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Analysis of super-allowed Fermi beta-decay

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Abstract

Analysis of the $J^\pi = 0^+ \to 0^+$ super-allowed Fermi transitions within isospin triplets is limited in the precision of its outcome not by the accuracy of the experimental input data nor by the confidence with which the radiative corrections can be applied but by knowledge of the nuclear mismatch: the subversion of the isospin symmetry along the multiplets by the charge-dependence of the forces, both Coulomb and specifically nuclear. Theoretical estimates of the mismatch differ considerably from author to author; their direct application results in clear violation of the hypothesis of conservation of the vector current and clear inconsistency with unitarity of the Cabibbo-Kobayashi-Maskawa matrix. This paper pursues and elaborates the previous suggestion that, in these unsatisfactory circumstances, the best procedure is to look to the experimental data themselves to determine and eliminate the mismatch by appropriate extrapolation to Z=0. This is done: (i) without any prior correction for mismatch; (ii) after correction for the full theoretical mismatch; (iii) after correction for case-to-case fluctuations in the theoretical mismatch. These three procedures are individually statistically satisfactory and mutually consistent in their extrapolation to $Z = 0$ despite the variety of the theoretical mismatches on which, in varying degrees, they are based. The resultant unitarity test for the CKM matrix is $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0003 \pm 0.0014$. The associated value for the operational vector coupling constant is: $G_V^*/(\hbar c)^3 = (1.15155 \pm 0.00064) \times 10^{-5}$ GeV^{-2}. If unitarity of the CKM matrix is alternatively assumed one may conclude, from a similar analysis, that the mean charge of the fermionic fields between which beta-decay takes place is $\overline{Q} = 0.172 \pm 0.060$ and that, at the 90% confidence level, $b_F < 2.6 \times 10^{-3}$ were b_F is the relative effective scalar coupling constant. Neutron decay is also discussed, with the provisional recommendations: $G_A^*/(\hbar c)^3 = (1.4557 \pm 0.0051) \times 10^{-5}$GeV^{-2}; $|G_A^*/G_V^*| = 1.2641 \pm 0.0017$.

(submitted to Nuclear Physics A)

1. Introduction

Beta-decay within $J^\pi = 0^+$ isotriplets, of which there are presently eight accurately-measured examples (see Table 1), is pure vector in nature (with the exception of a small, but very important, axial contribution associated with the radiative correction) provided that the effective scalar coupling constant is itself zero – as it is within the standard model. Such beta-decay therefore seems to offer an almost-direct determination of the vector coupling constant, subject to assumption of the conservation of the vector current (CVC) and, indeed, seems to offer a test of that assumption through the constancy, or otherwise, of the apparent vector coupling constant inferred from the eight decaying bodies, separately, under that assumption.

It may then be thought that, following CVC, the vector coupling constant G_V^*, inferred from such studies via the equation (see Appendix 1):

$$ft = \frac{K}{|M_V|^2 G_V^{*2}}, \tag{1}$$

becomes a "fundamental constant". There are, however complications relating to that view that imply that Eq. (1) is not a suitable starting point for a fundamental discussion: (i) the so-extracted constant G_V^* would be a purely *operational* constant that would include the radiative "inner" γ, Z-related corrections associated with the detailed W-anatomy of the weak interaction process; it furthermore would become a constant only after removal of a nuclear-structure-dependent (axial) element of the radiative correction to be discussed in detail below, and this element would then have to be replaced in the context of whatever beta-decay it was to which we subsequently wished to apply G_V^* in order to assess its rate; in this sense the *operational* vector coupling constant G_V^* would be nuclear-structure-sensitive and so not really a constant at all; (ii) there is no mechanism for removing from G_V^* the remaining "inner" radiative corrections, to which reference has just been made, in order to reveal some "truly-fundamental" G_V; on the other hand, no interest would reside in such a G_V, as opposed to G_V^*, except for purposes of comparison with a similar "truly-fundamental" muon coupling constant G_μ, similarly derived from the operational G_μ^*, in order to gain the element $|V_{ud}|$ of the Cabibbo-Kobayashi-Maskawa (CKM) matrix (see Appendix 2):

$$|V_{ud}| = G_V/G_\mu. \tag{2}$$

What is available (Ref.[6] and see further discussion below) is a combined radiative correction that directly relates G_V/G_μ to G_V^*/G_μ^* so there is no point in writing equations involving G_V and G_μ separately but we rather write:

$$(ft)^* = \frac{K}{2|V_{ud}|^2 G_\mu^{*2}} \tag{3}$$

where:

$$(ft)^* = ft(1 - \delta_c)(1 + \Delta^R) \tag{4}$$

and where now t is corrected only for branching, for orbital electron capture and for atomic excitation. In Eq. (4) δ_C corrects for the fact that isospin symmetry along the

isomultiplet is not exact but is subverted by the Coulomb force and by the charge-dependence of the specifically nuclear interaction itself; Δ^R is the overall radiative correction that : (i) supplements the effect of the Coulomb force as already allowed for in the reduction of the experimental Q-value for the beta-decay to the phase-space factor f through appropriate elaboration of the Fermi function $F(Z,W)$ and that: (ii) results from the detailed W,Z-anatomy of the beta-decay inner mechanism itself, there representing, as mentioned above, the *difference between* such effects for nucleon decay and for muon decay; G_μ^* is the operational Fermi coupling constant as determined from muon-decay[7,8]:

$$G_\mu^*/(\hbar c)^3 = (1.16639 \pm 0.00002) \times 10^{-5}\ \mathrm{GeV}^{-2} \tag{5}$$

where it has been corrected for both real and virtual radiative processes including all leading logarithmic corrections of the form $\alpha^{n+1} ln^n(m_\mu/m_e)$, $n \geq 0$, by use of the fine structure constant evaluated at the muon mass in the $\overline{MS}$ scheme; but it has *not* been corrected for those "inner" radiative processes involving the W,Z-anatomy of the weak interaction, these having been transferred, as remarked above, into the Δ^R of Eq. (4) (the detailed anatomy of the weak interaction is involved in the extraction of Eq. (5) from the muon lifetime only through the tiny factor $1 + \frac{3}{5}\frac{m_\mu^2}{m_W^2} \simeq 1.0000010$ induced by the W-propagator on passage from local V-A theory to W-mediated decay.)

2. The radiative correction Δ^R

As has been remarked, the overall radiative correction Δ^R of Eq. (4) comprises some elements that are sensitive to the weak interaction mechanism and some that are not. The most important, Z-independent, piece of Δ^R in fact contains both types of element [6]:

$$1 + \Delta = \left\{1 + \frac{\alpha}{2\pi}\left[ln\frac{m_p}{m_A} + 2C\right] + \frac{\alpha(m_p)}{2\pi}\left[\overline{g(W,W_0)}\right]\right\} S(m_p, m_Z)\,. \tag{6}$$

In Eq. (6) the last term in the curly brackets, involving $\overline{g(W,W_0)}$, is the well-known "outer" radiative correction of order α of Sirlin [9]. (The derivation of $g(W,W_0)$ ignores certain aspects of nucleon structure although it is largely insensitive to strong interaction effects. It also neglects terms of order $\alpha q/m_N$ and $\alpha(W/m_N)ln(m_N/W)$ which, taken across the beta-spectra, range up to about 0.003% and 0.02% respectively so that they are on the edge of meriting further consideration. Terms of order $\alpha v/c$, where v is a nucleon velocity within the nucleon, and that might be anticipated on dimensional grounds, could take the serious magnitude of 0.2 – 0.3%; fortunately, they do not occur – I am very grateful to Professor Sirlin for his elucidation of this point.)

As written in Eq. (6) this "outer" correction has been extended to higher (leading-log) orders in α through the use of $\alpha(m_p)$, the fine structure constant evaluated at the proton mass in the $\overline{MS}$ scheme, and the short-range factor $S(m_p, m_Z)$ that involves all the $\alpha(m_x)$ where $x = p, c, \tau, b, W$ and Z (see Ref. [1] for an explicit recipe for $S(m_p, m_Z)$. $S(m_p, m_Z)$ itself contains the vector component of the "inner" weak-interaction-mechanism-sensitive radiative correction, exact to order α by

CVC and containing higher orders in leading-log approximation, and also a portion of the quark (short-range) contribution to the axial component. The remainder of the short-range axial contribution is cut off at some "axial mass" m_A in the middle term in the curly brackets of Eq. (6) to avoid divergence of the responsible γW box diagram while the long-range nucleonic part of the axial contribution is represented by C.

Now although Eq. (6) is concerned primarily with the electroweak interaction there is a small influence of QCD; this QCD effect has been incorporated in the numerical evaluation of $S(m_p, m_Z)$ by F. Jergerlehner in his estimate, as earlier reported [10], using recent data bearing on the vacuum polarization function $\mathcal{R}e\pi^\gamma$ derived by studying $e^+e^- \to$ hadrons, and solving the renormalization group equations from the initial condition of the well-determined $\alpha(m_Z)$. Using $\alpha_s = 0.117$ at m_Z for the estimate of the QCD correction Jegerlehner finds:

$$S(m_p, m_Z) = 1.022284 \pm 0.000020\,. \tag{7}$$

Jegerlehner also gives:

$$\alpha^{-1}(m_p) = 133.85 \pm 0.11\,. \tag{8}$$

These values we use in what follows; they are very close to earlier estimates [1].

An alternative bookkeeping of the QCD effects places them not in $S(m_p, m_Z)$ but rather within the curly brackets of Eq. (6) where they are represented by $\frac{\alpha}{2\pi}\mathcal{A}$. With this bookkeeping (there setting $\alpha_s = 0$) Jegerleher finds $S(m_p, m_Z) = 1.022507 \pm 0.000020$ together with which we should then need [10] $\mathcal{A} = -0.19$; this compares closely with the expected $\mathcal{A} \simeq -\frac{\alpha_s}{\pi} \ln\frac{m_Z}{m_A} = -0.17$ in which we use the value for m_A adopted below and, following Jegerlehner, $\alpha_s = 0.117$ as at m_Z.

The uncertainties in the $S(m_p, m_Z)$ of Eqs. (7) and (8) and $\alpha^{-1}(m_p)$ together entrain a systematic uncertainty of only a little larger than $\pm 0.002\%$ in Δ. However, we must recognize that these uncertainties arise in the pursuit of particular procedures and that the definition of those procedures is not itself without ambiguity and is therefore a source of additional uncertainty that it is difficult to quantify. It is, in particular, useful to observe that the difference between the "old" combination of "outer" radiative correction of order α only [9] and "inner" correction of order α only (the first entry of Ref. [6]) and the "new" Eq. (6), that additionally incorporates leading-log terms in higher powers of α, ranges from 0.15% for ^{14}O to 0.13% for ^{54}Co, the "new" correction being the larger. (We may note that $\alpha^2(m_Z/m_p)^2 \simeq 0.11\%$.) It would seem prudent to associate an uncertainty of perhaps $\pm 20\%$ of this additional "leading log" 0.14% with this definition of procedure and therefore to attribute an addition uncertainty of $\pm 0.03\%$ to Δ.

Turning to the axial elements of Eq. (6), visible there only through the terms $ln\frac{m_p}{m_A}$ and C, we must admit at once that the take-over between these short-range and long-range axial terms, as parameterized by the axial mass m_A, is not well defined. Various values for axial masses are to be found in the literature: the lightest physical axial particle is the $T^G(J^{\pi C}) = 1^-(1^{++})$ a_1-meson of mass 1260 $\pm$30 MeV [7] but of width about 400 MeV (chiefly against $\rho\pi$): no other meson of appropriate quantum numbers appears likely to exist below at least 2300 MeV [7]; other axial mass estimates derive from the dipole parameterization of the axial form factor for

νN interactions at low energy: 1032 ± 36 MeV [11] and 1050 ± 30 MeV [12]; we also note that the Weinberg relation [13] $m_A \simeq \sqrt{2} m_V$, where m_A and m_V are, respectively, axial and vector masses, defined under a weak form of dominance, gives $m_A \simeq 1086$ MeV (setting $m_V = m_\rho$); an extreme range of 400 to 1600 MeV has been suggested [6] for the m_A of Eq. (6).

The last of these estimates, interpreting the suggested extreme range as a 90-95% confidence level, and taking the logarithmic mean of the values as is suggested by the form of the term in which m_A appears in Eq. (6), leads to $ln \frac{m_p}{m_A} = 0.16 \pm 0.40$ where the uncertainty is now at the 1σ - level; combining this with the other values just listed, namely $ln \frac{m_p}{m_A} = -0.29, -0.11$ and -0.14, from the a_1-mass, νN scattering and the Weinberg relation respectively, leads to our adopted value:

$$ln \frac{m_p}{m_A} = -0.10 \pm 0.40 . \tag{9}$$

We note that the uncertainty adopted in Eq. (9) implies an associated uncertainty of ±0.049% in Δ; in view of the experimental precision of today's ft-values, whose uncertainties average only ±0.063%, as seen from Table 1, an improvement in our understanding of the relationship between the two axial components of Δ is highly desirable.

