefore, s enough to study a small fluctuation of the pseudospin around a constant vector s^a. Due the invariance we may choose s^a arbitrary to make calculations simple. The simplest oice is the ground state $|g_0\rangle$. We may recover the SU(2) invariance afterwards.

We evaluate the energy of the coherent state $|\hat{\Phi}\rangle$ by expanding it as

$$\Delta E_C^+ = \langle\hat{\Phi}|\hat{H}_C^+|\hat{\Phi}\rangle - \langle g_0|\hat{H}_C^+|g_0\rangle \equiv \langle g_0|\Delta\hat{H}_C^+|g_0\rangle, \tag{5.29}$$

th

$$\Delta\hat{H}_C^+ = e^{-i\hat{\mathcal{O}}}\hat{H}_C^+ e^{i\hat{\mathcal{O}}} - \hat{H}_C^+ = -i[\hat{\mathcal{O}}, \hat{H}_C^+] - \frac{1}{2}[\hat{\mathcal{O}}, [\hat{\mathcal{O}}, \hat{H}_C^+]] + \cdots, \tag{5.30}$$

in $\mathcal{O}$, i.e., in f^a. The pseudospin configuration $s^a(\boldsymbol{x})$ is related to $f^a(\boldsymbol{x})$ as in (3.22) up to the first order in f^a: We may set $f^1 = 0$ in this order.

We evaluate the SU(2)-invariant term $\hat{H}_C^+$ to the nontrivial lowest order in f^a. The first order term is

$$[\hat{\mathcal{O}}, \hat{H}_C^+] = \pi \sum_a \int d^2q V_+(\boldsymbol{q}) \int d^2k f_{-\boldsymbol{k}}^a [\hat{S}_{\boldsymbol{k}}^a, \hat{\rho}_{\boldsymbol{q}}\hat{\rho}_{-\boldsymbol{q}}]. \tag{5.31}$$

Using the algebra (2.22) ~ (2.24) and the fact that $V_+(\boldsymbol{q}) = V_+(-\boldsymbol{q})$ we obtain

$$V_+(\boldsymbol{q})[\hat{S}_{\boldsymbol{k}}^a, \hat{\rho}_{\boldsymbol{q}}\hat{\rho}_{-\boldsymbol{q}}] = \frac{i}{\pi}V_+(\boldsymbol{q})\exp[\frac{\ell_B^2}{2}\boldsymbol{q}\boldsymbol{k}]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\{\hat{S}_{\boldsymbol{k}+\boldsymbol{q}}^a, \hat{\rho}_{-\boldsymbol{q}}\}. \tag{5.32}$$

Its matrix element vanishes due to the ground-state condition (5.2) since it yields a delta function $\delta(\boldsymbol{q})$.

The second order term arises from

$$[\hat{\mathcal{O}}, [\hat{\mathcal{O}}, \hat{H}_C^+]] = \pi \sum_{a,b=2}^{3} \int d^2q V_+(\boldsymbol{q}) \int d^2l \int d^2k f_{-\boldsymbol{l}}^b f_{-\boldsymbol{k}}^a [\hat{S}_{\boldsymbol{l}}^b, [\hat{S}_{\boldsymbol{k}}^a, \hat{\rho}_{\boldsymbol{q}}\hat{\rho}_{-\boldsymbol{q}}]], \tag{5.33}$$

where

$$\begin{aligned}
&V_+(\boldsymbol{q})[\hat{S}_{\boldsymbol{l}}^b, [\hat{S}_{\boldsymbol{k}}^a, \hat{\rho}_{\boldsymbol{q}}\hat{\rho}_{-\boldsymbol{q}}]] \\
&= \frac{1}{\pi^2}V_+(\boldsymbol{q})\exp[\frac{\ell_B^2}{2}\boldsymbol{q}(\boldsymbol{k}-\boldsymbol{l})]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\sin[\ell_B^2\frac{\boldsymbol{l}\wedge\boldsymbol{q}}{2}]\{\hat{S}_{\boldsymbol{l}-\boldsymbol{q}}^b, \hat{S}_{\boldsymbol{k}+\boldsymbol{q}}^a\} \\
&\quad -\frac{1}{2\pi^2}V_+(\boldsymbol{q})\varepsilon^{bac}\exp[\frac{\ell_B^2}{2}(\boldsymbol{q}\boldsymbol{l}+\boldsymbol{l}\boldsymbol{k}+\boldsymbol{k}\boldsymbol{q})]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\cos[\ell_B^2\frac{\boldsymbol{l}\wedge(\boldsymbol{k}+\boldsymbol{q})}{2}]\{\hat{\rho}_{-\boldsymbol{q}}, \hat{S}_{\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{q}}^c\} \\
&\quad -\frac{1}{4\pi^2}V_+(\boldsymbol{q})\delta^{ab}\exp[\frac{\ell_B^2}{2}(\boldsymbol{q}\boldsymbol{l}+\boldsymbol{l}\boldsymbol{k}+\boldsymbol{k}\boldsymbol{q})]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\sin[\ell_B^2\frac{\boldsymbol{l}\wedge(\boldsymbol{k}+\boldsymbol{q})}{2}]\{\hat{\rho}_{-\boldsymbol{q}}, \hat{\rho}_{\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{q}}\}.
\end{aligned} \tag{5.34}$$

The terms containing the density operator $\hat{\rho}_{-q}$ vanish due to the ground-state condition. The nontrivial anticommutator in (5.34) has been calculated in (5.23), which gives

$$\langle g_0|\{\hat{S}_{\boldsymbol{l}-\boldsymbol{q}}^a, \hat{S}_{\boldsymbol{k}+\boldsymbol{q}}^b\}|g_0\rangle = \delta^{ab}\rho_0\exp[-\frac{\ell_B^2}{2}(\boldsymbol{k}+\boldsymbol{q})^2]\delta(\boldsymbol{l}+\boldsymbol{k}). \tag{5.35}$$

for $a \neq 1$ and $b \neq 1$. Hence, we obtain

$$V_+(\boldsymbol{q})\langle g_0|[\hat{S}^b_{\boldsymbol{l}},[\hat{S}^a_{\boldsymbol{k}},\hat{\rho}_{\boldsymbol{q}}\hat{\rho}_{-\boldsymbol{q}}]]|g_0\rangle = -\frac{1}{\pi^2}\rho_0\delta^{ab}V_+(\boldsymbol{q})\exp[-\frac{\ell_B^2}{2}(\boldsymbol{q}^2+\boldsymbol{k}^2)]\sin^2[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\delta(\boldsymbol{k}+\boldsymbol{l}), \tag{5.36}$$

for $a \neq 1$ and $b \neq 1$, and

$$\langle g_0|[\hat{O},[\hat{O},\hat{H}^+_C]]|g_0\rangle = -2\rho_0\int\frac{d^2q}{2\pi}V_+(\boldsymbol{q})\exp[-\frac{\ell_B^2}{2}\boldsymbol{q}^2]\sum_{a=2,3}\int d^2k f^a_{\boldsymbol{k}}f^a_{-\boldsymbol{k}}\exp[-\frac{\ell_B^2}{2}\boldsymbol{k}^2]\sin^2[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]. \tag{5.37}$$

