Fermilab Library
0 1160 0027804 8

TRI-PP-93 80

TRI PP 93 80
Sep 1993

On the structure function of photon

Zaixin Xu*
TRIUMF, 4004 Wesbrook Mall, Vancouver, B.C., Canada V6T 2A3
and
Department of Physics and Astronomy, University of Victoria,
Victoria, B.C., Canada V8W 3P6

Abstract

A general solution of the modified Altarelli-Parisi equations in moment representation satisfied by the parton distributions of photon, including a general form of Born term, is obtained by using the variation coefficient procedure. The result shows that it is necessary to apply the general solution to find the x- and Q^2-dependence of photon structure function, especially for experimentally relevant Q^2 values.

NAME | I.D No. | M.S.
RETURN TO FERMILAB LIBRARY

(submitted to Physics Letters B)

FERMILAB
DEC 02 1993
LIBRARY

*On leave from Department of Physics, East China Normal University, Shanghai 200062, China

I. Introduction

For the past ten years or so, the photon structure function has considerably raised theoretical and experimental interest. For its pointlike portion, the strong interaction correction to the Fermi-Weizacker-Williams (FWW) approximation in QED was studied by using various methods including the operator product expansion [1], ladder techniques [2] and the modified Altarelli-Parisi (MAP) equations [3,4]. It has been found that the strong interaction effect in color space does not modify the Q^2-dependence, but it changes the x-dependence. Since then many interesting papers have been published [5,6].

However, it should be noticed that in general the FWW approximation includes non-logarithmic terms and these terms are quite important for experimentally relevant Q^2 values $\lesssim 100$ GeV^2 [7,8]. In this paper, we investigate the first-order strong interaction effect on the parton distributions in the photon by using the MAP scheme in which the general FWW approximation is considered. The master equations are listed in Sec. 2. In Sec. 3, we obtain the general solution for the MAP equation in moment representation. In a special case it can be reduced to the form which was found earlier [3 ~ 5]. The general solution shows that even taking the leading-logarithmic approximation of FWW for the photon structure function, the first-order strong interaction effect modifies not only its x-dependence but also its Q^2-dependence. In Sec. 4, we investigate carefully the singularity of the solution and list the procedure for the reproduction of x- dependence of the parton distributions in the photon. Finally, the numerical result and the discussion are given in Sec. 5.

II. Modified Altarelli-Parisi equations

The moment representation of the equations satisfied by the quark and gluon distributions in the photon can be written as

$$\frac{d}{dt}q_\gamma^{NS}(n,t) = d_n^{qq}q_\gamma^{NS}(n,t) + \frac{\alpha}{\pi}(e_q^2)^{NS}\frac{d}{dt}F^B_{q/\gamma}(n,t), \tag{1}$$

$$\begin{aligned}\frac{d}{dt}\Sigma_\gamma(n,t) &= d_n^{qq}\Sigma_\gamma(n,t) + d_n^{qg}G_\gamma(n,t) + \frac{\alpha}{\pi}2N_f < e_q^2 > \frac{d}{dt}F^B_{q/\gamma}(n,t),\\ \frac{d}{dt}G_\gamma(n,t) &= d_n^{gq}\Sigma_\gamma(n,t) + d_n^{gg}G_\gamma(n,t),\end{aligned} \tag{2}$$

where $t = ln(ln(Q^2/\Lambda^2)/ln(Q_0^2/\Lambda^2))$, the moments of the splitting functions or the anomalous dimension matrix elements can be found, for example, in Ref.[4]

In this paper, as an example, we use the FEP distribution [8] as the general Born term:

$$q_\gamma^B(x,t) = \frac{\alpha}{\pi}e_q^2F^B_{q/\gamma}(x,t) = \frac{\alpha}{\pi}e_q^2 ln\frac{Q^2}{\Lambda^2}f^B_{q/\gamma}(x,t), \tag{3}$$

and

$$F^B_{q/\gamma}(x,t) = N_c\left\{\left[x^2+(1-x)^2\right]ln\frac{W}{t_0}+[6x(1-x)-1](1-\frac{t_0}{W})\right\}, \tag{4}$$

where $W = Q^2/x$, $t_0 = 1$ GeV^2.