The long-range part C of the axial contribution splits into two parts:

$$C = C(1) + C_{\rm NS} \tag{10}$$

$C(1)$ is the domain of a single nucleon: for example the beta-decaying proton undergoes axial decay, flipping its spin which is flipped back again by virtual photon exchange with the departing positron rendering the overall decay $J^\pi = 0^+ \rightarrow 0^+$; but for a consideration about to be introduced $C(1)$ would be called a Born term; it was, indeed, earlier written $C_{\rm BORN}$; it will obviously depend upon $g_A(\mu_p + \mu_n)$. $C_{\rm NS}$, NS standing for "nuclear structure", recognizes that we are dealing with complex nuclei so that the nucleon whose spin is flipped by the virtual photon exchange with the positron in order to make the overall process $J^\pi = 0^+ \rightarrow 0^+$ need not be the one that emitted the positron by axial decay. The likelihood of this 2-nucleon term obviously depends upon the mutual disposition of the two nucleonic orbitals involved and so is nuclear-structure-sensitive hence the suffix NS. The first quantification of this important term was by Jaus and Rasche [14] using rather simple nuclear wave functions; however, the effect is of long range and so is not very sensitive to the finer details of the wave functions involved; subsequent computations using modern wave functions [15,16] give $C_{\rm NS}$- values not very different from the originals [14]. As might be expected, $C(1)$ and $C_{\rm NS}$ are numerically of comparable importance.

At this point we encounter a further nuclear structure subtlety of significance for the assessment of the long-range axial term. It is very well established (see, e.g., Ref. [17]) that when one uses the best modern many-body wave functions to describe nuclei such as those of our present concern, excellent agreement between theory and experiment for such matters as axial decay probabilities and magnetic moments is found but only if one, systematically and uniformly, replaces the free-nucleon values of g_A, μ_p and μ_n by effective or quenched values that may differ from the free-nucleon values by up to some tens of percent. It is not yet clear to what

degree this phenomenon is due to some authentic change in the intrinsic properties of nucleons on immersion in the nuclear medium or to what degree it is due to the use of an inadequate configurational basis for generating the many-body wave functions but the phenomenon itself is definite in the content of present many-body wave functions such as those used to generate $C_{\rm NS}$ [15,16] and it will also presumably manifest itself in the (ostensibly) 1-nucleon term $C(1)$ of Eq. (10), this influence of the medium explaining why $C(1)$ is no longer written as $C_{\rm BORN}$. This important consideration has been raised by I.S. Towner [18] who recommends the values for $C(1)$ and $C_{\rm NS}$ listed in Table 1. Also listed in Table 1 are the values for $C_{\rm NS}$ following Ref. [16] multiplied by 0.83, that figure being the mean impact of the quenching according to Towner [18], that we might expect, *pari passu*, there to apply also.

References [15] and [16] , as well as Ref. [18], give (somewhat impressionistic) estimates of the uncertainties that might attach to the $C_{\rm NS}$; for the most part these estimates bring the two sets of values into (adequate) accord. These uncertainties average about ±0.08 which corresponds to an impact of about ±0.6 sec on ft; as seen from Table 1, this is not negligible in relation to the experimental uncertainties but we do not here attempt to take these theoretical uncertainties explicitly into account nor list them in Table 1 but rather, as will be seen, allow their impact to be realized in the overall dispersion of the $(ft)^*$-values, following Eq. (5), to which they contribute. We may note, however, that a reasonable figure for the systematic uncertainty in $|V_{ud}|^2$ associated with uncertainty in the long-range axial terms might be taken from the above-mentioned ±0.06 sec in ft, namely about 0.02%; we take this into account in due course. (A further small uncertainty, worth about ±0.2 sec in ft, is associated with treatment of the nucleonic form factors [15]).

Table 1 shows Δ_T, being the evaluation of Eq. (6) following the $C(1)$ and $C_{\rm NS}$ of Ref. [18], and Δ_B which follows the $C(1)$ of Ref. [18] and the $C_{\rm NS}$ of Ref. [16] moderated by the above-mentioned factor of 0.83.

In addition to the Z-independent radiative corrections of all (leading-log) orders in α, now discussed and listed in Δ_T, Δ_B in Table 1, we have those involving Z. Naive vertex counting might suggest that huge terms of order $Z^2\alpha$ could arise but such fortunately vanish identically [19] and those of order $Z\alpha$ are also cancelled [19]; however there is no present demonstration that terms of order $(Z\alpha)^n$, $n > 1$, do not occur; now $(Z\alpha)^2 \simeq 0.04$ for ^{54}Co so such a term would be of major importance. As will be seen, the method of analysis advocated here could automatically compensate for such a term in Z^2 provided that it would not be strongly W_0-sensitive.

The most important Z-dependent terms that have been explicitly considered are those of order $Z\alpha^2(\delta_2)$ and $Z^2\alpha^3(\delta_3)$ but the possibility of others must also be recognized making up a total of δ_r such that:

$$\Delta^R = \Delta + \delta_r . \tag{11}$$

These Z-dependent radiative corrections δ_r have been discussed and evaluated at length [1] and will not be elaborated upon here; they are listed in Table 1; it is estimated [1] that they carry uncertainties ranging from about 0.006 to 0.034% across the range of Z in question.

In addition to δ_2 and δ_3 the δ_r of Table 1 include very rough but explicit assessments of all remaining terms $Z^n\alpha^m$ [1]. An alternative approach to these remaining terms is to write $\delta_r \simeq (\delta_2 + \delta_3)S(m_p, m_Z)$; the difference between these two versions of

δ_r ranges from 0.0022 to 0.0053%, significantly smaller than the other uncertainties in δ_r just noted.

In summary we may say that the radiative correction carries significant systematic uncertainties of: (i) ±0.03% associated with the procedure lying behind Eq. (6); (ii) ±0.05% associated with m_A; (iii) ±0.02% associated with C; (iv) ±0.01 to ±0.03% associated with δ_r making a total of ±0.062 to ±0.069% which will be applied as appropriate in what follows.

3. The nuclear mismatch δ_C

It has been clear since the earliest times of our concern with these $J^\pi = 0^+ \to 0^+$ super-allowed Fermi transitions that the isospin symmetry along the isotriplets must be subverted to the tune of at least some tenths of a percent by the charge-dependent forces, Coulomb and specifically nuclear [20]. Early quantitative estimates were based on the Coulomb perturbation of harmonic oscillator wave functions [21-24]; they gave values of δ_C (see Eq. (4)) ranging from about 0.03 to 0.06% for ^{14}O and from about 0.3 to 0.6% for ^{54}Co. Later work used many-body shell model wave functions generated by a Saxon-Woods potential [24-26] or a Hartree-Fock mean field [27]; this shell-model approach splits δ_C into two parts:

$$\delta_C = \delta_{CM} + \delta_{BE} \,. \tag{12}$$

In Eq. (12) the larger component, δ_{BE}, reflects the different binding energies, and so, together with the effect of the Coulomb potential, differing radial wave functions, of the decaying proton in the initial nucleus and of the neutron into which it transforms in the daughter nucleus; this difference of radial wave function implies a less-than-perfect overlap between initial and final states, hence a finite value of δ_{BE}. Furthermore the many-body nature of the wave functions means that each case will involve a range of effective binding energies reflecting the range of excitation of the parent states in the nuclei of A-1 so that δ_{BE} is determined not just by the nominal proton and neutron binding energies but also by the parentage spectrum and fractional parentage coefficients in the relevant nuclei of $T = \frac{1}{2}$ and $\frac{3}{2}$. The estimation of δ_{BE} is therefore dependent not only upon the method chosen for generating the single-nucleonic wave functions: what choice of Saxon-Woods potential; what choice of Skyrme force for the Hartree-Fock mean field?; but also upon the parentage structure for the states involved: what choice of basis states; what choice of 2-body matrix elements? None of these choices is unique and it is difficult to know what choice is optimal. There are also very significant uncertainties associated with: (i) the way in which the range of single-nucleonic binding energies referred to the various parent states is to be induced viz. by what recipe for modification of the relevant mean field; (ii) the overall artificiality of considering the excitation of a given parent state to be translated literally into an equal increase in the binding of the associated single nucleon involved in the beta-decay. It is not surprising that values for δ_{BE} range from about 0.2 to 0.3% for ^{14}O and from about 0.3 to 0.6% for ^{54}Co.

The additional term δ_{CM} in Eq. (12) arises from the T_3-dependence of the configuration mixing along the multiplet; it is smaller than δ_{BE} but important, rising to as much as 0.1%, but also depending quite strongly, being evidently a delicate

effect, on the mode of generating the configuration mixing and on the range of basis states admitted [24,26,27].

The present situation as to these shell-model-based estimates of δ_C, which one might hope to be more realistic than the harmonic-oscillator-based results, is summarized in Table 2. As we see from Table 2, the divergences in the estimates of δ_C, combining all values of δ_{BE} and δ_{CM} for each A-value, are 0.12, 0.17, 0.24, 0.21, 0.13, 0.16, 0.13 and 0.22% for our bodies from ^{14}O through ^{54}Co. This average divergence of 0.17% is much larger than the average uncertainty of ±0.063% in the individual experimental ft-values of Table 1 and the ±0.062 to 0.069% uncertainty in the overall radiative connection listed at the end of the previous section. Nor must we be tempted to split the difference and associate an uncertainty of, say, ±0.08% with the theoretical δ_C because we have no assurance that the right answer lies within the range of estimates represented by the combinations we have just considered.

The seriousness of the situation is further emphasized by recent estimates of δ_C using a very different, R-matrix-based, approach [29] that, in effect, combines the δ_{BE} and δ_{CM} of Eq. (12) and that, in effect, replaces the $p-n$ overlap just described, which has common core nuclei, by an overlap of different core nuclei with common nucleons outside. The results of this R-matrix approach are also given in Table 2; they raise the average divergence of the theoretical estimates of δ_C from the above mentioned 0.17% to an even-more-unacceptable 0.42%.

Now these divergences are most serious and we have very little guidance as to choice among the theoretical δ_C that give rise to them. (An exception to this remark concerns some of the δ_{CM}, where it appears [28] that the Chalk River estimates [26,28] are superior to those of Ormand and Brown [27] for the heaviest bodies of our concern on account of the broader basis employed by Chalk River; this is reflected in our construction of Table 2. However, this preference as to δ_{CM} has no bearing on the reliability of the δ_{BE} as is explicitly recognized [28]; we here make no preference as to δ_{BE}.) In these unhappy circumstances the best procedure is evidently democratic rather than eclectic namely to combine the various δ_{CM} and δ_{BE} in all possible ways into a range of 6 different δ_C, to combine these, with the addition of the δ_C of column B of Table 2, with the two Δ^R that stem from the combination of the Δ_T and Δ_H of Table 1 with the δ_r of Table 1, following Eq. (11), and to analyze the resultant $(ft)^*$, following Eq. (5). Table 3 quantifies the results of this procedure, listing the resultants of the 14 combinations of the 7 δ_C and 2 Δ^R according to the coding given in the caption.

The columns under Constant in Table 3 refer to straightforward averaging of the 8 $(ft)^*$-values from each of the 14 δ_C, Δ^R-sets taken separately and to the associated confidence levels $Q(\chi^2|\nu)$. It is evident that the results are strongly inconsistent with the CVC expectation of a constant value of $(ft)^*$, independent of Z, for all the δ_C, Δ^R-sets.

This inconsistency with a constant $(ft)^*$-value is not, however, due to a random fluctuation of $(ft)^*$ from Z-value to Z-value such as we might expect were CVC not to hold; this is demonstrated in Fig. 1 which displays the ft-values of Table 1 corrected only by (the two separate versions of) Δ^R with no δ_C-correction at all. It is seen that the data-points are regularly related and can be nicely fitted by quadratics in Z: using the $C_{NS(T)}$ to construct the Δ^R we find the extrapolation to $Z = 0$ (Fig. 1(a)):

$$(ft)^*_0 = 3139.7 \pm 5.2 \text{ sec} \tag{13}$$

with $Q(\chi^2|\nu) = 0.18$ while using the $C_{NS(B)}$ (Fig. 1(b)):

$$(ft)_0^* = 3135.2 \pm 5.2 \text{ sec} \tag{14}$$

with $Q(\chi^2|\nu) = 0.32$.

For further consideration we take forward, from this analysis:

$$(ft)_0^* = 3137.4 \pm 5.7 \text{ sec} \tag{15}$$

where the error is increased over that of Eqs. (13) and (14) to allow for the dispersion of the individual values.

We are therefore led to the conclusion that, most probably, the theoretical δ_C are simply deficient or incomplete as clearly must be the case for some, or all, of them owing to their wide divergences that we have already noted.