For a smooth pseudospin texture we may expand $\sin^2(\frac{1}{2}\ell_B^2\boldsymbol{k}\wedge\boldsymbol{q})$ and take the lowest order term. For any function $g(\boldsymbol{q}^2)$ which depends only on $\boldsymbol{q}^2 = q_x^2+q_y^2$ we may use the formula,

$$\int d^2q(\boldsymbol{k}\wedge\boldsymbol{q})^2 g(\boldsymbol{q}^2) = \frac{1}{2}\int d^2q(k_x^2+k_y^2)(q_x^2+q_y^2)g(\boldsymbol{q}^2). \tag{5.38}$$

As a result we have

$$\begin{aligned}\Delta E^+_C &= -\frac{1}{2}\langle g_0|[\hat{O},[\hat{O},\hat{H}^+_C]]|g_0\rangle \\ &= \frac{1}{8}\rho_0\ell_B^4\int\frac{d^2q}{2\pi}V_+(\boldsymbol{q})\boldsymbol{q}^2\exp[-\frac{\ell_B^2}{2}\boldsymbol{q}^2]\sum_{a=2,3}\int d^2k\boldsymbol{k}^2 f^a_{\boldsymbol{k}}f^a_{-\boldsymbol{k}} \\ &= \frac{1}{2}\rho^+_s\sum_{a=2,3}\int d^2x[\partial_k f^a(\boldsymbol{x})]^2,\end{aligned} \tag{5.39}$$

where ρ^+_s has been defined by (5.28). Using (3.22) we may rewrite this as

$$\Delta E^+_C = \frac{1}{2}\rho^+_s\sum_{a=1}^{3}\int d^2x[\partial_k s^a(\boldsymbol{x})]^2, \tag{5.40}$$

into a global SU(2) invariant form.

2. *SO(2)-invariant term*

We next evaluate the SO(2)-invariant term $\hat{H}^-_C$. Due to the lack of the SU(2) invariance the energy depends on the pseudospin itself in addition to its derivative. Indeed, the SO(2)-invariant term represents the capasitive Coulomb energy associated with the density difference s^3 between the two layers. We evaluate the Coulomb energy ΔE^-_C in the expansion as in (5.30), or

$$\Delta E^-_C = -i\langle g_0|[\hat{O},\hat{H}^-_C]|g_0\rangle - \frac{1}{2}\langle g_0|[\hat{O},[\hat{O},\hat{H}^-_C]]|g_0\rangle + \cdots, \tag{5.}$$

by choosing $|g_0\rangle$ as a ground state. We recover the SO(2) invariance afterwards.

The first order term involves

$$[\hat{O},\hat{H}^-_C] = \pi\sum_{a=2,3}\int d^2qV_-(\boldsymbol{q})\int d^2k f^a_{-\boldsymbol{k}}[\hat{S}^a_{\boldsymbol{k}},\hat{S}^3_{\boldsymbol{q}}\hat{S}^3_{-\boldsymbol{q}}], \tag{5.}$$

where

$$\begin{aligned}V_-(\boldsymbol{q})[\hat{S}^a_{\boldsymbol{k}},\hat{S}^3_{\boldsymbol{q}}\hat{S}^3_{-\boldsymbol{q}}] &= \frac{i}{2\pi}\varepsilon^{a3c}V_-(\boldsymbol{q})\exp[\frac{\ell_B^2}{2}\boldsymbol{q}\boldsymbol{k}]\cos[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\{\hat{S}^c_{\boldsymbol{k}+\boldsymbol{q}},\hat{S}^3_{-\boldsymbol{q}}\} \\ &+\frac{i}{4\pi}\delta^{a3}V_-(\boldsymbol{q})\exp[\frac{\ell_B^2}{2}\boldsymbol{q}\boldsymbol{k}]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\{\hat{\rho}_{\boldsymbol{k}+\boldsymbol{q}},\hat{S}^3_{-\boldsymbol{q}}\}.\end{aligned} \tag{5.}$$

We use the ground-state properties (5.2) and (5.23) to find that $\langle g_0|[\hat{O},\hat{H}^-_C]|g_0\rangle = 0$. T second order term is

$$\begin{aligned}\Delta E^-_C &= -\frac{1}{2}[\hat{O},[\hat{O},\hat{H}^-_C]] \\ &= -2\pi\sum_{a,b=2,3}\int d^2qV_-(\boldsymbol{q})\int d^2l\int d^2k f^b_{-\boldsymbol{l}}f^a_{-\boldsymbol{k}}[\hat{S}^b_{\boldsymbol{l}},[\hat{S}^a_{\boldsymbol{k}},\hat{S}^3_{\boldsymbol{q}}\hat{S}^3_{-\boldsymbol{q}}]],\end{aligned} \tag{5.}$$

where

$$\begin{aligned}&(2\pi)^2V_-(\boldsymbol{q})\sum_{a,b=2,3}f^b_{-\boldsymbol{l}}f^a_{-\boldsymbol{k}}[\hat{S}^b_{\boldsymbol{l}},[\hat{S}^a_{\boldsymbol{k}},\hat{S}^3_{\boldsymbol{q}}\hat{S}^3_{-\boldsymbol{q}}]] \\ &= -V_-(\boldsymbol{q})f^2_{-\boldsymbol{l}}f^2_{-\boldsymbol{k}}\exp[\frac{\ell_B^2}{2}\boldsymbol{q}(\boldsymbol{k}-\boldsymbol{l})]\cos[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\cos[\ell_B^2\frac{\boldsymbol{l}\wedge\boldsymbol{q}}{2}]\{\hat{S}^1_{\boldsymbol{l}-\boldsymbol{q}},\hat{S}^1_{\boldsymbol{k}+\boldsymbol{q}}\} \\ &+V_-(\boldsymbol{q})f^2_{-\boldsymbol{l}}f^2_{-\boldsymbol{k}}\exp[\frac{\ell_B^2}{2}(\boldsymbol{q}\boldsymbol{l}+\boldsymbol{l}\boldsymbol{k}+\boldsymbol{k}\boldsymbol{q})]\cos[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\cos[\ell_B^2\frac{\boldsymbol{l}\wedge(\boldsymbol{k}+\boldsymbol{q})}{2}]\{\hat{S}^3_{-\boldsymbol{q}},\hat{S}^3_{\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{q}}\} \\ &+V_-(\boldsymbol{q})f^3_{-\boldsymbol{l}}f^2_{-\boldsymbol{k}}\exp[\frac{\ell_B^2}{2}(\boldsymbol{q}\boldsymbol{l}+\boldsymbol{l}\boldsymbol{k}+\boldsymbol{k}\boldsymbol{q})]\cos[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\cos[\ell_B^2\frac{\boldsymbol{l}\wedge(\boldsymbol{k}+\boldsymbol{q})}{2}]\{\hat{S}^3_{-\boldsymbol{q}},\hat{S}^2_{\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{q}}\} \\ &-V_-(\boldsymbol{q})\sum_{b=2,3}f^b_{-\boldsymbol{l}}f^3_{-\boldsymbol{k}}\exp[\frac{\ell_B^2}{2}(\boldsymbol{q}\boldsymbol{l}+\boldsymbol{l}\boldsymbol{k}+\boldsymbol{k}\boldsymbol{q})]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\sin[\ell_B^2\frac{\boldsymbol{l}\wedge(\boldsymbol{k}+\boldsymbol{q})}{2}]\{\hat{S}^3_{-\boldsymbol{q}},\hat{S}^b_{\boldsymbol{l}+\boldsymbol{k}+\boldsymbol{q}} \\ &+\cdots.\end{aligned} \tag{5.}$$