III. Solution

Equation(1) for the non-singlet sector of the moment of quark distribution in a photon has the general solution:

$$q^{NS}_\gamma(n,t) = \frac{\alpha}{\pi}(e_q^2)^{NS}e^{d^{qq}_n}\left[\int dte^{-d^{qq}_n t}\frac{d}{dt}F^B_{q/\gamma}(n,t)+\beta_0\right]. \tag{5}$$

Equation(2) is a set of linear inhomogeneous equations. The general solution of the homogeneous equations corresponding to it can be expressed as

$$\begin{aligned}\Sigma_\gamma(n,t) &= C_1e^{\lambda^+_n t}+C_2e^{\lambda^-_n t},\\ G_\gamma(n,t) &= \frac{1}{d^{qg}_n}\left[C_1(\lambda^+_n-d^{qq}_n)e^{\lambda^+_n t}+C_2(\lambda^-_n-d^{qq}_n)e^{\lambda^-_n t}\right],\end{aligned} \tag{6}$$

where

$$\lambda^\pm_n = \frac{1}{2}\left\{d^{qq}_n+d^{gg}_n\pm\left[(d^{qq}_n-d^{gg}_n)^2+4d^{qg}_nd^{gq}_n\right]^{1/2}\right\}, \tag{7}$$

are the eigenvalues of the anomalous dimension matrix.

A solution of the inhomogeneous equations can be found by using the variation procedure. In this precedure, the coefficients C_1 and C_2 in Eq.(6) should be transformed into the functions of t, and they satisfy the following equations:

$$\begin{aligned}e^{\lambda^+_n t}\frac{dC_1(t)}{dt}+e^{\lambda^-_n t}\frac{dC_2(t)}{dt} &= \frac{\alpha}{\pi}2N_f<e_q^2>\frac{d}{dt}F^B_{q/\gamma}(n,t),\\ \frac{1}{d^{qg}_n}\left[(\lambda^+_n-d^{qq}_n)e^{\lambda^+_n t}\frac{dC_1(t)}{dt}+(\lambda^-_n-d^{qq}_n)e^{\lambda^-_n t}\frac{dC_2(t)}{dt}\right] &= 0.\end{aligned} \tag{8}$$

The solution of Eq.(8) is

$$\begin{aligned}C_1(t) &= \frac{\alpha}{\pi}2N_f<e_q^2>\left(\frac{\lambda^-_n-d^{qq}_n}{\lambda^-_n-\lambda^+_n}\right)\left[\int dte^{-\lambda^+_n t}\frac{d}{dt}F^B_{q/\gamma}(n,t)+\beta_1\right],\\ C_2(t) &= \frac{\alpha}{\pi}2N_f<e_q^2>\left(\frac{\lambda^+_n-d^{qq}_n}{\lambda^+_n-\lambda^-_n}\right)\left[\int dte^{-\lambda^-_n t}\frac{d}{dt}F^B_{q/\gamma}(n,t)+\beta_2\right].\end{aligned} \tag{9}$$

In Eqs.(5) and (9), β_0, β_1 and β_2 can be determined from the condition: $q_\gamma(n,t) = q^B_\gamma(n,t)$ if $Q \le Q_0$