In view of the smooth behaviour of the ft-values, corrected only for Δ^R and displayed in Fig. 1, we are encouraged, while abandoning the use of these $J^\pi = 0^+ \to 0^+$ transitions for testing CVC (see Appendix 3) to hope that the several theoretical versions of δ_C may each give a reasonable account of at least a part of the mismatch and then look to the data themselves to tell us the rest by appropriate extrapolation to $Z = 0$ at which point, presumably, the mismatch will have fallen away. (Although extrapolation to $Z = 0$ is a natural procedure it is not immediately obvious that it is there that the true "charge independent" $(ft)^*$-value $(ft)_0^*$ is to be found: (i) the *de facto* total mass-splitting within isomultiplets effectively vanishes closer to $Z = 1$ than to $Z = 0$; (ii) the procedure has no direct regard for the role of the charge-dependence of the specifically-nuclear force; however this effect impacts largely upon the estimate of the smaller component, δ_{CM}, of δ_C, its effect upon the larger Δ_{BE} being accounted for through the use there of experimental binding and excitation energies; furthermore, its effect upon δ_{CM} will tend to be Z-sensitive and so will tend to be removed by the method of analysis here favoured, about to be presented, which uses not the theoretical δ_C themselves but rather their fluctuations from case to case.

The question now arises as to the form, as a function of Z, that the residual mismatch might be expected to take after correction of the ft-values by the theoretical δ_C. We have very little guidance here because we do not know in what way the theoretical δ_C are deficient but it is obvious from Fig. 1 that an overall quadratic in Z, without any mismatch correction at all, fits quite well so we should not expect to have to go beyond that in the Maclaurin series in handling the residual mismatch after correction by the theoretical δ_C. We must also recall, at this stage, our earlier remark that there may be further radiative corrections in $(Z\alpha)^2$ not yet accounted for.

Table 3 now shows the effect of fitting the $(ft)^*$-values, as severally corrected under the various δ_C, Δ^R-sets, linearly and quadratically in Z, together with the respective extrapolations, $(ft)_0^*$, to $Z = 0$. The $Q(\chi^2|\nu)$ distribution is seen to be statistically acceptable for the quadratic fit but the linear fit is unsatisfactory. The distribution of $(ft)_0^*$-values from the quadratic fit is shown in Fig. 2 from which we carry forward the value:

$$(ft)_0^* = 3135.1 \pm 5.1 \text{ sec}. \tag{16}$$

The bulk of the uncertainty quoted in Eq. (16) is due to the experimental errors with a small addition due to the dispersion in the values deriving from the several δ_C, Δ^R-sets. It is a striking observation, from Table 3 and Fig. 2, that although, as we have seen, the δ_C themselves vary so widely from set to set nevertheless their associated $(ft)_0^*$-values show good mutual consistency. We also note that the result of Eq. (16), the satisfactory quadratic extrapolation to $Z = 0$ of the δ_C-corrected $(ft)^*$-values, is in good accord with Eq. (15) which involves no δ_C-correction at all.

The magnitude of the quadratic term resulting from these fits may be expressed as about $0.20(Z\alpha)^2$ which we may bear in mind in the context of our earlier remark that additional radiative corrections of order $(Z\alpha)^2$ may be present[19]. We also note, however, that the theoretical δ_C themselves display a similar quadratic form [10]; certainly no inference as to the radiative nature of this residual $0.20(Z\alpha)^2$ can be drawn.

Although, as we have seen, application of the full theoretical δ_C prior to extrapolation to $Z = 0$ results in consistent values for $(ft)_0^*$ across the δ_C, Δ^R-sets, we must recognize the possibility that there may be, concealed within the evident mutual inconsistencies of those sets themselves, a false common behaviour with Z that would invalidate our procedure of extrapolation to $Z = 0$. It was because of this that it was suggested, some time ago [33], when the trouble with the theoretical δ_C was becoming clearly recognized, that one should, as far as possible, effectively look to the data themselves for a determination of δ_C by the method of extrapolation to $Z = 0$ but that, prior to this, we should correct the data not by the δ_C themselves, as we have just done, but rather by the case-to-case ***fluctuations***, δ_{Cf}, in the δ_C about some empirical underlying form, δ_{Cs}, smooth in Z. This suggestion was predicated upon: (i) the observation that although the theoretical δ_C, variously derived, differed considerably from author to author their ***fluctuations*** δ_{Cf} were remarkably similar in form and in magnitude; (ii) the hope that although the bulk of the δ_C may be deficient or distorted by, for example, inadequate treatment of the mean field or by inadequate treatment of the contribution to the mismatch from the core and other spectator nucleons, yet the δ_{Cf}, being due largely to the valence nucleons, whose behaviour was relatively well understood (see e.g. Ref. [17]), might be more reliably estimated by the theoretical treatments.

We therefore make the ***Ansatz*** that the fluctuations δ_{Cf}, derived from the various δ_C of Table 2, are reliable so that the use of the δ_{Cf}, rather than δ_C, within Eq. (5) will result in a common set of $(ft)^*$ with a common extrapolation to $(ft)_0^*$ at $Z = 0$ apart, of course, from relatively small differences associated with different calculational procedures.

This approach will be convincing only if, indeed, the δ_{Cf} thus extracted from the various theoretical versions of δ_C show smaller dispersion, within each Z-value, than they show differences from Z-value to Z-value. That this is the case is displayed in Fig. 3 where the "errors" on the mean $\overline{\delta_{Cf}}$-values are given by the usual expression for a standard deviation but are otherwise purely illustrative. (In the present extraction of the δ_{Cf} from the various δ_C derived from Table 2 a smooth underlying form $\delta_{Cf} = aZ + bZ^2$ has been taken. Earlier [1] other reasonable forms for δ_{Cf} have been explored; the results from those alternative forms differ only inconsiderably as to $(ft)_0^*$ from those presented here.)

It seems that not only are the $\overline{\delta_{Cf}}$ well-defined in relation to their differences but that they have a regular behaviour with Z although no point is further made of this nor of the $\overline{\delta_{Cf}}$.

Our first step based on this fluctuation approach is to repeat the earlier analysis but using the δ_{CJ} rather than the δ_C, again using a free quadratic fit in Z. The results are given in Table 4 and in Fig. 4. The $Q(\chi^2|\nu)$ are very satisfactory and we find:

$$(ft)_0^* = 3136.4 \pm 5.1 \text{ sec} \tag{17}$$

where, as with Eq. (16), the bulk of the uncertainty is due to the experimental errors.

Equation (17), with its associated error, derives from the assembly of the 14 individual entries from Table 4. We may, alternatively, but not, of course, independently, perform the quadratic fit upon the averages of the $(ft)^*$-values, for each Z-value, over the 14 δ_C, Δ^R-sets. The result is shown in Fig. 5 from which we quote:

$$(ft)_0^* = 3134.7 \pm 5.2 \text{ sec}. \tag{18}$$

This fit has $Q(\chi^2|\nu) = 0.60$.

We note immediately that the $(ft)_0^*$-value resulting from this analysis using the δ_{CJ}, namely the $(ft)_0^* \simeq 3135.6$ sec of Eqs. (17) and (18), is very close to the $(ft)_0^* \simeq 3135.1$ sec of Eq. (16) which uses the full theoretical δ_C themselves and is, indeed, close to the $(ft)_0^* \simeq 3137.4$ sec of Eq. (15) which uses no theoretical δ_C at all.

Now we have so far performed free quadratic fits in Z to gain the results of Eqs. (17) and (18) but we may now, following our *Ansatz* that the δ_{CJ}-corrected data should all display the same basic form with Z, use the implied mutual consistency of the 14 sets of $(ft)^*$ to restrict the details of the fit with Z. We do this by imposing the condition of a common peaking, at some Z_{max}, on all the 14 data sets. This is done in two ways: (i) most simply, we take the Z_{max}=22.25 from the averaged fit of Fig. 5 and impose it upon the 14 separate sets of $(ft)^*$; this results in the constrained columns of Table 4 and in Fig. 6 which yield, again with very satisfactory $Q(\chi^2|\nu)$:

$$(ft)_0^* = 3135.5 \pm 3.7 \text{ sec}; \tag{19}$$

(ii) more generally, we derive Z_{max} from each of the 14 data sets separately and apply each such value to the other 13 data sets resulting in a total of 182 values of $(ft)_0^*$ from which we gain:

$$(ft)_0^* = 3136.8 \pm 4.0 \text{ sec}. \tag{20}$$

The χ^2 distribution for these 182 fits is displayed in Fig. 7; it is highly satisfactory. (There is, as seen, a small aberrant island in Fig. 7, having $12 < \chi^2 < 14$ and $3130 < (ft)_0^* < 3140$, containing 9 cases (all of which derived from TOB); these cases, of themselves, would give: $(ft)_0^* = 3134.3 \pm 3.5$ sec; their exclusion would make no sensible difference to the result of Eq. (20)).

We have now treated the experimental data of Table 1 in six ways based on the method of extrapolation to $Z = 0$; the resultant $(ft)_0^*$-values are collected in Table 5; although their uncertainties are dominated by their common experimental errors, from the ft-values of Table 1, we may combine their values with a conventional $1/\sigma^2$ weighting and quote our final:

$$(ft)_0^* = 3136.0 \pm 3.5 \text{ sec} \tag{21}$$

in which some slight reduction of the overall error has come about on account of a global analysis of the contribution to the error from the dispersion of the individual entries taken over the entire outcome of the six approaches.

Before proceeding to further discussion we swiftly examine the consequences of repeating the entire analysis that has just been presented but using, instead of the δ_{BE} of the shell-model-based treatment, values derived from the earlier harmonic oscillator estimates of the nuclear mismatch to which reference was made in passing. This we do in order to probe the possible sensitivity of our $(ft)_0^*$ to a very different way of estimating the major part of the δ_C. For this purpose we use as a starting point the four sets of harmonic oscillator mismatches [21–24] as listed in Ref. [24].

As stressed in Ref. [24] the harmonic oscillator mismatch is rather sensitive to the nuclear radius used for its evaluation; we have not, however, attempted to make any correction on this account since this cannot be done unambiguously, but have used the various authors' estimates as they stand; we must anticipate an exaggeration of the dispersion of the results of the analysis in consequence and be the more impressed if we find good internal consistency within the approach.

We now consider these harmonic oscillator mismatches as the analogues of the δ_{BE} of the shell-model approaches when the ground state of the nucleus of $A - 1$ is taken as unique parent. As a mock-up of the effect of a realistic parentage spectrum, which obviously cannot easily be incorporated into this harmonic oscillator approach itself, we multiply these initial mismatch values by the factors, taken from the shell-model approaches [24,25], by which the δ_{BE} of the shell model, using the full parentage expansions, exceed those using the $A - 1$ ground state as unique parent. We now combine these δ_{BE}-equivalents, so enhanced, with the δ_{CM} of Table 2 and the Δ^R as before into a harmonic-oscillator-shell-model hybrid and repeat all the earlier exercises.

Table 6 with Figs. 8, 9 and 10 present the detailed results of the analyses leading to the analogues of Eqs. (16), (17) and(19) respectively as summarized in Table 7 which also presents the outcome of analyses analogue as those of Eqs. (18) and (20). (All analyses are completely satisfactory statistically.)

Weighting the results presented in Table 7 as before, we quote the overall result of this hybrid treatment of the δ_C:

$$(ft)_0^* = 3139.3 \pm 3.6 \text{ sec}. \tag{22}$$

When we include, for consistency, the "no δ_C" result from the first entry of Table 5 we have:

$$(ft)_0^* = 3139.1 \pm 3.6 \text{ sec}. \tag{23}$$

The various results reported in Table 7 are in good accord and their overall resultant in Eq. (23) is in good accord with the earlier Eq. (21) from the purely shell-model analysis. As suggested earlier, we should not attempt to combine Eqs. (21) and (23) since the shell-model approach is clearly better-based as well as more general, but we note with satisfaction the robustness of the conclusions of the $Z = 0$ extrapolation method against major changes of the input leading to them.

3.1. *Note on ^{14}O and ^{10}C*

As is immediately obvious from a glance at Figs. 1 or 5, the ft-value for ^{14}O is critical for the method of extrapolation to $Z = 0$ advocated here; this is true both in respect of the inferred $(ft)_0^*$-value and in respect of its error. Specifically, for the sake of illustration, keep all the input values behind Fig. 5 as they are with the exception of that for ^{14}O and: (i) raise or lower the $(ft)^*$-value for ^{14}O to the limits of its present error bars (± 2.0 sec) keeping its error as at present; (ii) keep the $(ft)^*$-value for ^{14}O as at present but reduce its error from its present value to that for $^{26}Al^m$(± 1.2 sec). The result of operation (i) is to raise or lower $(ft)_0^*$ by 4.2 sec viz. an amplification of the change in $(ft)^*$ for ^{14}O by a factor of a little more than 2. The result of operation (ii) is to reduce the error in $(ft)_0^*$ from ± 5.2 sec to ± 3.8 sec. (Both operations (i) and (ii) being carried out within unconstrained quadratic fitting in Z.)