The dots $\cdots$ represent the terms containing the density operator $\hat{\rho}$ and vanish due to t ground-state condition. Using the relation (5.23), we obtain

$$V_-(\boldsymbol{q})\sum_{a,b=2,3}f^b_{-\boldsymbol{l}}f^a_{-\boldsymbol{k}}\langle g_0|[\hat{S}^b_{\boldsymbol{l}},[\hat{S}^a_{\boldsymbol{k}},\hat{S}^3_{\boldsymbol{q}}\hat{S}^3_{-\boldsymbol{q}}]]|g_0\rangle$$

$$= -2\rho_0^2 V_-(\boldsymbol{q}) f_{\boldsymbol{q}}^2 f_{-\boldsymbol{q}}^2 \exp[-\ell_B^2 \boldsymbol{q}^2]\delta(\boldsymbol{l}-\boldsymbol{q})\delta(\boldsymbol{k}+\boldsymbol{q}),$$
$$+\frac{1}{(2\pi)^2} V_-(\boldsymbol{q})\rho_0 f_{\boldsymbol{k}}^2 f_{-\boldsymbol{k}}^2 \exp[-\frac{\ell_B^2}{2}(\boldsymbol{q}^2+\boldsymbol{k}^2)]\cos^2[\ell_B^2 \frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\delta(\boldsymbol{k}+\boldsymbol{l})$$
$$+\frac{1}{(2\pi)^2} V_-(\boldsymbol{q})\rho_0 f_{\boldsymbol{k}}^3 f_{-\boldsymbol{k}}^3 \exp[-\frac{\ell_B^2}{2}(\boldsymbol{q}^2+\boldsymbol{k}^2)]\sin^2[\ell_B^2 \frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\delta(\boldsymbol{k}+\boldsymbol{l}). \tag{5.46}$$

ere, we make a momentum expansion of $\cos^2[\frac{1}{2}\ell_B^2 \boldsymbol{k}\wedge\boldsymbol{q}]$ and $\sin^2[\frac{1}{2}\ell_B^2 \boldsymbol{k}\wedge\boldsymbol{q}]$ to find that

$$\Delta E_C^- = \frac{1}{2}\rho_s^- \int d^2k f_{\boldsymbol{k}}^2 f_{-\boldsymbol{k}}^2 - \frac{1}{2}\rho_s^- \int d^2k f_{\boldsymbol{k}}^3 f_{-\boldsymbol{k}}^3 + E_{\mathrm{CAP}}, \tag{5.47}$$

here ρ_s^- is defined by (5.28). The term E_{CAP} represents the capacitance energy. In a smooth nit it is given by

$$E_{\mathrm{CAP}} = \frac{e^2\rho_0^2}{2C}\int d^2k f_{\boldsymbol{k}}^2 f_{-\boldsymbol{k}}^2, \tag{5.48}$$

ith the capacitance C given by

$$\frac{1}{C} = \frac{4}{e^2}\ell_B^2 \int d^2q[V_-(0) - \nu^{-1}V_-(\boldsymbol{q})]\exp[-\frac{\ell_B^2}{2}\boldsymbol{q}^2]. \tag{5.49}$$

/e may rewrite (5.47) as

$$\Delta E_C^- = -\frac{1}{2}\rho_s^- \sum_{a=1}^{2}\int d^2x[\partial_k s^a(\boldsymbol{x})]^2 + \frac{1}{2}\rho_s^- \int d^2x[\partial_k s^3(\boldsymbol{x})]^2 + \frac{e^2\rho_0^2}{2C}\int d^2x s^3(\boldsymbol{x}) s^3(\boldsymbol{x}), \tag{5.50}$$

to the global SO(2)-invariant form.

3. *Skyrmions Excitations*

We have already argued that there is a Skyrmion excitation as a pseudospin texre $|\hat{\Phi}\rangle = e^{i\hat{\phi}}|g_0\rangle$ in the lowest Landau level. Since it modulates the electron density contributes to the Coulomb energy. We calculate the Coulomb energy of Skyrmions, $E_S = \langle g_0|e^{-i\hat{\phi}}\hat{H}_C e^{i\hat{\phi}}|g_0\rangle$. We expect the Coulomb interaction between the Skyrmion denies to emerge. Indeed, the fourth order term gives

$$\Delta E_S = \pi \int d^2q V(\boldsymbol{q})\Delta\hat{\rho}_{\boldsymbol{q}}\Delta\hat{\rho}_{-\boldsymbol{q}}, \tag{5.51}$$

where $\Delta\hat{\rho}_{\boldsymbol{q}}$ is the LLL projected Skyrmion density (5.14). Namely, it just describes the Coulomb interaction between Skyrmions. We combine (5.39), (5.50) and (5.51) into the formula (5.26).

In the SO(2) invariant limit the Skyrmion energy is given by [6]

$$\Delta E_C = \frac{1}{2}\rho_s^+ \sum_{a=1}^{3}\int d^2x[\partial_k s^a(\boldsymbol{x})]^2 = 4\pi\rho_s^+|Q|, \tag{5.52}$$

where Q is the Pontryagin number. Thus, one Skyrmion with $Q = 2$ and two Skyrmions with $Q = 1$ have the same energy. However, the Coulomb energy (5.51) is proportional to Q^2. Hence, only Skyrmions with $Q = \pm 1$ are physically relevant. They carry the electric charge $\pm\nu e = \pm e/m$ at the filling factor $\nu = 1/m$.

The Skyrmion energy (5.52) is realized by the classical Skyrmion solution (4.32). However, as we have noticed before, its capacitance energy diverges. Now, we may modify the Skyrmion configuration appropriately so that it minimizes the total energy (5.26) while keeping its Pontryagin number unchanged. An obvious variational function suggested from (4.34) is

$$\phi_1 = f(r), \quad \phi_2 = \sqrt{1-f(r)^2}\cos\theta, \quad \phi_3 = \pm\sqrt{1-f(r)^2}\sin\theta, \tag{5.53}$$

which has the Pontryagin number $Q = \pm 1$, where $f(0) = -1$ and $f(\infty) = 1$. In the SO(2) invariant limit it is given by $f(r) = (r^2 - \kappa^2/r^2 + \kappa^2)$, but in the presence of the capacitance term it approaches to the asymptotic value $f(\infty) = 1$ much faster. As a result a Skyrmion excitation becomes a localized object. In the monolayer system with spin degrees of freedom the Zeemann energy plays a similar role.

E. Tunneling Energy

We proceed to include the tunneling interaction between the two layers. The Hamiltonian is given by (2.5), which is

$$H_T = \int d^2x \mathcal{H}_T(\boldsymbol{x}) = -4\pi\lambda S_0^1, \tag{5.54}$$

where S_0^1 is the zero-momentum component ($\boldsymbol{p} = 0$) of the generator $S^1(\boldsymbol{x})$. The energy density of the coherent state $|\Phi\rangle$ is

$$\langle\Phi|\mathcal{H}_T(\boldsymbol{x})|\Phi\rangle = -2\lambda\langle\Phi|S^1(\boldsymbol{x})|\Phi\rangle = -2\lambda\rho_0\sqrt{1-\sigma^2(\boldsymbol{x})}\cos\varphi(\boldsymbol{x}), \tag{5.55}$$

where we have used (3.20). Here, the variables $\sigma(\boldsymbol{x})$ and $\varphi(\boldsymbol{x})$ need not be small quantities although they should be sufficiently smooth.