If we take an integration by part on Eqs.(5) and (9), the solution, which satisfies the initial condition, of MAP moment equations can be written as

$$\begin{aligned}q_\gamma(n,t) &= \frac{\alpha}{\pi}\Big[e_q^2F^B_{q/\gamma}(n,Q^2)+(e_q^2)^{NS}A_0e^{d^{qq}_n t}\\ &\quad+<e_q^2>\left(\frac{\lambda^-_n-d^{qq}_n}{\lambda^-_n-\lambda^+_n}A_1e^{\lambda^+_n t}+\frac{\lambda^+_n-d^{qq}_n}{\lambda^+_n-\lambda^-_n}A_2e^{\lambda^-_n t}\right)\Big],\\ G_\gamma(n,t) &= \frac{\alpha}{\pi}2N_f<e_q^2>\frac{d^{gq}}{\lambda^+_n-\lambda^-_n}\left(A_1e^{\lambda^+_n t}-A_2e^{\lambda^-_n t}\right),\end{aligned} \tag{10}$$

where

$$\begin{aligned}ln\frac{Q^2}{\Lambda^2}A_0 &= d^{qq}_n\int_0^t dte^{-d^{qq}_n t}F^B_{q/\gamma}(n,t),\\ ln\frac{Q^2}{\Lambda^2}A_1 &= \lambda^+_n\int_0^t dte^{-\lambda^+_n t}F^B_{q/\gamma}(n,t),\\ ln\frac{Q^2}{\Lambda^2}A_2 &= \lambda^-_n\int_0^t dte^{-\lambda^-_n t}F^B_{q/\gamma}(n,t).\end{aligned} \tag{11}$$

It is obvious that the terms, other than $e_q^2F^B_{q/\gamma}(n,t)$, in Eq.(10) illustrate the strong interaction effect on the parton distributions of photon.

To take leading-logarithmic approximation for the Born term in Eq.(10), (11) and set $Q_0^2 = t_0$, we have

$$\begin{aligned}q_\gamma(n,t) &= \frac{\alpha}{\pi}ln\frac{Q^2}{\Lambda^2}C_n\Big\{\frac{(e_q^2)^{NS}}{1-d^{qq}_n}\left(1-e^{-(1-d^{qq}_n)t}\right)\\ &\quad+\frac{<e_q^2>}{(\lambda^-_n-\lambda^+_n)}\left[\frac{\lambda^-_n-d^{qq}_n}{1-\lambda^+_n}\left(1-e^{-(1-\lambda^+_n)t}\right)-(\lambda^+_n\leftrightarrow\lambda^-_n)\right]\Big\},\\ G_\gamma(n,t) &= \frac{\alpha}{\pi}ln\frac{Q^2}{\Lambda^2}2N_f\frac{<e_q^2>d^{gq}_n}{(\lambda^+_n-\lambda^-_n)}C_n\left[\frac{1-e^{-(1-\lambda^+_n)t}}{1-\lambda^+_n}-(\lambda^+_n\leftrightarrow\lambda^-_n)\right],\end{aligned} \tag{12}$$

where

$$C_n \equiv \left[f^B_{q/\gamma}(n,t)\right]_{L.A} = N_c\frac{2+n+n^2}{n(n+1)(n+2)}. \tag{13}$$

If we consider the case of $t\to\infty$ or $t_0 = \Lambda^2$, noting $\lambda^-_n<0$; $\lambda^+_n<0$ (if $n>2$), the apparent solution (12) can be reduced to

$$\begin{aligned}q_\gamma(n,t) &= \frac{\alpha}{\pi}ln\frac{Q^2}{\Lambda^2}C_n\left[\frac{(e_q^2)^{NS}}{1-d^{qq}_n}+<e_q^2>\frac{1-d^{qq}_n}{(1-\lambda^+_n)(1-\lambda^-_n)}\right],\\ G_\gamma(n,t) &= \frac{\alpha}{\pi}ln\frac{Q^2}{\Lambda^2}2N_fC_n\frac{<e_q^2>d^{gq}_n}{(1-\lambda^+_n)(1-\lambda^-_n)}.\end{aligned} \tag{14}$$

The above result has been obtained in the early discussion on the photon structure function [3 ~ 5] and it means that the strong interaction effect does not modify the

Q^2-dependence of the effective photon approximation in QED ($\sim ln Q^2/\Lambda^2$), but it changes the x-dependence [1 ~ 6]. However, in the general case the picture for the photon structure function is quite different. The solution in Eq.(10) and (12) show that even for the leading- logarithmic approximation the first-order strong interaction in color space affects not only its x-dependence but also its Q^2-dependence.