Improvements in the input data for ^{14}O are, therefore, much to be desired. The situation with respect to the lifetime is that two accurate (± 0.035 and $\pm 0.040\%$) and concordant values exist but the more recent of them dates from 1978 while the not inconsiderable correction (0.61%) for the Gamow-Teller branch to the ground state of ^{14}N dates from 1966 (see Ref. [1] for details). While there are no specific reasons for doubting these data the importance of the matter would certainly warrant remeasurement in both cases. The situation with respect to the Q_{EC}-value is unfortunate in that two ostensibly very accurate values (± 0.13 and ± 0.077 keV) may be alternatively derived from the corpus of data but they differ by as much as 0.8 keV depending on details of the analysis (see Refs: [1] and [10] for discussion); because of this the PDG prescription for inflation of errors [7] has been used in deriving the Q_{EC}-value, and its error [10], lying behind the entry for ^{14}O in Table 1. Experimental resolution of this conflict is highly desirable.

Given the considerable leverage, in the extrapolation method, of data at low Z-value, the benefit of an accurate point at $Z = 5$ viz. the super-allowed decay of ^{10}C to its mirror at 1.74 MeV in ^{10}B, cannot be overstressed. The problem is that, in contrast to the happy accident for ^{14}O, where the allowed Gamow-Teller transition to the ground state is "accidentally" suppressed, in the case of ^{10}C the allowed Gamow-Teller transition to the state in ^{10}B at 0.72 MeV is fast so that the desired super-allowed Fermi branch is of only about 1.5%. Present experimental accuracy in the ft-value has, after using considerable effort, reached about ± 9 sec (see Ref. [34]) which, as may be judged from Figs. 1 or 5, is not yet very useful. A good point for ^{10}C would be the most important single contribution to our present study; the tremendous experimental effort demanded would be well repayed. To quantify the effect on the extraction of the $(ft)_0^*$-value of a given accuracy in the $(ft)^*$-value for ^{10}C we consider: (i) the improvement in the accuracy of the extracted $(ft)_0^*$-value consequent upon the availability of an $(ft)^*$-value of a given accuracy for ^{10}C, that $(ft)^*$-value for ^{10}C being supposed to fall on the line shown in Fig. 5 for the present data-set of Table 1; (ii) the change in the $(ft)_0^*$-value brought about by a given $(ft)^*$-value for ^{10}C of a given accuracy but falling off the line of Fig. 5. The answer to (i) is given in Fig. 11 from which we see that an accuracy of ± 3 sec or so for ^{10}C is necessary in order to achieve a significant sharpening (by 20% or so) in the accuracy of our present knowledge of $(ft)_0^*$. The answer to (ii) is given in Fig. 12 from which we see that, for values of the ^{10}C $(ft)^*$-value falling within a resonable range, an accuracy of ± 4 sec or better would be necessary to shift the present $(ft)_0^*$-value to the limits of its present error bars. From both points of view, (i) and (ii), we see that an accuracy of something like ± 3 to 4 sec in $(ft)^*$ must be aimed for.

4. Unitarity of the Cabibbo-Kobayashi-Maskawa matrix

Before using our value for $(ft)_0^*$ given by Eq. (21) for extracting $|V_{ud}|^2$ via Eq. (3) we must attend to its uncertainty. The ± 3.5 sec of Eq. (21) has had no regard for the systematic uncertainties that we noted at the end of Section 2 as amounting to some ± 0.062 to $\pm 0.069\%$; taking these into account raises the uncertainty on $(ft)_0^*$ to ± 4.0 sec Using Eq. (5) we now have:

$$|V_{ud}|^2 = 0.9516 \pm 0.0012 . \tag{24}$$

Using the PDG [7] recommended values for $|V_{us}|(0.2205 \pm 0.0018)$, $|V_{ub}/V_{cb}|(0.08 \pm 0.02)$ and $|V_{cb}|$ (0.040 ± 0.005) we have:

$$|V_{us}|^2 + |V_{ub}|^2 = 0.04863 \pm 0.00079 \tag{25}$$

which combines with Eq. (24) to give:

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.0003 \pm 0.0014 . \tag{26}$$

Although we have not used the value of $(ft)_0^*$ given in Eq. (23) from the hybrid harmonic-oscillator-shell-model analysis we may note that it would, of itself, correspond to a unitarity test of 0.9993 ± 0.0014 to compare with Eq. (26).

We also note that if we were to persist in an analysis that did not recognize the manifest Z-dependence of the $(ft)^*$-values derived from the theoretical δ_C, and were to average the results of $\overline{(ft)}^*$ from the Constant column of Table 3, we should find: $\overline{\overline{(ft)}}^* = 3151.5 \pm 2.2$ sec where the error derives, somewhat arbitrarily, from the quadratic sum of the mean of the experimental errors in the ft-values and the standard deviation of the dispersion in the $\overline{(ft)}^*$-values. This value of $\overline{\overline{(ft)}}^*$ would correspond to a unitarity test of 0.9956 ± 0.0012 in clear violation of CKM unitarity.

These remarks complete the quantification of our assertion that a conventional analysis of the present $J^\pi = 0^+ \rightarrow 0^+$ ft-values, using just the theoretical δ_C on a case-by-case basis, is not only inconsistent with CVC but also, if one ignores that inconsistency, is then inconsistent with unitarity of the CKM matrix whereas the method of quadratic extrapolation to $Z = 0$, which takes CVC as given, results in excellent mutual consistency as between the several prescriptions for effecting that extrapolation, and is in excellent accord with CKM unitarity.

5. The value of the operational G_V^*

Our emphasis so far has been upon the extraction, from the data, of the value of $(ft)_0^*$ with a view to the making of the CKM unitarity test of the previous section. But there is also evident interest in deriving the best value for the operational G_V^* for use in ordinary nuclear physics. In doing this we must bear in mind the stricture that for work of the highest accuracy we must take into account C_{NS}, which will vary from

case to case, as also will $C(1)$, at the same time as using G_V^*. Specifically, as well as applying the usual Z-dependent "outer" radiative corrections we should, on using G_V^*, additionally multiply the calculated speed of a vector transition by the factor: $1 + \frac{\alpha}{\pi}\{C(1) - C_{\rm BORN} + C_{\rm NS}\}$. Since, as is seen from Table 1, $C(1) - C_{\rm BORN} \simeq -0.25$ while $C_{\rm NS}$ ranges between $+0.5$ and -1 for those cases investigated we may anticipate, in other cases, an uncertainty of the order of $\frac{\alpha}{\pi}$ in the radiative corrections if $C(1)$ and $C_{\rm NS}$ are not evaluated. We may here also note that the old "outer" radiative correction of order α, viz. $\frac{\alpha}{2\pi}\,\overline{g(W,W_0)}$, has become $\frac{\alpha(m_p)}{2\pi}\,\overline{g(W,W_0)}\,S(m_p,m_Z)$ in the new formulation of Eq. (6) which corresponds to a further increase in the speed of the transition, due to the introduction of higher powers of α, by $0.0054\ \overline{g(W,W_0)}\%$.

This being said, we may write:

$$G_V^*/(\hbar c)^3 = \left[\frac{K}{2(ft)_0^*}\left\{1 + \frac{\alpha}{2\pi}\left[ln\frac{m_p}{m_A} + 2C_{\rm BORN}\right]\right\} S(m_p,m_Z)\right]^{\frac{1}{2}}. \tag{27}$$

Using Eq. (21), other values as before and $C_{\rm BORN} = 0.881 \pm 0.031$ [15] we have:

$$G_V^*/(\hbar c)^3 = (1.15155 \pm 0.00064) \times 10^5\ {\rm GeV}^{-2}. \tag{28}$$

(Note that in using Eq. (21) here we do not have to increase its error on account of those systematic uncertainties, such as those residing in m_A and $S(m_p,m_Z)$, as was necessary in applying it in Section 4.)

6. Neutron decay

Germane to the questions of $|V_{ud}|$ and G_V^* that we have been discussing is the decay of the neutron. This has the enormous advantage that the decay does not suffer from the uncertainties entrained by δ_C that, as we have exposed at length, limit the precision with which we may extract those quantities from the decay of complex nuclei. (This is not strictly correct in respect of $|V_{ud}|$, as we note in Appendix B, because the neutron is a structured object, in the quark sense, just as a complex nucleus is a structured object in the nucleonic sense; however, as we see in Appendix B, these structural uncertainties in the case of the neutron are probably at the 0.01% level whereas for a complex nucleus they are some tens of times larger).

The disadvantage for the neutron is that its decay is mixed vector-axial:

$$ft = \frac{K}{G_V^{*2} + 3G_A^{*2}}; \tag{29}$$

since $|G_A^*/G_V^*| \simeq 1.26$ the contribution to the decay from the axial interaction is some 4.8 times larger than the vector.

(We write G_A^* to emphasize that, as with G_V^*, this is an *operational* constant which includes the axial "inner" radiative correction, whatever that might be; attempt to assess and remove this correction from G_A^* does not arise in our present context nor that of practical applications of G_A^*.)

The sorting-out of the G_V^*, G_A^* terms may be effected through various types of correlation study of which the best-pursued at the present time involves the asymmetry in the emission of the electrons from polarized neutrons.

Both the lifetime measurement and the asymmetry measurement are exceedingly delicate and it cannot yet be said that either has settled down to a generally consensual value. In the case of the lifetime it is best to exclude measurements pre-1986 which showed wide scatter and significant inconsistency with subsequent measurements. A survey down to 1990 but excluding those earlier measurements showed good consistency and yielded [35] a mean lifetime:

$$\tau_m = 887.6 \pm 2.7\ {\rm sec}. \tag{30}$$

Two measurements subsequent to this survey gave the values: from Ref. [36]:

$$\tau_m = 893.6 \pm 5.3\ {\rm sec} \tag{31}$$

and from Ref. [37]

$$\tau_m = 888.4 \pm 3.3\ {\rm sec}. \tag{32}$$

The results in Eqs. (30)-(32) are concordant at: $Q(\chi^2|\nu) = 0.60$ and together yield:

$$\tau_m = 888.7 \pm 1.9\ {\rm sec}. \tag{33}$$

More recently we have the further [38]:

$$\tau_m = 882.6 \pm 2.7\ {\rm sec}. \tag{34}$$

Equation (34) sits very uneasily with Eq. (33) at $Q(\chi^2|\nu) = 0.065$; we do not feel it wise to combine them but rather pursue their separate consequences.

The situation in respect of the asymmetry measurements that yield $|G_A^*/G_V^*|$ is somewhat similar. The survey of 1990 (Ref. [35]) yielded:

$$|G_A^*/G_V^*| = 1.262 \pm 0.004 \tag{35}$$

a result dominated by a measurement from ILL [39] but including other, concordant, work. However, a more recent result from Gatchina [40]:

$$|G_A^*/G_V^*| = 1.2544 \pm 0.0036 \tag{36}$$

is not in happy agreement with the ILL-dominated result of Eq. (35): $Q(\chi^2|\nu) = 0.16$. (The asymmetry results of Eqs. (35) and (36) have been corrected for recoil and other small effects, including weak magnetism, but no allowance has been made for radiative corrections. It is evident that these latter are small, changing the asymmetry coefficient by the order of 0.1% of its own value whereas the present experimental error on that quantity is a little over 1% of its own value. It is desirable that this correction be estimated with better accuracy than is currently available [41] since it is already on the edge of significance.)

In view of the importance of the issue it is again, as with the lifetime, not considered wise to combine the two results of Eqs. (35) and (36) but rather to treat them separately.

In view of our unwillingness, on such a critical matter, to average data-sets of near-incompatibility we combine the two lifetime values of Eqs. (33) and (34) and the two $|G_A^*/G_V^*|$ - values of Eqs. (35) and (36) in their four possible ways:

$$\begin{aligned} \text{I} &= \text{Eqs. (33) and (35)} && (37)\\ \text{II} &= \text{Eqs. (34) and (36)} && (38)\\ \text{III} &= \text{Eqs. (33) and (36)} && (39)\\ \text{IV} &= \text{Eqs. (34) and (35)} && (40) \end{aligned}$$

Figure 13 shows the results of these combinations I through IV together with our present result for G_V^* from Eq. (28). We see that we have compatibility between our value of G_V^* and neutron decay only for combination I above. Since, as we have seen in Eq. (26), our result is very close to expectation based on the unitarity of the CKM matrix we may express the results, in that regard, of combinations II, III and IV by saying that they violate that unitarity by about 3.6, 2.6 and 2.3 standard deviations respectively.