After the LLL projection the tunneling Hamiltonian reads $\hat{H}_T = -4\pi\lambda\hat{S}_0^1$. The energy of the pseudospin texture is

$$E_T = \langle\hat{\Phi}|\hat{H}_T|\hat{\Phi}\rangle = -4\pi\lambda\langle\hat{\Phi}|\hat{S}_0^1|\hat{\Phi}\rangle = -4\pi\lambda\bar{s}_0^1, \tag{5.56}$$

which has been estimated in (5.7). According to the result (5.11), we find that

$$\bar{s}_0^1 \cong s_0^1 = \int d^2x\langle\Phi|S^1(\boldsymbol{x})|\Phi\rangle, \tag{5.57}$$

apart from negligible derivative terms: They are negligible compared with the corresponding terms in the Coulomb energy (5.26). Therefore,

$$E_T = -2\lambda\rho_0\sqrt{1-\sigma^2(\boldsymbol{x})}\int d^2x\cos\varphi(\boldsymbol{x}). \tag{5.58}$$

The tunneling energy (5.56) turns out to be unaffected by the LLL projection.

F. Effective Hamiltonian for Goldstone Mode

In the (m, m, m) phase we have pointed out the existence of the Goldstone mode in association with a degeneracy of the ground state. We now derive its effective Hamiltonian. We consider the case when the density difference between the two layers is small enough compared with the total density, i.e., $s^3(\boldsymbol{x}) \ll 1$. Substituting the pseudospin configuration (3.20) into the energy change (5.26) and expanding it we rewrite as $\Delta E_C = \int d^2x\mathcal{H}$ with

$$\mathcal{H} = \frac{\rho_E}{2}(\partial_k\varphi)^2 + \frac{\rho_A}{2}(\partial_k\sigma)^2 + \frac{e^2\rho_0^2}{2C}\sigma^2. \tag{5.59}$$

Note that the angle φ is not assumed small in this derivation. Clearly, the field φ represents a Goldstone mode, which has arisen because the ground state $|g_0\rangle$ breaks t global SO(2) symmetry of the system spontaneously.

The fields $\varphi(\boldsymbol{x})$ and $\sigma(\boldsymbol{x})$ in the effective Hamiltonian are classical fields. However, now show that the canonical commutation relation should be imposed on them,

$$[\rho_0\sigma(\boldsymbol{x}), \theta(\boldsymbol{y})] = \delta(\boldsymbol{x}-\boldsymbol{y}), \tag{5.}$$

as is consistent with the commutator (3.13). To show this we evaluate the equation of moti

$$\begin{aligned} i\hbar\frac{d}{dt}\hat{S}_k^a &= [\hat{S}_k^a, \hat{H}_C] \\ &= \frac{1}{2}\int d^2qV_+(\boldsymbol{q})[\hat{S}_k^a, \hat{\rho}_q\hat{\rho}_{-q}] + 2\int d^2qV_-(q)[\hat{S}_k^a, \hat{S}_q^3\hat{\rho}_{-q}^3]. \end{aligned} \tag{5.}$$

We then take its expectation value by the state $|\hat{\Phi}\rangle = e^{i\hat{\theta}}|g_0\rangle$. We need to deal with co mutation relations such as $[\hat{S}_l^b, [\hat{S}_k^a, \hat{\rho}_q\hat{\rho}_{-q}]]$, which we have already evaluated before; s (5.36) and (5.46). For a smooth pseudospin texture, we may use the relation (3.22) togeth with the parametrization (3.20). As a result we obtain

$$\begin{aligned} \frac{d}{dt}\varphi &= \frac{\rho_A}{\rho_0}\nabla^2\sigma - \frac{e^2\rho_0}{C}\sigma, \\ \frac{d}{dt}\sigma &= -\frac{\rho_E}{\rho_0}\nabla^2\varphi, \end{aligned} \tag{5.}$$

which are the equations of motion for the Goldstone mode.

It is trivial to check that this set of equations agree precisely with the Heisenberg equ tions of motion derived from the Hamiltonian (5.59) together with the commutation relati (5.60). This proves that the angle $\varphi(x)$ and the density difference $\sigma(\boldsymbol{x})$ can be regarded the quantum fields describing the coherent mode between the two layers.

To see the eigenmodes we diagonalize the Hamiltonian (5.59). In the momentum spa the Hamiltonian density reads

$$\mathcal{H} = \frac{1}{2M_{\boldsymbol{k}}}\varphi_{\boldsymbol{k}}^\dagger\varphi_{\boldsymbol{k}} + \frac{M_{\boldsymbol{k}}}{2}E_{\boldsymbol{k}}^2\rho_0^2\sigma_{\boldsymbol{k}}^\dagger\sigma_{\boldsymbol{k}}, \tag{5.}$$

where

$$M_{\boldsymbol{k}} = \frac{1}{\rho_E \boldsymbol{k}^2}, \qquad E_{\boldsymbol{k}}^2 = \frac{\rho_E \boldsymbol{k}^2}{\rho_0}\Big(\frac{\rho_A \boldsymbol{k}^2}{\rho_0} + \frac{e^2\rho_0}{C}\Big). \tag{5.64}$$

hen we set

$$\alpha_{\boldsymbol{k}} = \frac{1}{\sqrt{2}}\Big(\sqrt{G_{\boldsymbol{k}}}\rho_0\sigma_{\boldsymbol{k}} + i\frac{1}{\sqrt{G_{\boldsymbol{k}}}}\varphi^\dagger_{-\boldsymbol{k}}\Big),$$
$$\alpha^\dagger_{\boldsymbol{k}} = \frac{1}{\sqrt{2}}\Big(\sqrt{G_{\boldsymbol{k}}}\rho_0\sigma^\dagger_{\boldsymbol{k}} - i\frac{1}{\sqrt{G_{\boldsymbol{k}}}}\varphi_{-\boldsymbol{k}}\Big), \tag{5.65}$$

ith $G_{\boldsymbol{k}} = M_{\boldsymbol{k}}E_{\boldsymbol{k}}$, the operators $\alpha_{\boldsymbol{k}}$ and $\alpha^\dagger_{\boldsymbol{l}}$ obey $[\alpha_{\boldsymbol{k}}, \alpha^\dagger_{\boldsymbol{l}}] = \delta(\boldsymbol{k} - \boldsymbol{l})$. The Hamiltonian is agonalized as

$$H = \int d^2k E_{\boldsymbol{k}} \alpha^\dagger_{\boldsymbol{k}}\alpha_{\boldsymbol{k}}, \tag{5.66}$$

ith (5.64). The excitation mode has a linear dispersion relation, $E_{\boldsymbol{k}} \cong e\sqrt{\rho_E/C}|\boldsymbol{k}|$, as $\boldsymbol{k} \to 0$: his is a superfluid mode for a finite capacitance C.