IV. Singularity and polynomial expansion

In order to find the x-dependent distribution, we use the method proposed earlier [3]. After factoring out the singular behavior at $x \simeq 0$ the distribution can be expressed in a series of Jacobi Polynomials.

From Eq.(14), the rightmost singularity in the complex n-plane of the moments occurs at $1 - \lambda_n^+ = 0$ or $n = 1.596$ ($N_f = 4$) [3]. However, to do this in the general case we should be very careful. In fact, according to Eq.(12), $q_\gamma(n,t)$ and $G_\gamma(n,t)$ do not possess singularity when $1 - \lambda_n^+ = 0$, because

$$\lim_{\lambda_n^+ \to +1} \frac{1 - e^{-(1-\lambda_n^+)t}}{1 - \lambda_n^+} = t\,. \tag{15}$$

However, if noting that $d_n^{qq} = 0^-$, $d_n^{gg} = 1/\pi b$, $d_n^{qg} = d_n^{gq} = +\infty$ and $\lambda_n^- = 0^-$ when $n \to +1$ or $\lambda_n^+ = +\infty$, we have

$$\begin{aligned}
&\lim_{n\to 1^+} \frac{(\lambda_n^- - d_n^{qq})}{(\lambda_n^- - \lambda_n^+)(1-\lambda_n^+)}\left(1 - e^{-(1-\lambda_n^+)t}\right) = \frac{e^{\lambda_n^+ t}}{(\lambda_n^+)^3} = +\infty\,,\\
&\lim_{n\to 1^+} \frac{d_n^{gq}}{(\lambda_n^+ - \lambda_n^-)(1-\lambda_n^+)}\left(1 - e^{-(1-\lambda_n^+)t}\right) = \frac{e^{\lambda_n^+ t}}{\lambda_n^+} = +\infty\,.
\end{aligned} \tag{16}$$

It means that the rightmost singularity in complex n-plane for the correction portion of parton distribution of photon occurs at $n = 1$. Thus the distribution can be written as

$$xq_\gamma(x,t) \equiv \frac{\alpha}{\pi} e_q^2 ln \frac{Q^2}{\Lambda^2} x f_{q/\gamma}(x,t) = \frac{\alpha}{\pi} e_q^2 ln \frac{Q^2}{\Lambda^2}\left[x f_{q/\gamma}^B(x,Q^2) + x f_{q/\gamma}^{s.c.}(x,Q^2)\right]\,, \tag{17}$$

and the strong interaction correction term can be obtained from the following quite simple formula [9]: ($i = q$ or g)

$$\begin{aligned}
f_{i/\gamma}^{s.c.}(x,Q^2) &= \sum_{n=0}^{N} C_n(Q^2) x^n\,,\\
C_n(Q^2) &= \sum_{l\geq n}^{N} b_n(Q^2) p(l,n)\,,\\
b_n(Q^2) &= \sum_{k=0}^{n} p(n,k) f_{i/\gamma}^{s.c.}(n,Q^2)\,,\\
p(n,k) &= (-1)^{k+n}(2n+1)^{1/2}\frac{(n+k)!}{(k!)^2(n-k)!}\,.
\end{aligned} \tag{18}$$