If one insisted upon an averaging procedure for both the lifetime and the asymmetry, with PDG-type inflation, there would result a considerable discrepancy between G_V^* derived, as in this paper, from the super-allowed Fermi transitions, or from the unitarity of the CKM matrix, and from neutron decay. Such a discrepancy might be read as a signal for, say, a right-handed W-boson but the view presented here is that such an interpretation would be premature and that we must wait for the experimental situation in respect of neutron decay to clarify. In the meantime it is clear from Fig. 13 and from our discussion that, within the standard model, we must favour combination I.

(Use of $\overline{(ft)}^*$=3151.5 sec that we saw would follow from a Z-independent analysis of the $(ft)^*$-values, averaging the $\overline{(ft)}^*$-values from the Constant column of Table 3, using only the theoretical δ_C for the nuclear mismatch, corresponds to $G_V^*/(hc)^3 = 1.14871 \times 10^{-5}$ GeV^{-2} which is 2.2 standard deviations away from the G_V^*-value of combination I (the corresponding discrepancies with combinations II, III and IV are 4.4, 3.5 and 3.3 standard deviations respectively); such analyses of the super-allowed Fermi data are therefore in severe conflict with *any* of the analyses of the neutron data.)

Although not germane to the chief thrust of this paper we may also, of course, use the data presented here to extract best values for G_A^* and $|G_A^*/G_V^*|$ (see Appendix 4).

7. Inferences following unitarity of the CKM matrix

As we have seen in Eq. (26), the present analysis of super-allowed Fermi decay is consistent with the unitarity of the CKM matrix. We may, alternatively, assume that unitarity and use our data to draw other inferences. In the notation of this paper, using Eqs. (3), (5) and (25), this assumption of unitarity amounts to:

$$(ft)_0^* = 3136.9 \pm 2.6 \text{ sec}. \qquad (41)$$

7.1. *The mean charge of the fermionic fields*

As we have remarked, a large part of the radiative correction of Eq. (6), namely the vector part of order α, is effectively exact by CVC but of the remainder of the "inner" sector of Δ, as there written, the axial part that we have discussed at length, is only approximate depending upon, among other things, the "axial mass" m_A, for which we accepted the value given by Eq. (9), and $\overline{Q}$, the mean charge of the fermionic fields between which the beta-decay takes place. Now within the standard model we have, of course, $\overline{Q} = \frac{1}{6}$, being the mean charge of the u and d quarks; this value is implied within Eq. (6). It is, however, of some interest to ask whether our beta-decay data, assuming CKM unitarity, can give us a value for $\overline{Q}$ (which would be $\overline{Q} = \frac{1}{2}$ for the elementary neutrons and protons of the pre-quark era with V_{us} effectively replaced by $\sin\theta_C$, θ_C being the Cabibbo angle, with negligible change to the inferred $(ft)_0^*$ since V_{ub} is so small (see Section 4)).

To approach $\overline{Q}$ we therefore re write Eq. (6) without the assumption that $\overline{Q} = \frac{1}{6}$:

$$\begin{aligned} \Delta &= \frac{3}{4}(S(m_p, m_Z) - 1)(1 + 2\overline{Q}) + S(m_p, m_Z)\left\{\frac{\alpha}{2\pi}\left[6\overline{Q}\, ln\, \frac{m_p}{m_A} + 2C\right]\right.\\ &\quad \left. + \frac{\alpha(m_p)}{2\pi}\, \overline{g(W, W_0)}\right\}. \end{aligned} \qquad (42)$$

We now perform the usual unconstrained quadratic fit in Z to the same δ_{Cf}-corrected data set as lies behind Fig. 5 but using the Δ of Eq. (42) in place of the Δ of Eq. (6) and including the $(ft)_0^*$ of Eq. (41) at $Z = 0$; we do this varying $\overline{Q}$ for the range of m_A-values shown in Fig. 14 with the results there shown. (The fit minimizes at $Q(\chi^2|\nu) = 0.60$.) Figure 14 also shows our adopted range of Eq. (9) for m_A and the resultant 1σ-limit in the $m_A - \overline{Q}$ plane.

From the ellipse of Fig. 14 we find:

$$\overline{Q} = 0.172 \pm 0.060 \qquad (43)$$

in good accord with the $\overline{Q} = 0.167$ expected for quarks and clearly excluding the $\overline{Q} = 0.5$ of "elementary nucleons".

7.2. *The value of m_A*

Instead of using our assumed range of m_A to determine $\overline{Q}$ we could reverse this procedure: we similarly fix $\overline{Q}$ at $\frac{1}{6}$ and minimize on m_A; this yields:

$$ln\, \frac{m_p}{m_A} = 0.0 \pm 1.6 \qquad (44)$$

which is within our adopted range of Eq. (9) but of so low an accuracy as to be uninteresting.

7.3. *The scalar coupling constant*

The context of our discussion so far has been the V-A of the standard model with which, under our present analysis, everything is in good accord, at least in the

vector domain. But, as has been emphasized [43,44] a possible scalar coupling, from outside the standard model, would modify the analysis and it is interesting to enquire as to the degree to which such a coupling can be excluded.

In the notation of Ref. [43] we expect an effective scalar coupling G_S, of strength relative to the vector G_V measured by:

$$b_F = G_S/G_V \,, \tag{45}$$

namely the Fierz interference factor, to reduce the lifetime of the predominantly vector transition:

$$ft \rightarrow (ft)_V\,(1 - 2b_F\gamma < W^{-1} >) \tag{46}$$

where $(ft)_V$ is that for a pure vector transition. (As pointed out in Ref. [44] this definition differs by a factor 2 from others in use; the handedness of the possible scalar interaction is an important question mark.)

In Eq. (46):

$$\gamma = [1 - (\alpha Z)^2)]^{\frac{1}{2}} \tag{47}$$

and $< W^{-1} >$ is the average of W^{-1} over the decay spectrum. Now $< W^{-1} >$ varies considerably over the eight bodies of our concern (from 0.4375 for ^{14}O to 0.1539 for ^{54}Co) which constitutes the leverage by which b_F might be approached.

The analysis of Ref. [43] resulted in $b_F = (0.6 \pm 2.5) \times 10^{-3}$; it was, however, as other analyses with very similar outcomes [45], based upon theoretical values for the nuclear mismatch δ_C which, as we have exposed here, are now known to be seriously discrepant. The analysis of Ref. [44] recognized the effective residual Z-dependence of the ft-values corrected by the theoretical δ_C and looked for the influence of a finite b_F on the form of that dependence, finding: $b_F < 1.6 \times 10^{-2}$ at 90% confidence level, not unexpectedly a significantly poorer limit than that derived earlier when trust reposed in the theoretical δ_C. However we have *a priori* no reason to expect any particular functional form for the effective residual Z-dependence after correction by the theoretical δ_C or after correction by the fluctuations δ_{cf} or after no correction at all; as we have exposed at length, all these procedures result in $(ft)^*$-distributions that are very satisfactorily fitted by a quadratic in Z with no call for a term that would follow $< W^{-1} >$.

If, however, we pin down $(ft)_0^*$ through the assumption of unitarity of the CKM matrix we can do much better if we continue to insist that the fit by a quadratic in Z remain adequate. Such insistence is, of course, itself arbitrary to some degree but it would smack of a conspiracy if the vector-plus-finite-scalar $(ft)^*$-distribution were fitted by a quadratic alone while the underlying pure vector contributions demanded higher powers of Z.

But before we introduce a finite value of b_F into our fitting of the $(ft)^*$, including the $(ft)_0^*$-value of Eq. (41) from CKM unitarity, we must make sure that the latter value is not itself significantly sensitive to the presence of a possible scalar coupling. That it is not is indeed the case.

The impact of possible scalar coupling on the partial beta-decay lifetimes of hyperons and K-mesons, from which $|V_{us}|$, and hence the $(ft)_0^*$ of Eq. (41), is extracted is very much less than the corresponding impact on the nuclear lifetimes, that

we relate to the $(ft)_0^*$ of Eq. (41) in the present analysis, owing to the $< W^{-1} >$-factor. Specifically: (i) from a standard tabulation [46] we find that $\Lambda \rightarrow pe\overline{\nu}$ and $\Sigma^- \rightarrow ne\overline{\nu}$ are, respectively, 109 and 171 times less sensitive to scalar admixture than is $n \rightarrow pe\overline{\nu}$; for comparison with the nuclear decays of Table 1 these factors become $166\gamma < W^{-1} >$ and $261\gamma < W^{-1} >$ respectively so that the nuclear decays are some 30 to 100 times the more sensitive; (ii) in the case of $K^+ \rightarrow \pi^0 e^+ \nu$ we assess the impact of a possible scalar coupling by integrating over the Dalitz plot density (for the latter see e.g. Eq. (2) of Appendix 1 of Ref. [47]); in the illustrative limit $m_e, m_\pi \rightarrow 0$ we find that K-decay is less affected by scalar admixture than nuclear decay by the factor $\frac{1}{3}\gamma m_K < W^{-1} >$ viz. factors of about 50 to 140 for the cases of Table 1.

We must also concern ourselves with the impact of possible scalar coupling on the rate of muon decay and hence on the inferred value of G_μ^* that figures in our equations. Assuming perfect neutrino helicity, muon decay is speeded by the factor (see e.g. Ref. [48]) $1 + \frac{1}{8}\frac{G_S^2}{G_V^2}$; the effect is here of second order in G_S and so is quite negligible. (This G_S for the muon refers to an intrinsic coupling and so is different from that figuring in Eq. (45) which effectively includes also a possible hadronically-induced scalar coupling (which is a second-class current, zero within the standard model). So long as we are concerned only with limits this finesse need not detain us.)

We may therefore use the $(ft)_0^*$-value of Eq. (41) with confidence as, within its error, a fixed point in our questioning of the effect of b_F on the nuclear super-allowed Fermi transitions which we do by performing an unconstrained quadratic fit in Z on the δ_{Cf}-corrected data set lying behind Fig. 5 now including the effect of b_F following Eq. (46). Following minimization we find:

$$b_F = (0.0 \pm 1.4) \times 10^{-3} \tag{48}$$

or $b_F < 2.6 \times 10^{-3}$ at the 90% confidence level. (We can, of course, carry out the present exercise without reference to a $(ft)_0^*$ derived from unitarity of the CKM matrix. As expected, the result, $b_F = 0.007 \pm 0.013$ or $b_F < 3.1 \times 10^{-2}$ at the 90% confidence level, is much weaker than that including the unitarity constraint; the associated $(ft)_0^* = 3106 \pm 54$ sec is also not useful but it does emphasize the degree to which our present analysis is predicated upon the V–A structure of the standard model.)

Appendix A. The ft formula

In Eq. (1) f is the phase-space factor inferred from the experimental Q-value for the beta-decay having regard for a number of detailed effects that have been the object of recent exposure [1].

Although it is not here appropriate to dwell upon the technicalities of the translation of the primary experimental data on the nuclear reaction Q-values into the f-factors it is of interest just to note the rough magnitudes of the various considerations that must be called into account for the eight bodies of our concern; in the most sensitive Z-case for each effect: (i) atomic excitations in the Q-value measurements (0.02%); (ii) the Coulomb field of the daughter nucleus (a factor 2); (iii) the screening of the Coulomb field by the orbital electrons (0.2%); (iv) the recoil of the Coulomb field (0.001%); (v) the finite size of the nucleus (4%); (vi) the finite mass of the nucleus (0.02%); (vii) the departure of the nucleus from a uniform sphere (0.1%); (viii) the deformation of the nucleus (0.01%); (ix) convolution of the leptonic and nucleonic wave functions (0.7%). We similarly note corrections that must be applied to the experimental lifetime measurements (additional, of course, to overt branching ratios) before use in the ft-values: (i) atomic excitation (0.01%); (ii) orbital electron capture (0.1%). In Eq. (1) t has additionally been defined as incorporating those radiative corrections that do not directly concern themselves with the W, Z-mechanism of the weak interaction process nor with the axial contribution entrained by the possibility that two spin flips, one due to the weak process and the other due to a virtual photon exchange, may cancel out.

The confidence with which these various corrections can be applied has been extensively discussed [1]; it depends partly upon experimental uncertainties (for example the $< r^2 >^{\frac{1}{2}}$-values of the daughter nuclei) and partly upon theoretical uncertainties: in the case of the f-values the corrections increase the uncertainty that is due solely to experimental error in the Q-values by amounts ranging from a few percent for the lighter nuclei to a factor 3 for the heaviest; in the overall ft-values the experimental uncertainties are similarly raised by amounts varying from about 5% to a factor of about 2.

K is a combination of natural constants having the numerical value: $K/(\hbar c)^6 = (8.120270 \pm 0.000010) \times 10^{-7}$ GeV^{-4} sec. M_V is the matrix element for the transition, having the value $\sqrt{2}$ in the ideal world of perfect charge-independence of the forces.