When we include the tunneling interaction, the Hamiltonian density for the Goldstone ode is modified as

$$\mathcal{H} = \frac{\rho_E}{2}(\partial_k\varphi)^2 + \frac{\rho_A}{2}(\partial_k\sigma)^2 + \frac{e^2\rho_0^2}{2C}\sigma^2 + 2\lambda\rho_0\sqrt{1-\sigma^2}\cos\varphi. \tag{5.67}$$

s diagonalization is trivial as far as small fluctuations of σ and φ around the classical ound state $\sigma = 0$ and $\varphi = \pi$ are concerned. It is clear that the energy density $E_{\boldsymbol{k}}$ in (5.64) modified as

$$M_{\boldsymbol{k}} = \frac{1}{\rho_E \boldsymbol{k}^2}, \qquad E_{\boldsymbol{k}}^2 = \Big(\frac{\rho_E \boldsymbol{k}^2}{\rho_0} + \lambda\Big)\Big(\frac{\rho_A \boldsymbol{k}^2}{\rho_0} + \frac{e^2\rho_0}{C} - \lambda\Big). \tag{5.68}$$

has a finite gap, $E_{\boldsymbol{k}} \cong \sqrt{(e^2\rho_0\lambda/C) - \lambda^2}$, as $\boldsymbol{k} \to 0$. The $\alpha_{\boldsymbol{k}}$ mode is no longer gapless, cause the tunneling interaction breaks the SO(2) symmetry explicitly. Indeed, due to is term the degeneracy of the ground states (5.4) is removed, and the Goldstone mode is rned into a gapful mode. Nevertheless, the $\alpha_{\boldsymbol{k}}$ mode gives the lowest-lying excitation the tunneling interaction is taken to be much smaller than the Coulomb interaction. In is case the tunneling interaction does not modify the hierarchy of the energy spectra gnificantly. The tunneling term breaks the SO(2) symmetry explicitly but only softly. ich a mode is called the pseudo-Goldstone mode.

VI. ELECTRIC CURRENTS

A. Definition of Currents

We analyze the electric current $J_i^\alpha \equiv -(e/M)\psi^\dagger_\alpha P_i \psi_\alpha$ in each layer in external electric field E_i^α. There are some attempts [18,8] on this problem but they are unsatisfactory since their currents are introduced in a rather ad hoc manner. We presents a systematic method. As is well known [9], the current J_i^α vanishes when it acts on the LLL state due to the LLL condition (2.12). We overcome this problem in the following way. Let us make an infinitesimal local phase transformation, $\psi_\alpha \to e^{if^\alpha(\boldsymbol{x})}\psi_\alpha$, with the gauge field fixed. Since this is not a symmetry, the Hamiltonian (2.1) is modified by $\Delta H = (1/e)\int d^2x J_i^\alpha \partial_i f^\alpha$. We can use this relation to define the current J_i^α. Since the LLL projection is modified as the momentum P_i is modified by $\partial_i f^\alpha$, it is interpreted that the current flows via a Landau-level mixing. The modification of the Hilbert space may be neglected in the limit $f^\alpha \to 0$.

It is convenient to define

$$J_i^\pm = -\frac{e}{M}(\psi_1^\dagger P_i \psi_1 \pm \psi_2^\dagger P_i \psi_2) = J_i^1 \pm J_i^2. \tag{6.1}$$

The current J_i^+ is associated with the in-phase transformation,

$$\psi_\alpha \to e^{i\gamma(\boldsymbol{x})}\psi_\alpha, \tag{6.2}$$

whose generator is

$$O_\gamma^+ = \int d^2x \gamma(\boldsymbol{x})\rho(\boldsymbol{x}) = \int d^2k \gamma_{-\boldsymbol{k}}\rho_{\boldsymbol{k}}. \tag{6.3}$$

The current J_i^- is associated with the out-of-phase transformation,

$$\psi_1 \to e^{i\gamma(\boldsymbol{x})}\psi_1, \qquad \psi_2 \to e^{-i\gamma(\boldsymbol{x})}\psi_2, \tag{6.4}$$

whose generator is

$$O_\gamma^- = 2\int d^2x \gamma(\boldsymbol{x}) S^3(\boldsymbol{x}) = 2\int d^2k \gamma_{-\boldsymbol{k}} S^3_{\boldsymbol{k}}, \tag{6.5}$$

where the existence of the factor 2 should be noticed since $f^3 = 2\gamma$ in (2.6). The local phase transformations modify the Hamiltonian by $\Delta H^{\pm}$. After the LLL projection, it is

$$\Delta\hat{H}^{\pm} = -i[\hat{O}^{\pm}_{\gamma}, \hat{H}] - \frac{1}{2}[\hat{O}^{\pm}_{\gamma}, [\hat{O}^{\pm}_{\gamma}, \hat{H}]] + \cdots. \tag{6.6}$$

The projected current $J_i^{\pm}$ is defined by the formula,

$$k_i \hat{J}_i^{\pm}(\boldsymbol{k}) = \frac{ie}{\hbar}\frac{\delta\Delta\hat{H}^{\pm}}{\delta\gamma_{-\boldsymbol{k}}}\bigg|_{\gamma=0} \tag{6.7}$$

where the limit $\gamma \to 0$ is taken after the derivative is taken. It is clear that only the first order term in the expansion (6.6) is relevant. We evaluate the matrix element $\bar{J}_i(\boldsymbol{k}) \equiv \langle\hat{\Phi}|\hat{J}_i(\boldsymbol{k})|\hat{\Phi}\rangle$ and denotes its Fourier transformation as $\bar{J}_i(\boldsymbol{x})$. We can justify the drift current discussed in Ref. [9] based on this formula.

B. Hall Current

The Hamiltonian consists of the electric-field term H_E, the Coulomb term H_C and the tunneling term H_T. We first analyze the current induced by the external electric field $E_i^{\alpha}(\boldsymbol{x})$. We choose a gauge in which $E_i^{\alpha}(\boldsymbol{x}) = \partial_i A_0^{\alpha}(\boldsymbol{x})$, with A_0^{α} the electric potential applied in the layer α. The Hamiltonian is given by

$$\begin{aligned}\hat{H}_E &= e\int d^2x \hat{A}_0^{\alpha}(\boldsymbol{x})\rho^{\alpha}(\boldsymbol{x}) \\ &= e\int d^2q (A_0^{+}(\boldsymbol{q})\hat{\rho}_{-\boldsymbol{q}} + 2A_0^{-}(\boldsymbol{q})\hat{S}^3_{-\boldsymbol{q}}),\end{aligned} \tag{6.8}$$

where $A_0^{\mp} = \frac{1}{2}(A_0^1 \pm A_0^2)$. The change of this Hamiltonian by the in-phase transformation is

$$\begin{aligned}\Delta H_E^{+} &= -i[\hat{O}^{+}_{\gamma}, \hat{H}_E] \\ &= -ie\int d^2k\gamma_{-\boldsymbol{k}}\int d^2q\left\{A_0^{+}(\boldsymbol{q})[\hat{\rho}_{\boldsymbol{k}}, \hat{\rho}_{-\boldsymbol{q}}] + 2A_0^{-}(\boldsymbol{q})[\hat{\rho}_{\boldsymbol{k}}, \hat{S}^3_{-\boldsymbol{q}}]\right\}.\end{aligned} \tag{6.9}$$