V. Numerical result and discussion

In the five-term approximation, the coefficients C_i ($i = 0 \sim 4$) for the first-order strong interaction correction of the quark and gluon distributions in a photon at $Q^2 = 10,\ 10^2,\ 10^4$ GeV^2 are listed in Table 1. We wish to point out that the five-term polynomial form can reproduce the values of the moments given by Eq.(10) accurately. For the first sixteen moments of $f_{q/\gamma}^{s.c.}(x,Q^2)$ ($q = 2/3$, $Q^2 = 10$ GeV^2), for example, the difference between the value given by the five-term expression and Eq.(10) is less than 1%. The full Born term $x f_{q/\gamma}^B(x,Q^2)$, correction distribution $x f_{q/\gamma}^{s.c.}(x,Q^2)$ and the total distribution $x f_{q/\gamma}(x,Q^2)$ for the case of $q = 2/3$, $Q^2 = 10$ GeV^2 are shown in Fig. 1. In Fig. 2, we plot the quark distribution $x f_{q/\gamma}(x,Q^2)$ for the case of $q = 2/3, 1/3$ and the gluon distribution $x f_{g/\gamma}(x,Q^2)$ at three energy scales. In our calculation we set $N_f = 4$. Fig. 2 shows that the strong interaction effect on the x- and Q^2-dependence should all be noticed, especially for the region of experimentally relevant Q^2 values ($\lesssim 100$ GeV^2). One of the most important factors which cause the larger effect is due to an important contribution of the non-logarithmic terms in Born term in Eq.(4). The correction distribution caused by strong interaction, illustrated in Fig. 1, has a small positive value in the small x region and a relatively large negative value in the large x region. Thus the total distribution increases in small x region and decreases in large x region due to the effect of strong interaction. This result can be interpreted from Fig. 3, which shows the ratio of moment of the strong interaction correction to Born distribution. The values of ratio increase as n increases. It implies that the strong interaction effect makes the quark distribution in the photon q_γ falling at large x but rising at small x. We can also see from Fig. 2 and Fig.3 that the correction from the strong interaction does increase with the increasing energy and at last approach to a limit case (see the curve for $Q^2 = 10^9$ GeV^2 in Fig. 3), which has been predicted in the early papers.

Acknowlegements

I would like to thank the University of Victoria, Prof. H. Fearing and other members of the TRIUMF theory group for their help and hospitality.

References

1. E. Witten, Nucl. Phys. **B120** (1977) 189.
2. C. Llewellyn Smith, Phys. Lett. **79B** (1978) 83.
3. A. Nicolaidis, Nucl.Phys. **B163** (1980) 156.
4. G. Altarelli, Phys. Rep. **81** (1982) 1.
5. J. Hassan and P. Pilling, Nucl. Phys. **B187** (1981) 563; M.Gluck and E.Reya, Phys. Rev. **D28** (1983) 2749; Zaixin Xu, Phys. Rev. **D30** (1984) 1440.

6. M. Fontannaz and E. Pilon, Phys. Rev. **D45** (1992) 382; M. Gluck and Reya and A. Vogt, Phys. Rev. **D45** (1992) 3986; **D46** (1992) 1973; J. Forshaw and P. Harriman, Phys. Rev. **D46** (1992) 3778.

7. for example: P. Kessler, *9th Int. Workshop on Photon-Photon Collisions, San Diego, Calif., USA* (March 22-26,1992).

8. J. Field, F. Kapusta and L. Poggiol, Phys. Lett. **B181** (1986) 362; Z. Phys. **C36** (1987) 121.

9. Zaixin Xu, TRIUMF preprint: *TRI-PP-93 56*, June 1993 (will be published in Phys. Rev. **D**).

Table 1: Coefficients of the five-term expansion for the strong interaction correction of parton distributions in a photon $f^{s.c.}_{i/\gamma}$ ($i = q$ or g) at three energy scales