Appendix B. Nucleon structure and quark masses

Equation (2), in respect of G_V, is not strictly correct for three reasons: (i) the nucleon is an extended object and so we must consider the momentum-transfer dependence of the coupling constant; this effect is, at most, about 0.02% in its influence upon G_V for the most sensitive of the beta-decays that we shall consider [1]; (ii) the nucleon is a structured object but that structure is not completely charge-independent: there is, for example, a 0.14% mass splitting between neutron and proton. This means that the intrinsic vector rate for $p \to ne^+\nu_e$ differs slightly from the corresponding intrinsic rate for the underlying $u \to de^+\nu_e$ for the "free quark" that we wish to access to gain $|V_{ud}|$. Discussion of this effect, which eventually comes down to the $u - d$ mass difference itself, may be couched in various languages for example, in microscopic terms, $\rho - \omega$ mixing [2,3], or, more generally, of chiral symmetry breaking [4]; the most recent estimates [3,4] are that the effect is very tiny, of the order of 0.01%; (iii) the CKM matrix is primitively unitary only in the approximation that the quark masses are zero; we should, in fact, consider the effect of these masses at the level of the one-loop corrections: the CKM matrix should be renormalized. This effect increases the rate of $W \to ud$ by only about 0.0017% [5].

Appendix C. The conserved vector current

Historically, these $J^\pi = 0^+ \to 0^+$ super-allowed Fermi transitions have been of great importance in probing the validity of CVC, as is well-recognized. However, as the present analysis demonstrates, we have now reached the point of refinement in experimentation and understanding of the radiative corrections where we might be tempted to claim, following the Constant column of Table 3, with its tiny $Q(\chi^2|\nu)$ entries, that CVC is violated. (There is, of course, danger of some logical confusion here because a major part of the radiative correction, the vector part of the "inner" radiative correction within the $S(m_p, m_Z)$ of Eq. (6), that is involved in constructing Table 3, itself depends upon CVC; however, it would remain true that overall consistency could not be reached within CVC). But such a claim would depend upon the adequacy of our command of the nuclear mismatch δ_C which it would not be wise; in view of the inconsistencies that we have exposed, to assert. The position adopted in this paper is rather to assume the validity of CVC and, in effect, to proceed from there to an assessment of δ_C by reference to the data themselves. This position is predicated upon the universal success of CVC wherever it comes into play elsewhere, most generally as the background to the standard model. The specific test of CVC most clearly related to the super-allowed Fermi transitions themselves is the beta-decay of the pion, $\pi^+ \to \pi^0 e^+\nu_e$, the CVC expectation for the rate of which exceeds the experimental rate by the factor 1.016 ± 0.034 [7,30]. In quoting this result the earlier analysis [30] has been updated on account of improved values for the $\pi^+ - \pi^0$ mass differences [7], for the operational G_V^*, as reported in this paper, and for the relevant radiative corrections. (The charge-dependence of particle structure that we noted affects, in principle, nucleon vector decay will also affect that of the pion where its effect will also, presumably, be slight although we note that the square of the relative mass splitting within the isomultiplet, that the Behrends-Sirlin-Ademollo-Gatto theorem might seem to bid us regard as an *a priori* gauge of the renormalization of the coupling constant due to the symmetry-breaking, is more than 500 times larger for the pion than for the nucleon.) This agreement between the beta-decay of the pion and the CVC expectation is very satisfactory but it is yet more impressive, in our present context, to look for a test of CVC in a field as remote as possible from that of nuclear beta-decay. A striking example is afforded by the decay: $\tau^- \to \pi^-\pi^0\nu_\tau$, which is linked by CVC to the isovector component of $e^+e^- \to \pi^+\pi^-$. The branching ratio for this decay, observed as $\tau^- \to h^-\pi^0\nu_\tau$, after a small correction for the measured decay: $\tau^- \to K^-\pi^0\nu_\tau$, is [31]: 0.2536 ± 0.0044. The CVC expectation is [32]: 0.246 ± 0.011 (with which a recent careful re-evaluation by R.J. Sobie is in essential agreement) so that the ratio of experimental to expected branching ratios is: 1.032 ± 0.050. Although this τ-decay, re-written as the physically-inaccessible: $\pi^+ \to \pi^0\tau^+\nu_\tau$, is effectively the same as pion

beta-decay, the operational link to CVC is very different in the two cases.

Although one abandons with reluctance the role of the super-allowed Fermi transitions in testing CVC it seems, at this juncture and in view of the strong other support for CVC just adduced, best to do so in order to gain more confident access to $(ft)_0^*$ and hence to $|V_{ud}|$ and to the testing of the unitarity of the CKM matrix.

Appendix D. G_A^* and $|G_A^*/G_V^*|$

We may extract G_A^* from the data presented in three ways: (i) from the neutron data alone: our provisionally-favoured solution, represented by combination 1, Eq. (37) yields: $G_A^*/(hc)^3 = (1.4556 \pm 0.0016) \times 10^{-5}$ GeV^{-2}; (ii) the $|G_A^*/G_V^*|$ ratio of Eq. (35) plus the G_V^*-value of Eq. (28) yield: $G_A^*/(hc)^3 = (1.4533 \pm 0.0046) \times 10^{-5}$ GeV^{-2}; (iii) the neutron lifetime of Eq. (33) plus the G_V^*-value of Eq. (28) yield: $G_A^*/(hc)^3 = (1.4562 \pm 0.0019) \times 10^{-5}$ GeV^{-2}. These values are concordant but not, of course, independent; we adopt: $G_A^*/(hc)^3) = (1.4557 \pm 0.0015) \times 10^{-5}$ GeV^{-2}. From (iii) above we extract $|G_A^*/G_V^*| = 1.2646 \pm 0.0019$, where the error allows for the common role of G_V^*, in good accord with Eq. (35) with which it may be combined to give: $|G_A^*/G_V^*| = 1.2641 \pm 0.0017$. (The numerical reductions here and in Section 6 use the procedures and recommendations of Ref. [42]).

Note added 8 December 1994

After this paper had been prepared for publication Chalk River [49] announced a remarkable improvement in the determination of the strength of the super-allowed branching in ^{10}C-decay that was discussed in detail in Section 3.1 above. Their result for the branching ratio, BR=(1.4625 ± 0.0025)% may be combined with earlier [34,50], concordant, measurements to yield BR=(1.4638 ± 0.0022)% at $Q(\chi^2|\nu)$=0.57. Using Q_{EC} = 1907.73 ± 0.09 keV (Ref. [51]), reduced by 90 ± 60 eV on account of atomic excitation in the (p,n) reaction upon which this Q_{EC}-value is based [1,10] we have: f = 2.2996 ± 0.0012. The measured lifetime of ^{10}C is [52]: 19.290 ± 0.012 sec; the correction for electron capture raises the lifetime against beta-decay by 0.289% while atomic excitation [1] effectively shortens it by 0.024% so that the corrected lifetime of the super-allowed branch becomes: 1321.3 ± 2.2 sec and: ft = 3038.5 ± 5.3 sec. Application of Eq. (6) with $C_{NS(T)} = -1.35$; $C_{NS(B)} = -1.43$; $C(1) = 0.69$ following the references and discussion as above, and with $\overline{g(W,W_0)} = 12.637$; δ_r=0.185%, leads to: $(ft)^*$ = 3153.4 ± 5.3 sec following Ref. [18] and $(ft)^*$ = 3152.8 ± 5.3 sec following Ref. [16], the latter modified for quenching as in Section 2; these values may be compared with the previsions of Figs. 1(a) and 1(b) respectively, namely 3149.1 and 3147.1 sec. This shows that ^{10}C falls in line with the expectation of the analysis of this paper; in particular, it enhances confidence in the ^{14}O point, the importance of which was stressed in Section 3.1. When the above $(ft)^*$-values for ^{10}C are incorporated into the quadratic fits displayed in Fig. 1 we find: $(ft)_0^*$ = 3141.3 ± 4.7 sec at $Q(\chi^2|\nu)$ = 0.15 to compare with Eq. (13) and $(ft)_0^*$ = 3137.3 ± 4.7 sec at $Q(\chi^2|\nu)$ = 0.23 to compare with Eq. (14); the changes are small and are in line with the expectations of Figs. 11 and 12. It is not possible to carry out the analyses based on the theoretical δ_C and the δ_{Cf} in the same way as in Section 3 because the appropriate material is not available. However, it is likely that $\overline{\delta_{Cf}}$ for ^{10}C would be quite small on account of its low Z-value so we may take δ_{Cf}=0 and repeat the analysis that leads to Eq. (18) finding: $(ft)_0^*$ = 3136.8 ± 4.7 sec at $Q(\chi^2|\nu)$=0.47 in close agreement with the earlier figure. We may also carry out a constrained fit using Z_{max}=23.32 which derives from the mean of the 14 Z_{max} values of the individual δ_{Cf} corrected data sets as discussed in the main text; this yields: $(ft)_0^*$ = 3137.8 ± 3.3 sec at $Q(\chi^2|\nu)$ = 0.70. If we combine the above $(ft)_0^*$ values as before, with a $1/\sigma^2$ weighting, we find: $(ft)_0^*$ = 3137.9 ± 3.3 sec in accord with Eq. (21). In arriving at a new final $(ft)_0^*$ value we must have regard for the necessarily incomplete nature of the analysis that incorporates ^{10}C; we adopt: $(ft)_0^*$ = 3136.8 ± 3.3 sec. The uncertainty is raised to ±3.8 sec by systematic uncertainties as in Section 4; this leads to: $|V_{ud}|^2$ = 0.9514 ± 0.0011 and so to the unitarity test of $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2$ = 1.0000 ± 0.0014.

We have seen that inclusion of the new result for ^{10}C [49] strengthens the conclusions of this paper namely that one should look to the data themselves to determine the magnitude of the nuclear mismatch and that, when this is done, excellent consistency is found between alternative methods of handling those data to reveal this mismatch and that the result, as to $|V_{ud}|$, is itself in close accord with unitarity of the CKM matrix.

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Table 1. The ft are the experimental values corrected for branching, for orbital electron capture and for atomic excitations; they derive from a great number of sources, largely as detailed in Ref. [1], most importantly A.E.R.E. Harwell (masses and lifetimes), Munich and Auckland (masses), Brookhaven National Laboratory (lifetimes) and, especially, Chalk River (masses and lifetimes); they are as quoted in Ref. [10] with very small corrections for ^{50}Mn and ^{54}Co on account of weak Gamow-Teller branches recently reported from Chalk River [28]. The stated uncertainties in these ft-values include not only those due to the experiments themselves but also those stemming from the reduction of the experimental data (chiefly in respect of the f-factors [1]. $C(1)$ is the nominally 1-nucleon contribution to the axial radiative correction in Eq. (6) following Towner [15,18]. $C_{NS(T)}$ is the associated 2-nucleon contribution following Towner and $C_{NS(B)}$ that following Ref. [16] moderated by the factor 0.83 representing the mean of the quenching factors as suggested by Towner [18]. Δ_T and Δ_B are the total radiative corrections independent of Z given in Eq. (6) incorporating $C_{NS(T)}$ and $C_{NS(B)}$ respectively. δ_r is the total Z-dependent radiative correction following Ref. [1] and $\pm$ the estimated uncertainty in δ_r. (Δ_T, Δ_B, δ_r and $\pm$ are in percent).

Body	ft sec	$C(1)$	$C_{NS(T)}$	$C_{NS(B)}$	Δ_T	Δ_B	δ_r	$\pm$
^{14}O	3039.2± 2.0	0.66	-0.88	-1.18	3.510	3.439	0.235	0.006
^{26}Al^m	3038.4± 1.2	0.64	0.20	0.35	3.578	3.614	0.346	0.011
^{34}Cl	3050.0± 2.2	0.61	-0.13	-0.11	3.382	3.386	0.418	0.016
^{38}K^m	3049.1± 3.4	0.59	-0.09	-0.08	3.347	3.350	0.451	0.018
^{42}Sc	3045.8± 1.3	0.62	0.40	0.51	3.446	3.472	0.487	0.022
^{46}V	3046.0± 1.9	0.62	0.14	0.26	3.348	3.377	0.513	0.025
^{50}Mn	3044.4± 1.6	0.61	0.14	0.26	3.316	3.345	0.540	0.029
^{54}Co	3049.9± 1.7	0.60	0.17	0.30	3.293	3.324	0.566	0.034

Table 2. Theoretical estimates of the nuclear mismatch $\delta_C = \delta_{CM} + \delta_{BE}$; under δ_{CM} the column headed T is from Ref. [24] for ^{14}O, ^{26}Al^m and ^{34}Cl, by private communication from Dr. I.S. Towner for ^{42}Sc and from Ref. [28] for the remainder; the column headed O is from Ref. [27] for the first five entries and by private communication from Dr. W.E. Ormand for ^{42}Sc the entries in brackets being taken from the T column following discussion in Ref. [28]; under δ_{BE} the column headed W is from Ref. [25] updated for ^{14}O by the use of current parameterization for the Saxon-Woods potential employed and for ^{50}Mn and ^{54}Co by the use of present more extensive experimental data on the relevant parent states; the column headed T is from Ref. [24] and that headed O is from Ref. [27]. The column headed B is from Ref. [29].