Therefore, the in-phase current is given by

$$\begin{aligned}k_i\hat{J}_i^{E+}(\boldsymbol{k}) = &-\frac{2ie^2}{\hbar}\int\frac{d^2q}{2\pi}A_0^{+}(\boldsymbol{q})\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\exp[-\frac{\ell_B^2}{2}\boldsymbol{k}\boldsymbol{q}]\hat{\rho}_{\boldsymbol{k}-\boldsymbol{q}} \\ &-\frac{4ie^2}{\hbar}\int\frac{d^2q}{2\pi}A_0^{-}(\boldsymbol{q})\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\exp[-\frac{\ell_B^2}{2}\boldsymbol{k}\boldsymbol{q}]\hat{S}^3_{\boldsymbol{k}-\boldsymbol{q}},\end{aligned} \tag{6.10}$$

where use was made of the density algebra (2.22) and (2.23). On the other hand, the o
of-phase transformation modifies the Hamiltonian as

$$\begin{aligned}\Delta H_E^{-} &= -i[\hat{O}^{-}_{\gamma}, \hat{H}_E] \\ &= -2ie\int d^2k\gamma_{-\boldsymbol{k}}\int d^2q\left\{A_0^{+}(\boldsymbol{q})[\hat{S}^3_{\boldsymbol{k}}, \hat{\rho}_{-\boldsymbol{q}}] + 2A_0^{-}(\boldsymbol{q})[\hat{S}^3_{\boldsymbol{k}}, \hat{S}^3_{-\boldsymbol{q}}]\right\}.\end{aligned} \tag{6.}$$

Therefore, the out-of-phase current is given by

$$\begin{aligned}k_i\hat{J}_i^{E-}(\boldsymbol{k}) = &-\frac{4ie^2}{\hbar}\int\frac{d^2q}{2\pi}A_0^{+}(\boldsymbol{q})\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\exp[-\frac{\ell_B^2}{2}\boldsymbol{k}\boldsymbol{q}]\hat{S}^3_{\boldsymbol{k}-\boldsymbol{q}} \\ &-\frac{2ie^2}{\hbar}\int\frac{d^2q}{2\pi}A_0^{-}(\boldsymbol{q})\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\exp[-\frac{\ell_B^2}{2}\boldsymbol{k}\boldsymbol{q}]\hat{\rho}_{\boldsymbol{k}-\boldsymbol{q}}.\end{aligned} \tag{6.}$$

It is notable that the formula (6.12) for $\hat{J}_i^{E-}$ is obtainable from the formula (6.10) for $\hat{J}_i^{E+}$
interchanging the symbols + and −.

For simplicity we assume that the electric field is uniform in each layer α, E_j^{α}
$-\partial_j A_0^{\alpha}(\boldsymbol{x})$ = constant, or

$$q_j A_0^{\pm}(\boldsymbol{q}) = 2\pi i E_j^{\mp}\delta(\boldsymbol{q}). \tag{6.}$$

Expanding the $\sin[\frac{1}{2}\ell_B^2(\boldsymbol{k}\wedge\boldsymbol{q})]$ in (6.12) and using the above relation, we obtain

$$k_i\hat{J}_i^{E\pm}(\boldsymbol{k}) = \frac{e^2\ell_B^2}{\hbar}\varepsilon_{ij}k_i(E_j^{\pm}\hat{\rho}_{\boldsymbol{k}} + 2E_j^{\mp}\hat{S}^3_{\boldsymbol{k}}), \tag{6.}$$

or

$$\hat{J}_i^{E\pm}(\boldsymbol{k}) = \frac{e^2\ell_B^2}{\hbar}\varepsilon_{ij}(E_j^{\pm}\hat{\rho}_{\boldsymbol{k}} + 2E_j^{\mp}\hat{S}^3_{\boldsymbol{k}}). \tag{6.}$$

We estimate this in a smooth pseudospin texture $|\hat{\Phi}\rangle$,

$$\bar{J}_i^{E\pm}(\boldsymbol{k}) \equiv \langle\hat{\Phi}|\hat{J}_i^{E\pm}(\boldsymbol{k})|\hat{\Phi}\rangle = \frac{e^2\ell_B^2}{\hbar}\varepsilon_{ij}(E_j^{\pm}\bar{\rho}_{\boldsymbol{k}} + 2\rho_0 E_j^{\mp}\bar{s}^3_{\boldsymbol{k}}), \tag{6.}$$

where $\bar{\rho}_{\boldsymbol{k}-\boldsymbol{q}}$ and $\bar{s}^3_{\boldsymbol{k}-\boldsymbol{q}}$ have been defined by (5.13) and (5.7), respectively, and have alrea
been estimated. First, $\bar{\rho}_{\boldsymbol{k}}$ consists of the uniform charge distribution and the Skyrmi
excitations $\Delta\bar{\rho}_{\boldsymbol{k}}$ on it, as we explained from (5.13) to (5.17). Next, $\bar{s}^3_{\boldsymbol{k}-\boldsymbol{q}}$ describes the dens

fference between the two layers as in (5.11). Taking the Fourier transformation, we obtain e Hall current,

$$\bar{j}_i^{E\pm}(\boldsymbol{x}) = \frac{e^2\nu}{2\pi\hbar}\varepsilon_{ij}\{E_j^{\mp} + E_j^{\mp}s^3(\boldsymbol{x}) + E_j^{\pm}\ell_B^2 Q_0(\boldsymbol{x})\}. \tag{6.17}$$

ne first term is the well-known form of the Hall current for the uniform distribution electrons. In the (m, m, m) phase a coherent density fluctuation is allowed, and the cond term describes this effect. The Hall current is proportional to the electron density each layer, which is $\rho^1(\boldsymbol{x}) = \rho_0(1 + s^3)$ and $\rho^2(\boldsymbol{x}) = \rho_0(1 - s^3)$. The third term comes from e Skyrmion excitations, where $Q_0(\boldsymbol{x})$ is the Skyrmion density or the Pontryagin number ensity. In particular, when an identical electric field E_j is applied to both of the layers, we tain

$$\bar{j}_i^{E+}(\boldsymbol{x}) = \frac{e^2\nu}{2\pi\hbar}\varepsilon_{ij}E_j\{1 + \ell_B^2 Q_0(\boldsymbol{x})\}. \tag{6.18}$$

y means of measuring the Hall current distribution the Skyrmion excitation is observable.

C. Supercurrent

We next analyze the current due to the Coulomb interaction. The in-phase transformon changes the Hamiltonian (5.25) as

$$\begin{aligned} H_C^+ &= -i[\hat{O}_\gamma^+, \hat{H}_C] \\ &= -\pi i\int d^2q\int d^2k V_+(\boldsymbol{q})\gamma_{-\boldsymbol{k}}[\hat{\rho}_{\boldsymbol{k}}, \hat{\rho}_{-\boldsymbol{q}}\hat{\rho}_{\boldsymbol{q}}] - 4\pi i\int d^2q\int d^2k V_-(\boldsymbol{q})\gamma_{-\boldsymbol{k}}[\hat{\rho}_{\boldsymbol{k}}, \hat{S}^3_{-\boldsymbol{q}}\hat{S}^3_{\boldsymbol{q}}], \end{aligned} \tag{6.19}$$