Q^2	$f^{s.c.}_{i/\gamma}$	C_0	C_1	C_2	C_3	C_4
	$q = 2/3$	0.0176	0.3134	-1.6728	2.4300	-1.3779
10 GeV^2	$q = 1/3$	0.0159	0.0186	-0.3162	0.5484	-0.3402
	g	0.1245	-0.7863	2.1093	-2.4846	1.0434
	$q = 2/3$	0.0426	0.2634	-1.9004	3.1434	-1.9827
10^2 GeV^2	$q = 1/3$	0.0429	-0.2004	0.3300	-0.2217	-0.0567
	g	0.2298	-1.7226	5.0577	-6.2412	2.6952
	$q = 2/3$	0.0843	-0.0351	-1.2903	2.9493	-2.3133
10^4 GeV^2	$q = 1/3$	0.0792	-0.5103	1.1976	-1.1448	0.2325
	g	0.3603	-2.9739	9.0720	-11.368	4.9443

Figure captions

Fig. 1. Full Born distribution $xf^{H}_{q/\gamma}$ (dashed line), correction distribution from the first-order strong interaction $xf^{s.c.}_{q/\gamma}$ (dash-dot line) and total distribution $xf_{q/\gamma}$ (solid line) for the case of $q = 2/3$, $Q^2 = 10$ GeV^2.

Fig. 2. Total quark distributions for $q = 2/3$ (above curves), $q = 1/3$ (lower curves) and gluon distributions (curves in small x region) in the photon at $Q^2 = 10$ (solid lines), 10^2 (dashed lines), 10^4 GeV^2 (dash-dot lines).

Fig. 3. The ratio of moment value of the correction distribution from strong interaction to the Born distribution for the cases of $Q^2 = 10$ (solid line) , 10^2 (dashed line), 10^4 (dash-dot line), and 10^9 GeV^2 (dash-dot-dot line).

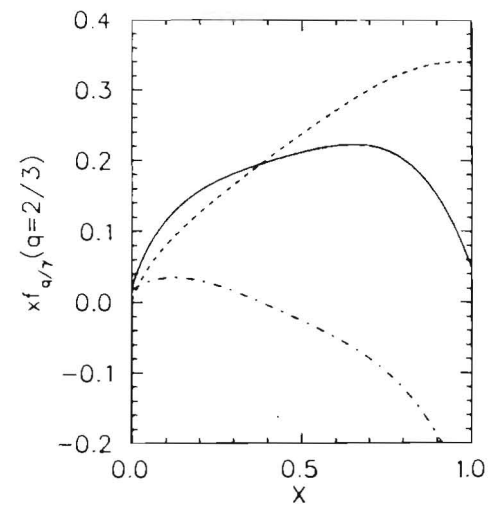

Fig. 1

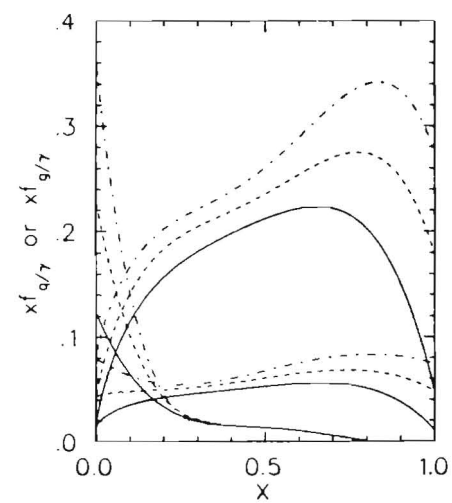

Fig. 2

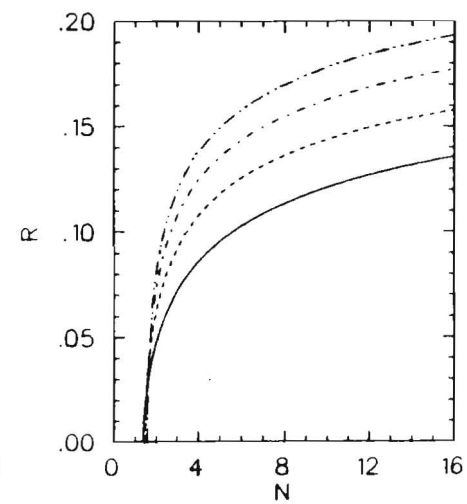

Fig. 3