Body	δ_{CM}		δ_{BE}			δ_C
	T	O	W	T	O	B
^{14}O	0.00	0.01	0.29	0.28	0.18	0.12
^{26}Al^m	0.06	0.01	0.35	0.27	0.23	0.05
^{34}Cl	0.02	0.06	0.57	0.62	0.42	0.11
^{38}K^m	0.10	0.11	0.34	0.54	0.38	0.10
^{42}Sc	0.05	0.11	0.31	0.35	0.28	0.12
^{46}V	0.09	0.09	0.31	0.36	0.20	0.05
^{50}Mn	0.07	(0.07)	0.41	0.40	0.28	0.05
^{54}Co	0.05	(0.05)	0.47	0.56	0.34	0.05

Table 3. Analysis of the ft-values of Table 1 reduced to $(ft)^*$-values following Eq. (5) using shell-model sources of the δ_C. The first letter of the group of three under Set, when W, T or O, signifies the use of the corresponding column under δ_{BE} of Table 2; the second letter, when T or O indicates the use of that column under δ_{CM}; when both first and second letters are B column δ_C of Table 2 is implied; the third letter indicates the use of the associated column under C_{NS} of Table 1 in constructing Δ^R. The columns headed Constant refer to the direct averaging of the $(ft)^*$-values from the 8 Z-values for each set with the associated confidence level $Q(\chi^2|\nu)$; the columns headed Linear and Quadratic refer to fits to the $(ft)^*$-values linear and quadratic, respectively, in Z (of the daughter nucleus) with the $(ft)_0^*$ the extrapolations to $Z = 0$ and the $Q(\chi^2|\nu)$ the respective confidence levels. ($x(-y)$ signifies $x \times 10^{-y}$).

Set	Constant		Linear		Quadratic	
	$\overline{(ft)^*}$	$Q(\chi^2\|\nu)$	$(ft)_0^*$	$Q(\chi^2\|\nu)$	$(ft)_0^*$	$Q(\chi^2\|\nu)$
WTT	3148.9	1.6(-6)	3140.5	6.2(-3)	3131.4	0.013
WTB	3149.5	4.3(-8)	3139.9	5.0(-3)	3127.0	0.039
TTT	3148.5	4.7(-3)	3143.9	0.029	3135.0	0.060
TTB	3149.1	1.0(-4)	3143.1	4.4(-3)	3130.2	0.037
OTT	3152.0	5.3(-4)	3143.9	0.46	3140.3	0.39
OTB	3152.6	1.6(-5)	3143.1	0.37	3135.9	0.51
WOT	3148.8	1.3(-3)	3141.8	0.18	3135.0	0.23
WOB	3149.3	2.3(-5)	3141.0	0.063	3130.3	0.21
TOT	3148.4	0.049	3145.2	0.088	3138.3	0.11
TOB	3149.0	7.3(-4)	3144.3	5.0(-3)	3133.6	0.018
OOT	3151.9	0.020	3145.0	0.89	3143.6	0.82
OOB	3152.5	5.0(-4)	3144.2	0.50	3139.2	0.50
BBT	3159.9	2.1(-10)	3146.9	0.29	3138.2	0.50
BBB	3160.5	1.1(-13)	3146.0	0.068	3133.8	0.37

Table 4. Analysis of the ft-values of Table 1 reduced to $(ft)^*$-values following Eq. (5) but using $(ft)^* = ft(1 + \Delta^R)(1 - \delta_{Cf})$. The notation is as for Table 3, the first two letters indicating the respective origins of the δ_{BE} and δ_{CM} from the fluctuations, δ_{Cf}, in whose $\delta_C = \delta_{BE} + \delta_{CM}$ the $(ft)^*$ are derived. The columns headed Unconstrained result from a free quadratic fit in Z; the columns headed Constrained require the quadratic fit to peak at $Z_{max} = 22.25$ as detailed in the text.

Set	Unconstrained		Constrained	
	$(ft)_0^*$	$Q(\chi^2\|\nu)$	$(ft)_0^*$	$Q(\chi^2\|\nu)$
WTT	3135.3	0.14	3136.3	0.21
WTB	3130.4	0.25	3132.7	0.33
TTT	3137.8	0.27	3137.6	0.38
TTB	3132.8	0.14	3134.0	0.20
OTT	3141.7	0.54	3135.9	0.39
OTB	3136.7	0.63	3132.2	0.57
WOT	3136.7	0.46	3137.5	0.58
WOB	3131.5	0.42	3133.7	0.50
TOT	3139.5	0.18	3139.0	0.26
TOB	3134.3	0.033	3135.2	0.058
OOT	3143.3	0.82	3137.2	0.58
OOB	3138.4	0.54	3133.6	0.46
BBT	3137.8	0.61	3138.0	0.73
BBB	3132.7	0.43	3134.3	0.54

Table 5. Results of treating the experimental ft-values of Table 1 using different methods of extrapolation to $(ft)_0^*$ at $Z = 0$ as described in the text using various shell-model δ_C-values. The column Eq. gives the equation reference to the result in the text. The letter U means that the fit is an unconstrained quadratic in Z; C means that the fit has been constrained by Z_{max}-values variously derived as explained in the text.

Method	Eq.	$(ft)_0^*$ sec
No δ_C-correction (U)	15	3137.4 ± 5.7
Full theoretical δ_C (U)	16	3135.1 ± 5.1
δ_{Cf} individual (U)	17	3136.4 ± 5.1
δ_{Cf} averaged (U)	18	3134.7 ± 5.2
δ_{Cf} individual (C)	19	3135.5 ± 3.7
δ_{Cf} individual (C)	20	3136.8 ± 4.0

Table 6. Analysis of the ft-values of Table 1 reduced to $(ft)^*$-values following Eq. (5) using hybrid harmonic-oscillator-shell-model sources for the δ_C. The notation under Set follows that described for Table 4 except that the first letter refers to the source of the harmonic oscillator mismatch used for deriving the analogue of δ_{BE} as described in the text: D = Ref. [21]; F = Ref. [22]; L = Ref. [23]; T = Ref. [24]. The entries are the $(ft)_0^*$-values in seconds. The column headed δ_C refers to the use of the full theoretical δ_C in Eq. (5); that headed $\delta_{Cf}(U)$ refers to the unconstrained quadratic fit in Z of the $(ft)^*$-values derived from the use of the δ_{Cf} in Eq. (5); that headed $\delta_{Cf}(C)$ derives from the same $(ft)^*$-values as for $\delta_{Cf}(U)$ but with the quadratic fits constrained by the Z_{max}-value given by the fit to the average of the 16 sets of $(ft)^*$-values for each Z-value.

Set	δ_C	$\delta_{Cf}(U)$	$\delta_{Cf}(C)$
DTT	3142.8	3140.6	3139.9
DTB	3138.3	3135.6	3137.2
DOT	3146.2	3143.0	3137.1
DOB	3141.6	3138.0	3134.5
LTT	3141.5	3140.3	3139.2
LTB	3137.0	3135.2	3136.5
LOT	3144.9	3141.5	3140.7
LOB	3140.3	3136.6	3138.0
FTT	3141.7	3139.4	3138.5
FTB	3137.1	3134.3	3135.9
FOT	3145.0	3140.5	3140.3
FOB	3140.5	3135.4	3137.6
TTT	3144.6	3141.6	3139.3
TTB	3140.1	3136.6	3136.6
TOT	3148.1	3144.2	3141.2
TOB	3143.5	3139.1	3138.6

Table 7. As Table 5 but using the hybrid harmonic-oscillator-shell-model δ_C-values as described in the text. (The first of the constrained entries refers to constraint by the Z_{max}-value derived from the average of the δ_{Cf}-corrected values; the second of the constrained entries derives from the 240 separate fits using the 16 values of Z_{max} as described in the text for the shell-model-based treatment.)

Method	$(ft)_0^*$ sec
Full theoretical δ_C (U)	3142.1 ± 5.2
δ_{Cf} individual (U)	3139.0 ± 5.3
δ_{Cf} averaged (U)	3138.9 ± 5.6
δ_{Cf} individual (C)	3138.2 ± 3.7
δ_{Cf} individual (C)	3139.2 ± 4.2

Figure Captions

1. The experimental ft-values of Table 1 adjusted only for the radiative correction, $\Delta^R = \Delta + \delta_r$, taken from Table 1: $(ft)^* = ft(1 + \Delta^R)$. For Fig. 1(a) Δ_T, with $C_{NS(T)}$ in Eqs. (6) and (10), is used for Δ in Δ^R; for Fig. 1(b) $C_{NS(B)}$ takes the place of $C_{NS(T)}$ to give Δ_B. The curves are free quadratic fits in Z resulting in the $(ft)_0^*$ intercepts at $Z = 0$ given in the text. (Z refers to the daughter body.)
2. The intercepts $(ft)_0^*$ at $Z = 0$ of fits freely quadratic in Z to the values $(ft)^* = ft(1 + \Delta^R)(1\text{-}\delta_C)$ where the shell-model δ_C, Δ^R- sets are as described in the text and in the heading to Table 3.
3. The mean fluctuations $\overline{\delta_{Cf}}$ resulting from fitting the various sets of δ_C, derived from all combinations given by Table 2, individually to the form $\delta_{C*} = aZ + bZ^2$ so that $\delta_{Cf} = \delta_C - \delta_{C*}$.
4. As for Fig. 2 but using $(ft)^* = ft(1 + \Delta^R)(1\text{-}\delta_{Cf})$.
5. Free quadratic fitting in Z to the averages of the 14 sets of $(ft)^* = ft(1+\Delta^R)(1-\delta_{Cf})$ for each Z-value.
6. As Fig. 4 but constraining the quadratic fitting by Z_{max} = 22.25 from the fit shown in Fig. 5.
7. The $(ft)_0^*$-χ^2 distribution for the 182 constrained quadratic fits given by imposing the Z_{max} derived from each δ_C, Δ^R-set upon the other 13 using $(ft)^* = ft(1 + \Delta^R)(1 - \delta_{Cf})$.
8. As Fig. 2 but using hybrid harmonic-oscillator-shell-model δ_C, Δ^R - sets. The labelling is as described in the heading to Table 6.
9. As Fig. 4 but using hybrid δ_c, Δ^R - sets as in Fig. 8.
10. As Fig. 6 but using hybrid δ_C, Δ^R-sets as in Figs. 8 and 9. constrained by Z_{max} from the hybrid analogue of Fig. 5.
11. The effect on the accuracy, $\pm(ft)_0^*$, of the extraction of the $(ft)_0^*$-value by unconstrained quadratic fitting in Z, as in Fig. 5, of adding a ^{10}C $(ft)^*$-value to the present data-set, of accuracy $\pm^{10}$C and falling on the line of Fig. 5. The dashed line shows the accuracy afforded by the present data-set of Table 1.
12. The effect on the inferred $(ft)_0^*$-value of a determination of the $(ft)^*$-value for ^{10}C of an accuracy given by the numbers (±sec) labelling the lines. Δ^{10}C is the amount by which the $(ft)^*$-value for ^{10}C exceeds that corresponding to $Z = 5$ on the line of Fig. 5 (3147.5 sec); $\Delta(ft)_0^*$ is the amount by which $(ft)_0^*$ is raised by such a determination.
13. Comparison of the G_V^*-value from the present analysis of super-allowed Fermi decay (the vertical band) following Eq. (28) with the ellipses of uncertainty in the $G_A^* - G_V^*$ plane resulting from the four combinations of neutron lifetime and decay-asymmetry data defined in the text by Eqs. (37) through (40). (The units for the ordinate are as for the abscissa.)
14. The mean charge, $\overline{Q}$, of the fermionic fields between which beta-decay takes place inferred as function of the axial mass m_A of Eq. (42). The full lines derive from a free quadratic fit to the $(ft)^*$-values from super-allowed Fermi decay together with the constraint $(ft)_0^*$ of Eq. (41) implied by unitarity of the CKM matrix. The dashed lines are at the 1σ-values of the axial mass m_A as adopted in Eq. (9).