hile the out-of-phase transformation changes it as

$$\begin{aligned} H_C^- &= -i[\hat{O}_\gamma^-, \hat{H}_C] \\ &= -2\pi i\int d^2q\int d^2k V_+(\boldsymbol{q})\gamma_{-\boldsymbol{k}}[\hat{S}^3_{\boldsymbol{k}}, \hat{\rho}_{-\boldsymbol{q}}\hat{\rho}_{\boldsymbol{q}}] - 8\pi i\int d^2q\int d^2k V_-(\boldsymbol{q})\gamma_{-\boldsymbol{k}}[\hat{S}^3_{\boldsymbol{k}}, \hat{S}^3_{-\boldsymbol{q}}\hat{S}^3_{\boldsymbol{q}}]. \end{aligned} \tag{6.20}$$

ne relevant commutators are

$$\begin{aligned} V_+(\boldsymbol{q})[\hat{\rho}_{\boldsymbol{k}}, \hat{\rho}_{-\boldsymbol{q}}\hat{\rho}_{\boldsymbol{q}}] &= \frac{i}{\pi}V_+(\boldsymbol{q})\exp[\frac{\ell_B^2}{2}\boldsymbol{q}\boldsymbol{k}]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\{\hat{\rho}_{-\boldsymbol{q}}, \hat{\rho}_{\boldsymbol{k}+\boldsymbol{q}}\}, \\ V_-(\boldsymbol{q})[\hat{\rho}_{\boldsymbol{k}}, \hat{S}^3_{-\boldsymbol{q}}\hat{S}^3_{\boldsymbol{q}}] &= \frac{i}{\pi}V_-(\boldsymbol{q})\exp[\frac{\ell_B^2}{2}\boldsymbol{q}\boldsymbol{k}]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\{\hat{S}^3_{-\boldsymbol{q}}, \hat{S}^3_{\boldsymbol{k}+\boldsymbol{q}}\}, \\ V_+(\boldsymbol{q})[\hat{S}^3_{\boldsymbol{k}}, \hat{\rho}_{-\boldsymbol{q}}\hat{\rho}_{\boldsymbol{q}}] &= \frac{i}{\pi}V_+(\boldsymbol{q})\exp[\frac{\ell_B^2}{2}\boldsymbol{q}\boldsymbol{k}]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\{\hat{\rho}_{-\boldsymbol{q}}, \hat{S}^3_{\boldsymbol{k}+\boldsymbol{q}}\}, \\ V_-(\boldsymbol{q})[\hat{S}^3_{\boldsymbol{k}}, \hat{S}^3_{-\boldsymbol{q}}\hat{S}^3_{\boldsymbol{q}}] &= \frac{i}{4\pi}V_-(\boldsymbol{q})\exp[\frac{\ell_B^2}{2}\boldsymbol{q}\boldsymbol{k}]\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\{\hat{S}^3_{-\boldsymbol{q}}, \hat{\rho}_{\boldsymbol{k}+\boldsymbol{q}}\}. \end{aligned} \tag{6.21}$$

Hence, the in-phase current is given by

$$\begin{aligned} k_i\hat{j}_i^{C+}(\boldsymbol{k}) &= \frac{ie}{\hbar}\int d^2qV_+(\boldsymbol{q})\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\exp[\frac{\ell_B^2}{2}\boldsymbol{k}\boldsymbol{q}]\{\hat{\rho}_{-\boldsymbol{q}}, \hat{\rho}_{\boldsymbol{k}+\boldsymbol{q}}\} \\ &\quad + \frac{4ie}{\hbar}\int d^2qV_-(\boldsymbol{q})\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\exp[\frac{\ell_B^2}{2}\boldsymbol{k}\boldsymbol{q}]\{\hat{S}^3_{-\boldsymbol{q}}, \hat{S}^3_{\boldsymbol{k}+\boldsymbol{q}}\}, \end{aligned} \tag{6.22}$$

while the out-of-phase current is given by

$$\begin{aligned} k_i\hat{j}_i^{C-}(\boldsymbol{k}) &= \frac{2ie}{\hbar}\int d^2qV_+(\boldsymbol{q})\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\exp[\frac{\ell_B^2}{2}\boldsymbol{k}\boldsymbol{q}]\{\hat{\rho}_{-\boldsymbol{q}}, \hat{S}^3_{\boldsymbol{k}+\boldsymbol{q}}\} \\ &\quad + \frac{2ie}{\hbar}\int d^2qV_-(\boldsymbol{q})\sin[\ell_B^2\frac{\boldsymbol{k}\wedge\boldsymbol{q}}{2}]\exp[\frac{\ell_B^2}{2}\boldsymbol{k}\boldsymbol{q}]\{\hat{S}^3_{-\boldsymbol{q}}, \hat{\rho}_{\boldsymbol{k}+\boldsymbol{q}}\}. \end{aligned} \tag{6.23}$$

We evaluate these currents in the smooth pseudospin texture $|\Phi\rangle$,

$$\begin{aligned} \langle\hat{\Phi}|\hat{j}_i^{C\pm}(\boldsymbol{k})|\hat{\Phi}\rangle &= \langle g_0|e^{-i\hat{O}}\hat{j}_i^{C\pm}(\boldsymbol{k})e^{i\hat{O}}|g_0\rangle \\ &= \langle g_0|\hat{j}_i^{C\pm}(\boldsymbol{k})|g_0\rangle - i\langle g_0|[\hat{O}, \hat{j}_i^{C\pm}(\boldsymbol{k})]|g_0\rangle - \frac{1}{2}\langle g_0|[\hat{O}, [\hat{O}, \hat{j}_i^{C\pm}(\boldsymbol{k})]]|g_0\rangle + \cdots. \end{aligned} \tag{6.24}$$

The term $\langle g_0|\hat{j}_i^{C\pm}(\boldsymbol{k})|g_0\rangle$ vanishes due to the ground-state properties (5.2) and (5.23) and due to the fact that $V_-(\boldsymbol{k})$ is an even function of $\boldsymbol{k}$. The first order term $\langle g_0|[\hat{O}, \hat{j}_i^{C+}(\boldsymbol{k})]|g_0\rangle$ of the in-phase current vanishes by the same reason.

The nontrivial term is the first order term $\langle g_0|[\hat{O}, \hat{j}_i^{C-}(\boldsymbol{k})]|g_0\rangle$ of the out-of-phase current. We examine the term,

$$\begin{aligned} \bar{j}_i^{T-}(\boldsymbol{k}) &\equiv -i\langle g_0|[\hat{O}, \hat{j}_i^{C-}(\boldsymbol{k})]|g_0\rangle = -i\int d^2l f^a_{-l}\langle g_0|[\hat{S}^a_{\boldsymbol{l}}, \hat{j}_i^{C-}(\boldsymbol{k})]|g_0\rangle \\ &\equiv \bar{j}_i^{(+)}(\boldsymbol{k}) + \bar{j}_i^{(-)}(\boldsymbol{k}), \end{aligned} \tag{6.25}$$

where we have denoted the contribution from the potential $V_\pm$ part as $\bar{j}_i^{(\pm)}$. We first evaluate the part $\bar{j}_i^{(+)}$. The relevant commutator is

$$[\hat{S}^a_l, \{\hat{\rho}_{-q}, \hat{S}^3_{k+q}\}] = -\frac{i}{\pi}\sin[\ell_B^2\frac{l\wedge q}{2}]\exp[-\frac{\ell_B^2}{2}lq]\{\hat{S}^a_{l-q}, \hat{S}^3_{k+q}\} + \cdots, \tag{6.26}$$