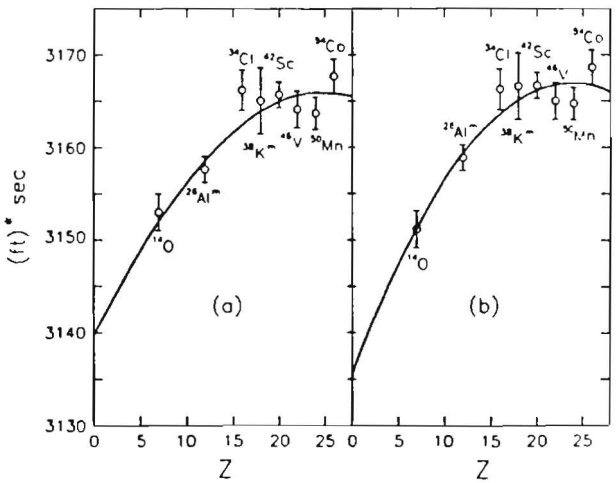

Fig. 1

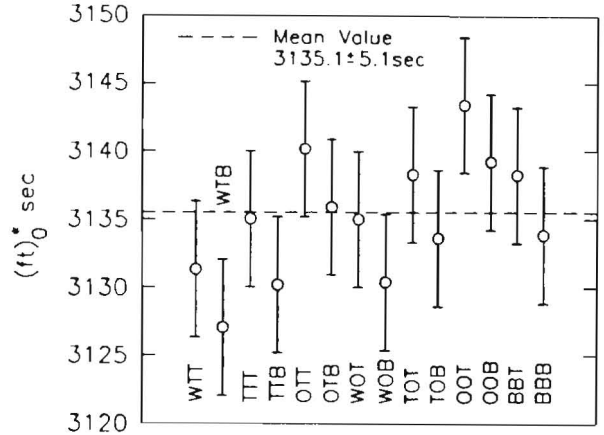

Fig. 2

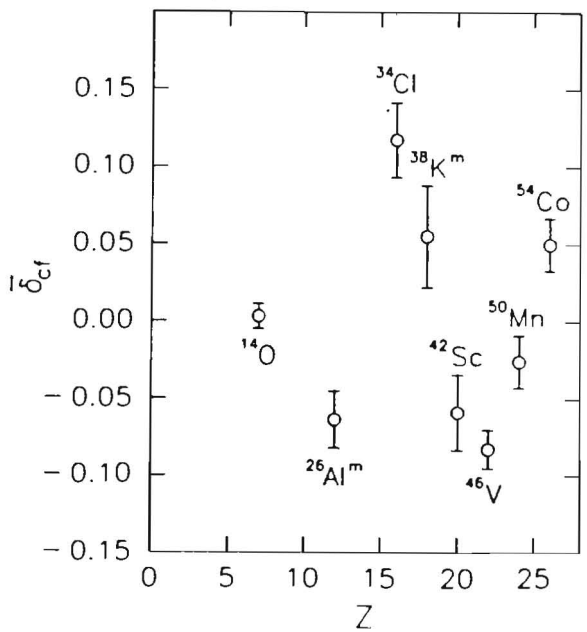

Fig. 3

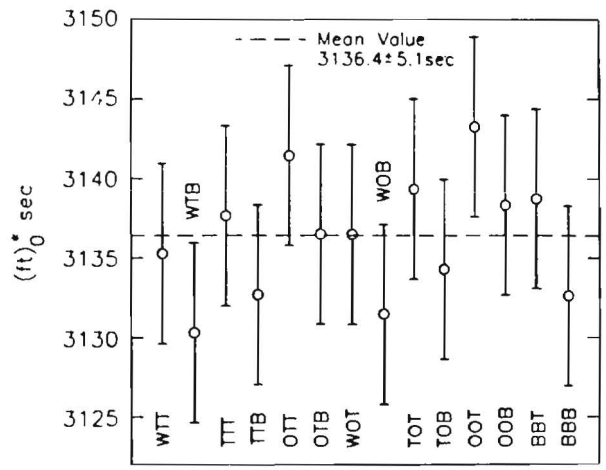

Fig. 4

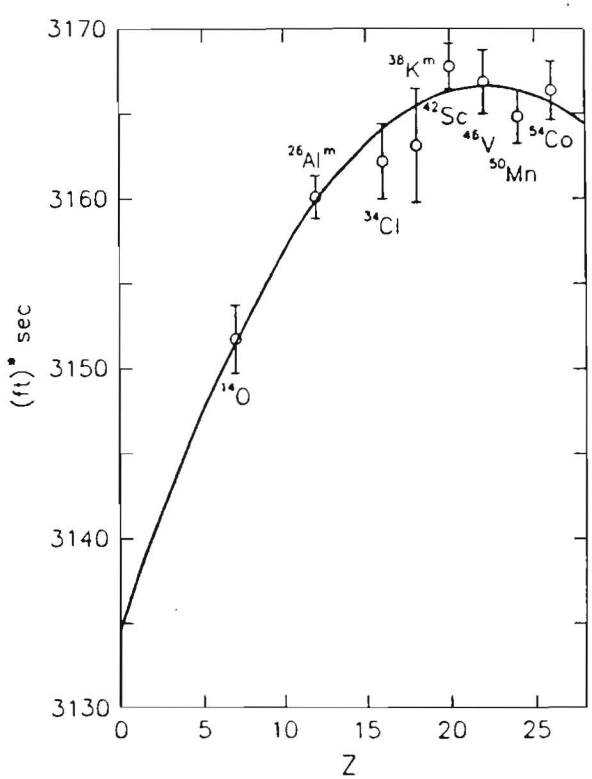

Fig. 5

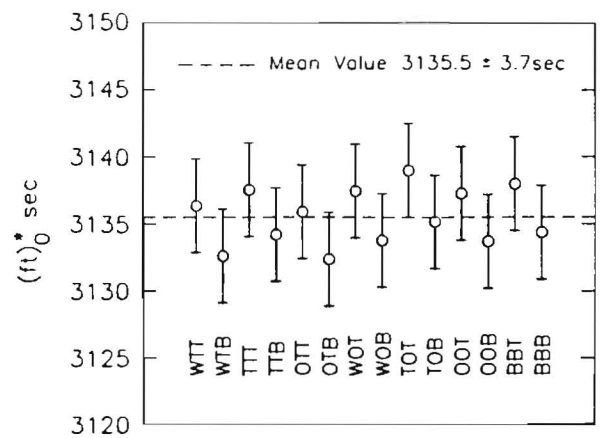

Fig. 6

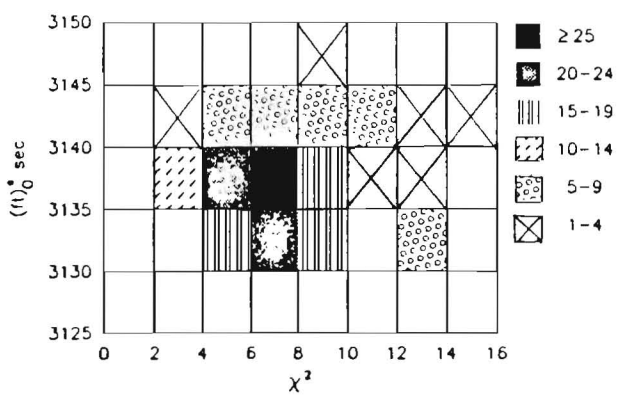

Fig. 7

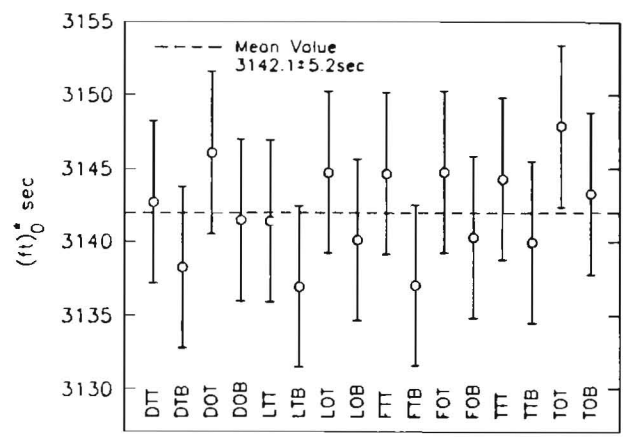

Fig. 8

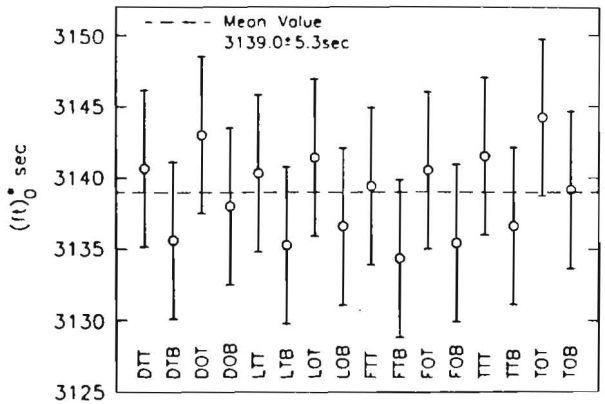

Fig. 9

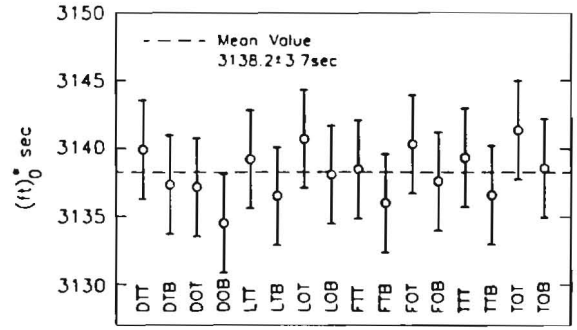

Fig. 10

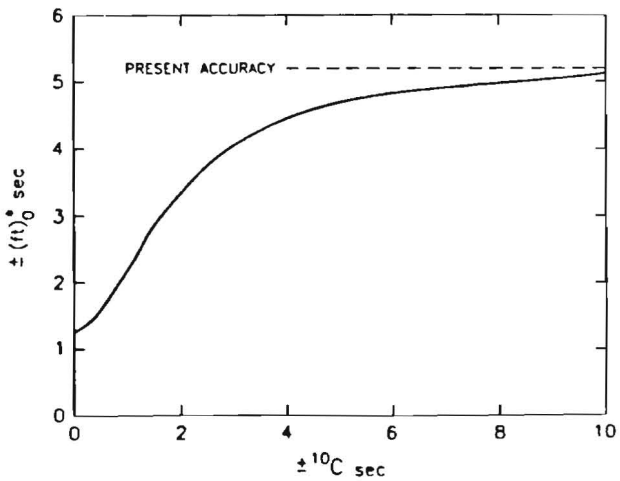

Fig. 11

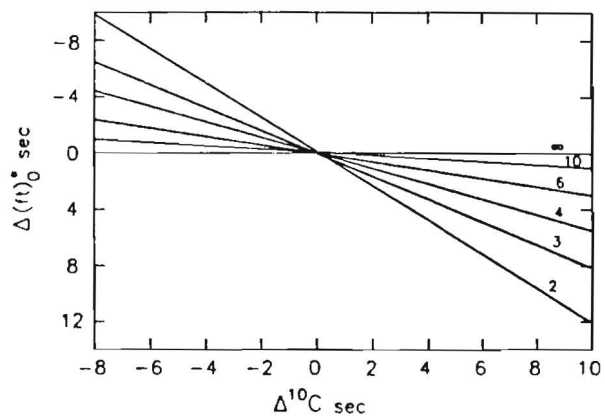

Fig. 12

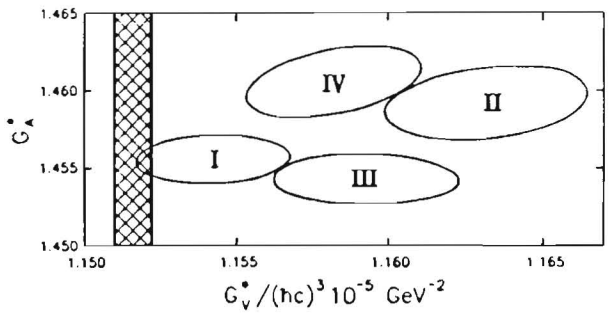

Fig. 13

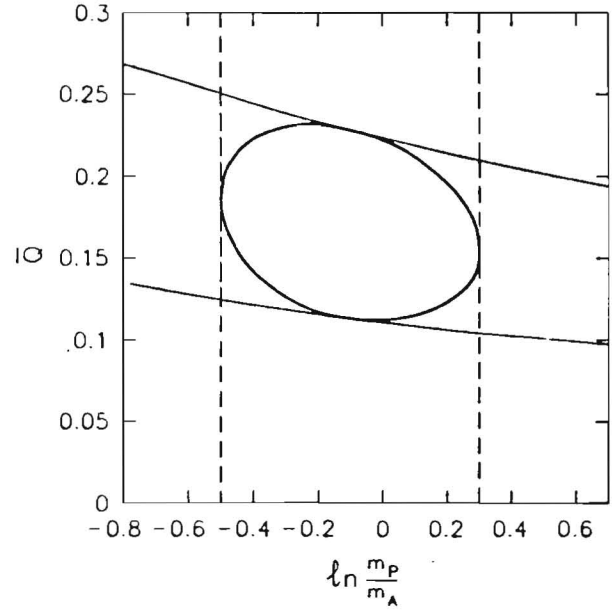

Fig. 14

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