The dots denote the term containing the operator ρ_{-q}. Since it yields $\delta(q)$ due to the ground-state condition (5.2) in the matrix element (6.25), the term does not contribute to the out-of-phase current (6.23). Using the ground-state property (5.23), or

$$\langle g_0|\{\hat{S}^a_{l-q}, \hat{S}^3_{k+q}\}|g_0\rangle = \delta^{a3}\rho_0\exp[-\frac{\ell_B^2}{2}(k+q)^2]\delta(l+k), \tag{6.27}$$

we obtain

$$k_i j_i^{(+)}(k) = \frac{4ie\rho_0}{\hbar}\int\frac{d^2q}{2\pi}\sin^2[\ell_B^2\frac{k\wedge q}{2}]V_+(q)f^3_k\exp[-\ell_B^2\frac{(k+q)^2}{2}]. \tag{6.28}$$

In a smooth pseudospin texture it is sufficient to take the lowest order term in k. We use the formula (5.38) to derive

$$j_i^{(+)}(k) = \frac{4ie\rho_0}{\hbar}\rho_s^+ k_i f^3_k, \tag{6.29}$$

where ρ_s^+ has been defined by (5.28). We can follow the same steps to evaluate the term $j_i^{(-)}$ which is the contribution from the potential V_- term. The main difference is the commutation relation

$$[\hat{S}^a_l, \{\hat{S}^3_{-q}, \hat{\rho}_{k+q}\}] = \frac{i}{\pi}\sin[\ell_B^2\frac{l\wedge(k+q)}{2}]\exp[\frac{\ell_B^2}{2}l(k+q)]\{\hat{S}^3_{-q}, \hat{S}^a_{l+k+q}\} + \cdots, \tag{6.30}$$

which replaces (6.26). Because of the difference of the overall sign between (6.26) and (6.30) we derive

$$j_i^{(-)}(k) = -\frac{4ie\rho_0}{\hbar}\rho_s^- k_i f^3_k, \tag{6.31}$$

where ρ_s^- has been defined by (5.28). Combining (6.29) and (6.31), and taking the Fourier transformation we find the out-of-phase current to be

$$j_i^{C-}(x) = \frac{2e}{\hbar}\rho_E\partial_i\varphi(x), \tag{6.32}$$

where ρ_E has been defined by (5.27). We have found that the phase difference $\varphi(x)$ induces a current to reduce itself. This current is what we expect naturally from the Coulomb energy (5.26). We have shown in the previous section that $\varphi(x)$ is a superfluid mode; hence, it is a supercurrent.

D. Tunneling Current

We finally analyze the current due to the tunneling interaction. After the LLL projecti the tunneling Hamiltonian is given,

$$\hat{H}_T = -4\pi\lambda\hat{S}^1_0, \tag{6.3}$$

where $\hat{S}^1_0$ is the zero-momentum component of the projected generator.

The in-phase transformation modifies the tunneling Hamiltonian as

$$\Delta H_T^+ = -i[\hat{O}_y^+, \hat{H}_T] = 4\pi\lambda i\int d^2k\gamma_{-k}[\hat{\rho}_k, \hat{S}^1_0], \tag{6.3}$$

which vanishes due to the density algebra (2.23). Hence, there exists no in-phase curre J_i^{T+}. The out-of-phase transformation modifies it as

$$\Delta H_T^- = -i[\hat{O}_y^-, \hat{H}_T] = 8\pi\lambda i\int d^2k\gamma_{-k}[\hat{S}^3_k, \hat{S}^1_0] = -4\lambda\int d^2k\gamma_{-k}\hat{S}^2_k, \tag{6.3}$$

where use was made of the density algebra (2.23). According to the formula (6.7), t out-of-phase current reads,

$$\hat{j}_i^{T-}(k) = -\frac{4ie\lambda}{\hbar k_i}\hat{S}^2_k. \tag{6.3}$$

We estimate $\hat{j}_i^{T-}(k)$ in the coherent state $|\hat{\Phi}\rangle$,

$$k_i\bar{j}_i^{T-}(k) \equiv k_i\langle\hat{\Phi}|\hat{j}_i^{T-}(k)|\hat{\Phi}\rangle = -\frac{4ie\lambda}{\hbar}\langle\hat{\Phi}|\hat{S}^2_k|\hat{\Phi}\rangle = -\frac{4ie\lambda}{\hbar}\bar{s}^2_k, \tag{6.3}$$

or

$$\partial_i\bar{j}_i^{T-}(x) = \frac{4e\lambda\rho_0}{\hbar}\bar{s}^2(x). \tag{6.3}$$

Since the in-phase current does not exist this yields

$$\partial_i\bar{j}_i^{T1}(x) = -\partial_i\bar{j}_i^{T2}(x) = \frac{2e\lambda\rho_0}{\hbar}\bar{s}^2(x). \tag{6.3}$$

At each point x this amount of charge is exchanged between the two layers. Hence, may identify

$$J^z(x) \equiv \partial_i\bar{j}_i^{T1}(x) = -\frac{2e\lambda\rho_0}{\hbar}\sqrt{1-\sigma(x)^2}\sin\varphi(x) \tag{6.4}$$

as the tunneling current, where we have replaced $\bar{s}^2(x)$ with $s^2(x)$ in a smooth pseudosp texture.

VII. CONCLUSION

Based on the bosonic CS gauge theory with the LLL projection, we have analyzed the terlayer coherence which is spontaneously developed in the (m, m, m) phase of the bilayer stem. The $W_\infty \times SU(2)$ algebra is the fundamental symmetry of the system. We have made perturbative analysis and an algebraic analysis, both of which have led to a trivial vacuum ctor in the in-phase density mode and revealed a rich gapless mode in the out-of-phase ensity fluctuation. In particular we have overcome the problem of the isolated Goldstone ode we encountered in our previous perturbative analysis [4].

It is one of our findings that the ground state is determined by the conditions (5.2) and .6). It is an eigenstate of the in- phase density operator and a coherent state of the out-of-hase density operator. When the representations of the $W_\infty \times SU(2)$ algebra are known, the round state conditions select one to describe the bilayer QH system with all its excitations. owever, since they are yet to be explored, we have employed a variational method. As a ariational state we have taken the state $|\hat{\Phi}\rangle = e^{i\hat{\mathcal{O}}}|g_0\rangle$, where $\hat{\mathcal{O}}$ is the generator of the local J(2) transformation with the LLL projection made.

Evaluating the Coulomb energy of the state $|\hat{\Phi}\rangle$, we have derived the effective Hamilnian governing the interlayer coherence. The result is given by (5.26), which allows both kyrmions and anti-Skyrmions as coherent excitations in the peudospin texture. It yields e effective Hamiltonian (5.67) for the Goldstone mode. We have already argued elsehere [5] that such a Hamiltonian leads to quantum coherent phenomena including the sephson-like effect.

In this paper we have analyzed the quantum coherence in the bilayer system in details. lmost all our results can be taken over to the monolayer QH system with spin degrees of eedom by replacing the capacitance energy with the Zeemann energy. Then, our results esent a field theoretical proof that it is a quantum ferromagnet with Skyrmion excitations a spin texture. In particular, Skyrmion excitations would be detectable by measuring the all current distribution. We believe that our field-theoretical method is a powerful tool to

analyze various aspects of the QH effect.

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